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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 78, №3-4, 2025 Механика
УДК 539.3 DOI: 10.54503/0002-3051-2025.78.3-4-103

REFLECTION AND TRANSMISSION OF SHEAR ELASTIC WAVES THROUGH PERIODICALLY STRATIFIED BI-MATERIAL ELASTIC LAYER

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Keywords: reflection and refraction, shear wave, periodic structure, frequency band gaps.

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Отражение и преломление сдвиговых упругих волн от периодически слоистого упругого слоя из двух материалов.

Ключевые слова: отражение, преломление, сдвиговая волна, периодическая структура, запрещенные зоны частот.

Исследовано поведение сдвиговых волн при взаимодействии с периодически расслаивающимся упругим слоем, расположенным между двумя идентичными упругими полупространствами. Анализ сосредоточен на определении характеристиках отражения и преломления этих волн от слоистой структуры. Коэффициент преломления определяется для количественной оценки доли волновой энергии, передаваемой через композитный слой. Показано, что когда частоты падающей сдвиговой волны в пределах запрещенной зоны частот соответствующей бесконечной периодической среды, коэффициент пропускания упругой волны близок к нулю. В этих частотных диапазонах периодическая структура демонстрирует почти полное отражение, а коэффициент отражения приближается к единице. И наоборот, когда частоты падающей волны лежат между любыми двумя соседними запрещенными зонами частот , существует конечный набор частот, при которых коэффициент пропускания упругих волн достигает единицы. На этих частотах отражательная способность полностью исчезает и волна идеально проходит через слоистую структуру. Кроме того, мы исследуем изменение фазового сдвига отраженной сдвиговой волной, когда слоистая структура ограничена с одной стороны упругим полупространством, а с другой - свободной от напряжений поверхностью.

Ղազարյան Կ.Բ., Ղազարյան Ռ.Ա., Թերզյան Ս.Հ.

Սահքի առաձգական ալիքների անդրադարձումը և բեկումը երկու նյութերից կազմված պարբերական շերտավոր առաձգական շերտից

Հիմնաբառեր՝ անդրադարձում, բեկում, սահքի ալիք, պարբերական կառուցվածք, հաճախությունների արգելված գոտիներ:

Հետազոտված է, պարբերական շերտավորվող առաձգական շերտի փոխազդեցությամբ պայմանավորված, սահքի ալիքների վարքը: Շերտը գտնվում է երկու նույնական առաձգական կիսատարածությունների միջև: Վերլուծությունը կենտրոնացված է շերտավոր կառուցվածքից այդ ալիքների անդրադարձման և բեկման բնութագրիչների որոշմանը: Բեկման գործակիցը որոշվում է կոմպոզիտային շերտի միջոցով փոխանցվող ալիքային էներգիայի մասնաբաժնի քանակական գնահատականի համար: Ցույց է տրված, որ երբ ընկնող սահքի ալիքի հաճախությունը գտնվում է համապատասխան անվերջ պարբերական միջավայրի արգելված գոտու հաճախության սահմաններում, առաձգական ալիքի բաց թողնման գործակիցը մոտ է զրոյին: Այդ հաճախական ընդգրկույթներում պարբերական կառուցվածքը համարյա լրիվ անդրադարձում է ցույց տալիս, իսկ անդրադարձման գործակիցը մոտենում է մեկին: Եվ հակառակը, երբ ընկնող ալիքի հաճախություններն ընկած են հաճախությունների արգելված, ցանկացած երկու հարևան, գոտիների միջև, գոյություն ունի հաճախությունների վերջավոր քանակ, որոնց դեպքում առաձգական ալիքների բաց թողնման գործակիցը հասնում է մեկին: Այդ հաճախություններում անդրադարձման հատկությունը ամբողջությամբ վերանում է, և ալիքը կատարելապես անցնում է շերտավոր կառուցվածքի միջով: Բացի այդ, ուսումնասիրում ենք սահքի ալիքով արտացոլված ֆազային շեղումը, երբ շերտավոր կառուցվածքը սահմանափակված է մի կողմից առաձգական կիսատարածությունով, իսկ մյուս կողմից՝ լարումներից ազատ մակերևույթով:

