ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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Features of Electroactive Shear Waves Controlling in a Cubic Symmetry Piezoelectric Waveguide with Mixed Electrical Surface Effects

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Keywords: electroacoustic wave, wave control, piezoelectric waveguide, electric field voltage, edge control, control function harmonics.

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Особенности управления электроактивной сдвиговой волной в пьезоэлектрическом волноводе кубической симметрии со смешанными электрическими поверхностными воздействами

Ключевые слова: электроакустическая волна, волновое управление, пьезоэлектрический волновод, напряжение электрического поля, фронтовое управление, гармоники функции управления.

Рассмотрена задача бесконтактного поверхностного управления распространением электроактивной поперечной волны в бесконечном волноводе из пьезоэлектрика кубической симметрии. Одна из поверхностей пьезоэлектрического волновода жестко заделана, а вторая свободна от механических напряжений. В качестве управляющих воздействий на поверхность волновода рассматриваются как перпендикулярное переменное электрическое смещение, приложенное к свободной поверхности волновода, так и параллельная переменная напряженность электрического поля вдоль его жестко заделанной поверхности.

Сформулирована неоднородная начально-краевая математическая задача со смешанными граничными условиями. В качестве решения краевой задачи электроупругости связанный с ним упругий сдвиг и сопутствующие колебания потенциала электрического поля представляются в виде рядов Фурье собственных мод электроупругих колебаний. Функции управления поверхностью представлены в виде рядов Фурье для соответствующих гармоник собственных форм электроупругих колебаний. Построены формы характеристик волнового поля в текущий момент времени. Проведены аналитические расчеты функций управления поверхностью в частном случае выбора начального и конечного состояний волнового процесса.

Ավետիսյան Լ.Վ.

Խորանարդային համաչափության պիեզոէլեկտրական ալիքատարում, էլեկտրական դաշտի խառը մակերևութային ազդեցություններով սահքի էլեկտրականորեն ակտիվ ալիքի կառավարման առանձնահատկությունները

Հիմնաբառեր էլեկտրաակուստիկ ալիք, ալիքի կառավարում, պիեզոէլեկտրական ալիքատար, էլեկտրական դաշտի լարում, մակերևութային կառավարում, կառավարման ֆունկցիայի հարմոնիկաներ

Դիտարկված է խորանարդային համաչափության պիեզոէլեկտրիկից պատրաստված ալիքատարում էլեկտրաակտիվ լայնակի ալիքի տարածման մակերևույթով անհպում կառավարման խնդիրը։ Պիեզոէլեկտրական ալիքատարի մակերևույթներից մեկը կոշտ ամրակցված է, իսկ երկրորդը զերծ է մեխանիկական լարումներից։ Ալիքատարի ազատ մակերևույթին կիրառվում է ուղղահայաց փոփոխական էլեկտրական դաշտ, իսկ դրա կոշտ ամրակցված մակերևույթի երկայնքով զուգահեռ փոփոխվող էլեկտրական դաշտ։

Ձևակերպված է անհամասեռ սկզբնական-եզրային մաթեմատիկական խնդիր՝ խառը եզրային պայմաններով: Որպես էլեկտրաառաձգականության սահմանային խնդրի լուծում, առաձգական սահքը և էլեկտրական դաշտի ուղեկցող տատանումները ներկայացված են էլեկտրաառաձգական տատանումների սեփական ալիքաձևերի Ֆուրիեի շարքի տեսքով: Մակերեւույթի ղեկավարման ազդեցությունները նույնպես ներկայացված են Ֆուրիեի շարքի տեսքով էլեկտրաառաձգական տատանումների սեփական ալիքաձևերի համապատասխան հարմոնիկաների միջոցով: Կառուցվել են ալիքային դաշտի բնութագրերի ձևերը ժամանակի ընթացիկ պահին: Ալիքային գործընթացի սկզբնական և վերջնական վիճակների ընտրության կոնկրետ դեպքում իրականացվել են մակերևույթի կառավարման ֆունկցիաների անալիտիկ հաշվարկներ:

The problem of contactless surface control of the propagation of an electroactive shear wave in an infinite waveguide made of a piezoelectric material of cubic symmetry is considered. One of the surfaces of the piezoelectric waveguide is rigidly embedded, and the second is free from mechanical stress. Both the perpendicular alternating electric displacement applied to the free surface of the waveguide and the parallel alternating electric field strength along its rigidly embedded surface are considered as control actions on the surface of the waveguide.

A non-homogeneous initial-boundary value mathematical problem with mixed boundary conditions is formulated. As a solution to the boundary value problem of electro elasticity, the associated elastic shear and accompanying oscillations of the electric field potential are represented in the form of Fourier series of the eigenmodes of electroelastic oscillations. The surface control functions are presented in the form of Fourier series for the corresponding harmonics of the natural modes of electroelastic vibrations. The forms of the characteristics of the wave field at the current moment in time have been constructed. Analytical calculations of surface control functions were carried out in the particular case of choosing the initial and final states of the wave process.

