ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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REGULATION BY LOCALIZATION OF WAVE ENERGY ALONG THE THICKNESS OF A PIEZOELECTRIC WAVEGUIDE: PROBLEM OF OPTIMAL CONTROL OF A THREE-COMPONENT ELECTROELASTIC WAVE

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Keywords: piezoelectric waveguide, electroelastic wave, wave energy, material anisotropy, electromechanical surface actions, controllability, initial-boundary value problem.

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Регулирование локализацией волновой энергии по толщине пьезоэлектрического волновода: задача оптимального управления трехкомпонентной электроупругой волной

Ключевые слова: пьезоэлектрический волновод, электроупругая волна, энергия волны, анизотропия материала, электромеханические воздействия, управляемость, начально-краевая задача**.**

Дана обобщающая постановка математической начально-краевой задачи о распространении электроупругой трехкомпонентной волны в слое пьезоэлектрического волновода, изготовленного из материала произвольной анизотропии. Начально-краевая математическая задача управления распространением электроупругой волны поверхностными электромеханическими воздействиями сводится к сходящейся системе задач управления собственными функциями и соответствующими собственными гармониками электромеханических характеристик распространяющейся волны. Даются определения точной управляемости трехкомпонентной электроупругой волны, а также определения регулировки локализацией волновой энергии электромеханическими поверхностными воздействиями как задача оптимального управления распределением трехкомпонентной электроупругой волны по толщине пьезоэлектрического волновода.

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Պիեզոէլեկտրական ալիքատարի հաստությամբ ալիքային էներգիայի տեղայնացման կարգավորումը. Եռաբաղադրիչ էլեկտրաառաձգական ալիքի օպտիմալ կառավարման խնդիրը

Հիմնաբառեր՝ պիեզոէլեկտրական ալիքատար, էլեկտրաառաձգական ալիք, ալիքի էներգիա, նյութի անիզոտրոպիա, մակերևութային ազդեցություններ, գործողություններ, կառավարելիություն, սկզբնական-եզրային խնդիր:

Բերված է կամայական անիզոտրոպիա ունեցող պիեզոէլեկտրական ալիքատար շերտում էլեկտրաառաձգական եռաբաղադրիչ ալիքի տարածումը նկարագրող մաթեմատիկական սկզբնական-եզրային խնդրի ընդհանրացնող ձևակերպումը: Մակերևութային էլեկտրամեխանիկական ազդեցություններով էլեկտրաառաձգական ալիքի տարածումը ղեկավարելու սկզբնական եզրային մաթեմատիկական խնդիրը վեր է ածվում սեփական ֆունկցիաների և տարածվող ալիքի էլեկտրամեխանիկական բնութագրերի համապատասխան սեփական հարմոնիկաների ղեկավարման խնդիրների կոնվերգենտ համակարգի: Տրված են եռաբաղադրիչ էլեկտրաառաձգական ալիքի ճշգրիտ կառավարելիության, ինչպես նաև էլեկտրամեխանիկական մակերևութային ազդեցությունների միջոցով, ալիքային էներգիայի տեղայնացման կարգավորման սահմանումներ՝ որպես պիեզոէլեկտրական ալիքատարի հաստությամբ եռաբաղադրիչ էլեկտրաառաձգական ալիքի էներգիայի բաշխման օպտիմալ ղեկավարման խնդիր:

A generalizing formulation of the mathematical initial-boundary value problem on the propagation of an electroelastic three-component wave in a piezoelectric waveguide layer made of a material of arbitrary anisotropy is given. The initial-boundary-value mathematical problem of controlling the propagation of an electroelastic wave by surface electromechanical influences is reduced to a convergent system of problems of controlling the eigenfunctions and the corresponding eigen harmonics of the electromechanical characteristics of the propagating wave. Definitions of the precise controllability of a three-component electroelastic wave are given, as well as definitions of regulation by the localization of wave energy by electromechanical surface influences as a problem of optimal control of the distribution of a three-component electroelastic wave over the thickness of a piezoelectric waveguide.

Introduction

The propagation of electroacoustic waves in piezoelectric waveguides is a process of changing the state of coupled elastic and electromagnetic fields and is considered as a state of a driving medium (system) with distributed parameters.

Developing a control theory for systems with distributed parameters is a much more complex task compared to a similar task for systems with lumped parameters. Moreover, the problems of optimality, controllability and observability for systems with distributed parameters are as complex as similar problems for systems with lumped masses [1].

In limited solid piezoelectric waveguides, in accordance with the anisotropy of the material and the fulfillment of boundary conditions, different types of electroactive normal electroelastic waves propagate along their surface, each of which is a set of longitudinal and/or transverse elastic waves and accompanying electric field oscillations [2, 3, 4]. An electroactive elastic wave in a piezoelectric medium is a multi-parameter process and can be controlled (regulated) by various possible electromechanical influences [4,5].

An important feature in problems of controlling electroacoustic processes is also the possibility of contactless influence on the surface of a piezoelectric medium [4]. This is caused by the inverse piezoelectric effect, where temporary fluctuations in the electric field at the surface of the piezoelectric body (surface electromotive force) create a surface-equivalent mechanical force.