In this study, we investigate the behaviour of incident shear waves as they interact with a periodically stratified bi-material elastic layer positioned between two identical elastic half-spaces. Our analysis focuses on the reflection and transmission characteristics of such waves as they encounter the layered structure. The transmission coefficient is defined to quantify the proportion of wave energy transmitted through the composite layer. It is demonstrated that, when the frequencies of the incident shear wave fall within the bandgaps of the corresponding infinite periodic medium, the transmissivity of the elastic wave approaches zero. In these frequency ranges, the structure exhibits almost complete reflection, with the reflectivity nearing unity. Conversely, when the frequencies of the incident wave lie between any two adjacent bandgaps of the infinite periodic medium, there exists a finite set of frequencies at which the elastic wave transmissivity reaches unity. At these specific frequencies, the reflectivity vanishes entirely, and the wave is perfectly transmitted through the layered structure. Additionally, we examine the phase shift experienced by the reflected shear wave when the layered structure is by an elastic half-space and the other by a free interface.

Furthermore, this study analyses the phase shift of the reflected shear wave occurring when one side of the layered structure is adjacent to an elastic half-space and the other side is bounded by a free interface.

Introduction.

The phenomena of reflection and transmission of elastic waves at the interface between two distinct media play a crucial role in a variety of scientific and engineering disciplines. In composites engineering, understanding how elastic waves interact at material boundaries is essential for the design and analysis of advanced composite structures. In geology and seismology, the propagation of elastic waves through different earth layers underpins the interpretation of seismic data, which is fundamental for studying the Earth's interior and for earthquake research. Seismic exploration also relies heavily on these phenomena, as the reflection and transmission of waves at subsurface interfaces are used to identify and characterise geological formations. Furthermore, in acoustics, the way elastic waves reflect and transmit at material boundaries determines the sound transmission properties of structures, influencing the design of materials and systems for noise control and sound insulation. The issues related to the reflection and transmission of elastic waves in layered media have been the focus of extensive analysis by numerous researchers. Their combined efforts established the basis for analysing wave behaviour at layered structure interfaces [1-11].

When two distinct elastic materials are joined together, it is typically assumed that the interface connecting them is perfectly bonded. This means that, at the boundary, certain fundamental physical quantities must remain continuous from one material to the other. In particular, both the displacement and the traction (which represents the mechanical forces acting across the interface) are required to be continuous. The continuity of displacement ensures that there is no separation or slip at the boundary, so the deformations are transmitted smoothly. Similarly, the continuity of traction guarantees that the mechanical forces are balanced and transferred seamlessly from one material to the other. This idealisation is commonly adopted in analytical and computational models, as it simplifies the study of how forces and deformations are transmitted and distributed at the interface between different elastic materials While the assumption of a perfectly bonded interface between two elastic materials simplifies analysis, it does not always represent actual conditions encountered in practice. In reality, interfaces may contain various defects, damages, or other imperfections. Several theoretical models have been developed to represent imperfect interfaces [11-16].

The most commonly used models include the strain-gradient model, which accounts for higher-order mechanical effects at the interface; the spring model, which introduces interfacial compliance; the mass model, which considers additional interfacial inertia; and the combined spring-mass model, which incorporates both compliance and inertia effects. Each of these models offers a different perspective on how imperfections at the interface can affect wave transmission and reflection, and they play a crucial role in the accurate analysis of layered composite structures [17-24].

This study focuses on the behaviour of incident shear waves as they encounter a periodically stratified bi-material elastic layer that is positioned between two identical elastic half-spaces. The research centres on analysing how these waves are reflected and transmitted when passing through the layered medium. By examining the reflection and transmission properties, the study aims to provide a detailed understanding of the mechanisms governing wave interaction within the composite structure. The findings contribute to a broader comprehension of elastic wave dynamics in layered materials, which is relevant for various applications in engineering and the physical sciences.

#

Problem statement and matrix approach for periodic structure.

