ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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STABILITY OF THE ELASTIC MULTY-SPAN PLATE RESTING ON PERIODICALLY ARRANGED EXTERNAL RIGID SUPPORTS

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Keywords: periodically structured bi material plate, stability analysis of the plate, matrix method, multi-span plate.

Аветисян А.С., Казарян К.Б.
Устойчивость упругой многопролетной пластины,
опирающейся на периодически расположенные внешние жесткие опоры

Ключевые слова: периодически структурированная пластина, упругая устойчивость, метод Блоха-Флоке, многопролетная пластина, анализ устойчивости пластины.

В работе исследуется потеря устойчивости периодически структурированной, двухкомпонентной пластины, подвергаемой равномерному сжатию. Сжимающие нагрузки приложены к противоположным граням пластины и действуют вдоль срединной поверхности. Конструкция пластины формируется чередующимися сегментами из двух различных упругих материалов и опирается на систему внешних жёстких линейных опор, расположенных с заданной периодичностью. Используя периодичность геометрии и нагружения пластины, анализ устойчивости сводится – посредством стандартных методов Блоха–Флоке – к решению соответствующих краевых задач для бесконечной двухкомпонентной и конечной однородной многопролетных пластин. Такое сведение обеспечивает вычислительно эффективную основу для оценки критических состояний потери устойчивости, сохраняя при этом ключевые механические особенности исходной периодической архитектуры.

Ավետիսյան Ա.Ս., Ղազարյան Կ.Բ.
Պարբերաբար դասավորված արտաքին կոշտ հենարանների վրա հենված
առաձգական բազմաթռիչք սալի կայունությունը

Հիմնաբառեր՝ պարբերական կառուցվածքով սալ, առաձգական կայունություն, Բլոխ-Ֆլոքեյի մեթոդ, բազմաթռիչք սալ, սալի կայունության վերլուծություն։

Աշխատանքում ուսումնասիրվում է իր հարթության մեջ հավասարաչափ սեղմման ենթարկվող պարբերաբար կառուցվածքով, երկնյութ սալի կայունությունը։ Սալը ձևավորված է տարբեր առաձգական նյութերից կազմված կրկնվող հատվածներից և հենվում է պարբերաբար տեղադրված արտաքին գծային հենարանների վրա։ Սեղմող բեռները հավասարաչափ ազդում են սալի միջին մակերևույթի երկայնքով, դրա հակադիր կողերեսների վրա։ Օգտագործելով սալի կառուցվածքային և բեռնման պարբերականությունը, սալի կայունության ուսումնասիրությունը Բլոխ-Ֆլոքեի պարբերականության արգումենտների միջոցով բերվում է երկնյութ, անվերջ կառուցվածքով և վերջավոր համասեռ բազմաթռիչք սալերի եզրային խնդիրների։ Այս մոտեցումը ապահովում է հաշվարկային արդյունավետ հիմք ծռման վարքագիծը գնահատելու համար՝ պահպանելով դրա հիմքում ընկած պարբերական կառուցվածքների ճարտարապետության հիմնարար մեխանիկական հատկությունները։

This work investigates the buckling of a periodically structured, piecewise bi-material plate subjected to uniform in-plane compression. The plate is formed by repeating segments composed of distinct elastic materials and supported by a rigid array of periodically spaced external line supports. Compressive loads act uniformly along the mid-surface on opposite faces. Exploiting the inherent geometric and loading periodicity, the stability analysis of the plate is reduced via standard Bloch–Floquet periodicity arguments to a boundary-value problems defining an

infinite bi-material and finite homogeneous multi-span plate. This reduction provides a computationally efficient basis for evaluating buckling behavior while preserving the fundamental mechanical properties of the underlying periodic architecture. This also supports the design and optimization of advanced multi-material structural systems.

1. Introduction.

The stability of thin-walled structural elements—such as **beams**, **plates**, and **shells**—has long been a central topic in engineering mechanics [1,2]. These components frequently appear in load-bearing systems where buckling and post-buckling behaviour govern design, safety, and performance. Among them, multi-span plates supported by internal constraints constitute an important class of structural systems encountered in bridges, building floors, aircraft fuselage panels, ship decks, and other engineering applications [3,4,7]. Their stability response is strongly influenced by the type, stiffness, and spatial arrangement of internal supports, making them a subject of sustained research interest.