Controllability of hyperbolic systems of equations is achieved in principle in three different ways: i) when the characteristics of a propagating wave are related to each other by Holmgren's uniqueness theorem, ii) by harmonic analysis of wave characteristics in connection with Ingham's lemma and its extensions, iii) the method of integrating factors. Moreover, the harmonic analysis method of wave characteristics, when applicable, gives very good results [1]. In a recently published work [6], the author presents the formulation and approaches to the study of problems of controllability and stabilization of the wave process under various types of influences on it.

Using the Green's function method, work [7] studies the controllability of linear and nonlinear mathematical initial-boundary value problems arising in many applied fields of science.

Without violating the generality of the approach to studying the controllability of the process and in order to avoid unnecessary mathematical difficulties, the article considers the issue of controllability of the mathematical initial-boundary value problem of electroelasticity for the case of a three-component electroacoustic quasi-static wave in a piezoelectric waveguide under various possible electromechanical surface influences [8].

1. Mathematical initial-boundary value problems for three-component electroelastic waves in piezoelectric waveguide under surface electromechanical influences

Piezoelectrics are essentially anisotropic crystalline materials. In an arbitrarily chosen sagittal plane $x_\alpha 0 x_\beta$, with a choice of indices $\{\alpha,\ \beta,\ \gamma\} \rightleftarrows \{1,\ 2,\ 3\}$, the tensor of electromechanical characteristics of the medium allows us to formulate a two-dimensional problem about a three-component electroelastic pure shear wave of the type $\{0,\ 0,\ u_\gamma(x_\alpha, x_\beta, t),\ \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha,\ \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta,\ 0\}$.

For piezoelectric materials that allow an anti-plane deformable state, the material relations of non-zero characteristics of the electromechanical field can be represented in the form

$$\begin{pmatrix} \sigma_{\beta\gamma}(x_\alpha, x_\beta, t) \\ \sigma_{\gamma\alpha}(x_\alpha, x_\beta, t) \\ D_\alpha(x_\alpha, x_\beta, t) \\ D_\beta(x_\alpha, x_\beta, t) \end{pmatrix} = \begin{pmatrix} 0 & c_{44}^* & e_{14}^* & e_{24}^* \\ c_{55}^* & 0 & e_{15}^* & e_{25}^* \\ e_{15}^* & e_{14}^* & -\varepsilon_{11} & 0 \\ e_{25}^* & e_{24}^* & -\varepsilon_{22}^* & 0 \end{pmatrix} \times \begin{pmatrix} \partial u_\gamma(x_\alpha, x_\beta, t)/\partial x_\alpha \\ \partial u_\gamma(x_\alpha, x_\beta, t)/\partial x_\beta \\ \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha \\ \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta \end{pmatrix} \tag{1.1}$$

Consequently, the quasistatic equations of electro elasticity will be different for the corresponding piezoelectrics, depending on the choice of anisotropy of the material and the crystallographic cross section in it [4, 5].

Representations of material connections of non-zero characteristics of the electromechanical field in a unified form (1.1) formally allow a unified representation of the mathematical initial-boundary value problem for piezoelectrics of all symmetry classes, in which the electroactive problem of antiplane deformation is possible.

Without violating the generality of reasoning, in a piezoelectric waveguide connected to the coordinate system $0xyz$ piezoelectric waveguide $\Omega_*(x, y, z) = \{|x| < \infty; |y| \le H_0; |z| < \infty\}$, the system of quasi-static electro elasticity equations for three-component electroelastic waves (pure shear electroactive elastic waves) will be written in the form of an invariant matrix system of linear differential homogeneous equations

$$\begin{pmatrix} \Lambda_{1\mathrm{w}}[*] - \rho[\partial^2/\partial t^2] & \Lambda_{1\varphi}[*] \\ \Lambda_{2\mathrm{w}}[*] & \Lambda_{2\varphi}[*] \end{pmatrix} \times \begin{pmatrix} \mathrm{w}(x, y, t) \\ \varphi(x, y, t) \end{pmatrix} = 0 \tag{1.2}$$

$$\begin{aligned}
&\Lambda_{1\mathrm{w}}[*]=c_{55}^{*}[\partial^2/\partial x^2]+c_{44}^{*}[\partial^2/\partial y^2],\\
&\Lambda_{2\mathrm{w}}[*]=e_{15}^{*}[\partial^2/\partial x^2]+e_{24}^{*}[\partial^2/\partial y^2]+(e_{25}^{*}+e_{14}^{*})\cdot[\partial^2/\partial x\partial y],\\
&\Lambda_{1\varphi}[*]=e_{15}^{*}[\partial^2/\partial x^2]+e_{24}^{*}[\partial^2/\partial y^2]+(e_{25}^{*}+e_{14}^{*})\cdot[\partial^2/\partial x\partial y],\\
&\Lambda_{2\varphi}[*]=-\varepsilon_{11}^{*}[\partial^2/\partial x^2]-\varepsilon_{22}^{*}[\partial^2/\partial y^2].
\end{aligned} \tag{1.3}$$

Linear operators (1.3) are introduced into the matrix system of equations (1.2). This also includes the shear displacement $\mathrm{w}(x,y,t)$ and electric potential $\varphi(x,y,t)$ of a three-component electroelastic wave of antiplane strain in the piezoelectric layer.