Consider two homogeneous elastic half-spaces, composed of identical materials, separated by a periodically in perfect interfacial contact. This stratified layer consists of $n(n=1,2,\dots,N)$ repeating unit cells, with each cell comprising two distinct sub-layers formed from different elastic materials A and B of widths a, b. Each sub-layer is assumed to be perfectly bonded to its adjacent sub-layers. The composite layer extends from the top surface $x=0$ to the bottom surface $x=Nd$ and $d=a+b$.

Shear waves propagation in stratified bi-material elastic layer obey to anti-plane equations of motion. Choosing the anti-plane deformation in the z - direction one has

$$\partial_x\sigma_{xz}+\partial_y\sigma_{yz}=\rho\partial_{t,t}u_z,\quad \sigma_{xz}=\mu\partial_x u_z,\sigma_{yz}=\mu\partial_y u_z \tag{1}$$

where $u_z(x,y,t)$ is the displacement in z - direction.

Considering a steady SH-wave propagation $\sim\exp[i(ky-\omega t)]$, where k,ω are the wave number and frequency, the solutions for amplitude functions $u(x),\mathrm{v}(x)$ within each the sub-layers material $x\in((n-1)d,(n-1)d+a), x\in(nd-b,nd)$ can be found as

$$u(x)=A_1\sin(px)+A_2\cos(px),\qquad \mathrm{v}(x)=B_1\sin(qx)+B_2\cos(qx). \tag{2}$$

Here

$$p=\sqrt{\frac{\omega^2}{c_a^2}-k^2},q=\sqrt{\frac{\omega^2}{c_b^2}-k^2},c_a=\sqrt{\frac{G_a}{\rho_a}},c_b=\sqrt{\frac{G_b}{\rho_b}}, \tag{3}$$

G_a,G_b are the shear modulus, ρ_a,ρ_b bulk density of sub-layers materials, correspondingly.

Enforcing the continuity of tractions and displacement at the contact interface $x_0 = nd - b$ of the two materials

$$u(x_0) = v(x_0)$$

$$G_a \frac{du(x_0)}{dx} = G_b \frac{dv(x_0)}{dx} \tag{4}$$

we can express B_1, B_2 via A_1, A_2

$$B_1 = A_1\left(\frac{G_a p\cos(px_0)\cos(qx_0)}{G_b q} + \sin(px_0)\sin(qx_0)\right) + $$
$$+A_2\left(\cos(px_0)\sin(qx_0) - \frac{G_a p\sin(px_0)\cos(qx_{0)}}{G_b q}\right) \tag{5}$$

$$B_2 = A_1\left(\sin(px_0)\cos(qx_0) - \frac{G_a p\cos(px_0)\sin(qx_0)}{G_b q}\right) + $$
$$+A_2\left(\frac{G_a p\sin(px_0)\sin(qx_0)}{G_b q} + \cos(px_0)\cos(qx_0)\right); \tag{6}$$

Introducing field vectors and constant vector

$$\mathbf{u}(x) = \begin{pmatrix} u(x) \\ G_a \dfrac{du(x)}{dx} \end{pmatrix}, \mathbf{v}(x) = \begin{pmatrix} \mathrm{v}(x) \\ G_b \dfrac{d\mathrm{v}(x)}{dx} \end{pmatrix}, \; \mathbf{A} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \tag{7}$$

the expressions of the vector $\mathbf{u}\big((n-1)d\big), \mathbf{v}\big(nd\big)$ can be expressed via the vector $\mathbf{A}$ in the following matrix form

$$\mathbf{u}\big((n-1)d\big) = \mathbf{PA} \tag{8}$$

$$\mathbf{v}\big(nd\big) = \mathbf{QA}$$

$$\mathbf{P} = \begin{pmatrix} \sin\big(p(n-1)d\big) & \cos\big(p(n-1)d\big) \\ G_a p\cos\big(p(n-1)d\big) & -G_a p\sin\big(p(n-1)d\big) \end{pmatrix} \tag{9}$$

$$\mathbf{Q} = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$$

$$q_{11} = \frac{G_a p\sin(bq)\cos(px_0)}{G_b q} + \cos(bq)\sin(px_0);$$

$$q_{12} = \cos(bq)\cos(px_0) - \frac{G_a p\sin(bq)\sin(px_0)}{G_\mu q}$$

$$q_{21}=G_a p\cos(bq)\cos(px_0)-G_b q\sin(bq)\sin(px_0);$$
$$q_{22}=-qG_b\sin(bq)\cos(px_0)-G_a p\cos(bq)\sin(px_0) \tag{10}$$

