

Reflection and refraction of multi-component electro-magneto-elastic waves from the interface of two *bmm* class piezoelectrics with different polarization of the medium^{*)}

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*The work is dedicated to the memory of our teacher:
Professor Mels Belubekyan*

Keywords: piezoelectric texture, waves reflection, waves refraction, multi-component waves, electro-magneto-elastic waves, plane deformation problem, non-acoustic contact.

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Отражение и преломление многокомпонентных электро-магнито-упругих волн от границы раздела двух пьезоэлектриков класса *bmm* с различной поляризацией среды

Ключевые слова: пьезоэлектрическая текстура, отражение волны, преломление волны, многокомпонентная волна, электромагнитная упругая волна, задача плоского деформирования, не акустический контакт.

Рассмотрено отражение и преломление многокомпонентных медленных электро-магнито-упругих волн от границы раздела граничащих изотропной и анизотропной пьезоэлектрических полуплоскостей класса *bmm* с различной поляризацией среды. В случае естественной поляризации кристалла, в плоскости изотропии полное электро-магнито-упругое поле распадается на пятикомпонентное электро-магнито-упругое поле плоской деформации и на четырехкомпонентное электро-магнито-упругое поле антиплоской деформации. При этом медленные упругие волны плоской деформации сопровождаются только медленной поперечно поляризованной волной электрического поля, а медленные упругие сдвиговые волны сопровождаются поляризованной в заданной плоскости медленной волной электрического поля. В случае другой поляризации кристалла в плоскости анизотропии также суммарное электро-магнито-упругое поле разделяется на пятикомпонентное электро-магнито-упругое поле плоской деформации и четырехкомпонентное электро-магнито-упругое поле антиплоской деформации. Но, при этом медленные упругие плоские волны деформации сопровождаются медленной волной электрического поля, поляризованной в заданной плоскости, а медленные упругие сдвиговые волны – медленной поперечно поляризованной волной электрического поля.

На поверхности не акустического контакта раздела разно поляризованных пьезоэлектрических полупространств, генерированное в одном из полупространств пьезоэлектрика многокомпонентное электро-магнито-упругое поле одного из типов трансформируется в гибрид многокомпонентных электро-магнито-упругих волн плоской и антиплоской деформаций.

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Բազմաբաղադրիչ էլեկտրա-մագնիսա-առաձգական ալիքների արդրադարձումը և բեկումը միջավայրի տարբեր բևեռացումով *bmm* դասի երկու պիեզոէլեկտրիկների միջերեսից

Հիմնաբառեր. պիեզոէլեկտրական կառուցվածք, ալիքի անդրադարձում, ալիքի բեկում, բազմաբաղադրիչ ալիք, էլեկտրամագնիսական առաձգական ալիք, հարթ դեֆորմացիայի խնդիր, ոչ ակուստիկ հիմունք:

Դիտարկված է բազմաբաղադրիչ դանդաղ էլեկտրա-մագնիսա-առաձգական ալիքների անդրադարձումը և բեկումը *bmm* դասի իզոտրոպ և անիզոտրոպ պիեզոէլեկտրական կիսահարթությունների միջերեսից՝ միջավայրի տարբեր բևեռացումների դեպքում: Պլեզոբյուրեղի բնական բևեռացման դեպքում, իզոտրոպային հարթությունում ընդհանուր էլեկտրամագնիսական առաձգական դաշտը տրոհվում է հարթ դեֆորմացիայի հինգ-բաղադրիչ էլեկտրամագնիսաառաձգական դաշտի և հակահարթ դեֆորմացիայի չորս-բաղադրիչ էլեկտրամագնիսաառաձգական դաշտի: Այս դեպքում դանդաղ առաձգական հարթ դեֆորմացիայի ալիքները ուղեկցվում են լայնակի բևեռացված դանդաղ էլեկտրական դաշտի ալիքով, իսկ դան-

դադ հակահարթ ղեֆորմացիայի ալիքները ուղեկցվում են տվյալ հարթությունում բևեռացված դանդաղ էլեկտրական դաշտի ալիքով: Անիզոտրոպային հարթությունում, քյուրեդի ըստ այլ առանցքի բևեռացման դեպքում, ընդհանուր էլեկտրամագնիսական առաձգական դաշտը նույնպես տրոհվում է հարթ ղեֆորմացիայի հինգ-բաղադրիչ էլեկտրամագնիսաառաձգական դաշտի և հակահարթ ղեֆորմացիայի չորս-բաղադրիչ էլեկտրամագնիսաառաձգական դաշտի: Բայց, այս դեպքում, դանդաղ առաձգական հարթ ղեֆորմացիայի ալիքները ուղեկցվում են յայնակի բևեռացված դանդաղ էլեկտրական դաշտի ալիքով, իսկ դանդաղ հակահարթ ղեֆորմացիայի ալիքները ուղեկցվում են տվյալ հարթությունում բևեռացված դանդաղ էլեկտրական դաշտի ալիքով:

Տարբեր բևեռացվածությամբ պիեզոէլեկտրական կիսատարածությունների միջև ոչ ակուստիկ հպման մակերևույթի վրա դրանցից մեկում զենեքացված մեկ տեսակի էլեկտրամագնիսաառաձգական ալիքը փոխակերպվում է հարթ և հակահարթ ղեֆորմացիաների բազմաբաղադրիչ էլեկտրամագնիսաառաձգական ալիքների հիբրիդի:

The reflection and refraction of multicomponent slow electro-magneto-elastic waves from the interface between isotropic and anisotropic piezoelectric half-planes of class $6mm$ with different polarization of the medium is considered. In the case of natural polarization of the crystal, in the isotropy plane, the total electro-magneto-elastic field is divided into a five-component electro-magneto-elastic field of plane strain and a four-component electro-magneto-elastic field of antiplane strain. In this case, slow elastic plane deformation waves are accompanied only by a slow transversely polarized electric field wave, and slow elastic shear waves are accompanied by a slow electric field wave polarized in a given plane. In the case of a different polarization of the crystal in the anisotropy plane, the total electro-magneto-elastic field is also divided into a five-component electro-magneto-elastic field of plane strain and a four-component electro-magneto-elastic field of antiplane strain. But, in this case, slow elastic plane deformation waves are accompanied by a slow electric field wave, polarized in a given plane, and slow elastic shear waves are accompanied by a slow transversely polarized electric field wave.

On the surface of the non-acoustic contact between differently polarized piezoelectric half-spaces, a multicomponent electro-magneto-elastic field of one type generated in one of the piezoelectric half-spaces is transformed into a hybrid of multicomponent electro-magneto-elastic waves of plane and antiplane deformations.

