ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 77, №3, 2024 Механика

УДК 539.3 DOI: 10.54503/0002-3051-2024.77.3-18

BROADENING FREQUENCY BANDGAPS IN A BEAM WITH PERIODIC INTERNAL HINGES, EXTERNAL SUPPORTS AND ATTACHED MASSES

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Key words: periodic structure, Floquet theory, multi span beams, hinges, bandgaps, attached masses.

Казарян К. Б.

Расширение запретных зон частот в балке с периодическими внутренними шарнирами, внешними опорами и присоединенными массами

Представлен сравнительный анализ механизма формирования запретных зон частот в однородных предварительно напряженных балках с периодическими внешними опорами, шарнирами и прикрепленными массами. На основе теории Флоке получены аналитические выражения для функции, определяющей структуру запретных зон. Рассмотрено несколько периодических топологических систем: мета балка с промежуточными внешними опорами и прикрепленными массами, мета балка с внутренними шарнирами в паре с массами. Для периодических структур выведены дисперсионные уравнения запретных зон, построены и проанализированы дисперсионные кривые. В статье новизной являются результаты, касающиеся расширения резонансной полосы фононных запрещенных зон мета балки за счет слияния раздельных множеств запретных зон, генерируемых внутренними шарнирами или внешними промежуточными опорами с прикрепленными массами.

Ключевые слова: периодические структуры, теория Флоке, многопролетные балки, шарниры, запретные зоны, присоединенные массы.

Ղազարյան Կ. Բ.

Պարբերական ներքին հոդակապերով, արտաքին հենարաններով և կցված զանգվածներով հեծանում հաճախությունների արգելված գոտիների ընդլայնումը

Ներկայացված է հաճախությունների արգելված գոտիների ձևավորման մեխանիզմի համեմատական վերլուծությունը նախապես լարված համասեռ, պարբերական հենարաններով, հոդակապերով և կցված զանգվածներով հեծաններում:

Դիտարկված է մի քանի պարբերական տոպոլոգիական համակարգեր. մետա հեծան միջանկյալ արտաքին հենարաններով և կցված զանգվածներով և մետահեծաններ՝ ներքին հոդակապերով՝ զանգվածների հետ:

Պարբերական կառուցվածքների համար դուրս են բերված արգելված գոտիների դիսպերսիոն հավասարումները, կառուցված և վերլուծված են դիսպերսիոն կորերը: Հոդվածում նորություն են այն արդյունքները, որոնք վերաբերում են մետա հեծանի ֆոնոն արգելված գոտիների ռեզոնանսային շերտի ընդլայնմանը՝ այն իրարից բաժան արգելված գոտիների միաձուլման հաշվին, որոնց առաջացումը պայմանավորված է ներքին հոդակապերով, միջանկյալ արտաքին հենարաններով և կցված զանգվածներով:

Բանալի բառեր. պարբերական կառուցվածքներ, Ֆլոկեի տեսություն, բազմաթռիչք հեծաններ, հոդակապեր, արգելված գոտիներ, կցված զանգվածներ:

A comparative analysis is presented of bandgap formation mechanism in homogeneous prestressed beams with periodic external supports, hinges and attached local masses. Based on the Floquet theory the analytical expressions are derived for deviation functions defining bandgap structure. Several periodic topological structures are considered: meta beams with intermediate external supports and attached masses, internal hinges paired with masses. For periodic structures the band gap dispersion equations are derived, dispersion curves are plotted and analyzed. The innovation in this paper is the results concerning widening of the resonant bandwidth of a meta beam phononic bandgaps by merging of multiple separated bandgaps generated by the internal hinges or external intermediate supports with attached masses.

Introduction

In this paper, we investigate the problem of bandgaps for flexural waves in infinite homogeneous pre tensioned meta beam with periodically attached masses, internal hinges and external support for the purpose of using such meta beams in energy harvesting applications.

In recent years has been growing interest in meta material thin-walled structures as plates and beams widely used in piezoelectric energy harvesting systems [1-13].

