ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 77, №4, 2024 Механика

УДК 539.3 DOI: 10.54503/0002-3051-2024.77.4-70

ON THE CHARACTER OF FORCED VIBRATIONS OF PLATES HAVING A PLANE OF ELASTIC SYMMETRY

Aghalovyan L.A., Aghalovyan M.L., Zakaryan T.V., Tovmasyan A.B.

Keywords: the anisotropic plate, 3D vibrations, resonance, asymptotic solution.

The spatial problem on forced vibrations of composite plates, having a plane of elastic symmetry, is solved. It is considered that on the facial surface of the plate normal and tangential loads act, which change harmonically over time, and the lower facial surface of the plate is rigidly fixed. For solving such type of problems, the hypotheses of classical and refined theories of plates and shells are not applicable. A fundamentally new asymptotics for the components of the stress tensor and the displacement vector was established, which made it possible to find an asymptotic solution to the problem, which becomes mathematically exact if the external loads are algebraic polynomials from tangential coordinates. The conditions for the occurrence of resonance are derived. It is shown that in such a plate the vibrations are purely flat and anti-flat (transverse). The amplitudes of these oscillations have been determined. The amplitudes of these oscillations are determined. The conditions for the occurrence of resonance are derived, and the values of resonance frequencies are determined.

Աղալովյան Լ.Ա., Աղալովյան Մ.Լ., Զաքարյան Տ.Վ., Թովմասյան Ա.Բ.

Առաձգական սիմետրիայի հարթություն ունեցող սալի ստիպողական տատանումների բնույթի մասին

Հիմնաբառեր. անիզոտրոպություն, սալ, 3D տատնումներ, ռեզոնանս, ասիմպտոտիկ լուծում:

Առաձգական սիմետրիայի հարթություն ունեցող սալերի համար լուծված է ստիպողական տատանումների տարածական խնդիր: Համարվում է, որ սալի դիմային մակերևույթի վրա ազդում են նորմալ և տանգենցիալ, ժամանակի ընթացքում հարմոնիկ փոփոխվող ուժեր: Ստորին դիմային մակերևույթը կոշտ ամրակցված է: Սալերի դասական և գոյություն ունեցող ճշգրտված տեսությունները տվյալ խնդրի լուծման համար կիրառելի չեն: Գտնված է լարումների թենզորի և տեղափոխման վեկտորի բաղադրիչների համար դասականից սկզբունքորեն տարբերվող ասիմպտոտիկա, որը թույլ է տվել գտնել խնդրի ասիմպտոտիկ լուծումը: Այդ լուծումը դառնում է մաթեմատիկորեն ճշգրիտ, երբ արտաքին ազդող ուժերը տանգենցիալ կոորդինատներից հանրահաշվական բազմանդամներ են: Ցույց է տրված, որ տատանումները հանդիսանում են զուտ հարթ և հակահարթ: Որոշված են տատանումների ամպլիտուդները, արտածված են ռեզոնանսի առաջացման պայմանները, որոշված են ռեզոնանսային հաճախությունները:

Агаловян Л.А.,Агаловян М.Л., Закарян Т.В., Товмасян А.Б.

О характере вынужденных колебаний пластин имеющих плоскость упругой симметрии

Ключевые слова: анизотропная пластина, 3D колебания, резонанс, асимптотическое решение.

Решена пространственная задача о вынужденных колебаниях композитных пластин, имеющих плоскость упругой симметрии. Считается, что на лицевую поверхность пластины действуют нормальные и тангенциальные нагрузки, которые по времени изменяются гармонически, а нижняя лицевая поверхность пластины жёстко закреплена. Классическая и существующие уточнённые теории пластин для решения задачи неприменимы. Установлена принципиально новая асимптотика для компонент тензора напряжений и вектора перемещения, позволившая найти асимптотическое решение задачи, которое становится математически точным если внешние нагрузки являются алгебраическими многочленами от тангенциальных координат. Выведены условия возникновения резонанса. Показано, что в такой пластине колебания являются сугубо плоскими и антиплоскими (поперечными). Определены амплитуды этих колебаний. Выведены условия возникновения резонанса, определены значения резонансных частот.

Introduction

Depending on the winding angle or reinforcement method, often the corresponding composite material is anisotropic and has a plane of elastic symmetry [1,2]. There are relatively few works devoted to the study of stress-strain states and the solution of static and dynamic problems of plates with a plane of elastic symmetry (13 independent constants of elasticity).

The classical and refined theories of plates and shells consider only one class of problems: it is assumed that the values of the corresponding components of the stress tensor are given on the facial surfaces of the plates and shells (the first boundary value problem of the theory of elasticity). These theories are not applicable for solving the second (the values of the components of the displacement vector are given on the facial surfaces) and mixed boundary value problems. As it follows from mathematically precise solutions of individual even simple such problems, normal displacement depends on the transverse coordinate, which contradicts to one of the basic conditions of classical theory. Below it will be shown, that in similar problems the components of the stress tensor have the same intensity, which is absent in the classical theory of plates and shells.

In last decades for solving spatial static and dynamic problems of plates and shells, especially anisotropic ones, an effective turned out the asymptotic method for solving singularly perturbed differential equations. The first works in this area are the works [3-6]. Asymptotic theories of isotropic plates and shells [6,7] and anisotropic plates and shells [8] have been constructed. A fundamentally new asymptotics, in comparison with the classical theory, for the components of the stress tensor and displacement vector has been established [8, 9], which allow to find solutions to the second and mixed static and dynamic boundary value problems for single-layered and multilayered plates. To the solution of the static spatial problems of single-layered and multilayered isotropic and orthotropic plates and shells are devoted the monographs [7, 8, 10]. The method turned out particularly effective for solving dynamic problems of orthotropic plates and shells [8, 11-14].

Waves, localized and interface oscillations in isotropic thin bodies by asymptotic method were studied in [15-18]. The asymptotic method was used in [19,20] to study the stress-strain states of layered structures.

In this paper are studied forced vibrations of anisotropic plates, which have a plane of elastic symmetry. The asymptotic solution for a three-dimensional dynamic mixed problem of theory of elasticity on forced vibrations of an anisotropic plate is obtained.