Introduction

Back in 1949, the propagation of plane waves in the isotropy plane of an infinite piezoelectric medium was studied, in which the electromagnetic and elastic modes are coupled, and the multicomponent electro-magneto-elastic wave has five phase velocities for both electromagnetic and acoustic waves [1]. Slower electromagnetic waves represent field changes that accompany elastic waves, and fast elastic waves are mechanical deformations that accompany an electromagnetic wave. The piezoelectric effect contributes to the reflection of electromagnetic waves at the interface. This introduces a small correction to the familiar expressions for reflection coefficients. This correction can be used to determine piezoelectric constants, provided that the reflection coefficients and other constants involved can be accurately measured.

Also, at certain frequencies the crystal can be brought into resonance. With the resonance of an elastic wave, an almost perfect reflection of the incident electromagnetic wave occurs.

Considering this fundamental principle and based on the lattice dynamics model, the propagation of a Rayleigh-type surface wave in a micropolar piezoelectric medium [2] and the features of the propagation of a bending wave in one-dimensional phononic crystals were studied. [3], propagation of electromagnetic waves in nanocomposite materials consisting of chiral nano inclusions [4], interphase shear-horizontal acoustic waves along the boundary of two half-spaces, which are piezoelectric crystals of cubic symmetry [5], generation and propagation of ultrasonic waves in a single-layer piezoelectric graphene nanoribbon [6].

There is extensive literature on the reflection and refraction of electro elastic waves at the interface of the layered structure of piezoelectrics [6÷22]. Previously, different authors considered both the problems of reflection and refraction of multicomponent electroacoustic waves from the surface of a homogeneous piezoelectric half-space with

different electromechanical conditions on it [6÷9], and the problems of reflection and refraction of such waves from the interface of two different piezoelectrics under different electromechanical contact conditions [10÷17]. Solutions to problems of reflection and refraction of electroacoustic waves are also known for cases of complete electromechanical contact or without acoustic contact of two piezoelectric half-spaces of different symmetry classes [18÷22].

The proposed work examines the propagation of a three-component electroelastic wave (the case of an electroactive antiplane deformation SH wave) considering the accompanying transversely polarized magnetic field. The possibility of converting a transversely polarized magnetic field into a transversal transversely polarized electric field and vice versa will be demonstrated.

1. Basic relations of electro-magneto-elasticity for piezoelectrics of class 6mm at different polarizations of the piezocrystal

The physical and mechanical constants describing the properties of a homogeneous piezoelectric medium: elastic rigidity, piezoelectric coefficients, dielectric constant and magnetic field permeability form a generalized electro-magneto-elastic tensor of piezoelectric materials $(\hat{\gamma}_{jn})_{12 \times 12} = (\hat{c}_{ij})_{6 \times 6} \cup (\hat{e}_{nm})_{3 \times 6} \cup (\hat{\epsilon}_{ik})_{3 \times 3} \cup (\hat{\mu}_{ik})_{3 \times 3}$. In wave processes in which temperature changes can be neglected, the material equations of state for a piezo dielectric medium are presented in a significantly simplified form

$$\begin{aligned} \sigma_{ij}(u_{nm}, E_k, H_n) &= c_{ijnk} u_{nk} - e_{ijm} E_m, & D_m(u_{nm}, E_k) &= e_{nmk} u_{nk} + \epsilon_{mk} E_k, \\ B_m(H_n) &= \mu_{nm} H_n \end{aligned} \quad (1.1)$$

When formulating the equations of motion of a medium for piezoelectric materials, mass forces of an electromagnetic nature arising as a result of the interaction of induced currents with an electromagnetic field are usually not considered. Therefore, the equations of motion of the medium take the well-known simple form

$$\partial \sigma_{ij} / \partial x_j = \rho (\partial^2 u_i / \partial t^2). \quad (1.2)$$

In the absence of free electric charges and conduction current, the electromagnetic field in these media is described by Maxwell's dynamic equations

$$\begin{aligned} \epsilon_{ijk} (\partial E_j / \partial x_k) + \partial B_i / \partial t &= 0, & \epsilon_{ijk} (\partial H_j / \partial x_k) - \partial D_i / \partial t &= 0, \\ \partial B_j / \partial x_j &= 0, & \partial D_n / \partial x_n &= 0. \end{aligned} \quad (1.3)$$

In equations (1.3), the components of the Levi-Civitan tensor $\epsilon_{ijk} = [\vec{e}_i \vec{e}_j \vec{e}_k]$ is defined as a mixed product of the basis vectors of the piezo texture $\{\vec{e}_k\}$.

Physic mechanical constants of the material and the generalized linear tensor of electro-magneto-elasticity as a whole, for each piezo texture, are determined in accordance with the geometric scheme of piezo textures, according to the rules for installing crystals according to the syngonies and the rules for choosing crystallographic axes $Ox_1x_2x_3$ in them (Fig. 1) [21, 22].

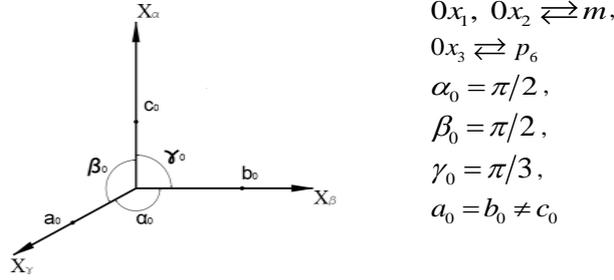


Fig. 1. Geometric scheme of piezo texture class $6mm$ of hexagonal symmetry with a choice of crystallographic coordinate axes, angles and unit vectors

Piezoelectrics of the $6mm$ class have transversally isotropic textures, and three significantly different axial polarizations are possible in them:

- i) the case of natural polarization, when the axis of crystalline polarization $0x_3 \rightleftharpoons \vec{p}_6$ is combined with the axis $0z$ of the Cartesian coordinate system,
- ii) the case when the axis of crystal polarization $0x_3 \rightleftharpoons \vec{p}_6$ lies in the isotropy plane of the crystal (for clarity, we assume $\vec{p}_6 \parallel 0x$).
- iii) the case when the axis of crystal polarization $0x_3 \rightleftharpoons \vec{p}_6$ lies in the plane of isotropy of the crystal (for clarity, we assume $\vec{p}_6 \parallel 0y$).