Omitting the vector $\mathbf{C}=\mathbf{P}^{-1}\mathbf{u}((n-1)d)$ leads to the following relation connecting values of the field vectors at a cells interface

$$\mathbf{v}(nd)=\mathbf{M}\mathbf{u}((n-1)d) \tag{11}$$

Here $\mathbf{M}=\mathbf{Q}\mathbf{P}^{-1}$ is a unimodal matrix

$$\mathbf{M}=\begin{pmatrix} \cos(ap)\cos(bq)-\dfrac{G_a p\sin(ap)\sin(bq)}{G_b q} & \dfrac{\sin(ap)\cos(bq)}{G_a p}+\dfrac{\cos(ap)\sin(bq)}{G_b q} \\ -G_a p\sin(ap)\cos(bq)-G_b q\cos(ap)\sin(bq) & \cos(ap)\cos(bq)-\dfrac{G_b q\sin(ap)\sin(bq)}{G_a p} \end{pmatrix} \tag{12}$$

Let note that elements of matrix $\mathbf{M}$ do not depend of cell number n. Repeating this procedure the n -th times the propagator unimodal matrix $\mathbf{M}^n$ can be found. The matrix $\mathbf{M}^n$ links the field vectors at $x=0$ and $x=nd$ surfaces of the waveguide.

$$\mathbf{M}^n\mathbf{u}(0)=\mathbf{v}(nd), \qquad n=1,2,....N \tag{13}$$

According to Sylvester's matrix polynomial theorem [28] for 2x2 matrices the elements of the n -th power of an unimodal matrix $\mathbf{M}^n$ can be cast as

$$\mathbf{M}^n=\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{14}$$

and can be simplified using the following matrix identity

$$M_{11}=m_{11}S_{n-1}(\eta)-S_{n-2}(\eta); \quad M_{12}=m_{12}S_{n-1}(\eta)$$
$$M_{21}=m_{21}S_{n-1}(\eta); \qquad M_{22}=m_{22}S_{n-1}(\eta)-S_{n-2}(\eta) \tag{15}$$

where $S_{n-1}(\eta)$ are the Chebyshev polynomials of second kind, namely

$$S_n(\eta)=\frac{\sin((n+1)\phi)}{\sin(\phi)};\cos(\phi)=\eta;$$
$$\eta=\frac{1}{2}Tr(\hat{M})=\frac{1}{2}(m_{11}+m_{22}); \tag{16}$$

In addition to the finite sized bi-material layer we consider a corresponding infinite bi-material layer. By applying the Floquet conditions at the unit cell boundaries, $x=(n-1)d$, $x=nd$ we obtain:

$$\mathbf{v}(nd)=\exp(\kappa d)\mathbf{u}((n-1)d) \tag{17}$$

Using (11) leads to the Floquet equation

$$\cos(\kappa d)=\eta(\omega) \tag{18}$$

Here κ is the complex Bloch wave number, and $\eta(\omega)$ is defined in (16).

Thus, the deviation function $\eta(\omega)$ – argument of the Chebyshev polynomials, as applied to the corresponding infinite periodic medium, defines the frequency "stopbands" under $|\eta(\omega)| > 1$ and identifies the "passband" frequencies under $|\eta(\omega)| < 1$.

Shear elastic wave reflection and transmission.

Consider now the reflection and transmission of the shear wave through periodically stratified bi-material elastic layer positioned between two identical elastic half-spaces.