A substantial body of work has examined the vibration characteristics of rectangular plates equipped with internal point supports, line supports, or mixed boundary conditions. These studies have clarified how discrete or continuous internal constraints modify natural frequencies, mode shapes, and dynamic response under various loading scenarios [5,8–11, 14]. Parallel investigations have focused on the static bending behavior of plates with elastic or rigid internal supports, revealing significant effects on deflection patterns, stress distributions, and load-carrying capacity [3,4,6,7,9,13].

In recent decades, research on periodically arranged elastic and piezoelastic structures has expanded rapidly, driven by developments in phononic and photonic materials. These studies have primarily addressed wave-propagation phenomena, including dispersion characteristics, band-gap formation, and dynamic filtering effects in periodic media [15]. Periodic structural configurations—whether arising from material heterogeneity, geometric repetition, or discrete support patterns—have proven essential in the design of advanced metamaterials and vibration-control systems.

Despite this extensive literature, the static stability of periodically supported structural members has received comparatively limited attention. To the best of our knowledge, only few works have addressed the stability of periodically arranged elastic beams. Using Floquet theory, these studies analyze the buckling behavior of a compressed piecewise bi-material beam with periodic supports and a periodically pinned beam interacting with an elastic foundation [11,16,17]. Their findings demonstrate that periodicity in support conditions can fundamentally alter the stability landscape, producing non-classical buckling modes and sensitivity to geometric and material periodicity.

However, no systematic investigation appears to exist on the static stability of bi-material multi-span plates resting on periodically arranged rigid line supports. The interplay between bi-material composition, multi-span geometry, and periodic support distribution introduces a rich structural configuration whose stability characteristics remain unexplored.

The present paper addresses this gap by developing a comprehensive stability analysis for such plates. We formulate the governing equations, construct the appropriate boundary and continuity conditions, and employ a Floquet-based framework to characterize the buckling behavior of bi-material multi-span plates with periodic rigid line supports. The results provide new insights into the stability mechanisms of periodically constrained plate systems and establish a foundation for further analytical and computational developments in this area.

2. Statement of problem, stability equation of a bi–material plate

Let consider a piecewise bi-material rectangular plate constituted by periodically repeated two sub-plates made from different elastic materials. Each of these sub-plates of widths $d_1, d_2, \left(d = d_1 + d_2\right)$ is labelled by the index $(s) = 1,\ (s) = 2$ within sub cells $x \in \left((n-1)d, nd - d_2\right)$ and $x \in \left(nd - d_2, nd\right)$ of the plate unit cell $x \in \left((n-1)d, nd\right), y \in \left(0, a\right)$.

The sub-plates rested at interfaces $x = (n-1)d;\ \ x = nd - d_2;\ \ x = nd,$ on the rigid supports. At these locations, the plate displacements vanish, reflecting the kinematic constraint imposed by the supports. In contrast, the bending moments and rotation angles remain continuous across the interface. This continuity follows directly from the compatibility requirements of classical plate theory: even when the transverse displacement is fixed to zero, the plate must still transmit bending actions and maintain smooth rotational behavior to avoid artificial singularities in curvature.

The plate is compressed in its middle surface by forces uniformly distributed along opposite edges $y = 0, y = a$ (Fig.1). Magnitude of these forces per unit of edge length be denoted by P.

In sub-plate regions we have the following equations of plate stability

$$D_j \Delta^2 W_j + P \frac{\partial^2 W_j}{\partial y^2} = 0 \tag{1}$$

where $W_j(x, y)$ are the plate transversal displacements, D_j is the bending stiffness.