The problems of localization of electro elastic waves in periodic piezoelectric beams and applications in vibration energy harvesting are studied in [11,12] where it is shown that the flexural vibration energy within bandgaps may be localized in the cells and can be used for energy harvesting.

Locally resonant meta beams usually have narrow bandgaps, which significantly limits its applications in engineering devices. Various approaches have been proposed to broaden the frequency range of meta materials 'bandgaps.

Among them is an optimization method based on the genetic algorithm proposed in [14] to broaden bandgaps in multi-resonant piezoelectric metamaterial through the merging of multiple separated bandgaps.

The hybrid metastructure consists of a piezoelectric bimorph cantilever with segmented electrodes shunted to resonant circuits and flexural resonators is considered in [15] were is fixed the possibility of increasing gap bandwidth by merging separate mechanical and electromechanical bandgaps.

The new approach suggested in [16] concerning widening of the resonant bandwidth of a piezoelectric harvester based on phononic band gaps generated by internal hinges.

In [17 it is shown that in a multi-span beam rested on periodic rigid and elastic supports the tensive and compressive axial forces sufficiently widening the multiple bandgaps of the flexural waves. The widening of the bandgaps occurs also with increasing the stiffness of the rotational spring attached to an elastic support.

The problem of reducing vibration transmittance in low-frequency regimes and over a broad frequency range through the resonance-Bragg band gap coupling phenomena in metamaterial rod is considered in [18].

Flexural frequency multiple bandgaps in metamaterial beams can be generated in many ways.

Bandgap formation due to Bragg's scattering in periodic beam consisting of two or more kinds of materials are discussed in [19,20].

Bandgap formation in strings and beams with periodic local resonators or periodic supports are considered in [21-23].

Effects caused by bandgaps in the piezoelectric periodic meta beams are studied in [24,25].

The review of the most recent developments in piezoelectric energy harvesting methods for converting localized mechanical wave energy into electrical energy using artificially designed mechanical structures are given in [26].

Governing equations and solutions

This section presents the basic dynamics equations, interface relations and solutions of an Euler meta beam tensioned by an axial force. Tthree configurations of a periodic infinite meta beam are considered: beam with periodic attached masses (**M**), beam with periodic internal hinges paired with masses (**MH**), beam rested on periodic external supports when at equal distance from suppoorts attached masses are located (**MS**).

Transverse vibration of the Euler beam is given by the following equation

$$EI\frac{\partial^4 W}{\partial z^4}-Q\frac{\partial^2 W}{\partial z^2}+\rho A\frac{\partial^2 W}{\partial t^2}=0; \tag{1}$$

where $W(z,t)$ is the defection of beam, EI,ρ,A denote the flexural rigidity, the mass density per unit volume and the cross-sectional area, respectively, $Q>0$ is the axial tension force .

Consider $W(z,t)$ in the form

$$W(z,t)=U(z)\exp(i\omega t) \tag{2}$$

where ω is the circular frequency, $U(z)$ is the amplitude function.

Introducing the dimensionless coordinate $x=zd^{-1}$, solutions for amplitude functions in a beam repeated elementary unit cell $x\in(n-1,n)$ can be written as

$$U_{\pm}(x)=A_{\pm1}\sin(px)+A_{\pm2}\sinh(qx)+A_{\pm3}\cos(px)+A_{\pm4}\cosh(qx)$$
$$q=\frac{\sqrt{\sqrt{F^2+4\Omega^2}+F}}{\sqrt{2}},\; p=\frac{\sqrt{\sqrt{F^2+4\Omega^2}-F}}{\sqrt{2}}; \tag{3}$$

Here Ω and F

$$\Omega^2=\frac{\omega^2 d^4\rho A}{EI};F=\frac{d^2 Q}{EI} \tag{4}$$

are dimensionless notations of frequency and tensile force.