1.**Statement of the problem, basic equations and relationships**

It is required to find in the area $D = \{(x, y, z): 0 \le x \le a, 0 \le y \le b, -h \le z \le h, 2h << l,\ l = \min(a, b)\}$, which is occupied by a plate (Fig. 1) solutions to the equations of motion of the three-dimensional problem of elasticity theory:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{1}$$
$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2},$$

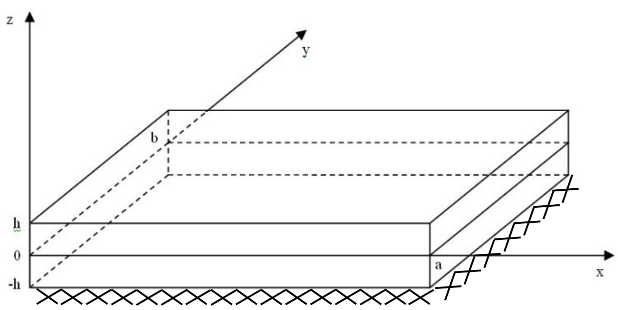

Fig.1. The structure of plate

and the relations of an anisotropic body, which have a plane of elastic symmetry [1,2]:

$$\frac{\partial u}{\partial x} = a_{11}\sigma_{xx} + a_{12}\sigma_{yy} + a_{13}\sigma_{zz} + a_{16}\sigma_{xy}$$
$$\frac{\partial v}{\partial y} = a_{12}\sigma_{xx} + a_{22}\sigma_{yy} + a_{23}\sigma_{zz} + a_{26}\sigma_{xy}$$
$$\frac{\partial w}{\partial z} = a_{13}\sigma_{xx} + a_{23}\sigma_{yy} + a_{33}\sigma_{zz} + a_{36}\sigma_{xy} \tag{2}$$
$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = a_{44}\sigma_{yz} + a_{45}\sigma_{xz}$$
$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = a_{45}\sigma_{yz} + a_{55}\sigma_{xz}$$
$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a_{16}\sigma_{xx} + a_{26}\sigma_{yy} + a_{36}\sigma_{zz} + a_{66}\sigma_{xy}$$

at the following boundary conditions at $z = h$

$$\sigma_{xz}(x, y, h, t) = \sigma_{xz}^{+}(\xi, \eta)\exp(i\Omega t),$$
$$\sigma_{yz}(x, y, h, t) = \sigma_{yz}^{+}(\xi, \eta)\exp(i\Omega t), \tag{3}$$
$$\sigma_{zz}(x, y, h, t) = -\sigma_{zz}^{+}(\xi, \eta)\exp(i\Omega t),$$
$$\xi = \frac{x}{l}, \eta = \frac{y}{l}, \zeta = \frac{z}{h},$$

and at $z = -h$

$$u(x, y, -h, t) = 0,\ v(x, y, -h, t) = 0,\ w(x, y, -h, t) = 0 \tag{4}$$

where Ω – the frequency of forced action.

The conditions on the lateral surfaces of the plate we will not specify for now; by them is caused the appearance of boundary layer .

2.The asymptotic solution of the problem.

The solution of the formulated problem will be sought in the form

$$\sigma_{\alpha\beta}(x,y,z,t) = \sigma_{ij}(\xi,\eta,\zeta)\exp(i\Omega t)\,, \alpha,\beta = x,y,z, i,j = 1,2,3, \quad (5)$$
$$u(x,y,z,t) = u_x(\xi,\eta,\zeta)\exp(i\Omega t)\,, (u,v,w; u_x,u_y,u_z)$$

Substituting (5) in equations (1) and elasticity relations (2) and in the newly obtained system, moving to dimensionless coordinates and displacements

$$\xi = \frac{x}{l}\,,\ \eta = \frac{y}{l},\ \zeta = \frac{z}{h},\ U = \frac{u_x}{l},\ \ V = \frac{u_y}{l},\ \ W = \frac{u_z}{l}\,, \quad (6)$$

as a result we will obtain the system singularly perturbed by the small parameter $\varepsilon = \frac{h}{l}$:

$$\frac{\partial\sigma_{11}}{\partial\xi} + \frac{\partial\sigma_{12}}{\partial\eta} + \varepsilon^{-1}\frac{\partial\sigma_{13}}{\partial\zeta} + \varepsilon^{-2}\Omega_*^2 U = 0,$$
$$\frac{\partial\sigma_{12}}{\partial\xi} + \frac{\partial\sigma_{22}}{\partial\eta} + \varepsilon^{-1}\frac{\partial\sigma_{23}}{\partial\zeta} + \varepsilon^{-2}\Omega_*^2 V = 0,$$
$$\frac{\partial\sigma_{13}}{\partial\xi} + \frac{\partial\sigma_{23}}{\partial\eta} + \varepsilon^{-1}\frac{\partial\sigma_{33}}{\partial\zeta} + \varepsilon^{-2}\Omega_*^2 W = 0, \quad (7)$$
$$\frac{\partial U}{\partial\xi} = a_{12}\sigma_{11} + a_{12}\sigma_{22} + a_{13}\sigma_{33} + a_{16}\sigma_{12},$$
$$\frac{\partial V}{\partial\eta} = a_{12}\sigma_{11} + a_{22}\sigma_{22} + a_{23}\sigma_{33} + a_{26}\sigma_{12},$$
$$\varepsilon^{-1}\frac{\partial W}{\partial\zeta} = a_{13}\sigma_{13} + a_{23}\sigma_{22} + a_{33}\sigma_{33} + a_{36}\sigma_{12},$$
$$\frac{\partial W}{\partial\eta} + \varepsilon^{-1}\frac{\partial V}{\partial\zeta} = a_{44}\sigma_{23} + a_{45}\sigma_{13},$$
$$\frac{\partial W}{\partial\xi} + \varepsilon^{-1}\frac{\partial U}{\partial\zeta} = a_{45}\sigma_{23} + a_{55}\sigma_{13},$$
$$\frac{\partial V}{\partial\xi} + \frac{\partial U}{\partial\eta} = a_{16}\sigma_{11} + a_{26}\sigma_{22} + a_{36}\sigma_{33} + a_{66}\sigma_{12},$$
$$\Omega_*^2 = \rho h^2\Omega^2,$$

The solution of the singularly perturbed system (7) is the sum of the solutions of the external problem (I^{out}) and the boundary layer (I_b)[8]: $I = I^{out} + I_b$.

The solution to the external problem we will seek out in the form of

$$\sigma_{ij}^{out} = \varepsilon^{-1+s}\sigma_{ij}^{(s)}(\xi,\eta,\zeta), i,j = 1,2,3, s = \overline{0,N}$$
$$(U^{out}, V^{out}, W^{out}) = \varepsilon^s\big(U^{(s)}, V^{(s)}, W^{(s)}\big), \quad (8)$$

where notation $s = \overline{0,N}$ means summation by repeating (umbral) index s from 0 to number of approximations N. From (8) it follows that the stresses must have the same intensity.