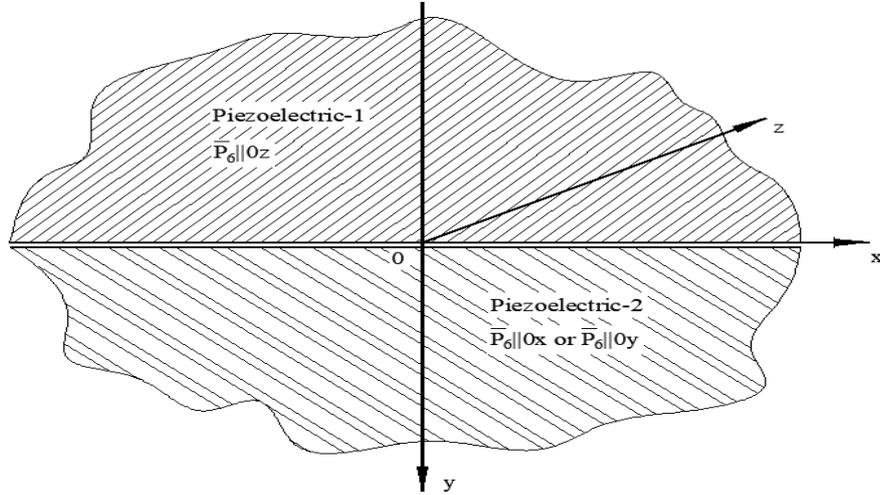


Fig. 2. Composite space of two differently polarized piezoelectrics of class $6mm$

To study the reflection and breaking of an electro-magneto-elastic wave from the contact surface of a two-layer piezoelectric space, we assign it to a single Cartesian coordinate system $0xyz$ so that the components of the half-space in it border the coordinate

plane Ozx and the coordinate axis Oy perpendicular to this plane (Fig. 2). As follows from the geometric diagram of the $6mm$ class piezo texture with the choice of crystallographic coordinate axes and angles (Fig. 1), the last two axial polarizations make an angle between themselves $\pi/2 \rightarrow 90^\circ$.

In the case of the direction of the axis of crystal polarization $\vec{p}_6 \parallel Ox_3$, along the coordinate axis Oz lying in the isotropy plane of the piezocrystal of the Cartesian system, the coordinate systems $Ox_1x_2x_3 \leftrightarrow Oxyz$ coincide, and therefore the planes $Ox_1x_2 \leftrightarrow Oxy$ coincide. The functional connections of the material, the equations of motion of the elastic medium and the equations of electrodynamics, for the separated problems of plane and antiplane deformations in the anisotropy plane $Ox_1x_2 \leftrightarrow Oxy$, subject to $\partial[*]/\partial z = 0$, will respectively be written in the form:

a) Functional connections between the medium and the electro-magneto-elasticity equation in the plane deformation problem

$$\begin{aligned} \sigma_{xx} &= c_{11} \frac{\partial u}{\partial x} + c_{12} \frac{\partial v}{\partial y} - e_{13} E_z, & \sigma_{yy} &= c_{12} \frac{\partial u}{\partial x} + c_{11} \frac{\partial v}{\partial y} - e_{13} E_z, \\ \sigma_{xy} &= 2(c_{11} - e_{12}) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \end{aligned} \quad (1.4)$$

$$D_z = e_{13} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \varepsilon_3 E_z, \quad B_x = \mu_1 H_x, \quad B_y = \mu_2 H_y \quad (1.5)$$

Considering (1.4) and (1.5) as well as two-dimensional equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \quad (1.6)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \frac{\partial D_z}{\partial t}, \quad \frac{\partial E_z}{\partial y} = -\frac{\partial B_x}{\partial t}, \quad \frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} \quad (1.7)$$

the equations of the electroactive plane strain problem are reduced to the form

$$\begin{aligned} c_{11} \frac{\partial^2 u}{\partial x^2} + \frac{c_{11} - c_{12}}{2} \frac{\partial^2 u}{\partial y^2} + \frac{c_{11} + c_{12}}{2} \frac{\partial^2 v}{\partial x \partial y} - e_{13} \frac{\partial E_z}{\partial x} &= \rho \frac{\partial^2 u}{\partial t^2} \\ \frac{c_{11} - c_{12}}{2} \frac{\partial^2 v}{\partial x^2} + c_{11} \frac{\partial^2 v}{\partial y^2} + \frac{c_{11} + c_{12}}{2} \frac{\partial^2 u}{\partial x \partial y} - e_{13} \frac{\partial E_z}{\partial y} &= \rho \frac{\partial^2 v}{\partial t^2} \\ \Delta E_z = \mu_1 \varepsilon_3 \frac{\partial^2 E_z}{\partial t^2} + \mu_1 \frac{\partial^2}{\partial t^2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right), & \frac{\partial H_x}{\partial t} = -\frac{1}{\mu_1} \frac{\partial E_z}{\partial y}, \quad \frac{\partial H_y}{\partial t} = \frac{1}{\mu_1} \frac{\partial E_z}{\partial x}, \end{aligned} \quad (1.8)$$

b) Functional connections between the medium and the equation in the anti-plane deformation problem

$$\begin{aligned}\sigma_{zx} &= c_{44} \frac{\partial w}{\partial x} - e_{15} E_x, & \sigma_{yz} &= c_{44} \frac{\partial w}{\partial y} - e_{15} E_y \\ D_x &= e_{15} \frac{\partial w}{\partial x} + \varepsilon_1 E_x, & D_y &= e_{15} \frac{\partial w}{\partial y} + \varepsilon_1 E_y\end{aligned}\quad (1.9)$$

$$\tilde{c}_{t1}^2 \Delta w = \frac{\partial^2 w}{\partial t^2}, \quad \Delta H_z = \frac{1}{c^2} \frac{\partial^2 H_z}{\partial t^2} \quad (1.10)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_1} \frac{\partial H_z}{\partial x} - \frac{e_{15}}{\varepsilon_1} \frac{\partial^2 w}{\partial x \partial t}, \quad \frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon_1} \frac{\partial H_z}{\partial y} - \frac{e_{15}}{\varepsilon_1} \frac{\partial^2 w}{\partial y \partial t} \quad (1.11)$$

In the first equation (1.10) the elastic shear wave velocity $\tilde{c}_{t1} = \sqrt{(1 + \chi^2)c_{44}/\rho}$ appears.