In the case of the unidirectional propagation along the waveguide of normal electroelastic waves $\vec{F}(x,y,t)=\vec{f}(y,t)\cdot\exp(\pm ikx)$, the antiplane electroelastic state will be described with respect to the amplitude functions of the electromechanical characteristics $\mathrm{w}(y,t)$ and $\varphi(y,t)$ in two-dimensional form

$$\begin{pmatrix}\Lambda_{1\mathrm{w}}[*]-\rho[\partial^2/\partial t^2] & \Lambda_{1\varphi}[*]\\ \Lambda_{2\mathrm{w}}[*] & \Lambda_{2\varphi}[*]\end{pmatrix}\times\begin{pmatrix}\mathrm{w}(y,t)\\ \varphi(y,t)\end{pmatrix}=0 \tag{1.4}$$

$$\Lambda_{1\mathrm{w}}[*]=c_{44}^{*}\cdot[\partial^2/\partial y^2]-c_{55}^{*}k^2,\quad \Lambda_{2\mathrm{w}}[*]=e_{24}^{*}\cdot[\partial^2/\partial y^2]+ik(e_{25}^{*}+e_{14}^{*})\cdot[\partial/\partial y]-e_{15}^{*}k^2,$$
$$\Lambda_{1\varphi}[*]=e_{24}^{*}\cdot[\partial^2/\partial y^2]+ik(e_{25}^{*}+e_{14}^{*})\cdot[\partial/\partial y]-e_{15}^{*}k^2,\quad \Lambda_{2\varphi}[*]=-\varepsilon_{22}^{*}\cdot[\partial^2/\partial y^2]+\varepsilon_{11}^{*}k^2. \tag{1.5}$$

On the surface of a piezoelectric waveguide $y=\pm H_0$, the conjugation conditions of the electric and mechanical fields are always satisfied. Mechanical boundary conditions impose restrictions on shear displacement $\mathrm{w}(x,y,t)$, or shear stress $\sigma_{zx}(x,y,t)$ and/or $\sigma_{yz}(x,y,t)$. Electrical boundary conditions impose restrictions on the normal component of the electrical displacement $D_y(x,y,t)$, or on the tangential component of the electrical intensity $E_x(x,y,t)=-\partial\varphi(x,y,t)/\partial x$.

In the case of the unidirectional propagation along the waveguide of normal electroelastic waves, the inhomogeneous electromechanical boundary conditions are written in the form of different mathematical linear combinations of four physically independent, mechanical and/or electrical surface actions [2,3,4]

$$\mathrm{w}(y,t)\big|_{y=\pm H_0}=\mu_{\pm}(t);\qquad \varphi(y,t)\big|_{y=\pm H_0}=\phi_{\pm}(t), \tag{1.6}$$

$$\mathrm{w}(y,t)\big|_{y=\pm H_0}=\mu_{\pm}(t);\qquad \left[\partial\varphi(y,t)/\partial y\right]\big|_{y=\pm H_0}=\delta_{\pm}(t), \tag{1.7}$$

$$\left[\partial\mathrm{w}(y,t)/\partial y\right]_{y=\pm H_0}=\tau_{\pm}(t);\qquad \varphi(y,t)\big|_{y=\pm H_0}=\phi_{\pm}(t), \tag{1.8}$$

$$\left[\mathrm{w}(y,t)/\partial y\right]_{y=\pm H_0}=\tau_{\pm}(t);\qquad \left[\varphi(y,t)/\partial y\right]_{y=\pm H_0}=\delta_{\pm}(t), \tag{1.9}$$

Included in the inhomogeneous boundary conditions, the time functions $\mu_{\pm}(t);\ \phi_{\pm}(t);\ \tau_{\pm}(t);\ \delta_{\pm}(t)$ are defined on the time axis $t>0$ and belong to the class $\mathbb{C}_2[*]$ of functions.

In the general case of the dynamic formulation of the problem of electro elasticity, at the initial and final moments of the time interval $t\in[0;T_0]$, four functions of the elastic and electrical characteristics of the wave field or four different combinations thereof, the conditions of the initial and final states is specified.

In the quasi-static formulation of electroacoustic problems, the propagation of an elastic wave with accompanying oscillations of the electric field is considered. Then the electroelastic state is determined by two known functions of the initial and two other known functions of the final states, or by two different combinations of these four existing characteristics of the electroelastic field $\mathrm{w}(x,y,t)$, $\varphi(x,y,t)$, $\dot{\mathrm{w}}(x,y,t)$ and $\dot{\varphi}(x,y,t)$, which are interconnected by two equations (1.2).

In the case of the unidirectional propagation along the waveguide of normal electroelastic waves, the electroelastic state during the propagation of normal electroelastic waves, in the waveguide layer is determined by two known functions of the initial state

$$\mathrm{w}(y,0)=\xi(y); \qquad \dot{\mathrm{w}}(y,0)=\zeta(y), \tag{1.10}$$

or

$$\varphi(y,0_0)=\eta(y); \qquad \dot{\varphi}(y,0_0)=\gamma(y), \tag{1.11}$$

and two known functions of the final state

$$\mathrm{w}(y,T_0)=\tilde{\xi}(y); \qquad \dot{\mathrm{w}}(y,T_0)=\tilde{\zeta}(y), \tag{1.12}$$

or

$$\varphi(y,T_0)=\tilde{\eta}(y); \qquad \dot{\varphi}(y,T_0)=\tilde{\gamma}(y) \tag{1.13}$$

for an electroelastic wave.