In (3) superscripts $(\pm)$ denote segments

$$(-)\to x\in(n-1,n-1/2),\qquad (+)\to x\in(n-1/2,n)$$

Beam with periodic attached masses (M)

Consider the beam in the basic unit cell $x \in (n-1, n)$ with masses m located at points $x_0 = n - 1/2$. Contact conditions at point $x_0 = n - 1/2$ are the conditions of the continuity in displacement, slope and moment of beam

$$U_+(x_0) = U_-(x_0), \qquad \frac{dU_+(x_0)}{dx} = \frac{dU_-(x_0)}{dx}, \quad \frac{d^2U_+(x_0)}{dx^2} = \frac{d^2U_-(x_0)}{dx^2} \tag{5}$$

and the balance of shear force

$$\frac{d^3U_+(x_0)}{dx^3} - \frac{d^3U_-(x_0)}{dx^3} = \frac{md^3\omega^2}{EY} U_\pm(x_0) = \mu\Omega^2 U_\pm(x_0) \tag{6 }$$

Here the dimensionless parameter $\mu = m(\rho A d)^{-1}$ determines the ratio of the attached mass to the mass of a beam of length d .

At the end points of the periodic basic unit cell the Floquet conditions will be applied [19]

$$\begin{gathered} \frac{d^3U_+(n)}{dx^3} = \lambda \frac{d^3U_-(n-1)}{dx^3}, \qquad \frac{d^2U_+(n)}{dx^2} = \lambda \frac{d^2U_-(n-1)}{dx^3} \\ \frac{dU_+(n)}{dx} = \lambda \frac{dU_-(n-1)}{dx}, \qquad U_+(n) = \lambda U_-(n-1) \end{gathered} \tag{7}$$

Here $\lambda = \exp(ikd)$, k is the Floquet wave number.

Applying to the solutions (3) the contact (5,6) and the Floquet conditions (7) we get the equation determining the Floquet wave number

$$\beta(1+\lambda^4) + \gamma(\lambda^3 + \lambda) + \alpha\lambda^2 = 0 \tag{8}$$

where

$$\alpha = -2p\left(p^2 q + \mu\Omega^2 \cos(p)\sinh(q) + q^3\right) - 4pq\left(p^2+q^2\right)\cos(p)\cosh(q) +$$

$$+2\mu q\Omega^2 \sin(p)\cosh(q),$$

$$\beta = -pq\left(p^2+q^2\right),$$

$$\gamma = 2pq\left(p^2+q^2\right)\cos(p) + 2pq\left(p^2+q^2\right)\cosh(q) + \mu(-q)\Omega^2\sin(p) + \mu p\Omega^2 \sinh(q);$$

A similar type of this equation has been obtained and discussed in [27] for a vibrating piecewise bi-material periodic beam.

Taking into account $\lambda = \exp(ikd)$ the equations (8) can be written as

$$\alpha + 2\beta\cos(2kd) + 2\gamma\cos(kd) = \alpha - 2\beta + 4\beta\cos^2(kd) + 2\gamma\cos(kd) = 0$$

Solving it we get

$$\begin{gathered} \cos(kd) = \eta(\Omega, Q, \mu) \\ \eta(\Omega, Q, \mu) = \frac{-\gamma \pm \sqrt{-4\alpha\beta + 8\beta^2 + \gamma^2}}{4\beta} \end{gathered} \tag{9}$$

Solutions (9) define the two Floquet spectrum of beam frequencies. When there are no masses $\mu = 0$ then solutions are $\cos(kd) \to \cos(p), \cos(kd) \to \cosh(q)$

Since the Euler–Bernoulli beam vibration equation is not hyperbolic, one of the spectrum is the Floquet pseudo spectrum [27] which corresponds to the limiting case $\cos(kd) \to \cosh(q)$.

Beam with periodic internal hinges paired with attahed masses (MH)

Consider the beam in the basic unit cell $x \in (n-1, n)$ with internal hinges and paired masses m located at points $x_0 = n - 1/2$.