At the plate material separation lines, where the rigid supports are located $y \in (0, a),$ $x = (n-1)d, x = nd - d_2, x = nd,$ the plate displacements are equal to zero

$$W_1 = 0; \quad W_2 = 0, \tag{2a}$$

bending moments and rotation angels are continuous

$$\frac{\partial W_1}{\partial x} = \frac{\partial W_2}{\partial x}; \quad D_1 \frac{\partial^2 W_1}{\partial x^2} = D_2 \frac{\partial^2 W_2}{\partial x^2}. \tag{2b}$$

At $y = 0, y = a$ we assume the conditions of the simply supported edges

$$W_j = 0; \qquad \frac{\partial^2 W_j}{\partial y^2} = 0 \tag{3}$$

Assuming that under action of compressive force the plate buckling in sinusoidal modes and noting that $P/D_j > m^2\pi^2/a^2$ [2] we take the solutions in the elementary unit cell of plate in the form

$$W_j(x, y) = \sum_m U_{mj}(x) \sin\left(\frac{m\pi y}{a}\right); \qquad (m = 1, 2, 3.....) \tag{4}$$

$$U_j(x) = A_{1j}\sin(q_j x) + A_{2j}\cos(q\ x) + A_{3j}\sinh(p_j x) + A_{4j}\cosh(p_j x)$$

$$q_j = \frac{1}{a}\sqrt{m\pi\sqrt{\frac{Pa^2}{D_j}} - m^2\pi^2}; \qquad p_j = \frac{1}{a}\sqrt{m\pi\sqrt{\frac{Pa^2}{D_j}} + m^2\pi^2} \tag{5}$$

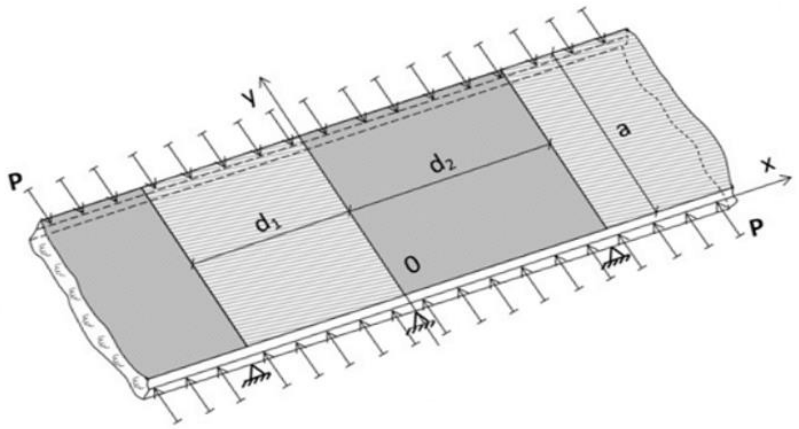


Fig. 1 *Scheme of laterally compressed finite graded plate consisting of periodically repeating bi-material rectangular plates of different elastic materials resting on rigid supports.*

Solutions (5) satisfying to conditions at the rigid supports

$$U_1\left((n-1)d\right) = 0;\ U_1\left((n-1)d + d_1\right) = 0; \quad U_2(nd) = 0,\ U_2(nd - d_2) = 0; \tag{6}$$

can be cast as

$$U_1(x) = A_{11} f_{11}\left(x\right) + A_{12} f_{12}\left(x\right) \tag{7}$$

where the following notation are used

$$f_{11}\left(x\right) = \sin(q_1 x) + \operatorname{csch}(p_1 d_1)\sinh\left(p_1 d(n-1) - p_1 x\right)\sin(p_1 d(n-1) + p_1 d_1) -$$
$$-\sin(q_1 d(n-1))\sinh(d(n-1) + p_1 d_1 - p_1 x),$$
$$f_{12}\left(x\right) = \cos(q_1 x) + \operatorname{csch}(p_1 d_1)\cos(q_1 d(n-1) + q_1 d_1)\sinh(p_1 d(n-1) - p_1 x) -$$
$$-\cos(d(n-1)q_1)\sinh(p_1 d(n-1) + p_1 d_1 - p_1 x),$$
$$f_{21}\left(x\right) = \sin(q_2 x) + \operatorname{csch}(d_2 p_2)\sin(q_2 d_2 - q_2 dn)\sinh(dp_2 n - p_2 x) -$$
$$-\sin(dnq_2)\sinh(-dp_2 n + p_2 d_2 + p_2 x),$$
$$f_{22} = \cos(q_1 x) - \operatorname{csch}(d_2 p_2)\cos(q_2 d_2 - q_2 dn)\sinh(p_2 dn - p_2 x) +$$
$$+\cos(dnq_2)\sinh(-dnp_2 + p_2 d_2 + p_2 x)$$