According to the obtained relations (1.4) ÷ (1.11), in the isotropy plane of the piezocrystal-1 $Ox_1x_2 \leftrightarrow Oxy$ (Fig. 2), the total electro-magneto-elastic field into $\{\mathbf{u}(x, y, t); \mathbf{v}(x, y, t); 0; 0; 0; E_z(x, y, t); H_x(x, y, t); H_y(x, y, t); 0\}$ a five-component electro-elastic field of plane strain and into a four-component electro-magneto-elastic field $\{0; 0; w(x, y, t); E_x(x, y, t); E_y(x, y, t); 0; 0; 0; H_z(x, y, t)\}$ anti-plane deformation is divided. Moreover, according to the identified structures of multicomponent waves, slow elastic plane strain waves (waves with the speed of sound) are accompanied only by $\Delta E_z(x, y, t) = \mu_1 \varepsilon_3 \frac{\partial^2 E_z(x, y, t)}{\partial t^2} + \mu_1 \frac{\partial^2}{\partial t^2} \left(\frac{\partial u(x, y, t)}{\partial x} + \frac{\partial v(x, y, t)}{\partial y} \right)$ a slow transversely polarized electric field wave. Fast magnetic fields are generated by slow electrical waves $[\partial H_x(x, y, t)/\partial t] = -\mu_1^{-1} \cdot [\partial E_z(x, y, t)/\partial y]$ and $[\partial H_y(x, y, t)/\partial t] = \mu_1^{-1} \cdot [\partial E_z(x, y, t)/\partial x]$

According to the identified structures of multicomponent waves, slow elastic waves of antiplane deformation are accompanied only by a slow electric field wave $\partial E_x(x, y, t) = -(e_{15}/\varepsilon_1) \cdot (\partial w(x, y, t)/\partial x)$ and $\partial E_y(x, y, t) = -(e_{15}/\varepsilon_1) \cdot (\partial w(x, y, t)/\partial y)$ polarized in a given plane. Also, fast electric field are generated by fast magnetic waves $[\partial E_x(x, y, t)/\partial t] = \varepsilon_1^{-1} \cdot [\partial H_z(x, y, t)/\partial x]$ and $[\partial E_y(x, y, t)/\partial t] = -\varepsilon_1^{-1} \cdot [\partial H_z(x, y, t)/\partial y]$.

In the case of the direction of the axis of crystal polarization $\bar{p}_6 \parallel Ox_3$, along the coordinate axis Ox lying in the isotropy plane of the piezocrystal of the Cartesian system, the coordinate systems $Ox_1x_2x_3 \rightleftharpoons Oyzx$ coincide, and therefore the planes $Ox_3x_1 \leftrightarrow Oxy$ coincide. The functional connections of the material, the equations of motion of the elastic medium and the equations of electrodynamics, for the separated problems of plane and antiplane deformations in the anisotropy plane $Ox_3x_1 \leftrightarrow Oxy$, subject to $\partial[*]/\partial z = 0$, will respectively be written in the form:

a) Functional connections between the medium and the electro-magneto-elasticity equation in the plane deformation problem

$$\begin{aligned}\sigma_{xx} &= c_{33} \frac{\partial \mathbf{u}}{\partial x} + c_{13} \frac{\partial \mathbf{v}}{\partial y} - e_{33} E_x, & \sigma_{yy} &= c_{13} \frac{\partial \mathbf{u}}{\partial x} + c_{11} \frac{\partial \mathbf{v}}{\partial y} - e_{31} E_x, \\ \sigma_{xy} &= c_{44} \left(\frac{\partial \mathbf{u}}{\partial y} + \frac{\partial \mathbf{v}}{\partial x} \right) - e_{15} E_y,\end{aligned}\quad (1.12)$$

$$D_x = e_{15} \left(\frac{\partial \mathbf{u}}{\partial y} + \frac{\partial \mathbf{v}}{\partial x} \right) + \varepsilon_1 E_x, \quad D_y = e_{31} \frac{\partial \mathbf{v}}{\partial y} + e_{33} \frac{\partial \mathbf{u}}{\partial x} + \varepsilon_3 E_y, \quad B_z = \mu_1 H_z, \quad (1.13)$$

Considering (1.12) and (1.13) as well as two-dimensional equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \quad \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = \rho \frac{\partial^2 \mathbf{v}}{\partial t^2} \quad (1.14)$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0, \quad \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} = -\mu_1 \frac{\partial H_z}{\partial t}, \quad -\frac{\partial H_z}{\partial y} = \frac{\partial D_x}{\partial t}, \quad \frac{\partial H_z}{\partial x} = \frac{\partial D_y}{\partial t}, \quad (1.15)$$

the equations of the electroactive plane strain problem are reduced to the form

$$\begin{aligned}c_{33} \frac{\partial^2 \mathbf{u}}{\partial x^2} + c_{44} \frac{\partial^2 \mathbf{u}}{\partial y^2} + (c_{13} + c_{44}) \frac{\partial^2 \mathbf{v}}{\partial x \partial y} - e_{33} \frac{\partial E_x}{\partial x} - e_{15} \frac{\partial E_y}{\partial y} &= \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \\ c_{44} \frac{\partial^2 \mathbf{v}}{\partial x^2} + c_{11} \frac{\partial^2 \mathbf{v}}{\partial y^2} + (c_{13} + c_{44}) \frac{\partial^2 \mathbf{u}}{\partial x \partial y} - e_{15} \frac{\partial E_y}{\partial x} - e_{31} \frac{\partial E_x}{\partial y} &= \rho \frac{\partial^2 \mathbf{v}}{\partial t^2} \\ e_{15} \frac{\partial^2 \mathbf{v}}{\partial x^2} + e_{31} \frac{\partial^2 \mathbf{v}}{\partial y^2} + (e_{33} + e_{15}) \frac{\partial^2 \mathbf{u}}{\partial x \partial y} + \varepsilon_1 \frac{\partial E_x}{\partial x} + \varepsilon_3 \frac{\partial E_y}{\partial y} &= 0\end{aligned}\quad (1.16)$$

$$\mu_1 \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}, \quad \mu_1 \frac{\partial^2 H_z}{\partial t^2} = \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_1} \right) \frac{\partial^2 H_z}{\partial x \partial y} - \frac{\partial}{\partial t} \left(\frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} \right). \quad (1.17)$$

b) Functional connections between the medium and the equation in the anti-plane deformation problem

$$\begin{aligned}\sigma_{xz} &= \frac{c_{11} - c_{12}}{2} \frac{\partial \mathbf{w}}{\partial x}, & \sigma_{yz} &= c_{44} \frac{\partial \mathbf{w}}{\partial y} - e_{15} E_z, \\ D_z &= e_{15} \frac{\partial \mathbf{w}}{\partial x} + \varepsilon_1 E_z, & B_x &= \mu_3 H_x, & B_y &= \mu_1 H_y,\end{aligned}\quad (1.18)$$

$$\begin{aligned}c_{44} \frac{\partial^2 \mathbf{w}}{\partial x^2} + (c_{11} - c_{12}) \frac{\partial^2 \mathbf{w}}{\partial y^2} - e_{15} \frac{\partial E_z}{\partial y} &= \rho \frac{\partial^2 \mathbf{w}}{\partial t^2} \\ \frac{\partial^2 E_z}{\partial t^2} &= \frac{1}{\varepsilon_1 \mu_3} \cdot \frac{\partial^2 E_z}{\partial x^2 \partial t} + \frac{1}{\varepsilon_1 \mu_1} \cdot \frac{\partial^2 E_z}{\partial y^2 \partial t} - \frac{e_{15}}{\varepsilon_1} \frac{\partial^3 \mathbf{w}}{\partial x \partial t^2}\end{aligned}\quad (1.19)$$