In this case the contact conditions at point $x_0 = n - 1/2$ are conditions of the continuity in displacement, namely the first of equation (5), and the balance of shear force (6). Besides these conditions we have the conditions that at the hinges are zero moments

$$\frac{d^2U_+(x_0)}{dx^2} = 0, \ \frac{d^2U_-(x_0)}{dx^2} = 0 \tag{10}$$

Applying to the solutions (3) contact conditions (5,6) and the Floquet conditions (7) and condition (10) we get the equation determining the Floquet wave number λ

$$\begin{aligned} &\left(\lambda^2+1\right)pq\left(pq^3\sinh(q)-p^3q\sin(p)\right)+ \\ &+\lambda\left(2p^4q\sin(p)\cosh(q)+\mu\Omega^2\left(p^2+q^2\right)\sin(p)\sinh(q)-2pq^4\cos(p)\sinh(q)\right)=0 \end{aligned} \tag{11}$$

Solving it we get

$$\cos(kd) = \eta(\Omega, Q, \mu)$$

$$\eta(\Omega,Q,\mu) = \frac{2p^4q\sin(p)\cosh(q) - 2pq^4\cos(p)\sinh(q) + \mu\Omega^2\left(p^2+q^2\right)\sin(p)\sinh(q)}{2p^4q\sin(p) - 2pq^4\sinh(q)} \tag{12}$$

Beam rested on periodic external supports with periodic masses (MS)

Consider the beam in the basic unit cell $x \in (n-1, n)$ rested at points $x = n-1, x = n$ on external supports, when at $x_0 = n - 1/2$ the masses are attached.

In this case the contact conditions (5,6) are valid, together with these conditions we have to consider the following Floquet conditions

$$\begin{aligned} &U_+(n) = 0, U_-(n-1) = 0 \\ &\frac{dU_+(n)}{dx} = \lambda\frac{dU_-(n-1)}{dx}, \frac{d^2U_+(n)}{dx^2} = \lambda\frac{d^2U_-(n-1)}{dx^3} \end{aligned} \tag{13}$$

Applying conditions (5,6,13) to solutions (3) we have the equation determining the Floquet wave number

$$\left(\lambda^2+1\right)f + g\lambda = 0 \tag{14}$$

Solving (14) we get

$$\cos(kd) = \eta(\Omega, Q, \mu),\ \eta(\Omega, Q, \mu) = -\frac{g}{2f} \tag{15}$$

where

$$f = 2pq\left(p^2+q^2\right)\left(p\sinh(q) - q\sin(p) + 2\mu\Omega^2\left(q\sin\left(\frac{p}{2}\right) - p\sinh\left(\frac{q}{2}\right)\right)\right)$$

$$g = 4pq\left(p^2+q^2\right)\left(q\sin(p)\cosh(q) - p\cos(p)\sinh(q)\right) +$$

$$+2\mu\Omega^2\left(\cosh(q)\left((q^2-p^2)\cos(p) - q^2\right) + p\left(-4q\sin\left(\frac{p}{2}\right)\sinh\left(\frac{q}{2}\right) + 2q\sin(p)\sinh(q) + p\cos(p)\right)\right);$$

Deviations functions $\eta(\Omega, Q, \mu)$ in (9),(12),(15) define the bandgaps of eigenfrequencies Ω were the flexural waves cannot propagate, when $|\eta(\Omega, Q, \mu)| > 1$ (values of k are complex). The stopband edges of eigenfrequencies are given by condition $|\eta(\Omega, Q, \mu)| = 1$

Discussion, numerical results

The imaginary parts of the Floquet wave number $\mathrm{Im}(kd)$ define the attenuation of the flexible waves whose frequencies are inside the bandgaps, while the real part of the Floquet wave number $\mathrm{Re}(kd)$ defines the dispersion of the flexible waves, whose frequencies are outside the bandgaps. The lowest contours of the attenuation curves, where $\mathrm{Im}(kd) \to 0$ define the maps of the first bandgap frequencies.