Since interface continuity conditions are imposed on the functions $\frac{\partial U_j}{\partial x}, D_j \frac{\partial^2 U_j}{\partial x^2}$ we consider the following column vectors

$$\mathbf{U}_j = \left(\frac{dU_j}{dx}, D_j \frac{d^2 U_j}{dx^2} \right)^T, \qquad \mathbf{A}_j = \left(A_{1j}, A_{2j} \right)^T \tag{8}$$

Using solutions (7), the vectors $\mathbf{U}_j$ in a matrix form can be cast as

$$\mathbf{U}_j(x) = \mathbf{F}_j(x) \cdot \mathbf{A}_j \tag{9a}$$

where

$$\mathbf{F}_j(x) = \begin{pmatrix} \frac{df_{j1}}{dx} & \frac{df_{j2}}{dx} \\ D_j \frac{d^2 f_{j1}}{dx^2} & D_j \frac{d^2 f_{j2}}{dx^2} \end{pmatrix} \tag{9b}$$

Eliminating vectors $\mathbf{A}_s$ from (9), using the matrix method procedure

$$\begin{aligned} &\mathbf{U}_1((n-1)d) = \mathbf{F}_1((n-1)d)\mathbf{A}_1; \ \mathbf{A}_1 = \mathbf{F}_1^{-1}((n-1)d)\mathbf{U}_1((n-1)d); \\ &\mathbf{U}_1((n-1)d + d_1) = \mathbf{F}_1((n-1)d + d_1)\mathbf{A}_1 \\ &\mathbf{M}_1 = \mathbf{F}_1((n-1)d + d_1)\mathbf{F}_1^{-1}((n-1)d); \end{aligned} \tag{10a}$$

the link relation for $\mathbf{U}_1(x)$ vector within first material can be found as

$$\mathbf{U}_1((n-1)d + d_1) = \mathbf{M}_1 \mathbf{U}_1((n-1)d) \tag{10b}$$

Repeating the same procedure for vector $\mathbf{U}_2(x)$ we get

$$\mathbf{U}_2(nd) = \mathbf{M}_2 \mathbf{U}_2(nd - d_2) \tag{11}$$

Herein $\mathbf{M}_s$ is the unimodal propagator matrix. The explicit expressions of the unimodal propagator matrix $\mathbf{M}_s$ elements can be derived as

$$\mathbf{M}_j = \begin{pmatrix} \eta_j & \theta_j \\ \vartheta_j & \eta_j \end{pmatrix}$$

$$\eta_s = \frac{p_j \cosh(d_j p_j) \sin(d_j q_j) - q_j \cos(d_j q_j) \sinh(d_j p_j)}{p_j \sin(d_j q_j) - q_j \sinh(d_j p_j)}$$

$$\theta_s = \frac{2 p_j q_j \left(1 - \cos(d_j q_j) \cosh(d_j p_j)\right) + (p_j^2 - q_j^2) \sin(d_j q_j) \sinh(d_j p_j)}{D_j (p_j^2 + q_j^2)\left(p_j \sin(d_j q_j) - q_s \sinh(d_j p_j)\right)}$$

$$\vartheta_s = \frac{D_j\left(p_j^2+q_j^2\right)\sin\left(d_j q_j\right)\sinh\left(d_j p_j\right)}{p_j\sin\left(d_j q_j\right)-q_j\sinh\left(d_j p_j\right)}; \tag{12}$$

Let note that elements of matrix $\mathbf{M}_j$ do not depend of the unit cell number n.