According to the obtained relations (1.12) ÷ (1.19), in the anisotropy plane of the piezocrystal-2 $Ox_3x_1 \leftrightarrow Oxy$ (Fig. 2), the total electro-magneto-elastic field is divided into a five-component electro-magneto-elastic field of plane strain $\{u(x, y, t); v(x, y, t); 0; E_x(x, y, t); E_y(x, y, t); 0; 0; 0; H_z(x, y, t)\}$ and into a two-component electro-elastic field $\{0; 0; w(x, y, t); 0; 0; E_z(x, y, t)\}$ antiplane deformation field, as well as a two-component plane magnetic field $\{H_x(x, y, t); H_y(x, y, t); 0\}$. Slow elastic plane waves of deformation (waves with the speed of sound) are accompanied by a slow electric field wave polarized in a given plane (1.16) and a transversely polarized magnetic field wave $\mu_1[\partial H_z(x, y, t)/\partial t] = [\partial E_x(x, y, t)/\partial y] - [\partial E_y(x, y, t)/\partial x]$. Slow elastic waves of antiplane deformation (waves with the speed of sound) are accompanied only by a fast electric transversely polarized wave (1.19).

In the case of the direction of the axis of crystal polarization $\vec{p}_6 \parallel Ox_3$, along the coordinate axis Oy lying in the isotropy plane of the piezocrystal of the Cartesian system, the coordinate systems $Ox_1x_2x_3 \rightleftharpoons Ozxy$ coincide, and therefore the planes $Ox_2x_3 \leftrightarrow Oxy$ coincide. The functional connections of the material, the equations of motion of the elastic medium and the equations of electrodynamics, for the separated problems of plane and antiplane deformations in the anisotropy plane $Ox_3x_1 \leftrightarrow Oxy$, subject to $\partial[*]/\partial z = 0$, will respectively be written in the form:

a) Functional connections between the medium and the electro-magneto-elasticity equation in the plane deformation problem

$$\sigma_{xx} = c_{11} \frac{\partial u}{\partial x} + c_{13} \frac{\partial v}{\partial y} - e_{13} E_y, \quad \sigma_{yy} = c_{13} \frac{\partial u}{\partial x} + c_{33} \frac{\partial v}{\partial y} - e_{33} E_y, \quad (1.20)$$

$$\sigma_{xy} = c_{44} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - e_{15} E_x$$

$$D_x = e_{15} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \varepsilon_1 E_x, \quad D_y = e_{13} \frac{\partial u}{\partial x} + e_{33} \frac{\partial v}{\partial y} + \varepsilon_3 E_y, \quad B_z = \mu_1 H_z, \quad (1.21)$$

$$c_{11} \frac{\partial^2 u}{\partial x^2} + c_{44} \frac{\partial^2 u}{\partial y^2} + (c_{13} + c_{44}) \frac{\partial^2 v}{\partial x \partial y} - e_{13} \frac{\partial E_y}{\partial x} - e_{15} \frac{\partial E_x}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2}$$

$$c_{44} \frac{\partial^2 v}{\partial x^2} + c_{33} \frac{\partial^2 v}{\partial y^2} + (c_{13} + c_{44}) \frac{\partial^2 u}{\partial x \partial y} - e_{15} \frac{\partial E_x}{\partial x} - e_{33} \frac{\partial E_y}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} \quad (1.22)$$

$$e_{15} \frac{\partial^2 v}{\partial x^2} + e_{33} \frac{\partial^2 v}{\partial y^2} + (e_{15} + e_{31}) \frac{\partial^2 u}{\partial x \partial y} + \varepsilon_1 \frac{\partial E_x}{\partial x} + \varepsilon_3 \frac{\partial E_y}{\partial y} = 0$$

$$\mu_1 \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}, \quad \mu_1 \frac{\partial^2 H_z}{\partial t^2} = \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_1} \right) \frac{\partial^2 H_z}{\partial x \partial y} - \frac{\partial}{\partial t} \left(\frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} \right) \quad (1.23)$$

b) Functional connections between the medium and the equation in the anti-plane deformation problem

$$\sigma_{xz} = c_{44} \frac{\partial w}{\partial x} - e_{15} E_z ; \quad \sigma_{zy} = (c_{11} - c_{12}) \frac{\partial w}{\partial y}, \quad D_z = e_{15} \frac{\partial w}{\partial x} + \varepsilon_1 E_z. \quad (1.24)$$

$$c_{44} \frac{\partial^2 w}{\partial x^2} + (c_{11} - c_{12}) \frac{\partial^2 w}{\partial y^2} - e_{15} \frac{\partial E_z}{\partial x} = \rho \frac{\partial^2 w}{\partial t^2} \quad (1.25)$$

$$\frac{\partial^2 E_z}{\partial t^2} = \frac{1}{\varepsilon_1 \mu_3} \frac{\partial E_z}{\partial y^2 \partial t} + \frac{1}{\varepsilon_1 \mu_1} \frac{\partial E_z}{\partial x^2 \partial t} - \frac{e_{15}}{\varepsilon_1} \frac{\partial w}{\partial x \partial t^2}$$

According to the obtained relations (1.20) ÷ (1.25), as in the previous case, in the anisotropy plane of the piezocrystal-2 $Ox_2x_3 \leftrightarrow Oxy$ (Fig. 2), the total electro-magneto-elastic field is divided into a five-component electromagnetoelastic field of plane strain and into a two-component electroelastic field, a field of antiplane strains, and also two-component plane magnetic field. Here too, slow elastic plane strain waves (waves with the speed of sound) are accompanied by a slow electric field wave polarized in a given plane (1.22) and a transversely polarized magnetic field wave. Slow elastic waves of antiplane deformation (waves with the speed of sound) are accompanied only by a fast electric transversely polarized wave (1.25).

It is interesting to note that both in the case of natural polarization of a piezoelectric, a plane strain wave is accompanied by a magnetic plane wave polarized in this plane and a transversely polarized electric field wave, and in other cases of polarization of a piezoelectric, an antiplane strain wave is accompanied by them. Just as in the case of natural polarization of a piezoelectric, an antiplane deformation wave is accompanied by an electric plane wave polarized in this plane and a transversely polarized magnetic field wave, so in other cases of polarization of a piezoelectric, a plane deformation wave is accompanied by them. From the above it follows that in the case of complete contact of differently polarized piezoelectric half-spaces (Fig. 2), the generation of an elastic wave of one type in one of the half-spaces after reflection and refraction from the interface is transformed into a complete package of electro-magneto-elastic waves in both half-spaces. In the case of non-acoustic contact of differently polarized piezoelectric half-spaces (Fig. 2), the generation of an elastic wave of one type in one of the half-spaces, after reflection and refraction from the interface, is transformed into an identical wave packet in its half-space and into an electro-magneto-elastic wave packet with another elastic component in the adjacent half-space.