On Figures 1,2,3 the attenuation curves $\mathrm{Im}(kd)$ versus frequency Ω are plotted, illustrating the variation of bandgap widths for **M**, **MH** and **MS** configurations. For these configurations the two different plots are presented when tension is $Q = 0$ or $Q = 40$.

(*For interpretation of the references to color in the figure's legend, the reader is referred to the web version of this article.*)

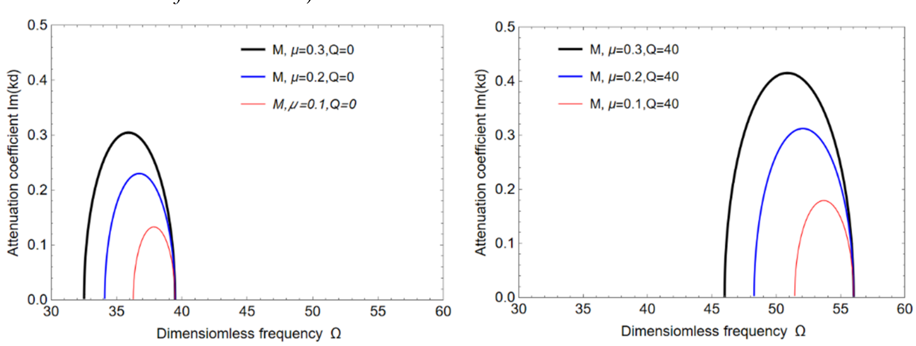

Fig1. Maps of the first bandgaps of **M** *beams*

As follows from the plots of Fig.1 in the case of **M** beams the increase of attached mass sufficiently widens the first gap widths (approximately two times) shifting the gap to the low frequency band. The increase of tensile force also expands the first gap widths but dislocating it to the high frequency band.

In the cases of **MH** and **MS** beams the increase of the attached mass slightly widens the first gap widths. Increasing the tensile force expands the first gap width shifting it to the high frequency band.