The system of homogeneous second-order differential equations (1.4), with the notation of linear operators (1.5), together with surface influences of the type (1.6) - (1.9) and representations of the initial and final states (1.10) ÷ (1.13), constitute a complete mathematical initial-boundary problem for studying the control of wave formation and propagation of a three-component electroacoustic wave in a piezoelectric waveguide.

In problems of this type, surface control of waves comes down to studying the controllability of wave formation of their own waveforms and changing the corresponding harmonics. Therefore, research on wave control can be carried out by harmonic analysis of wave characteristics, since it is applicable and gives very good results [1].

For this purpose, surface electromechanical influences acting on the propagation of electroacoustic waves are reduced to the corresponding volumetric influences. Then the boundary value problem with inhomogeneous boundary conditions (1.4) - (1.9) is written in the form of inhomogeneous equations with volumetric influences

$$\begin{pmatrix} \Lambda_{1\mathrm{v}}[*]-\rho[\partial^2/\partial t^2] & \Lambda_{1\psi}[*] \\ \Lambda_{2\mathrm{v}}[*] & \Lambda_{2\psi}[*] \end{pmatrix} \times \begin{pmatrix} \mathrm{v}(y,t) \\ \psi(y,t) \end{pmatrix} = \begin{pmatrix} f_{\mathrm{w}}(y,t) \\ f_{\varphi}(y,t) \end{pmatrix} \tag{1.14}$$

and with the corresponding (1.6) - (1.9), homogeneous surface conditions in each case

$$\mathrm{v}(y,t)\big|_{y=\pm H_0}=0; \qquad \psi(y,t)\big|_{y=\pm H_0}=0, \tag{1.15}$$

$$\mathrm{v}(y,t)\big|_{y=\pm H_0}=0; \qquad \left[\partial\psi(y,t)/\partial y\right]\big|_{y=\pm H_0}=0, \tag{1.16}$$

$$\left[\partial\mathrm{v}(y,t)/\partial y\right]_{y=\pm H_0}=0; \qquad \psi(y,t)\big|_{y=\pm H_0}=0, \tag{1.17}$$

$$\left[\mathrm{v}(y,t)/\partial y\right]_{y=\pm H_0}=0; \qquad \left[\psi(y,t)/\partial y\right]_{y=\pm H_0}=0, \tag{1.18}$$

Linear operators in the matrix equations (1.14) is already transformed to

$$\Lambda_{1\mathrm{v}}[*]\rightleftarrows\Lambda_{1\mathrm{w}}[*], \quad \Lambda_{2\mathrm{v}}[*]\rightleftarrows\Lambda_{2\mathrm{w}}[*], \quad \Lambda_{1\psi}[*]\rightleftarrows\Lambda_{1\psi}[*], \quad \Lambda_{2\psi}[*]\rightleftarrows\Lambda_{2\varphi}[*]. \tag{1.19}$$

For various types of surface influences (1.6) - (1.9) the transformation functions can be formally written in a uniform

$$\begin{Bmatrix} \mathrm{v}(y,t) \\ \psi(y,t) \end{Bmatrix} = \begin{Bmatrix} \mathrm{w}(y,t) \\ \varphi(y,t) \end{Bmatrix} - \begin{Bmatrix} \mathrm{W}_+(y,t)+\mathrm{W}_-(y,t) \\ \Phi_+(y,t)+\Phi_-(y,t) \end{Bmatrix} \tag{1.20}$$

In which the terms of the transformation are predetermined by the nature of surface influences (1.6) – (1.9)

$$\mathrm{W}_{\pm}(y,t)\in\left\{[(H_0\pm y)/2H_0]\cdot\mu_{\pm}(t);\ [(y\pm H_0)^2/4H_0]\cdot\tau_{\pm}(t)\right\} \tag{1.21}$$

$$\Phi_{\pm}(y,t)\in\left\{[(H_0\pm y)/2H_0]\cdot\phi_{\pm}(t);\ [(y\pm H_0)^2/4H_0]\cdot\delta_{\pm}(t)\right\} \tag{1.22}$$

In equations (1.14), volumetric influences of the type $f_{\mathrm{w}}(y,t)$ and $f_{\varphi}(y,t)$ are represented by transformation terms $\mathrm{W}(y,t)=\mathrm{W}_{+}(y,t)+\mathrm{W}_{-}(y,t)$ and $\Phi(y,t)=\Phi_{+}(y,t)+\Phi_{-}(y,t)$ as

$$f_{\mathrm{w}}(y,t)=\left[\Lambda_{1\mathrm{v}}[*]-\rho[\partial^2/\partial t^2]\right]\times\left[\mathrm{W}(y,t)\right]+\Lambda_{1\psi}[*]\times\left[\Phi(y,t)\right] \tag{1.23}$$

$$f_{\varphi}(y,t)=\Lambda_{2\mathrm{v}}[*]\times\left[\mathrm{W}(y,t)\right]+\Lambda_{2\psi}[*]\times\left[\Phi(y,t)\right] \tag{1.24}$$