By substituting (8) into (7) and in each equation equating coefficients at the same powers ε, we will obtain the following consistent system for determining unknown functions $\sigma_{ij}^{(s)}, U^{(s)}, V^{(s)}, W^{(s)}$:

$$\frac{\partial\sigma_{11}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{12}^{(s-1)}}{\partial\eta} + \frac{\partial\sigma_{13}^{(s)}}{\partial\zeta} + \Omega_*^2 U^{(s)} = 0,$$
$$\frac{\partial\sigma_{12}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{22}^{(s-1)}}{\partial\eta} + \frac{\partial\sigma_{23}^{(s)}}{\partial\zeta} + \Omega_*^2 V^{(s)} = 0$$
$$\frac{\partial\sigma_{13}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{23}^{(s-1)}}{\partial\eta} + \frac{\partial\sigma_{33}^{(s)}}{\partial\zeta} + \Omega_*^2 W^{(s)} = 0,$$

$$
\begin{aligned}
&\frac{\partial U^{(s-1)}}{\partial \xi} = a_{11}\sigma_{11}^{(s)} + a_{12}\sigma_{22}^{(s)} + a_{13}\sigma_{33}^{(s)} + a_{16}\sigma_{12}^{(s)},\\
&\frac{\partial V^{(s-1)}}{\partial \eta} = a_{12}\sigma_{11}^{(s)} + a_{22}\sigma_{22}^{(s)} + a_{23}\sigma_{33}^{(s)} + a_{26}\sigma_{12}^{(s)},\\
&\frac{\partial W^{(s)}}{\partial \zeta} = a_{13}\sigma_{11}^{(s)} + a_{23}\sigma_{22}^{(s)} + a_{33}\sigma_{33}^{(s)} + a_{36}\sigma_{12}^{(s)},\\
&\frac{\partial W^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s)}}{\partial \zeta} = a_{44}\sigma_{23}^{(s)} + a_{45}\sigma_{13}^{(s)},\\
&\frac{\partial W^{(s-1)}}{\partial \xi} + \frac{\partial U^{(s)}}{\partial \zeta} = a_{45}\sigma_{23}^{(s)} + a_{55}\sigma_{13}^{(s)},\\
&\frac{\partial V^{(s-1)}}{\partial \xi} + \frac{\partial U^{(s-1)}}{\partial \eta} = a_{16}\sigma_{11}^{(s)} + a_{26}\sigma_{22}^{(s)} + a_{36}\sigma_{33}^{(s)} + a_{66}\sigma_{12}^{(s)},
\end{aligned} \tag{9}
$$

From the elasticity relations of system (9) all stresses can be expressed through displacements by formulas

$$
\begin{aligned}
&\sigma_{13}^{(s)} = \frac{1}{\Delta_5}\left(a_{44}\frac{\partial U^{(s)}}{\partial \zeta} - a_{45}\frac{\partial V^{(s)}}{\partial \zeta} + \sigma_{13*}^{(s)}\right),\\
&\sigma_{23}^{(s)} = \frac{1}{\Delta_5}\left(-a_{45}\frac{\partial U^{(s)}}{\partial \zeta} + a_{55}\frac{\partial V^{(s)}}{\partial \zeta} + \sigma_{23*}^{(s)}\right),\\
&\sigma_{33}^{(s)} = \frac{1}{\Delta}\left(A_{33}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{33*}^{(s)}\right),\ \sigma_{11}^{(s)} = \frac{1}{\Delta}\left(A_{13}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{11*}^{(s)}\right),\\
&\sigma_{22}^{(s)} = \frac{1}{\Delta}\left(A_{23}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{22*}^{(s)}\right), \sigma_{12}^{(s)} = \frac{1}{\Delta}\left(A_{36}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{12*}^{(s)}\right),\\
&\sigma_{13*}^{(s)} = a_{44}\frac{\partial W^{(s-1)}}{\partial \xi} - a_{45}\frac{\partial W^{(s-1)}}{\partial \eta}, \sigma_{23*}^{(s)} = -a_{45}\frac{\partial W^{(s-1)}}{\partial \xi} + a_{55}\frac{\partial W^{(s-1)}}{\partial \eta},\\
&\sigma_{33*}^{(s)} = \frac{\partial}{\partial \xi}\left(A_{13}U^{(s-1)} + A_{63}V^{(s-1)}\right) + \frac{\partial}{\partial \eta}\left(A_{63}U^{(s-1)} + A_{23}V^{(s-1)}\right),\\
&\sigma_{11*}^{(s)} = \frac{\partial}{\partial \xi}\left(A_{11}U^{(s-1)} + A_{16}V^{(s-1)}\right) + \frac{\partial}{\partial \eta}\left(A_{16}U^{(s-1)} + A_{12}V^{(s-1)}\right),\\
&\sigma_{22*}^{(s)} = \frac{\partial}{\partial \xi}\left(A_{12}U^{(s-1)} + A_{26}V^{(s-1)}\right) + \frac{\partial}{\partial \eta}\left(A_{26}U^{(s-1)} + A_{22}V^{(s-1)}\right),\\
&\sigma_{12*}^{(s)} = \frac{\partial}{\partial \xi}\left(A_{16}U^{(s-1)} + A_{66}V^{(s-1)}\right) + \frac{\partial}{\partial \eta}\left(A_{66}U^{(s-1)} + A_{26}V^{(s-1)}\right),\\
&\sigma_{ij}^{(m)} = 0, U^{(m)} = V^{(m)} = W^{(m)} = 0 \text{ at } m<0
\end{aligned} \tag{10}
$$

where

$$
\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & a_{16}\\ a_{12} & a_{22} & a_{23} & 0 & 0 & a_{26}\\ a_{13} & a_{23} & a_{33} & 0 & 0 & a_{36}\\ 0 & 0 & 0 & a_{44} & a_{45} & 0\\ 0 & 0 & 0 & a_{45} & a_{55} & 0\\ a_{16} & a_{26} & a_{36} & 0 & 0 & a_{66} \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{16}\\ a_{12} & a_{22} & a_{23} & a_{26}\\ a_{13} & a_{23} & a_{33} & a_{36}\\ a_{16} & a_{26} & a_{36} & a_{66} \end{vmatrix}, \Delta_4 = \begin{vmatrix} a_{11} & a_{12} & a_{16}\\ a_{12} & a_{22} & a_{26}\\ a_{16} & a_{26} & a_{66} \end{vmatrix},
$$

$$
\Delta_5 = a_{44}a_{55} - a_{45}^2, \quad \Delta = \Delta_3\Delta_5, \quad A_{33} = \Delta_4\Delta_5, \tag{11}
$$

A_{ij} – cofactor, corresponding to element a_{ij} of determinant Δ.