2. Reflection and refraction of shear electro-magneto-elastic waves from the interface of differently polarized piezoelectric half-spaces

For simplicity of calculations, as an example, we consider the reflection and refraction of a shear electro-magneto-elastic wave from the interface of a non-acoustic contact of two differently polarized piezoelectric half-spaces of class $6mm$ of hexagonal symmetry with different natural polarizations.

The two-layer piezoelectric space is assigned to the Cartesian coordinate system $Oxyz$ so that the components of the half-space in it are bordered by the coordinate plane Ozx ,

and the coordinate axis Oy is perpendicular to this plane. The half-space $-\infty < x < \infty$, $-\infty < y \leq 0$, $-\infty < z < \infty$ polarized along the axis Oz , is occupied by a piezo active elastic medium of class **6mm** Piezoelectric - 1, and the half-space $-\infty < x < \infty$, $0 \leq y < \infty$, $-\infty < z < \infty$, polarized along the axis Oy , is occupied by a piezo active elastic medium of class **6mm** Piezoelectric - 2 (Fig. 3).

From the half-space Piezoelectric - 1 at an angle α_0 , an elastic shear wave is applied to the interface of non-acoustic contact media $y = 0$

$$w_{01}(x,y,t) = A \cdot \exp[i(\omega t - k_{11}x - k_{12}y)] \quad (2.1)$$

According to the system of equations (1.10) and (1.11), the wave signal will be accompanied by a plane polarized electric wave $\{E_{01}(x,y,t), E_{02}(x,y,t), 0\}$

$$\begin{aligned} E_{0x}(x,y,t) &= ik_{11}(e_{15}/\varepsilon_1) \cdot A \cdot \exp[i(\omega t - k_{11}x - k_{12}y)] \\ E_{0y}(x,y,t) &= -ik_{12}(e_{15}/\varepsilon_1) \cdot A \cdot \exp[i(\omega t - k_{11}x - k_{12}y)] \end{aligned} \quad (2.2)$$

A transversely polarized slow magnetic wave together with a wave signal $w_{01}(x,y,t)$ is not induced. According to the first equation (1.10), the direction cosines will be equal

$$k_{11} = k_1 \cdot \sin \alpha_0, \quad k_{12} = k_1 \cdot \cos \alpha_0, \quad k_1^2 = \omega^2 / \tilde{c}_{11}^2 \quad (2.3)$$

A similar electroelastic wave is reflected from a mechanically free surface deep into the half-space of Piezoelectric - 1

$$\begin{aligned} w_{11}(x,y,t) &= B \cdot \exp[i(\omega t - k_{11}x + k_{12}y)] \\ E_{1x}(x,y,t) &= ik_{11}(e_{15}/\varepsilon_1) \cdot B \cdot \exp[i(\omega t - k_{11}x + k_{12}y)] \\ E_{1y}(x,y,t) &= -ik_{12}(e_{15}/\varepsilon_1) \cdot B \cdot \exp[i(\omega t - k_{11}x + k_{12}y)] \end{aligned} \quad (2.4)$$

The complete electro elastic wave in the half-space Piezoelectric - 1 is accept the form

$$\begin{aligned} w_1(x,y,t) &= [A \cdot \exp[(-ik_{12}y)] + B \cdot \exp(ik_{12}y)] \cdot \exp[i(\omega t - k_{11}x)] \\ E_x^{(1)}(x,y,t) &= ik_{11}(e_{15}/\varepsilon_1) \cdot [A \cdot \exp[(-ik_{12}y)] + B \cdot \exp(ik_{12}y)] \cdot \exp[i(\omega t - k_{11}x)] \\ E_y^{(2)}(x,y,t) &= -ik_{12}(e_{15}/\varepsilon_1) \cdot [A \cdot \exp[(-ik_{12}y)] + B \cdot \exp(ik_{12}y)] \cdot \exp[i(\omega t - k_{11}x)] \end{aligned} \quad (2.5)$$

Non-acoustic contact at the interface of piezoelectric half-spaces is realized under the following contact conditions

$$\begin{aligned} \sigma_{yz}^{(1)}(x,0,t) = 0, \quad \sigma_{yy}^{(2)}(x,0,t) = 0, \quad \sigma_{xy}^{(2)}(x,0,t) = 0, \\ E_x^{(1)}(x,0,t) = E_x^{(2)}(x,0,t), \quad D_y^{(1)}(x,0,t) = D_y^{(2)}(x,0,t), \end{aligned} \quad (2.6)$$

Under conditions of acoustic-free contact (2.9), electromechanical stresses $\sigma_{yz}^{(1)}(x,0,t) = 0$, $\sigma_{yy}^{(2)}(x,0,t) = 0$, $\sigma_{xy}^{(2)}(x,0,t) = 0$ and electric field displacements $D_y^{(1)}(x,0,t)$, $D_y^{(2)}(x,0,t)$ are presented in the form of material relations (1.9), (1.20) and (1.21)

$$\begin{aligned}
\sigma_{yz}^{(1)}(x,0,t) &= c_{44} \frac{\partial w_1(x,0,t)}{\partial y} - e_{15} E_y^{(1)}(x,0,t), \\
\sigma_{yy}^{(2)}(x,0,t) &= c_{13} \frac{\partial u_2(x,0,t)}{\partial x} + c_{33} \frac{\partial v_2(x,0,t)}{\partial y} - e_{33} E_y^{(2)}(x,0,t), \\
\sigma_{xy}^{(2)}(x,0,t) &= c_{44} \left(\frac{\partial u_2(x,0,t)}{\partial y} + \frac{\partial v_2(x,0,t)}{\partial x} \right) - e_{15} E_x^{(2)}(x,0,t), \\
D_y^{(1)}(x,0,t) &= e_{15} \frac{\partial w_1(x,0,t)}{\partial y} + \varepsilon_1 E_y^{(1)}(x,0,t), \\
D_y^{(2)}(x,0,t) &= e_{13} \frac{\partial u_2(x,0,t)}{\partial x} + e_{33} \frac{\partial v_2(x,0,t)}{\partial y} + \varepsilon_3 E_y^{(2)}(x,0,t),
\end{aligned} \tag{2.7}$$

According to the system of equations (1.22) and (1.23), a slow electroelastic wave is induced in the half-space Piezoelectric – 2

$$\begin{aligned}
u_2(x,y,t) &= C_u \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + a_v C_v \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + \\
&+ b_{ux} C_{e2x} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + b_{uy} C_{e2y} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] \\
v_2(x,y,t) &= a_u C_u \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + C_v \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + \\
&+ b_{vx} C_{e2x} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + b_{vy} C_{e2y} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] \\
E_x^{(2)}(x,y,t) &= a_{2x} C_u \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + b_{2x} C_v \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + \\
&+ C_{e2x} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + b_{2x} C_{e2y} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] \\
E_y^{(2)}(x,y,t) &= a_{2y} C_u \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + b_{2y} C_v \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + \\
&+ b_{2y} C_{e2x} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)] + C_{e2y} \cdot \exp[i(\omega t - k_{21}x - k_{22}y)]
\end{aligned} \tag{2.8}$$