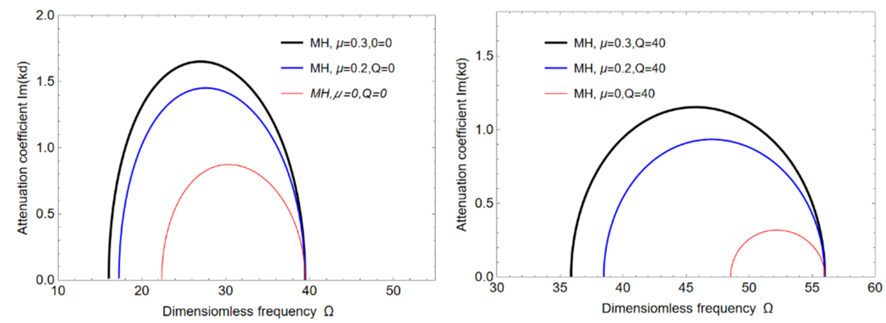

Fig2. Maps of the first bandgaps of **MH** *beams*

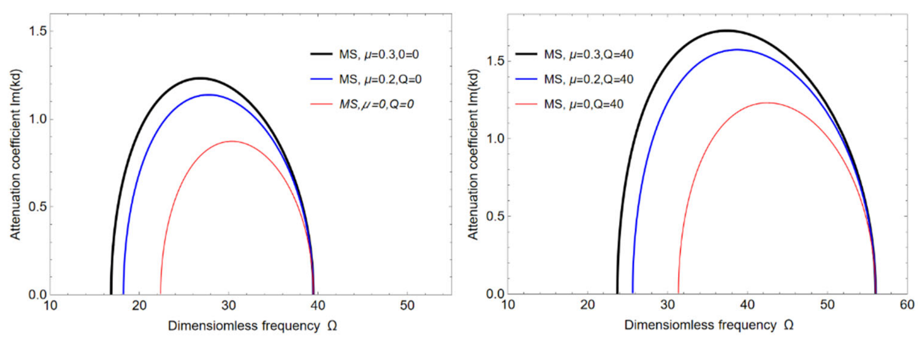

Fig.3 Maps of the first bandgaps of ***MS*** *beams*

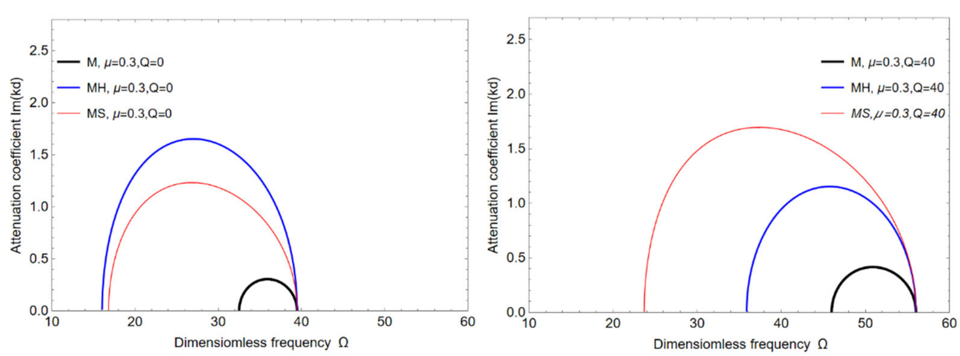

Fig.4 Maps of the first bandgaps of M,MH, MS beams

On the Fig.4 for comparison the maps of first bandgaps of **M**, **MH** and **MS** beams are presented when tensions are $Q=0$ and $Q=40$. On Fig.5 maps of the first, second and third bandgaps of **M,MH, MS** beams are presented.

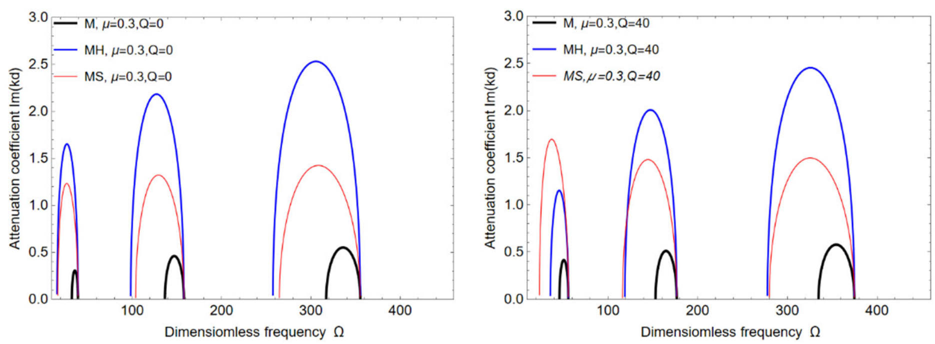

*Fig.5 Maps of the first, second and third multiple bandgaps of **M,MH, MS** beams*

Obviously as follows from Fig.5 the results obtained for first bandgap are valid for all multiple subsequent gaps of M, MS, MH beams

Conclusions

The analysis of the band gaps structures of the meta beams based on the plots of Fig. 1-5 can be summarized as follows:

- In **M** beams increasing the attached mass sufficiently widening the gap bandwidth and shifting the vibration band gaps to low-frequency regions.
- Increasing tensile force magnitude slightly increases the bandwidth of **M**, **MH** and **MS** beams shifting the vibration band gaps to high-frequency regions.
- The bandwidth of the **MS** beams can be wider than the bandwidth of the **MH** beams depending of tensile force magnitude.
- All gaps of the **M** beams are located within the gaps of the **MH** and **MS** beams
- Widening the resonant bandwidths of a meta beam harvester with phononic band gaps generated by internal hinges and external supports is more significant than widening of the resonant bandwidths due to increase of the attached masses values.
- The impact of the masses on the gap formation is insignificant in this meta structure with internal hinges and support.
- All results obtained for first bandgap are valid for all subsequent bandgaps of **M**, MS and **MH** beams

Acknowledgments

The work was supported by the State Committee of Science of RA, in the framework of the research project 21T-2C299.

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Received 12.08.2024