By substituting $\sigma_{13}^{(s)}, \sigma_{23}^{(s)}$ into two first equations of system (9), for determining $U^{(s)}, V^{(s)}$ we will obtain the system

$$a_{44}\frac{\partial^2 U^{(s)}}{\partial\zeta^2} - a_{45}\frac{\partial^2 V^{(s)}}{\partial\zeta^2} + \Delta_5\Omega_*^2 U^{(s)} = R_u^{(s)}(\xi,\eta,\zeta)$$

$$-a_{45}\frac{\partial^2 U^{(s)}}{\partial\zeta^2} + a_{55}\frac{\partial^2 V^{(s)}}{\partial\zeta^2} + \Delta_5\Omega_*^2 V^{(s)} = R_v^{(s)}(\xi,\eta,\zeta) \tag{12}$$

$$R_u^{(s)} = -\Delta_5\left(\frac{\partial\sigma_{11}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{12}^{(s-1)}}{\partial\eta}\right) - a_{44}\frac{\partial^2 W^{(s-1)}}{\partial\xi\partial\zeta} + a_{45}\frac{\partial^2 W^{(s-1)}}{\partial\eta\partial\zeta},$$

$$R_v^{(s)} = -\Delta_5\left(\frac{\partial\sigma_{12}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{22}^{(s-1)}}{\partial\eta}\right) + a_{45}\frac{\partial^2 W^{(s-1)}}{\partial\xi\partial\zeta} - a_{55}\frac{\partial^2 W^{(s-1)}}{\partial\eta\partial\zeta},$$

By substituting the value of $\sigma_{33}^{(s)}$ into the third equation of system (9), for determination of $W^{(s)}$ we will obtain the equation

$$\frac{\partial^2 W^{(s)}}{\partial\zeta^2} + \frac{\Delta_3}{\Delta_4}\Omega_*^2 W^{(s)} = \frac{1}{\Delta_4\Delta_5}R_w^{(s)}(\xi,\eta,\zeta) \tag{13}$$

$$R_w^{(s)} = -\Delta_5\left(\frac{\partial\sigma_{12}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{22}^{(s-1)}}{\partial\eta}\right) - \frac{1}{\Delta_3}\left[\frac{\partial^2}{\partial\xi\partial\zeta}\left(A_{13}U^{(s-1)} + A_{63}V^{(s-1)}\right) + \right.$$

$$\left. + \frac{\partial^2}{\partial\eta\partial\zeta}\left(A_{63}U^{(s-1)} + A_{23}V^{(s-1)}\right)\right],$$

From the system (12) it follows that from the very beginning $(s = 0)$ displacements $U^{(s)}, V^{(s)}$ are dependent, independent is the equation for $W^{(s)}$. And for orthotropic plates $(a_{45} = 0, \Delta_5 = a_{55}a_{44})$ they are independent, to them are correspond the shear vibrations ($a_{44} = \frac{1}{G_{23}}, a_{55} = \frac{1}{G_{13}}, G_{13}, G_{23}$ −modulus of shear). To the equation (13) correspond the longitudinal vibrations. We can state that for plates with a plane of elastic symmetry, shear vibrations are dependent from the very beginning, but they in the initial approximation do not depend on longitudinal vibrations. For orthotropic plates, in the initial approximation, all three vibrations are independent.

From system (12) $V^{(s)}$can be expressed through $U^{(s)}$ by formula

$$V^{(s)} = -\frac{1}{a_{45}\Omega_*^2}\frac{\partial^2 U^{(s)}}{\partial\zeta^2} - \frac{a_{55}}{a_{45}}U^{(s)} + R_v^{*(s)} \tag{14}$$

$$R_v^{*(s)} = \frac{1}{\Delta_5\Omega_*^2}\left(\frac{a_{55}}{a_{45}}R_u^{(s)} + R_v^{(s)}\right)$$

and for determining $U^{(s)}$taking into acount (14) we will obtain the equation

$$\frac{\partial^4 U^{(s)}}{\partial\zeta^4} + (a_{44}+a_{55})\Omega_*^2\frac{\partial^2 U^{(s)}}{\partial\zeta^2} + \Delta_5\Omega_*^2 U^{(s)} = \bar{R}_u^{(s)} \tag{15}$$

$$\bar{R}_u^{(s)} = \frac{1}{\Delta_5}\left(a_{55}\frac{\partial^2 R_u^{(s)}}{\partial\zeta^2} + \Delta_5\Omega_*^2 R_u^{(s)} + a_{45}\frac{\partial^2 R_v^{(s)}}{\partial\zeta^2}\right)$$

The solution to the equation (15) is $U^{(s)} = U_0^{(s)} + U_\tau^{(s)}(\xi,\eta,\zeta)$, where $U_\tau^{(s)}$ −is particular solution. The characteristic equation of the homogeneous equation (15) is

$$\lambda^4 + (a_{44} + a_{55})\Omega_*^2\lambda^2 + \Delta_5\Omega_*^4 = 0,$$

$$\text{or } k^2 + (a_{44} + a_{55})k + \Delta_5 = 0, k = \left(\frac{\lambda}{\Omega_*}\right)^2 \tag{16}$$

The roots of this equation are

$$k_{1,2}=\frac{-(a_{44}+a_{55})\pm\sqrt{(a_{44}+a_{55})^2-4\Delta_5}}{2}=\frac{-(a_{44}+a_{55})\pm\sqrt{(a_{44}-a_{55})^2+4a_{45}^2}}{2}$$

Since $\Delta_5>0 \Rightarrow k_{1,2}<0$, $\lambda_{1,2}=\pm b_1\Omega_* i, \lambda_{3,4}=\pm b_2\Omega_* i$ consequently

$$U_0^{(s)}=D_1^{(s)}\cos b_1\Omega_*\zeta+D_2^{(s)}\sin b_1\Omega_*\zeta+D_3^{(s)}\cos b_2\Omega_*\zeta+D_4^{(s)}\sin b_2\Omega_*\zeta \quad (17)$$

$$b_{1,2}=\sqrt{\frac{(a_{44}+a_{55})\mp\sqrt{(a_{44}-a_{55})^2+4a_{45}^2}}{2}}$$