By introducing transformation functions (1.20), descriptions of the initial and final states (1.10) - (1.13) acquire a new entry

$$\mathrm{v}(y,0)=\xi(y)-\left[\mathrm{W}_{+}(y,0)+\mathrm{W}_{-}(y,0)\right];\quad \dot{\mathrm{v}}(y,0)=\zeta(y)-\left[\dot{\mathrm{W}}_{+}(y,0)+\dot{\mathrm{W}}_{-}(y,0)\right] \tag{1.25}$$

or

$$\psi(y,0)=\eta(y)-\left[\Phi_{+}(y,0)+\Phi_{-}(y,0)\right];\quad \dot{\psi}(y,0)=\gamma(y)-\left[\dot{\Phi}_{+}(y,0)+\dot{\Phi}_{-}(y,0)\right] \tag{1.26}$$

and two known functions of the final state

$$\mathrm{v}(y,T_0)=\tilde{\xi}(y)-\left[\mathrm{W}_{+}(y,T_0)+\mathrm{W}_{-}(y,T_0)\right];\ \dot{\mathrm{v}}(y,T_0)=\tilde{\zeta}(y)-\left[\dot{\mathrm{W}}_{+}(y,T_0)+\dot{\mathrm{W}}_{-}(y,T_0)\right] \tag{1.27}$$

or

$$\psi(y,T_0)=\tilde{\eta}(y)-\left[\Phi_{+}(y,T_0)+\Phi_{-}(y,T_0)\right];\ \dot{\psi}(y,T_0)=\tilde{\gamma}(y)-\left[\dot{\Phi}_{+}(y,T_0)+\dot{\Phi}_{-}(y,T_0)\right] \tag{1.28}$$

for an electroelastic wave.

By introducing transformation functions (1.20), the initial-boundary value mathematical problem with a system of inhomogeneous equations (1.14) and homogeneous surface conditions (1.15) - (1.18), together with descriptions of the initial and final states (1.25) - (1.28) constitute the initial-boundary value problem control of a three-component electroelastic wave in a coordinate rectangle $Q_T=[-H_0\le y\le H_0]\times[0\le t\le T_0]$.

2. Three-component electroelastic wave formation and propagation in a piezoelectric waveguide

In a homogeneous mathematical boundary value problem formed from homogeneous equations of the corresponding system (1.14), when $f_{\mathrm{w}}(y,t)\equiv 0$, $f_{\varphi}(y,t)\equiv 0$ and the homogeneous boundary conditions of the type (1.15) ÷ (1.18), separation of variables is possible. Then the solutions to this homogeneous mathematical boundary value problem are represented as a function

$$\begin{pmatrix}\mathrm{v}(y,t)\\ \psi(y,t)\end{pmatrix}=\begin{pmatrix}\mathrm{V}(y)\\ \psi(y)\end{pmatrix}\cdot\theta(t)=\sum_{n=0}^{n=\infty}\begin{pmatrix}\mathrm{V}_n(y)\\ \psi_n(y)\end{pmatrix}\cdot\theta_n(t), \tag{2.1}$$

expanded in Fourier series $\mathrm{V}(y)=\sum_{n=0}^{n=\infty}\mathrm{V}_n(y)$, $\theta(t)=\sum_{n=0}^{n=\infty}\theta_n(t)$.

It is important to pay attention to the fact that in the expansions of both sought functions the time regime will be the same $\theta_{\psi}(t)=\theta_{\mathrm{w}}(t)=\theta(t)$. This follows from the homogeneity of the second equation of the matrix system (1.4) and is a consequence of the quasi-static formulation of the problem, in which the harmonics of the accompanying electric field coincide with the harmonics of elastic vibrations.

The difference in surface conditions (1.15) ÷ (1.18) in the formulation of the boundary value problem leads to different possible formations of the proper forms and structure of the overall electroelastic wave along the thickness of the waveguide.

The matrix system of homogeneous equations is then written in the form

$$\begin{pmatrix} \Lambda_{1\mathrm{w}}[*] & \Lambda_{1\varphi}[*] \\ \Lambda_{2\mathrm{w}}[*] & \Lambda_{2\varphi}[*] \end{pmatrix} \times \begin{pmatrix} \mathrm{V}(y) \\ \psi(y) \end{pmatrix} = \frac{\rho\ddot{\theta}(t)}{\theta(t)} = -\omega_{\theta}^{2} . \tag{2.2}$$

In accordance with the boundary conditions of the first kind (1.15) and the second kind (1.18), in piezoelectric waveguides made from materials of the classes ***6mm*** of hexagonal symmetry, ***4mm*** of tetrogonal symmetry, ***mm2*** of rhombic symmetry the resulting signals can be represented by their own shapes corresponding to different oscillation frequencies

$$\begin{Bmatrix} \mathrm{V}(y) \\ \psi(y) \end{Bmatrix} = \sum_{m=0}^{\infty} \begin{Bmatrix} A_{\mathrm{w}m} \cos[k_m \alpha_{1m}(\omega, kh)\cdot y] + B_{\mathrm{w}m} \sin[k_m \alpha_{1m}(\omega, kh)\cdot y] \\ (e_{15}/\varepsilon_{11})\cdot\left[A_{\mathrm{w}m} \cos[k_m \alpha_{1m}(\omega, kh)\cdot y] + B_{\mathrm{w}m} \sin[k_m \alpha_{1m}(\omega, kh)\cdot y]\right] \end{Bmatrix} \tag{2.3}$$

Here $\alpha_{1m}(\omega, kh) = \sqrt{\omega_{\theta m}^2 / k_m^2 C_{1t}^2 - 1} = m\pi/2k_m H_0$ are the eigenvalues of the accompanying electroelastic oscillations corresponding to the oscillation frequencies the eigenmodes $\omega_{\theta m} = k_m C_{1t}\cdot\sqrt{1+\left(m\pi/2k_m H_0\right)^2}$, $C_{1t} = \sqrt{c_{44}/\rho}$ shear volumetric wave velocity.