In which normalizing factors are determined from the following systems of algebraic equations

$$\begin{aligned}
\begin{pmatrix} -(c_{13} + c_{44})k_{21}k_{22} & -k_{22}e_{15} & k_{21}e_{13} \\ c_{44}k_{21}^2 + c_{33}k_{22}^2 - \rho\omega^2 & k_{21}e_{15} & -k_{22}e_{33} \\ e_{15}k_{21}^2 + e_{33}k_{22}^2 & -k_{21}\varepsilon_1 & k_{22}\varepsilon_3 \end{pmatrix} \times \begin{pmatrix} a_u \\ a_{2x} \\ a_{2y} \end{pmatrix} &= \begin{pmatrix} c_{11}k_{21}^2 + c_{44}k_{22}^2 - \rho\omega^2 \\ (c_{13} + c_{44})k_{21}k_{22} \\ (e_{15} + e_{31})k_{21}k_{22} \end{pmatrix} \\
\begin{pmatrix} k_{21}^2c_{11} + k_{22}^2c_{44} - \omega^2\rho & k_{22}e_{15} & k_{21}e_{13} \\ k_{21}k_{22}(c_{13} + c_{44}) & k_{21}e_{15} & k_{22}e_{33} \\ k_{21}k_{22}(e_{15} + e_{31}) & -k_{21}\varepsilon_1 & -k_{22}\varepsilon_3 b_{2y} \end{pmatrix} \times \begin{pmatrix} a_v \\ b_{2x} \\ b_{2y} \end{pmatrix} &= \begin{pmatrix} k_{21}k_{22}(c_{13} + c_{44}) \\ k_{21}^2c_{44} + k_{22}^2c_{33} - \omega^2\rho \\ k_{21}^2e_{15} + k_{22}^2e_{33} \end{pmatrix}
\end{aligned} \tag{2.9}$$

$$\begin{pmatrix} k_{21}^2 c_{11} + k_{22}^2 c_{44} - \rho \omega^2 & k_{21} k_{22} (c_{13} + c_{44}) & -k_{21} e_{13} \\ k_{21}^2 c_{44} + k_{22}^2 c_{33} - \omega^2 \rho & k_{21} k_{22} (c_{13} + c_{44}) & -k_{22} e_{33} \\ k_{21}^2 e_{15} + k_{22}^2 e_{33} & k_{21} k_{22} (e_{15} + e_{31}) & k_{22} \varepsilon_3 \end{pmatrix} \times \begin{pmatrix} b_{ux} \\ b_{vx} \\ b_{2y} \end{pmatrix} = \begin{pmatrix} k_{22} e_{15} \\ k_{21} e_{15} \\ k_{21} \varepsilon_1 \end{pmatrix} \quad (2.10)$$

$$\begin{pmatrix} k_{21}^2 c_{11} + k_{22}^2 c_{44} - \omega^2 \rho & k_{21} k_{22} (c_{13} + c_{44}) & -k_{22} e_{15} \\ k_{21} k_{22} (c_{13} + c_{44}) & k_{21}^2 c_{44} + k_{22}^2 c_{33} - \omega^2 \rho & -k_{21} e_{15} \\ k_{21} k_{22} (e_{15} + e_{31}) & k_{21}^2 e_{15} + k_{22}^2 e_{33} & k_{21} \varepsilon_1 \end{pmatrix} \times \begin{pmatrix} b_{uy} \\ b_{vy} \\ b_{2x} \end{pmatrix} = \begin{pmatrix} k_{21} e_{13} \\ k_{22} e_{33} \\ -k_{22} \varepsilon_3 \end{pmatrix}$$

A slow magnetic wave of transverse polarization together with electroelastic wave transformation is not induced.

Fast magnetoelastic waves accompanying slow electroacoustic waves, determined from equations (1.10), (1.11) and (1.23), are infinitesimal (of the order of $\approx 10^{-5}$) and are not taken into account in problems of electro-magneto-elasticity in quasi-static elasticity.

In a refracted multicomponent wave (2.8), the direction cosines are represented as

$$k_{21} = k_2 \cdot \sin \beta, \quad k_{22} = k_2 \cdot \cos \beta, \quad k_2^2 = \omega^2 / \tilde{c}_{l2}^2 \quad (2.11)$$

The speed $\tilde{c}_{l2}(k_2, \omega)$ of a slow electroactive plane strain wave (**P&SV**) is determined from the characteristic equation of the system (2.12) and (2.13).

Substituting the relations of electroelastic waves (2.5) and (2.8) into the boundary conditions (2.6), taking into account representations (2.7), (2.9) and (2.10), we find the amplitudes both of reflected $B(A, \alpha_0)$ and refracted $C_u(A, \alpha_0)$, $C_v(A, \alpha_0)$, $C_{e_{2x}}(A, \alpha_0)$, $C_{e_{2y}}(A, \alpha_0)$ multicomponent waves in the corresponding half-spaces.

The angle of transformed waves is found based on the same flow of the hybrid in the direction Ox : $k_{11} = k_{21}$. From which it turns out

$$\sin \beta = (\tilde{c}_{l2} / \tilde{c}_{l1}) \cdot \sin \alpha_0 \quad (2.12)$$

The study of the behavior of a refracted wave and a wave hybrid as a whole comes down to a numerical analysis of the behavior of the amplitude functions both of reflected $B(A, \alpha_0)$ and refracted $C_u(A, \alpha_0)$, $C_v(A, \alpha_0)$, $C_{e_{2x}}(A, \alpha_0)$, $C_{e_{2y}}(A, \alpha_0)$ multicomponent waves in the corresponding half-spaces and the refraction angle function

$$\beta((c_{44}/c_{11}), \alpha_0) = \arcsin[(\tilde{c}_{l2} / \tilde{c}_{l1}) \cdot \sin \alpha_0]. \quad (2.13)$$

An analysis of this kind was carried out in [15,16,17,19] and others, in the problem of the emergence of a hybrid during the propagation of an electroelastic shear wave.

Conclusions.

In the sagittal planes of differently polarized piezoelectric half-spaces of a 6mm class piezoelectric, heterogeneous multicomponent electro-magneto-elastic waves are formed. On the surface of the non-acoustic contact between differently polarized piezoelectric half-spaces, a multicomponent electro-magneto-elastic field of one type generated in one of the piezoelectric half-spaces is transformed into a hybrid of multicomponent electro-magneto-elastic waves of plane and antiplane deformations.