According to (14),(17) we have

$$V^{(s)}=D_1^{(s)}d_1\cos b_1\Omega_*\zeta+D_2^{(s)}d_1\sin b_1\Omega_*\zeta+ \\ +D_3^{(s)}d_2\cos b_2\Omega_*\zeta+D_4^{(s)}d_2\sin b_2\Omega_*\zeta+\bar{V}_\tau^{(s)} \quad (18)$$

$$d_1=\frac{1}{a_{45}}(b_1^2-a_{55}), \qquad d_2=\frac{1}{a_{45}}(b_2^2-a_{55})$$

$$\bar{V}_\tau^{(s)}=-\frac{1}{a_{45}\Omega_*^2}\frac{\partial^2 U_\tau^{(s)}}{\partial\zeta^2}-\frac{a_{55}}{a_{45}}U_\tau^{(s)}+\frac{1}{\Delta_5\Omega_*^2}\left(\frac{a_{55}}{a_{45}}R_u^{(s)}+R_v^{(s)}\right)$$

According to (10),(17), (18)

$$\sigma_{13}^{(s)}=\frac{1}{\Delta_5}(D_1^{(s)}d_{11}\sin b_1\Omega_*\zeta-D_2^{(s)}d_{11}\cos b_1\Omega_*\zeta+ \\ +D_3^{(s)}d_{12}\sin b_2\Omega_*\zeta-D_4^{(s)}d_{12}\cos b_2\Omega_*\zeta+\sigma_{13\tau}^{(s)}) \quad (19)$$

$$\sigma_{23}^{(s)}=\frac{1}{\Delta_5}\Big(D_1^{(s)}d_{21}\sin b_1\Omega_*\zeta-D_2^{(s)}d_{21}\cos b_1\Omega_*\zeta+ \\ +D_3^{(s)}d_{22}\sin b_2\Omega_*\zeta-D_4^{(s)}d_{22}\cos b_2\Omega_*\zeta+\sigma_{23\tau}^{(s)}\Big)$$

$$d_{11}=\Omega_* b_1(b_1^2-a_{55}-a_{44}), \quad d_{12}=\Omega_* b_2(b_2^2-a_{55}-a_{44})$$

$$d_{21}=\Omega_* b_1(a_{45}-a_{55}d_1), \qquad d_{22}=\Omega_* b_2(a_{45}-a_{55}d_2)$$

$$\sigma_{13\tau}^{(s)}=a_{44}\frac{\partial U_\tau^{(s)}}{\partial\zeta}-a_{45}\frac{\partial V_\tau^{(s)}}{\partial\zeta}+\sigma_{13*}^{(s)},$$

$$\sigma_{23\tau}^{(s)}=-a_{45}\frac{\partial U_\tau^{(s)}}{\partial\zeta}+a_{55}\frac{\partial V_\tau^{(s)}}{\partial\zeta}+\sigma_{23*}^{(s)},$$

The solution to the equation (13) is

$$W^{(s)}=W_0^{(s)}+W_\tau^{(s)}(\xi,\eta,\zeta)$$

$$W_0^{(s)}=D_5^{(s)}\cos\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*\zeta+D_6^{(s)}\sin\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*\zeta \quad \text{at } \frac{\Delta_3}{\Delta_4}>0, \quad (20)$$

$$W_0^{(s)}=D_5^{(s)}\operatorname{ch}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*\zeta+D_6^{(s)}\operatorname{sh}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*\zeta \quad \text{at } \frac{\Delta_3}{\Delta_4}<0, \quad (21)$$

$$\sigma_{33}^{(s)}=\frac{1}{\Delta}\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_* A_{33}\left(-D_5^{(s)}\sin\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*\zeta+D_6^{(s)}\cos\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*\zeta\right)+\sigma_{33\tau}^{(s)}, \text{at } \frac{\Delta_3}{\Delta_4}>0, \quad (22)$$

$$\sigma_{33}^{(s)}=\frac{1}{\Delta}\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_* A_{33}\left(D_5^{(s)}\operatorname{sh}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*\zeta+D_6^{(s)}\operatorname{ch}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*\zeta\right)+\sigma_{33\tau}^{(s)}, \text{at } \frac{\Delta_3}{\Delta_4}<0, \quad (23)$$

$$\sigma_{33\tau}^{(s)}=\frac{1}{\Delta}\left(A_{33}\frac{\partial W_\tau^{(s)}}{\partial\zeta}+\sigma_{33*}^{(s)}\right),$$

The general solution of the external problem contains 6 yet unknown functions $D_1^{(s)}(\xi,\eta)-D_6^{(s)}(\xi,\eta)$, which are uniquely determined using boundary conditions (3),(4).

3.The determination of unknown functions of the solution

For determining the values of $D_1^{(s)}, D_2^{(s)}, D_3^{(s)}, D_4^{(s)}$ we satisfy conditions (3),(4) with respect to $\sigma_{xz}, \sigma_{yz}, u, v$. Using formulas (8), (17) - (19) we have

$$\begin{aligned} &D_1^{(s)} d_{11} c_1 - D_2^{(s)} d_{11} c_2 + D_3^{(s)} d_{12} c_3 - D_4^{(s)} d_{12} c_4 = e_1^{(s)} \\ &D_1^{(s)} d_{21} c_1 - D_2^{(s)} d_{21} c_2 + D_3^{(s)} d_{22} c_3 - D_4^{(s)} d_{22} c_4 = e_2^{(s)} \\ &D_1^{(s)} c_2 - D_2^{(s)} c_1 + D_3^{(s)} c_4 - D_4^{(s)} c_3 = e_3^{(s)} \\ &D_1^{(s)} d_1 c_2 - D_2^{(s)} d_1 c_1 + D_3^{(s)} d_2 c_4 - D_4^{(s)} d_2 c_3 = e_4^{(s)} \end{aligned} \qquad (24)$$

Where

$$e_1^{(s)} = \Delta_5 \sigma_{13}^{+(s)} - \sigma_{13*}^{(s)},\ \ e_2^{(s)} = \Delta_5 \sigma_{23}^{+(s)} - \sigma_{23*}^{(s)},$$

$$\sigma_{13}^{+(0)} = \varepsilon \sigma_{13}^{+},\quad \sigma_{13}^{+(s)} = 0,\quad s \neq 0$$

$$e_3^{(s)} = -U_\tau^{(s)}(\xi,\eta,-1),\qquad e_4^{(s)} = -\bar{V}_\tau^{(s)}(\xi,\eta,-1)$$

$$c_1 = \sin b_1 \Omega_*,\ \ c_2 = \cos b_1 \Omega_*,\ \ c_3 = \sin b_2 \Omega_*,\quad c_4 = \cos b_2 \Omega_*$$