In cases of mixed boundary conditions like (1.16) and (1.17) in the same piezoelectric waveguides, no proper forms are formed.

Conversely, in accordance with mixed boundary conditions of type (1.16) and (1.17) in piezoelectric waveguides made of materials of classes ***43m/23*** cubic symmetry, ***222*** rhombic symmetry, ***622*** of hexagonal symmetry, $\overline{\mathbf{4}}$***2m*** tetragonal symmetry, the resulting signals can be represented by their own shapes corresponding different vibration frequencies

$$\begin{Bmatrix} \mathrm{V}(y) \\ \psi(y) \end{Bmatrix} = \sum_{m=0}^{\infty} \begin{Bmatrix} A_{\mathrm{w}m} \cos[k_m \alpha_{2m}(\omega, kh)\cdot y] + B_{\mathrm{w}m} \sin[k_m \alpha_{2m}(\omega, kh)\cdot y] \\ \dfrac{ikm\pi e_{14}}{h\varepsilon_{11}}\cdot\left[B_{\mathrm{w}m} \cos[k_m \alpha_{2m}(\omega, kh)\cdot y] - A_{\mathrm{w}m} \sin[k_m \alpha_{2m}(\omega, kh)\cdot y]\right] \end{Bmatrix} \tag{2.4}$$

Here $\alpha_{2m}(\omega, kh) = \sqrt{\omega_{\theta m}^2 / k_m^2 C_{2t}^2 - \vartheta_*^2} = m\pi/2k_m H_0$ are the eigenvalues of the accompanying electroelastic oscillations corresponding to the oscillation frequencies the eigenmodes $\omega_{\theta m} = kC_{2t}\cdot\sqrt{\vartheta_*^2 + \left(m\pi/2k_m H_0\right)^2}$ and $\vartheta_*^2 = c_{55}/c_{44}$ is a shear anisotropy coefficient, $C_{2t} = C_{1t}\sqrt{\vartheta} = \sqrt{c_{55}/\rho}$ shear volumetric wave velocity in a waveguide of a different anisotropy.

In cases of the boundary conditions of the first kind (1.15) and the second kind (1.18), in the same piezoelectric waveguides, no proper forms are formed.

The function of the harmonics of the accompanying electric field coincides with the function of the harmonics of the propagating elastic vibrations and is represented as

$$\theta(t) = \sum_{m=0}^{\infty} \theta_m(t) = \sum_{m=0}^{\infty} \left[A_{\theta m} \sin(\omega_{\theta m} t) + B_{\theta m} \cos(\omega_{\theta m} t)\right] . \tag{2.5}$$

From (2.3) it follows that in the case of transversal anisotropy of the piezoelectric, changes in the shapes of the distribution of amplitudes of accompanying electrical vibrations are consistent with changes in the shapes of the distribution of elastic shear amplitudes.

From (2.4) it follows that in the case of a different anisotropy of the material, changes in the distribution of the ***m^th^*** form of amplitudes of accompanying electrical vibrations lag behind the change in the corresponding form of distribution of elastic shear amplitudes by $\alpha_{0m} = (2m+1)\pi/2$.

Having found the eigenforms $\{V_m(y);\ \psi_m(y)\}$ and their corresponding harmonics $\theta_m(y)$, by expanding the inhomogeneous initial-boundary value problem (1.14), (1.15), and (1.20)÷(1.25) into a Fourier series in the coordinate rectangle $Q_T = [-H_0 \le y \le H_0]\times[0 \le t \le T_0]$, in a generalized form we obtain the control equation for the eigenforms of the electroelastic waves

$$\left[\ddot{\theta}_m(t) - \Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\ddot{\mu}_{\pm m}(t)\right] + \omega_{\theta m}^2\cdot\left[\theta(t) - \Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\mu_{\pm m}(t)\right] = = -\Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\left(\omega_{\mu m}^2 - \omega_{\theta m}^2\right)\cdot\mu_{\pm m}(t) + \Gamma_{1\phi_\pm}\cdot\phi_{\pm m}(t) \tag{2.6}$$

With expansion coefficients in Fourier series in eigenforms

$$\Lambda_{1\mu_\pm} = \frac{1}{H_0}\int_0^{H_0}\left[\left(H_0 \pm y\right)\cdot V_m(y)\right]dy,$$

$$\Gamma_{1\phi_\pm} = \frac{1}{H_0}\int_0^{H_0}\left[\left(e_{15}^* k^2\left(H_0 \pm y\right) \mp ik(e_{25}^* + e_{14}^*)\right)\cdot V_m(y)\right]dy \tag{2.7}$$

$$\Lambda_{2\mu_\pm} = \frac{1}{H_0}\int_0^{H_0}\left[\left(e_{15}^* k^2\left(H_0 \pm y\right) \mp ik(e_{25}^* + e_{14}^*)\right)\cdot V_m(y)\right]dy,$$

$$\Gamma_{2\phi_\pm} = \frac{1}{H_0}\int_0^{H_0}\left[\varepsilon_{22}^* k^2\left(H_0 \pm y\right)\cdot V_m(y)\right]dy, \qquad \mu_\pm(t) = \left[\Gamma_{2\phi_\pm}/\Lambda_{2\mu_\pm}\right]\cdot\phi_\pm(t) \tag{2.8}$$