References

1. Kyame J.J., Wave Propagation in Piezoelectric Crystals. The Journal of the Acoustical Society of America, (1949), vol.21(3), pp.159-167, <https://doi.org/10.1121/1.1906490>
2. Wu H., Kuang Y., Propagation Characteristics of Flexural Wave in One-Dimensional Phononic Crystals Based on Lattice Dynamics Model, *Journal of Applied Mathematics and Physics*, (2022), vol.10 No.5, <https://doi.org/10.4236/jamp.2022.105100> ,
3. Hillion P., Wave Propagation in Nanocomposite Materials, *Journal of Electromagnetic Analysis and Applications*, (2010), vol.2, No.7, <https://doi.org/10.4236/jemaa.2010.27053>,
4. Zakharenko A. A., New Interfacial Shear-Horizontal Waves in Piezoelectric Cubic Crystals, *Journal of Electromagnetic Analysis and Applications*, (2010), vol.2, No.11, <https://doi.org/10.4236/jemaa.2010.211083>,
5. Rabiou M., Mensah S. Y., Abukari S. S., Amekpewu M., Sefa-Ntiri B., Twum A., Generation and Propagation of Ultrasonic Waves in Piezoelectric Graphene Nanoribbon, *Open Journal of Acoustics*, (2013), vol.3 No.3A, <https://doi.org/10.4236/oja.2013.33A007>,
6. Белубекян М.В., Гараков В.Г., Отражение нормально падающей сдвиговой электроупругой волны от плоской границы раздела двух пьезоактивных сред. Докл. НАН Армении, 2013, т.113, N 4. с. 364-369.
7. Avetisyan Ara S., Electroacoustic Waves in Piezoelectric Layered Composites, *Advanced Structured Materials*, (2023), vol.182, Springer Cham., p. 225, <https://doi.org/10.1007/978-3-031-26731-4> ,
8. Singh B., Yadav A.K. and Kaushal S., Effect of Impedance Boundary on Reflection of Plane Waves from free Surface of a Rotating Thermoelastic Solid Half Space, *Research J. Engineering and Tech.*, (2017), vol. 8(4), pp. 405-413, <https://doi.org/10.5958/2321-581X.2017.00071.X>
9. Singh B., Reflection of Elastic Waves from Plane Surface of a Half-space with Impedance Boundary Conditions, *Geosciences Research*, (2017), Vol. 2, No. 4, pp. 242-253, <https://dx.doi.org/10.22606/gr.2017.24004> ,
10. Rajneesh Chattopadhyay A., Reflection and refraction of waves at the interface of an isotropic medium over a highly anisotropic medium, *Acta Geophysica*. (2006), Vol.54, pp. 239–249, <https://doi.org/10.2478/s11600-006-0022-y> ,
11. Klenow B., Nisewonger A., Batra R.C., Brown A., Reflection and transmission of plane waves at an interface between two fluids, *Computers & Fluids* (2007), vol. 36, pp. 1298–1306, <https://doi.org/10.1016/j.compfluid.2007.03.014> ,
12. Pang Y., Wang Y.-S., Liu Jin-Xi, Fang D.-N., Reflection and refraction of plane waves at the interface between piezoelectric and piezomagnetic media, *International Journal of Engineering Sciences*, (2008), vol. 46, Iss. 11, pp. 1098-1110, <https://doi.org/10.1016/j.ijengsci.2008.04.006> ,
13. Rajneesh Kumar, Rajeev Kumar, "Reflection and refraction of elastic waves at the interface of an elastic half-space and initially stressed thermoelastic with voids half-

- space", *Multidiscipline Modeling in Materials and Structures*, (2012), vol. 8 Iss: 3 pp. 355 – 379, <http://dx.doi.org/10.1108/15736101211269159> ,
14. Vinh P.C., Tuan T.T., Tung D.X., Kieu N.T., Reflection and transmission of SH waves at a very rough interface and its band gaps, *Journal of Sound and Vibration*, (2017), vol. 411, pp. 422-434, <https://doi.org/10.1016/j.jsv.2017.08.046> ,
 15. Ghazaryan K.B., Papyan A.A., and Ohanyan S.K., "Reflection, refraction, and transmission of SH waves at a micropolar layer separating two elastic media." *Journal of Physics: Conference Series*, (2018), vol. 991, No. 1, IOP Publishing,.
 16. Avetisyan A.S., Khachatryan V.M., Galichyan T.A., Reflection and Transmission of Electro-Elastic Waves at Plane Non-Acoustic Contact Interface of Two Different Piezoelectric Half-Spaces, *Materials Int. Scientific-Practical. conf. "Multiferroics: preparation, properties, application"* Vitebsk, (2019), / ed. Rubanik V.V., 92-95 p., <http://doi.org/10.26201/ISSP.2019.45.557/MFerro.37>
 17. Джилавян С.А., Саргсян А.С., Дифракция плоской волны сдвига в составном пьезоэлектрическом пространстве. *Известия НАН РА. Механика* 2019, 72(1), с.35-48, <http://doi.org/10.33018/72.1.3>,
 18. Sahu S.A., Nirval S., Mondal S., Reflection and transmission of quasi-plane waves at the interface of piezoelectric semiconductors with initial stresses, *Appl. Math. and Mech. (Engl. Ed.)*, (2020), vol. 41(1), pp. 1-18, <https://doi.org/10.1007/s10483-021-2738-9> ,
 19. Singh B., Sangwan A., Singh J., Reflection and Transmission of Plane Wave at an Interface Between Two Rotating Micropolar Piezoelectric Solid Half-Spaces, *Archives of Acoustics*, (2021), vol. 46, No. 4, pp. 623–635, <https://doi.org/10.24425/aoa.2021.138155> ,
 20. Avetisyan A.S., Khachatryan V.M., and Mkrtchyan M.H., Formation of a hybrid of electroacoustic waves in piezoelectric layered composites, *J. Phys.: Conf. Ser.* (2022), vol. 2231 012025, <https://doi.org/10.1088/1742-6596/2231/1/012025> ,
 21. Kumar S., Pal P.C., and Majhi S., "Reflection and transmission of SH-waves at a corrugated interface between two semi-infinite anisotropic magnetoelastic half-spaces," *Waves Random Complex Media* (2023), vol. 27 (2), pp.339–358 <https://doi.org/10.1080/17455030.2016.1245454> ,
 22. Akshaya A., Kumar S. & Hemalatha K., Behaviour of Transverse Wave at an Imperfectly Corrugated Interface of a Functionally Graded Structure. *Phys. Wave Phen.* (2024), vol. 32, pp.117–134, <https://doi.org/10.3103/S1541308X24700067> ,

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