By Cramer’s formula

$$D_j^{(s)} = \frac{\Delta_{ju}^{(s)}}{\Delta_u},\ j=1,2,3,4 \qquad (25)$$

$$\Delta_u = \begin{vmatrix} d_{11}c_1 & -d_{11}c_2 & d_{12}c_3 & -d_{12}c_4 \\ d_{21}c_1 & -d_{21}c_2 & d_{22}c_3 & -d_{22}c_4 \\ c_2 & -c_1 & c_4 & -c_3 \\ d_1 c_2 & -d_1 c_1 & d_2 c_4 & -d_2 c_3 \end{vmatrix} = (d_2 - d_1)(d_{11}d_{22} - $$
$$ - d_{12}d_{21})\cos 2b_1\Omega_* \cos 2b_2\Omega_*$$

$\Delta_{ju}^{(s)}$ is obtained from Δ_u by replacing j-th column with column from free terms $e_i^{(s)}, i = 1,2,3,4$.

According to (25) we have

$$D_1^{(s)} = \frac{c_1}{\delta_1}\left(e_1^{(s)} d_{22} - e_2^{(s)} d_{12}\right),\qquad D_2^{(s)} = \frac{c_2}{\delta_1}\left(e_1^{(s)} d_{22} - e_2^{(s)} d_{12}\right),$$
$$D_3^{(s)} = \frac{c_3}{\delta_2}\left(e_1^{(s)} d_{21} - e_2^{(s)} d_{11}\right),\qquad D_4^{(s)} = \frac{c_4}{\delta_2}\left(e_1^{(s)} d_{22} - e_2^{(s)} d_{12}\right), \qquad (26)$$
$$\delta_1 = (d_{11}d_{22} - d_{12}d_{21})\cos 2b_1\Omega_*$$
$$\delta_2 = (d_{11}d_{22} - d_{12}d_{21})\cos 2b_2\Omega_*$$

At $\cos 2b_1\Omega_* = 0$ or $\cos 2b_2\Omega_* = 0$ the resonance will occur. The resonant frequencies are

$$\Omega = \frac{1}{2h}\sqrt{\frac{\pi}{b_1\rho}(2n-1)},\qquad \Omega = \frac{1}{2h}\sqrt{\frac{\pi}{b_2\rho}(2n-1)},\quad n \in N.$$

Using (20), (22) and satisfying to conditions (3), (4) relatively σ_{zz}, w, we will obtain the system

$$D_5^{(s)}c_5 - D_6^{(s)}c_6 = e_5^{(s)} \tag{27}$$
$$-D_5^{(s)}c_6 + D_6^{(s)}c_5 = e_6^{(s)}$$
from which is follows

$$D_5^{(s)} = \frac{e_5^{(s)}c_5 + e_6^{(s)}c_6}{\cos 2\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*}, \qquad D_6^{(s)} = \frac{e_6^{(s)}c_5 + e_5^{(s)}c_6}{\cos 2\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*},$$
$$e_5^{(s)} = -W_\tau^{(s)}(\xi,\eta,-1), \quad e_6^{(s)} = -\frac{\Omega_*\Delta}{A_{33}}\sqrt{\frac{\Delta_4}{\Delta_3}}\left(\sigma_{33}^{+(s)} + \sigma_{33\tau}^{(s)}\right)_{\zeta=1} \tag{28}$$
$$\sigma_{33}^{+(0)} = \varepsilon\sigma_{33}^{+}, \quad \sigma_{33}^{+(s)} = 0, \; s \neq 0$$
$$c_5 = \cos\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*, \qquad c_6 = \sin\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*, \text{ at } \frac{\Delta_3}{\Delta_4} > 0$$

The resonant frequencies are

$$\Omega = \frac{1}{2h}\left(\frac{\Delta_4}{\Delta_3}\right)^{\frac{1}{4}}\sqrt{\frac{\pi}{\rho}(2n-1)}, \; n \in N$$

To the case (21),(23) correspond

$$D_5^{(s)} = \frac{e_5^{(s)}c_5 + e_6^{(s)}c_6}{ch2\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*}, \quad D_6^{(s)} = \frac{e_6^{(s)}c_5 - e_5^{(s)}c_6}{ch2\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*}, \quad e_5^{(s)} = -W_\tau^{(s)}(\xi,\eta,-1),$$
$$e_6^{(s)} = -\frac{\Omega_*\Delta}{A_{33}}\sqrt{\left|\frac{\Delta_4}{\Delta_3}\right|}\left(\sigma_{33}^{+(s)} + \sigma_{33\tau}^{(s)}\right)_{\zeta=1} \tag{29}$$
$$c_5 = \mathrm{ch}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*, \quad c_6 = \mathrm{sh}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*, \quad \text{at} \quad \frac{\Delta_3}{\Delta_4} < 0$$

Since $\mathrm{ch}2\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_* \neq 0$ there is no resonance, i.e. longitudinal vibrations in this case aren't resonant.

4. On mathematically precise solutions

If functions in $\sigma_{xz}^{+}, \sigma_{yz}^{+}, \sigma_{zz}^{+}$ included in the boundary conditions (3) are polynomials from ξ, η , the iteration breaks at the certain approximation, which depends on the degree of polynomial. As a result, we will have the mathematically exact solution in the external problem. Particularly, at $\sigma_{xz}^{+} = \mathrm{const}, \sigma_{yz}^{+} = \mathrm{const}, \sigma_{zz}^{+} = \mathrm{const}$ the process breaks at initial approximation. For this case we have

$$u = lU^{(0)}\exp(i\Omega t), \quad v = lV^{(0)}\exp(i\Omega t),$$
$$\sigma_{xz} = \varepsilon^{-1}\sigma_{13}^{(0)}\exp(i\Omega t), \quad \sigma_{yz} = \varepsilon^{-1}\sigma_{23}^{(0)}\exp(i\Omega t)$$
$$U^{(0)} = D_1^{(0)}\cos b_1\Omega_*\zeta + D_2^{(0)}\sin b_1\Omega_*\zeta + D_3^{(0)}\cos b_2\Omega_*\zeta + D_4^{(0)}\sin b_2\Omega_*\zeta$$
$$V^{(0)} = D_1^{(0)}d_1\cos b_1\Omega_*\zeta + D_2^{(0)}d_1\sin b_1\Omega_*\zeta +$$
$$+D_3^{(0)}d_2\cos b_2\Omega_*\zeta + D_4^{(0)}d_2\sin b_2\Omega_*\zeta \tag{30}$$