Introduction

The propagation of an electroacoustic three-component wave in a homogeneous two-dimensional waveguide in the case of an oscillating crystallographic plane of a piezoelectric and the distribution of wave characteristics over the thickness of the waveguide is a well-studied problem in the linear theory of electro elasticity [1], [2], [3]. The possible generation of an electroacoustic three-component wave, as well as the localization of wave energy of high-frequency (short) waves, are determined by the anisotropy of the piezoelectric material and the electromechanical conditions on the surfaces of the piezoelectric waveguide [4].

Moreover, different anisotropies of the piezoelectric material and different electromechanical conditions on the surfaces of a homogeneous waveguide lead, accordingly, to different localization along these surfaces [4], [5].

The controllability of electroactive acoustic waves and accompanying electric field oscillations using boundary effects expands the possibilities of studying the nature of the wave process in a piezoelectric medium [6,7]. The variety of conditions for coupling electromechanical fields on the surface of a piezoelectric allows us to formulate various mathematical initial-boundary value problems of electro acoustics. A new possibility of surface exposure appears-exposure to electromechanical fields without acoustic contact [7].

The presented work examines the problem of contactless surface control of the propagation of an electroactive SH shear wave in an infinite piezoelectric waveguide made of an anisotropic piezoelectric material classes ***43m*** or ***23*** of cubic symmetry is considered. In contrast to the problems of controlling the propagation of shear waves in a piezoelectric waveguide of the ***6mm*** class with hexagonal symmetry, here the boundary value problem is formulated from a system of non-separable homogeneous quasi-static electro elasticity equations with mixed electromechanical surface conditions [8].

1. Problem modeling and formulation of a mathematical boundary value problem

Let us consider the propagation of the signal $F(x,y,t)=f(y)\cdot\exp[i(kx-\theta(t))]$ of a normal, three-component electroelastic wave $\{\mathrm{w}(x,y,t);\ \mathrm{e}_x(x,y,t);\ \mathrm{e}_y(x,y,t)\}$, in which $\mathrm{e}_x(x,y,t)=-(\partial\varphi(x,y,t)/\partial x)$ and $\mathrm{e}_y(x,y,t)=-(\partial\varphi(x,y,t)/\partial y)$, induced in a homogeneous piezoelectric waveguide $\Omega(x,y)=\{|x|<\infty,\ y\in[0;h],\ |z|<\infty\}$ associated with an orthogonal coordinate system $0xyz$ (Figure 1).

The case of a homogeneous anisotropic piezoelectric of classes ***43m*** or ***23*** cubic symmetry is considered as a waveguide material.

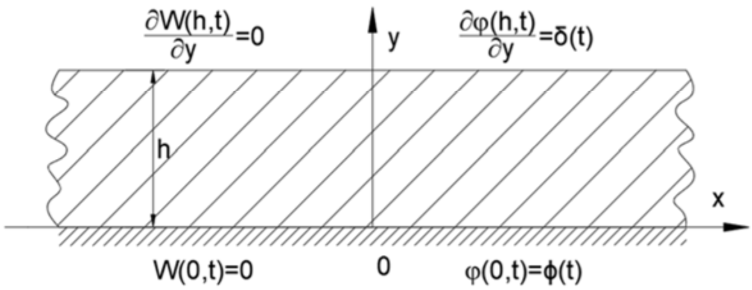

Fig. 1. Scheme of Homogeneous piezoelectric waveguide with the mixed electromechanical surface actions.

In case of cubic symmetry piezoelectric classes ***43m*** or ***23,*** the system of quasi-static equations for a three-component electroelastic wave has the form

$$\begin{aligned}&\mathrm{w}''(y)\cdot\theta(t)-k^2\cdot\mathrm{w}(y)\cdot\theta(t)+2i(e_{14}/c_{44})k\cdot\varphi'(y)\cdot\theta(t)=C_t^{-2}\cdot\mathrm{w}(y)\cdot\ddot{\theta}(t)\\&-2i(e_{14}/\varepsilon_{11})k\cdot\mathrm{w}'(y)+\varphi''(y)-k^2\cdot\varphi(y)=0\end{aligned}\qquad(1.1)$$

In equations (1.5), $C_t=\sqrt{c_{44}/\rho}$ is the velocity of the volumetric non-electroactive elastic shear wave, c_{44} is the shear rigidity, e_{14} is the piezoelectric modulus, ε_{11} is the relative dielectric constant and ρ is the density of the cubic symmetry, piezoelectric classes ***43m*** or ***23*** material.