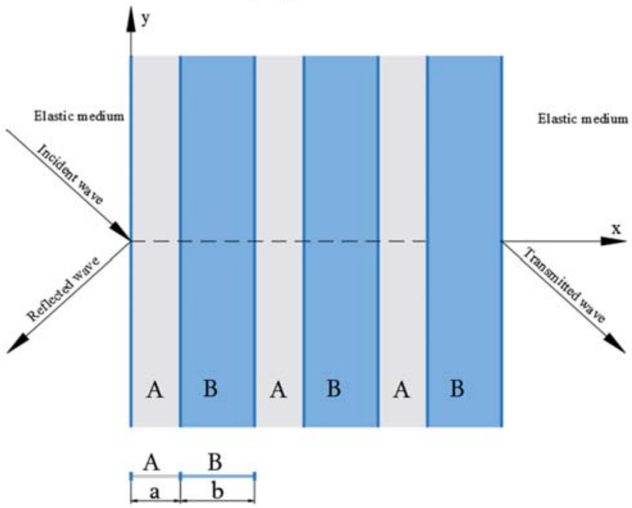

Figure 1: Reflection and transmission of shear wave through layer sandwiched into elastic medium

The shear displacements and shear stresses in the top $-\infty < x < 0$ and in the bottom $Nd < x < \infty$ half-spaces can be cast as

$$V(x,y,t) = V_0(x)\exp\left[i(ky - \omega t)\right]; \;\; U(x,y,t) = U_0(x)\exp\left[i(ky - \omega t)\right]$$

$$V_0(x) = \left(A_i \exp(irx) + A_r \exp(-irx)\right); \;\; U_0(x) = A_t \exp(irx) \tag{19}$$

$$\sigma_{xz1} = G\frac{dV_0}{dx}, \;\; \sigma_{xz2} = G\frac{dU_0}{dx};$$

Here

$$r = \sqrt{\frac{\omega^2}{c^2} - k^2} = d^{-1}\sqrt{\Omega^2 - K^2}; \Omega = \frac{\omega d}{c}, c = \sqrt{\frac{G}{\rho}}; \qquad K = kd$$

A_i, A_r, A_t, stand for the amplitudes of incident, reflected and transmitted shear waves, respectively, G is the shear modulus, ρ is the bulk density of semi-space material

The conditions for perfect contact between displacements and shear stresses in an elastic medium and a bi-material stratified layer at $x = 0, x = Nd$ can be expressed as follows:

$$\mathbf{v}(0) = \mathbf{U}(0), \qquad \mathbf{u}(Nd) = \mathbf{U}(Nd) \tag{20}$$

where

$$\mathbf{V}(0) = \begin{pmatrix} V_0(0) \\ G\dfrac{dV_0(0)}{dx} \end{pmatrix}, \ \mathbf{U}(Nd) = \begin{pmatrix} U_0(Nd) \\ G\dfrac{dU_0(Nd)}{dx} \end{pmatrix}; \tag{21}$$

Taking into account (21) the amplitudes $\mathrm{A}_r, \mathrm{A}_t$ via A_i can be found by solving the matrix equation

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_i + A_r \\ Gir(A_i - A_r) \end{pmatrix} = \begin{pmatrix} A_t \\ GirA_t \end{pmatrix} \exp(irNd) \tag{22}$$

$$A_r = -\frac{A_i(M_{21} + Gr\left(GrM_{12} - i(M_{11} - M_{22})\right)}{M_{21} - Gr\left(GrM_{12} + i(M_{11} + M_{22})\right)},$$
$$A_t = -\frac{2iA_i Gre^{-idNq}}{M_{21} - Gr\left(GrM_{12} + i(M_{11} + M_{22})\right)} \tag{23}$$

Defining reflection and transmission functions (coefficients) as

$$R_r = \left|\frac{A_r}{A_i}\right|^2, R_t = \left|\frac{A_t}{A_i}\right|^2 \tag{24}$$

from (23) we get

$$R_r = \frac{G^2r^2\left(G^2M_{12}^2r^2 + M_{11}^2 + M_{22}^2 - 2\right) + M_{21}^2}{G^2r^2\left(G^2M_{12}^2q^2 + M_{11}^2 + M_{22}^2 + 2\right) + P_{21}^2};$$
$$R_t = \frac{4G^2r^2}{G^2r^2\left(G^2M_{12}^2q^2 + M_{11}^2 + M_{22}^2 + 2\right) + M_{21}^2}; \tag{25}$$