$$\sigma_{13}^{(0)} = \frac{1}{\Delta_5}\Big(D_1^{(0)} d_{11}\sin b_1\Omega_*\zeta - D_2^{(0)} d_{11}\cos b_1\Omega_*\zeta + $$
$$+ D_3^{(0)} d_{12}\sin b_2\Omega_*\zeta - D_4^{(0)} d_{12}\cos b_2\Omega_*\zeta\Big),$$
$$\sigma_{23}^{(0)} = \frac{1}{\Delta_5}\Big(D_1^{(0)} d_{21}\sin b_1\Omega_*\zeta - D_2^{(0)} d_{21}\cos b_1\Omega_*\zeta + $$
$$+ D_3^{(0)} d_{22}\sin b_2\Omega_*\zeta - D_4^{(0)} d_{22}\cos b_2\Omega_*\zeta\Big)$$
$$D_1^{(0)} = \frac{\Delta_5 c_1 \varepsilon}{\delta_1}(\sigma_{13}^{+} d_{22} - \sigma_{23}^{+} d_{21})$$
$$D_2^{(0)} = \frac{\Delta_5 c_2 \varepsilon}{\delta_1}(\sigma_{13}^{+} d_{22} - \sigma_{23}^{+} d_{12})$$
$$D_3^{(0)} = \frac{\Delta_5 c_3 \varepsilon}{\delta_2}(\sigma_{13}^{+} d_{21} - \sigma_{23}^{+} d_{11})$$
$$D_4^{(0)} = \frac{\Delta_5 c_4 \varepsilon}{\delta_2}(\sigma_{13}^{+} d_{21} - \sigma_{23}^{+} d_{11})$$

for the remaining values we have

$$w = lW^{(0)} exp(i\Omega t), \qquad \sigma_{xx} = \varepsilon^{-1}\sigma_{11}^{(0)} exp(i\Omega t),$$
$$\sigma_{xy} = \varepsilon^{-1}\sigma_{12}^{(0)} exp(i\Omega t), \quad \sigma_{yy} = \varepsilon^{-1}\sigma_{22}^{(0)} exp(i\Omega t),$$
$$\sigma_{zz} = \varepsilon^{-1}\sigma_{33}^{(0)} exp(i\Omega t),$$

at $\frac{\Delta_3}{\Delta_4} > 0$

$$W_0^{(0)} = D_5^{(0)}\cos\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*\zeta + D_6^{(0)}\sin\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*\zeta,$$
$$\sigma_{33}^{(0)} = \frac{A_{33}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta}, \quad \sigma_{11}^{(0)} = \frac{A_{13}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta}, \tag{31}$$
$$\sigma_{22}^{(0)} = \frac{A_{23}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta}, \quad \sigma_{12}^{(0)} = \frac{A_{36}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta},$$
$$D_5^{(0)} = \frac{e_6^{(0)} c_6}{cos2\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*}, \quad D_6^{(0)} = \frac{e_6^{(0)} c_5}{cos2\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*},$$
$$e_6^{(0)} = \frac{\Delta\Omega_*}{A_{33}}\varepsilon\sigma_{33}^{+}\sqrt{\frac{\Delta_4}{\Delta_3}}, \quad c_5 = \cos\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*, \quad c_6 = \sin\sqrt{\frac{\Delta_3}{\Delta_4}}\Omega_*$$

at $\frac{\Delta_3}{\Delta_4} < 0$

$$W_0^{(0)} = D_5^{(0)}\mathrm{ch}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*\zeta + D_6^{(0)}\mathrm{sh}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*\zeta,$$
$$\sigma_{33}^{(0)} = \frac{A_{33}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta}, \quad \sigma_{11}^{(0)} = \frac{A_{13}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta}, \tag{32}$$
$$\sigma_{22}^{(0)} = \frac{A_{23}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta}, \quad \sigma_{12}^{(0)} = \frac{A_{36}}{\Delta}\frac{\partial W^{(0)}}{\partial\zeta},$$
$$D_5^{(0)} = \frac{e_6^{(0)} c_6}{ch2\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*}, \quad D_6^{(0)} = \frac{e_6^{(0)} c_5}{\mathrm{ch}2\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\Omega_*},$$

$$e_6^{(0)} = -\frac{\Delta\Omega_*}{A_{33}}\varepsilon\sigma_{33}^{+}\sqrt{\left|\frac{\Delta_4}{\Delta_3}\right|}, \qquad c_5 = ch\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\,\Omega_*, \qquad c_6 = \mathrm{sh}\sqrt{\left|\frac{\Delta_3}{\Delta_4}\right|}\,\Omega_*.$$

Obtained in the work the solution to the external problem satisfies the equations of motion, elasticity relations and boundary conditions (3), (4) on the facial surfaces of the plate. It, as a rule, will not satisfy the boundary conditions on the lateral surface of the plate. The arising discrepancy, according to the asymptotic method for solving singularly perturbed differential equations, is eliminated by constructing the solution for the boundary layer [3, 21]. All component of stresses and displacements decrease rapidly (exponentially) with distance from the lateral surface into the inside of the plate. This solution is constructed separately (autonomously) and is conjugated with the solution to the external problem in the manner described in [6,8], we do not stop on this. In elasticity theory this is known as end effects [22].

Conclusions

The asymptotic solution to the spatial dynamic problem of forced vibrations of anisotropic plates, which have a plane of elastic symmetry, is determined One of the facial surfaces of the plate is rigidly fixed, and the opposite facial surface is subject to normal and tangential loads, which are harmonically changed over time. It is shown, that for solving this class of problems (the second and mixed boundary value problems of elasticity theory) the hypotheses of classical and refined theories of plates are not applicable. The asymptotics for all components of the stress tensor and displacement vector was established, which made it possible to find the asymptotic solution to the three-dimensional problem.