The general solution to the control equation (2.6) for the ***m^th^*** true harmonic $g_m(\omega_{\theta m}t) = \theta_m(\omega_{\theta m}t) - \Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\mu_{\pm m}(\omega_{\theta m}t)$, is obtained by the method of variation of parameters for the eigenforms harmonics of the electroelastic waves and forced elastic vibrations

$$g_m(\omega_{\theta m}t) = A_{mg}\cdot\sin(\omega_{\theta m}t) + B_{mg}\cdot\cos(\omega_{\theta m}t) + +\Lambda_{1\mu_\pm}\frac{\omega_{\theta m}^2 - \omega_{\mu m}^2}{\tilde{c}_{*t}^2}\cdot\int_0^t \mu_{\pm m}(t)\left[A_{mg}\cdot\sin[\omega_{\theta m}(t-\tau)] + B_{mg}\cdot\cos[\omega_{\theta m}(t-\tau)]\right]\cdot d\tau + +\Gamma_{1\phi_\pm}\cdot\int_0^t \phi_{\pm m}(t)\cdot\left[A_{mg}\cdot\sin[\omega_{\theta m}(t-\tau)] + B_{mg}\cdot\cos[\omega_{\theta m}(t-\tau)]\right]\cdot d\tau \tag{2.9}$$

In which harmonics of the reduced eigenforms and the harmonics of surface actions are represented respectively, as

$$g_m(\omega_{\theta m}t) = \theta_m(\omega_{\theta m}t) - \Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\mu_{\pm m}(\omega_{\theta m}t), \tag{2.10}$$

$$\mu_{\pm n}\left(t\right) = A_{n\mu}^\pm\cdot\sin(\omega_{\mu n}t) + B_{n\mu}^\pm\cdot\cos(\omega_{\mu n}t), \tag{2.11}$$

$$\phi_{\pm n}\left(t\right) = A_{n\phi}^\pm\cdot\sin(\omega_{\phi n}t) + B_{n\phi}^\pm\cdot\cos(\omega_{\phi n}t). \tag{2.12}$$

Taking into account the representation of solutions (2.1) with expansions (2.3) or (2.4), as well as descriptions of the initial and final states (1.8), (1.9) and surface influences (1.5), they have the form of an infinite system of four algebraic equations regarding the amplitudes of true harmonics of electroelastic vibrations waves and harmonics of surface control actions

$$\begin{cases} g_m(0)+\Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\left(\omega_{\theta m}^2-\omega_{\mu m}^2\right)\cdot\mu_{\pm m}(0)+\Gamma_{1\phi_\pm}\cdot\phi_{\pm m}(0)=\gamma_m \\ \dot{g}_m(0)+\Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\left(\omega_{\theta m}^2-\omega_{\mu m}^2\right)\cdot\dot{\mu}_{\pm m}(0)+\Gamma_{1\phi_\pm}\cdot\dot{\phi}_{\pm m}(0)=\delta_m \\ g_m(\omega_{\theta m}T_0)+\Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\left(\omega_{\theta m}^2-\omega_{\mu m}^2\right)\cdot\mu_{\pm m}(\omega_{\mu m}T_0)+\Gamma_{1\phi_\pm}\cdot\phi_{\pm m}(\omega_{\phi m}T_0)=\tilde{\gamma}_m \\ \dot{v}(y,T_0)+\Lambda_{1\mu_\pm}(1/\tilde{c}_{*t}^2)\cdot\left(\omega_{\theta m}^2-\omega_{\mu m}^2\right)\cdot\dot{\mu}_{\pm m}(\omega_{\mu m}T_0)+\Gamma_{1\phi_\pm}\cdot\dot{\phi}_{\pm m}(\omega_{\phi m}T_0)=\tilde{\delta}_m \end{cases} \tag{2.13}$$

The period $T>0$ during which surface influences $\mu_\pm(t)$ and $\phi_\pm(t)$ lead the wave process from the initial state to the final state is determined from the conditions for the existence of nontrivial solutions of systems of type (2.13)

$$T_0=\min_{m\in\mathbb{N}^+}\{T_{0m}>0\}. \tag{2.14}$$

3. The Exact controllability problem for three-component electroelastic wave.

Let us present the initial boundary value problem of controlling a three-component electroactive wave process in a piezoelectric waveguide layer in the following form

$$\begin{cases} \left[\Lambda_{1v}[*]-\rho[\partial^2/\partial t^2]\right]v(y,t)=f_w(y,t) & \text{in} \quad \bar{Q}_T=\left(-H_0<y<H_0\right)\times\left(0<t<T_0\right) \\ \left[\partial v(y,t)/\partial y\right]_{y=\pm H_0}=0; & \text{in} \quad \Sigma=\Gamma\times\left(0<t<T_0\right) \\ v(y,0)=\xi(y)-\left[W_+(y,0)+W_-(y,0)\right] & \\ \dot{v}(y,0)=\zeta(y)-\left[\dot{W}_+(y,0)+\dot{W}_-(y,0)\right] & \text{in} \quad \Omega=\left(-H_0<y<H_0\right) \end{cases} \tag{3.1}$$

Under adequate conditions of regularity and compatibility of the initial data $\{v(y,0),\ \dot{v}(y,0)\}$ and the reduced influence $f_w(y,t)$, system (3.1) admits a unique solution $v(y,t)$, in the energy functional space $\mathbb{C}\left([0,T];H_0^1(\Omega)\right)\cap\mathbb{C}^1\left([0,T];L^2(\Omega)\right)$.