Energy flux conservation is expressed via reflection and transmission coefficients in the following transmissivity and reflectivity identity

$$R_r + R_t = 1 \tag{26}$$

Using (25) and the following recurrence identity formula for the Chebyshev polynomials of the second kind

$$S_n^2(\eta) + S_{n-1}^2(\eta) - 2\eta S_n(\eta)S_{n-1}(\eta) = 1 \tag{27}$$

the relations (25) can be transform as

$$R_t = \frac{4G^2r^2}{4G^2r^2 + S_{N-1}^2(\eta)\left(G^2r^2\left(G^2m_{12}^2r^2 + m_{11}^2 + m_{22}^2 - 2\right) + m_{21}^2\right)}; \tag{28}$$

Since $S_{N-1}(\eta) \to 0$ at $N \to \infty$ when $|\eta(\omega)| > 1$ the incident wave frequencies are in the "stopbands" of the stratified layer, the transmission function decreases as the number of cells increases. In contrast, within the "passbands," $|\eta(\omega)| < 1$ the transmission function exhibits periodic behaviour.

Consider now the wave reflection from a finite-length stratified layer bounded on one side by an elastic half-space and on the other by a mechanically free surface.

The amplitude of a reflection wave A_r can be found by solving the following matrix equation where $U(Nd)$ is the unknown displacement at the layer traction free surface.

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} A_i + A_r \\ Gir(A_i - A_r) \end{pmatrix} = \begin{pmatrix} U(Nd) \\ 0 \end{pmatrix}\exp(irNd) \tag{29}$$

From solution of (29) we have

$$A_r = -A_i\left(\frac{M_{21} + iGrM_{22}}{M_{21} - iG\,rRM_{22}}\right) \tag{30}$$

Since the complex conjugate of the reflection coefficient $T_r = A_r / A_i$ is equal its reciprocal, the magnitude $|T_r|$ equal unity at any frequency Ω of incident wave.

Writing T_r in the polar form of complex number we have

$$T_r = -\exp(2i\phi), \tag{31}$$

where $$\phi(\Omega) = \operatorname{arctg}\left(\frac{GrM_{22}}{M_{21}}\right) = \operatorname{arctg}\left(\frac{Gr\left(m_{22}S_{n-1}(\eta) - S_{n-2}(\eta)\right)}{m_{21}S_{n-1}(\eta)}\right) \tag{32}$$

Formula (32) gives the phase shift for a shear elastic wave reflected from a finite-length stratified layer, which is bonded to an elastic half-space on one surface and mechanically free on the other.#

Numerical analysis and discussions.

We here will illustrate the obtained theoretical results providing numerical analysis concerning bandgaps of the stratified layer (counterpart infinite media) , reflection and transmission coefficients as well as reflection wave phase shifts. Numerical calculations have been carried out for materials listed in Table 1

Table 1

Substance	Bulk density $\rho\left(\mathrm{kgm}^{-3}\right)$	Shear modulus $G(\mathrm{GPa})$	Transversal velocity $c\left(\mathrm{m\,s}^{-1}\right)$
Aluminium	2700	25	3040

Copper	8900	46	2320
Titanium	4500	42	3120

Based on (28), the transmission and reflection of the elastic wave are considered when material of elastic semi-spaces is the titanium, the elastic reflector made of the aluminium and copper materials. Numerical analysis of counterpart infinite piezoelectric media will be carried out for the aluminium and copper materials.

The condition $|\eta(\Omega)|>1$, where $\Omega=\omega d\sqrt{G^{-1}\rho}$ stand for titanium material defines bandgaps of counterpart infinite piezoelectric media.

On Figure 2 for counterpart infinite media of the layer made of aluminium and copper materials the deviation function curves versus frequency Ω are plotted.