Enforcing the continuity of bending moments and rotations at the interfaces of two materials where the rigid supports are placed

$$\mathbf{U}_1\left((n-1)d+d_1\right)=\mathbf{U}_2\left(nd-d_2\right) \tag{13}$$

we come to the following relation linking the vectors of bending moments and rotation angles at the edges of the n-th unit cell

$$\mathbf{U}_2\left(nd\right)=\mathbf{M}\mathbf{U}_1\left((n-1)d\right); \qquad \mathbf{M}=\mathbf{M}_2\mathbf{M}_1 \tag{14}$$

3**. Solution for periodic infinite multi-span plate**

In the framework of Floquet theory applying the quasi-periodic conditions at the ends of the plate unit cell

$$\mathbf{U}_1\left((n-1)d\right)=\lambda\mathbf{U}_2\left(nd\right) \tag{15a}$$

Here $\lambda=\exp(ikd)$, k is the Floquet- Bloch wave number

Taking into account (13) the following matrix equation is obtained

$$\left(\mathbf{M}-\lambda\mathbf{I}\right)\mathbf{U}_2\left((n-1)d\right)=0 \tag{15b}$$

here $\mathbf{I}$ is the identity matrix.

From the condition of non-trivial solutions of (15) equating to zero the determinant of matrix $\boldsymbol{M}-\lambda\boldsymbol{I}$ one can find the following equation providing the dependence between critical force P_0 and the Floquet- Bloch wave number

$$\cos\left(kd\right)=\frac{1}{2}\mathrm{tr}(\mathbf{M})=\eta_1\eta_2+\frac{\theta_1\vartheta_2+\theta_2\vartheta_1}{2} \tag{16a}$$

Regions of the compressive force where $\left|\mathrm{tr}(\mathbf{M}\right|>2$ (k is complex) correspond to plate stability regions. The plate instability regimes are regions where $\left|\mathrm{tr}(\mathbf{M})\right|<2$ (k is real).

The critical value P will be evaluated from equations

$$2\eta_1\eta_2+\theta_1\vartheta_2+\theta_2\vartheta_1=2 \tag{16b}$$

4. Stability of a finite length homogeneous multi span plate

For homogeneous plate $D_s=D, d_s=d, \eta_s=\eta, \theta_s=\theta, \vartheta_s=\vartheta$ we have that

$$\cos\left(2kd\right)=\eta^2+\theta\vartheta, \qquad \cos^2\left(kd\right)=\frac{1+\eta^2+\theta\vartheta}{2} \tag{17}$$

Taking into account that $\det\mathbf{M}=\eta^2-\theta\vartheta=1$, we come to the following Floquet characterizing equation for compressed homogeneous plate resting on periodically arranged rigid supports

$$\cos(kd) = \eta(P) = \frac{p\cosh(dp)\sin(dq) - q\cos(dq)\sinh(dp)}{p\sin(dq) - q\sinh(dp)} \tag{18}$$

Let now consider the homogeneous plate on periodically arranged $N+1$ rigid supports.

The relation linking at the $\mathbf{U}(nd), \mathbf{U}((n-1)d)$ at the edges of the n-th unit cell is given by equation (12)

Repeating this procedure the n-th times the propagator unimodal matrix $\mathbf{M}^n$ can be found. The matrix $\hat{\mathrm{M}}^N$ links the vectors at $x = 0$ and $x = Nd$ edges of the plate

$$\mathbf{M}^N\mathbf{U}(0) = \mathbf{U}(Nd), \quad n = 1, 2, ...N \tag{19}$$

According to Sylvester's matrix polynomial theorem for 2x2 matrices the elements of the unimodal matrix $\mathbf{M}^n$ can be cast as

$$\mathbf{M}^n = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{20}$$

$$M_{11} = \eta S_{n-1}(s) - S_{n-2}(s); \qquad M_{12} = \theta S_{n-1}(s)$$

$$M_{21} = \vartheta S_{n-1}(s); \qquad M_{22} = \eta S_{n-1}(s) - S_{n-2}(s)$$

where $S_n(s)$ are the Chebyshev polynomials of second kind, namely

$$S_n(s) = \frac{\sin((n+1)\arccos(s))}{\arccos(s)}; \qquad s = \eta(P)$$

The first Chebyshev polynomials of second kind are

$$S_0(s) = 1, \qquad S_1(s) = 2s; \qquad S_2(s) = 4s^2 - 1$$

Subsequent polynomials may be obtained from the recurrence relation of Chebyshev polynomials of second kind

$$S_n(s) = 2sS_{n-1}(s) - S_{n-2}(s)$$

The relation (18) enables us to consider the three boundary value stability problems for compressed plate