All components of the stress tensor expressed in terms of the components of the displacement vector. For determining the tangential components of the displacement vector, the system of two ordinary differential equations of the second order is derived, which is reduced to the solution of the ordinary differential equation of the fourth order. To it corresponds plane shear vibrations, which in the case of orthotropic plates break down into two independent shear vibrations. For determining the normal component of the displacement vector, the ordinary differential equation of the second order is derived; to which are correspond the longitudinal vibrations.

Analytical solutions to all equations have been found The conditions for the occurrence of resonance have been established, and the values of the resonant frequencies have been determined. The case of anisotropy has been established at which in the longitudinal vibrations of the plate resonance is impossible.

References

1. Lekhnitsky S. G. Theory of Elasticity of Anisotropic Body. - Moscow: Nauka, 1977. - 416 p.
2. Malmayster A. K., Tamuzh V. P., Teters G. A. The Strength of Polymer and Composite Materials.- Riga: Zinatne, 1980. - 572 p.
3. Friedrichs K. O. Asimptotic Phenomena in Mathematical Physics. //Bull. Amer. Math. Soc. - 1955. -Vol.61 - P.485.

4. Friedrichs K.O. and Dressler R.F. A Boundary-Layer Theory for Elastic Plates.// Comm. Pure and Appl. Math. - 1961. -Vol. 14.- №1. – P. 114-122.

5. Green A.E. On the Linear Theory of Thin Elastic Shells // Proc. Roy. Soc. Ser. A.-1962. -Vol. 266.- №1325.- P. 143-161.

6. Goldenweiser A.L. Derivation of an Approximate Theory of Bending of a Plate by the Method of Asymptotic Integration of the Equations of the Theory of Elasticity // J. Appl. Math. Mech. - 1962. -Vol. 26.- Issue. 4. - P. 1000-1025 .

7. Goldenweiser A.L. Theory of Thin Elastic Shells. Moscow: Nauka.- 1976.- 512 p.

8. Aghalovyan L.A. Asymptotic Theory of Anisotropic Plates and Shells —Singapore. World Scientific Publishing. - 2015. - 376 p. (Russian Edition: Moscow, Nauka, Fizmatlit. - 1997. - 414 p.).

9. Aghalovyan L.A. On the Structure of Solution of One Class of Plane Problems of the Theory of Elasticity of Anisotropic Body //The Inter-Universities Collection Mechanics: Yerevan: Publishing House of Yerevan State University - 1982. - Issue 2. - P. 7-12.

10. Aghalovyan L.A., Gevorkyan R.S. Non-Classical Boundary Value Problems of Anisotropic Layered Beams, Plates and Shells.- Yerevan: Publishing House "Gitutyun" of NAS RA. -2005. - 468 p.

11. Aghalovyan L.A. On the Asymptotic Method for Solving Dynamic Mixed Problems of Anisotropic Strips and Plates //Procedings of the Universities of the Russian Federation. North-Caucasian Region. Natural Sciences. - 2000. - №. 3. - P.8-11.

12. Aghalovyan M.L., Zakaryan T.V. Asymptotic Solution of the First 3D Dynamic Elasticity Theory Problem on Forced Vibrations of a Three-Layer Plate with an Asymmetric Structure // Mechanics of Composite Materials. - 2019. - Vol. 55.- № 1. - P. 3-18 .

13. Aghalovyan L.A., Ghulgazaryan L.G. On the Determination of Solutions of One Class of Dynamic Spatial Problems of the Mathematical Theory of Elasticity for Orthotropic Shells // Scientific Notes of the Armenian State Pedagogical University named after Kh. Abovyan. - 2012. - №2 (17). - P. 29-42.

14. Aghalovyan L.A., Aghalovyan M.L., Zakaryan T.V. Asymptotic Analysis of the Forced Oscillations of Double-Layered Plates with Viscous Resistance//Mechanics of Solids. - 2020. -Vol.55. -№5. -P.1062-1070.

15. Kaplunov J.D., Kossovich L.Yu., and Nolde E.V. Dynamics of Thin Walled Elastic Bodies. San-Diego: Academic Press. -1998. -226p.

16. Mikhasev G.I., Tovstik P.E. Localized Vibrations and Waves in Thin Shells. Asymptotic methods.- Moscow: Fizmatlit - 2009. -292 p.

17. Vilde M.V., Kaplunov Yu.D., Kossovich L.Yu. Edge and Interface Resonance Phenomena in Elastic Bodies. - Moscow: Fizmatlit - 2010 - 280 p. (in Russian).

18. Kaplunov J., Prikazchikov D.A., Prikazchikova L.A. Dispersion of Elastic Waves in a Strongly Inhomogeneous Three-Layered Plate. //International Journal of Solids and Structures.- 2017. -113. -P. 169-179.

19. Altenbach H., Eremeyev V.A., Naumenko K. On the Use of the First Order Shear Deformation Plate Theory for the Analysis of Three-layer Plates with Thin Soft Core Layer.// ZAMM-J. Appl. Math. Mech./ Zeitsch. Ange. Mathe. Mech. - 2015. - 95(10). - P. 1004-1011.

20. Boutin C. Linear Pantographic Sheets. Asymptotic Micro-macro Models Identification.//Mathematics and Mechanics of Complex Systems. - 2017. - 5(2). - P.127-162.

21. Naife A.Kh. The Perturbations Methods.- Moscow: “Mir”-1976. - 455 p.

22. Timoshenko S.P., Goodyer J. Theory of Elasticity. -Moscow: “Nauka” - 1979. -560 p.

Information about the authors:

Aghalovyan Lenser Abgar- Dr. of Sc., Professor, Head of Department of Thin-Walled Systems, Institute of Mechanics of NAS Armenia
Marshal Bagramyan ave.24/2, 0019, Yerevan, Armenia
Tel.(+37410)529630, E-mail: lagal@sci.am

Aghalovyan Mher Lenser- Dr. of Sc., Lecturer, Armenian State University of Economics, Leading Researcher, Institute of Mechanics of NAS Armenia,
Nalbandyan St. 128, 025 Yerevan, Armenia
Tel.(+37493)055070, E-mail: mheraghalovyan@yahoo.com

Zakaryan Tatevik Vladik-PhD, Researcher, Institute of Mechanics of NAS Armenia,
Marshal Bagramyan ave.24/2, 0019, Yerevan, Armenia
Tel.(+37494)638882, E-mail: zaqaryantatevik@mail.ru

Tovmasyan Artur Babken- PhD, Researcher, Institute of Mechanics of NAS Armenia,
Marshal Bagramyan ave.24/2, 0019, Yerevan, Armenia
Tel.(+37497)319999

Received 20.12.2024