Then, the wave energy for the process corresponding to the system of mathematical initial-boundary value problem (3.1) represented by the functional

$$E(t)=\frac{1}{2}\int_\Omega\left[\left|c_{44}^*\cdot[\partial^2 v(y,t)/\partial y^2]-c_{55}^*k^2v(y,t)\right|^2+\left|\dot{v}(y,t)\right|^2\right]\cdot dy \tag{3.2}$$

The task of precise controllability is to bring the wave state to equilibrium in a uniform time, regardless of the initial data, by external influence or control, which in the case under consideration is a reduced force $f_w(y,t)$.

More precisely, the problem of exact controllability will be formulated as: the existence of a time $T_0>0$ such that for each pair of initial data $\{v(y,0),\ \dot{v}(y,0)\}$ there is a control $f_w(y,t)$ such that the solution $v(y,t)$ of equation (3.1) satisfies the relations

$$v(y,T_0)=0, \qquad \dot{v}(y,T_0)=0. \tag{3.3}$$

This formulated as follows: *For arbitrary* $T_0>0$*, for each pair of input data* $\{v(y,0),\ \dot{v}(y,0)\}\in H_0^1(\Omega)\times L^2(\Omega)$ *there is a control* $f_w(y,t)\in\mathbb{C}\left([0.T_0];\ H^{-2}(\Omega)\right)$ *such* *that the solution of the mathematical initial-boundary value problem (3.1) satisfies relations (3.3).*

4. Optimal control for three-component electroelastic wave. Localization of wave energy along the thickness of the waveguide.

In the problem of propagation of a three-component electroelastic wave, the flow of electroelastic energy through the thickness $y \in [-H_0; H_0]$ of a piezoelectric waveguide determined by the expression

$$U_{em}^0 = \int_{-H_0}^{H_0} \begin{bmatrix} \sigma_{yz}(y,t)\cdot\varepsilon_{yz}(y,t)+\sigma_{zx}(y,t)\cdot\varepsilon_{zx}(y,t)+ \\ +E_x(y,t)\cdot D_x(y,t)+E_y(y,t)\cdot D_y(y,t) \end{bmatrix}\cdot dy \tag{3.4}$$

Solutions of the mathematical initial-boundary value problem (1.4)-(1.6), (1.10) and (1.12), taking into account relations (1.20)-(1.22), amplitude distributions (2.3) and (2.4), as well as solutions to the harmonic equation (2.9), represented in the form

$$\begin{Bmatrix} \mathrm{w}(y,t) \\ \varphi(y,t) \end{Bmatrix} = \sum_{n=0}^{n=\infty} \begin{pmatrix} \mathrm{V}_n(y)\cdot\theta_n(t) \\ \psi_n(y)\cdot\theta_n(t) \end{pmatrix} + \begin{Bmatrix} [(H_0+y)/2H_0]\cdot\mu_+(t)+[(H_0-y)/2H_0]\cdot\mu_-(t) \\ [(H_0+y)/2H_0]\cdot\phi_+(t)+[(H_0-y)/2H_0]\cdot\phi_-(t) \end{Bmatrix} \tag{3.5}$$

Considering the obtained solutions of the mathematical initial-boundary value problem, as well as the material relations of the piezoelectric medium, we obtain expressions for the energy integral depending on the surface influences: $U_{em}\left(\mathrm{w}(y,t),\ \mu_\pm(t),\ \phi_\pm(t)\right)$.

Localization of wave energy in a thin strip along the thickness of the waveguide is the problem of optimal control of wave energy along the selected thin strip.

This strip can be near surface $y \in [H_0-2\lambda; H_0] \cup [-H_0; -H_0+2\lambda]$.

$$\begin{aligned} U_{em}\left(\mathrm{w}(y,t),\mu_\pm(t),\phi_\pm(t)\right) = &\int_{-H_0}^{-H_0+2\lambda} \left(\sigma_{yz}\cdot\varepsilon_{yz}+\sigma_{zx}\cdot\varepsilon_{zx}+E_x\cdot D_x+E_y\cdot D_y\right)\cdot dy \\ &+\int_{H_0-2\lambda}^{H_0} \left(\sigma_{yz}\cdot\varepsilon_{yz}+\sigma_{zx}\cdot\varepsilon_{zx}+E_x\cdot D_x+E_y\cdot D_y\right)\cdot dy = 0.9\cdot U_{em}^0 \end{aligned} \tag{3.6}$$

The localized energy in the inner thin strip $y \le |2\lambda|$ along the thickness of the waveguide represented as the functional

$$U_{em}\left(\mathrm{w}(y,t),\mu_\pm(t),\phi_\pm(t)\right) = \int_{-2\lambda}^{2\lambda} \left(\sigma_{yz}\cdot\varepsilon_{yz}+\sigma_{zx}\cdot\varepsilon_{zx}+E_x\cdot D_x+E_y\cdot D_y\right)\cdot dy = 0.9\cdot U_{em}^0 \tag{3.7}$$

In relations (4.3) and (4.4), $\lambda \ll H_0$ is the wavelength.

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