1) The edges $x = 0$, $x = Nd$ are simply supported

2) The edges $x = 0$, $x = Nd$ are clamped

3) The edge $x = 0$ is clamped, the edge $x = Nd$ is simply supported

4) The edge $x = 0$ is simply supported, the edge $x = Nd$ is clamped

For the case 1) using the equation (19)

$$\left.\begin{pmatrix} \frac{dU}{dx} \\ 0 \end{pmatrix}\right|_{x=Nd} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \left.\begin{pmatrix} \frac{dU}{dx} \\ 0 \end{pmatrix}\right|_{x=0} \tag{21a}$$

we come to the characterizing equations

$$\vartheta S_{n-1}(s) = 0 \tag{21b}$$

For the case 2) we have that

$$\left.\begin{pmatrix} 0 \\ \dfrac{d^2U}{dx} \end{pmatrix}\right|_{x=Nd} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \left.\begin{pmatrix} 0 \\ \dfrac{d^2U}{dx} \end{pmatrix}\right|_{x=0} \tag{22a}$$

which result in

$$\theta S_{n-1}(s) = 0 \tag{22b}$$

For the case 3) we have finally

$$\left.\begin{pmatrix} \dfrac{dU}{dx} \\ 0 \end{pmatrix}\right|_{x=Nd} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \left.\begin{pmatrix} 0 \\ \dfrac{d^2U}{dx} \end{pmatrix}\right|_{x=0} \tag{23a}$$

$$\eta S_{n-1}(s) - S_{n-2}(s) = 0 \tag{23b}$$

Using the recurrence relation of Chebyshev polynomials we can rewrite the equation (22) as

$$T_n(s) = 0 \tag{24b}$$

where $T_n(s)$ are the Chebyshev polynomials of the first kind.

The first Chebyshev polynomials of the first kind are

$$T_1(s) = s, T_2(s) = 2s^2 - 1; T_3(s) = 4s^3 - 3s$$

The equations $\vartheta = 0$, $\theta = 0$ and $T_1(s) = 0$ correspond to classical stability equations of single plate with clamped edges, simply supported edge and clamped – simply supported edges, respectively.

The minimal roots of equation (23) $(N \geq 1)$ define critical forces of the plate with clamped– simply supported edges.

The minimal critical forces P for simply supported plate must be determined by comparison of minimal roots of equations $\vartheta = 0$, $S_{n-1}(s) = 0$. The minimal critical forces P for clamped plate must be determined by comparison of minimal roots of equations $\theta = 0$, $S_{n-1}(s) = 0$.

The equations (23) $(N > 1)$ has $(N-1)$ roots on the range $s \in (-1,1)$ and its zeroes are given by

$$\eta(P) = \cos\left(\frac{(2j+1)\pi}{2N}\right), j = 0,1,2.....N-1 \ . \tag{25a}$$

The equation $S_{N-1}(s) = 0$ has $(N-1)$ roots on the range $s \in (-1,1)$ and its zeroes are given by

$$\eta(P)=\cos\left(\frac{j\pi}{N}\right),\ j=1,2.....(N-1). \tag{25b}$$

Numerical analysis shows that for any given value m,β the function $\eta(P)$ is the negative and the monotonously increasing function in the range $Dd^{-2}m\pi\beta < P < Q_0$, where Q_0 is the smallest root of the equation $\eta(P)=-1$.

So in calculating the critical values of compressed force the smallest negative roots of the equations (23), (24) must be considered.

The critical force corresponding to the Floquet characterizing equation (16) must be calculating at

$$\cos(kd)=-1. \tag{26}$$

The magnitude of critical force can be represented by the formula

$$P=Da^{-2}K_n(\beta,m), \tag{27a}$$

where K_n is a numerical factor, $\beta=d/a$, index n stand by for the number of supports.