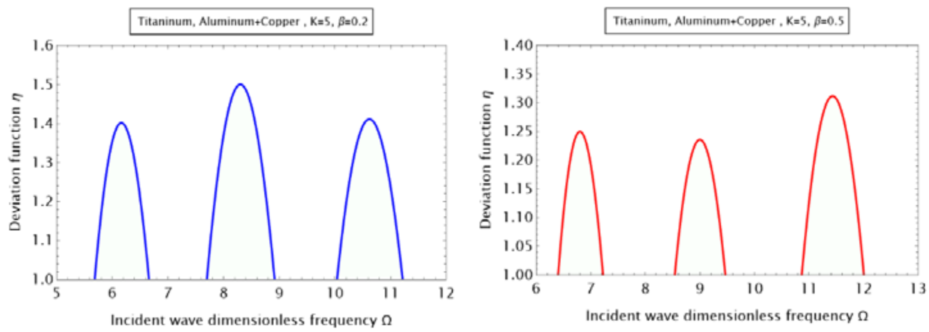

Figure 2. *Plots of a first and subsequent bandgaps for material at different values of parameter* β

Analysis of the function $\eta(\Omega)$ shows that that variations in material arrangement within the unit cell, as characterized by the filling coefficient $\beta=a/d$, slightly affect the lengths of the first bandgaps.

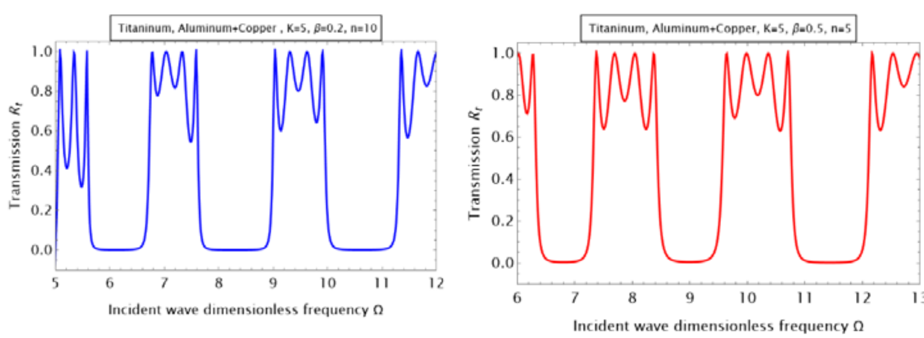

Figure 3. *Plots of a transmission functions at different values of parameter* β

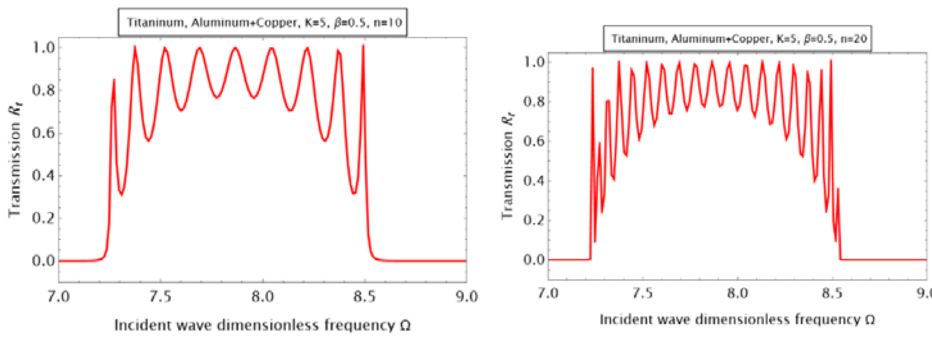

Figure 4. *Plots of a transmission functions at different values of cell numbers* $n = 10, n = 20$ *between first and second bandgaps*

Analysis of Fig. 4 illustrates that when the frequencies of the incident elastic wave fall within the bandgaps of the corresponding infinite periodic medium, the transmissivity of the elastic wave decreases significantly, approaching zero. In these frequency intervals, the layer becomes highly reflective, with the reflectivity nearing unity. This phenomenon occurs even in layers that contain a relatively small number $n = 5$ of unit cells, demonstrating the effect of bandgap formation on wave propagation in such structured materials.

As indicated by the transmission function plots, there exists a set of frequencies between any two bandgaps where the transmission function reaches a value of one, meaning that reflectivity effectively disappears. This phenomenon occurs consistently in the frequency ranges that lie between adjacent bandgaps. The presence of these specific frequencies can be explained by examining the behaviour of the transmission function outside of these intervals. In the range between two bandgaps, the transmission function possesses a number of zeroes, which are determined by the underlying physical properties of the system. The existence of a set of such frequencies can be explained as follows. Between two any bandgaps $|\eta| < 1$ the function has $N - 1$ zeros given by

$$\eta\left(\Omega_{0m}\right) = \cos\left(m\pi N^{-1}\right), m = 1, 2.....(N-1) .$$

From (14), it follows that at these frequencies,#

$$S_{N-1}\left(\eta\right) = 0, R_t = 1, (R_r = 0) .$$

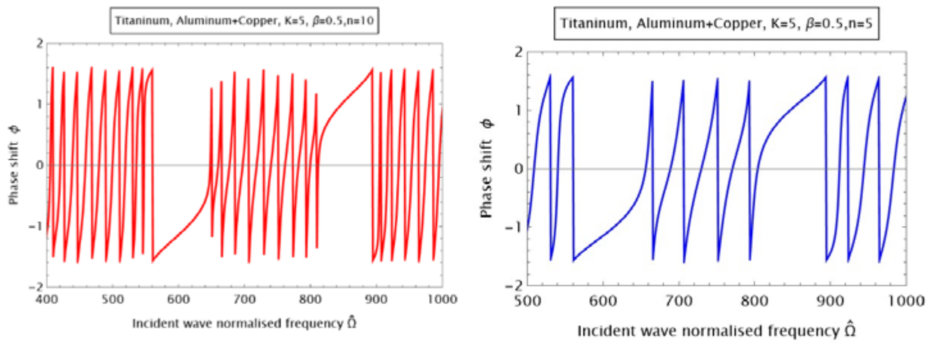

Figure 5. Phase shift plots of the reflected wave

The graph in Figures 5 based on (40) shows how the phase shift $\phi(\Omega)$ changes with frequency. Blue curves correspond to $n=10$, blue curves to $n=5$ structures. The phase shift is a function of the normalized frequency $\hat{\Omega}=100\Omega$.

As it follows from the plots on Figure 5 within the first bandgap the phase shifts $\phi(\Omega)$ reach its minimal and maximal values at the first and second gap boundaries $\phi=-\pi/2,\pi/2$. Outside of the gaps between the first and the second gaps there are the $N-1$ phase shift "resonance" curves each exhibiting repeated transitions from $-\pi/2$ to $\pi/2$.

Conclusions.

This study investigates the interaction between incident shear waves and a periodically layered bi-material elastic layer positioned between two identical elastic half-spaces. The analysis demonstrates that when the frequency of the incident shear waves falls within the bandgaps inherent to the equivalent infinite periodic medium, the transmissivity of the elastic waves becomes nearly zero. As a result, the waves are almost completely reflected, with the reflectivity approaching unity. This behaviour highlights the strong reflective nature of the layer within the bandgap regions. In contrast, for frequency ranges located between adjacent bandgaps, there exist certain discrete frequencies where the transmissivity of the elastic waves reaches unity. At these frequencies, perfect transmission occurs, and reflectivity is negligible. This phenomenon underscores the existence of transmission windows between bandgaps, where wave propagation through the layered structure is unhindered.

This phenomenon underscores the existence of transmission windows between bandgaps, where wave propagation through the layered structure is unhindered. Specifically, within these frequency intervals situated between two adjacent bandgaps, the transmission function attains a value of one. As a result, the reflectivity effectively drops to zero, allowing elastic waves to pass through the structure without significant attenuation or reflection. These transmission windows are a direct consequence of the physical properties of the periodic system, and their presence is consistently observed in the plots of the transmission function. Thus, between the bandgaps, the structure transitions from being highly reflective to fully transparent to incident elastic waves, which is a key characteristic in the analysis of wave propagation in such media. #

The phase shift of the reflected shear wave is analysed for the scenario in which one side of the layered structure is adjacent to an elastic half-space, while the opposite side is bounded by a free interface. Within the first bandgap, it is observed that the phase shift values attain their minimum and maximum precisely at the boundaries of the gap. Furthermore, in the frequency interval between the first and second bandgaps, phase shift resonance curves emerge, each exhibiting repeated transitions.

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Received 05 November, 2025