The smallest value of critical force must be found in dependence of parameter β and buckling mode numbers m

$$K^*(\beta)=\min_m K_n(\beta,m) \tag{27b}$$

The stability equation of a single simply supported plate admits analytical solution

$$\sin(dq)=0,\quad dq=\pi \tag{28}$$

$$K_n^*(\beta)=\pi^2\beta^{-1}\min_m\left(\frac{1}{m\beta}+\beta m\right)^2 \tag{29}$$

The Floquet characterizing equation (18) at $kd=\pi$ can be written as

$$\frac{p(1+\cosh(dp))\sin(dq)-q(1+\cos(dq))\sinh(dp)}{p\sin(dq)-q\sinh(dp)}=0 \tag{30}$$

Minimal root of this equation is equal to $dq=\pi$.

Numerical results

The numerical analysis of equations (23) , (24) shows that $K_{n+1}^*(\beta)<K_n^*$ and with increasing j the numerical factor K_n^* approach to $K_n^*(\beta)$ defined by equation (26) (See Table 1).

The corresponding numerical data for factor $K_j^*(\beta)$ are given in Table 1.

Note that according to (27b) $K_n^*(1)=4\pi^2\cong 39.4$.

Table 1

Cell numbers	Clamped-hinged edges			Clamped edges		
	$K^*(\beta)$					
	$\beta = 2$	$\beta = 1$	$\beta = 0.5$	$\beta = 2$	$\beta = 1$	$\beta = 0.5$
1	16.9	56.6	218.2	18.9	85.6	274.5
2	15.8	43.8	193.2	16.9	56.6	256.9
3	15.5	41.5	186.3	16.1	47.3	230.3
4	15.5	40.6	180.5	15.8	43.8	200.4
5	15.4	40.0	176.3	15.6	42.3	189.2
6	15.4	39.4	169.4	15.4	41.4	171.3
∞	15.3	**39.4**	158.06	15.4	**39.4**	158.06

For bi-material infinite plates the numerical data of the factor $K_n^*(\beta,\delta,\gamma)$, $\delta = d_1/d_2$, $\gamma = D_1/D_2$, $P = Da^{-2}K_n^*(\beta,\delta,\gamma)$ are presented in the Table 2.

The results of $K_j^*(\beta,\delta,\gamma)$ are obtained from the numerical calculations of the roots of the Floquet characterizing equation (16b).

Table 2.

$K_j^*(\beta,1,\gamma)$					
γ	$\beta = 3$	$\beta = 2$	$\beta = 1$	$\beta = 0.5$	$\beta = 0.2$
0.05	12.4	19.1	73.9	182.2	210.2
0.3	10.5	17.8	61.5	146.7	155.4
0.5	8.8	16.3	53.7	114.2	132.6
0.7	6.7	15.9	46.8	89.2	102.6
0.95	5.7	15.32	39.1	58.06	75.2

Conclusions

The article investigates the stability characteristics of a periodically structured, piecewise bi-material plate subjected to in-plane compression. The plate consists of repeating segments composed of distinct elastic materials and is supported by a system of rigid, periodically arranged line supports. Uniform compressive loads act along the mid-surface on opposite faces. Owing to the geometric and loading periodicity, the stability analysis of the infinite plate is rigorously reduced—via standard Bloch–Floquet-type periodicity

arguments—to the corresponding boundary-value problem for a rectangular two-component representative cell.

The principal results obtained for a homogeneous compressed plate resting on periodically distributed rigid supports can be stated with greater precision. Analytical derivations, supported by numerical computations, demonstrate that the minimum critical buckling load for a simply supported plate is invariant with respect to the number of supports. For an infinite plate with periodic rigid supports, this minimum critical load is identical to the critical load of a single simply supported plate, reflecting the absence of additional stabilizing or destabilizing effects induced by periodic repetition.

In contrast, for plates with clamped boundary conditions, or with mixed boundary conditions (one clamped edge and one simply supported edge), the minimum critical load exhibits a monotonic trend: as the number of supports increases, the critical load asymptotically approaches that of the simply supported configuration. This behavior underscores the dominant influence of boundary constraints relative to the periodic support pattern in governing global buckling response.

These findings clarify the interplay between boundary conditions, material heterogeneity, and periodic support arrangements, and they provide a rigorous foundation for stability assessment in periodically structured compressed plate.

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