On a mechanically rigidly fixed and electrically conducting surface $y=0$ of the waveguide, electromechanical conditions are written in the form

$$\mathrm{w}(y,t)\big|_{y=0}=0,\qquad\qquad \varphi(y,t)\big|_{y=0}=\phi(t).\qquad(1.2)$$

On a mechanically free waveguide surface $y=h$, loaded with a perpendicular electrical displacement, the electromechanical conditions will be written as

$$\left[c_{44}\frac{\partial w(y,t)}{\partial y}+ie_{14}k\cdot\varphi(y,t)\right]_{y=h}=0$$
$$\left[ie_{14}k\cdot w(y,t)-\varepsilon_{11}\frac{\partial\varphi(y,t)}{\partial y}\right]_{y=h}=\delta(t) \quad (1.3)$$

It is known that in the quasi-static formulation of the problem of electro elasticity, the dynamics of changes in oscillations of the accompanying electric field is uniquely determined by the dynamics of elastic waves [2], [9]. Therefore, to describe the initial and final states of the control process, pairs are selected from the conditions:
The initial state of the electroacoustic wave at the moment of time $t=0$ is written in the form of one of the pairs of conditions

$$w(y,0)=\xi(y),\quad \dot{w}(y,0)=\zeta(y), \quad (1.4)$$

$$w(y,0)=\xi(y),\quad \dot{\varphi}(y,0)=\vartheta(y), \quad (1.5)$$

$$\varphi(y,0)=\eta(y),\quad \dot{w}(y,0)=\zeta(y), \quad (1.6)$$

$$\varphi(y,0)=\eta(y),\quad \dot{\varphi}(y,0)=\vartheta(y). \quad (1.7)$$

The final state of the electroacoustic wave at the moment of time $t=T_0$ is written in the form of one of the pairs of conditions

$$w(y,T_0)=\tilde{\xi}(y),\qquad \dot{w}(y,T_0)=\tilde{\zeta}(y), \quad (1.8)$$

$$w(y,T_0)=\tilde{\xi}(y),\qquad \dot{\varphi}(y,T_0)=\tilde{\vartheta}(y), \quad (1.9)$$

$$\varphi(y,T_0)=\tilde{\eta}(y),\qquad \dot{w}(y,T_0)=\tilde{\zeta}(y), \quad (1.10)$$

$$\varphi(y,T_0)=\tilde{\eta}(y),\qquad \dot{\varphi}(y,T_0)=\tilde{\vartheta}(y). \quad (1.11)$$

The mathematical initial-boundary value problem of controlling the propagation of an electroelastic transverse wave under the influence of surface influences of an electric field is formed by a system of equations (1.1), boundary conditions (1.2), (1.3), as well as pairs of initial state conditions from (1.4)-(1.7) and pairs of conditions final state from (1.8-(1.11).
It determines the control by the surface effects of the electric field on both the distribution of wave characteristics over the thickness of the waveguide and on the nature of the propagation of the electroactive transverse wave. It is determined by what surface influences the wave process can be brought from the initial state at the moment of influence $t=0$ to the final state over a period of time $[0;T_0]$.

2. Solutions of the problem.

2.1 In a homogeneous piezoelectric waveguide made from a piezoelectric of cubic symmetry of classes ***43m*** or ***23***, an electroacoustic three-component wave is represented as a solution to a system of homogeneous equations (1.1), with homogeneous surface conditions

$$\mathrm{w}(y)\big|_{y=0}=0, \qquad \varphi(y)\big|_{y=0}=0. \tag{2.1}$$

on a mechanically rigidly fixed and electrically conducting surface $y=0$ of the waveguide, and

$$\left[\mathrm{c}_{44}\left[\partial\mathrm{w}(y)/\partial y\right]+ie_{14}k\cdot\varphi(y)\right]_{y=h}=0, \quad \left[ie_{14}k\cdot\mathrm{w}(y)-\varepsilon_{11}\left[\partial\varphi(y)/\partial y\right]\right]_{y=h}=0. \tag{2.2}$$

On a mechanically free waveguide surface $y=h$.

The components of a high-frequency electroacoustic wave (in the short-wave approximation, when $2\pi/k=\lambda\ll h$) are written in the form

$$\begin{aligned}\mathrm{w}(x,y,t)&=A_w\exp(kq_1y)\cdot\exp[i(kx-\omega t)+\\&+[2(e_{14}/c_{44})q_2/(q_2^2-\alpha_t^2)]\cdot A_\varphi\exp(kq_2y)\cdot\exp[i(kx-\pi/2-\omega t)\end{aligned} \tag{2.3}$$

$$\begin{aligned}\varphi(x,y,t)&=-A_\varphi\exp(kq_2y)\cdot\exp[i(kx-\omega t)+\\&+[2(e_{14}/\varepsilon_{11})q_1/(q_1^2-1)]\cdot A_w\exp(kq_1y)\cdot\exp[i(kx-\pi/2-\omega t)\end{aligned}. \tag{2.4}$$

In these decisions

$$q_1(\omega)=\left[[1+\alpha_t^2(k,\omega)+4\chi^2]/2-\sqrt{[1+\alpha_t^2(k,\omega)+4\chi^2]^2/4-\alpha_t^2(k,\omega)}\right]^{1/2} \tag{2.5}$$

$$q_2(\omega)=\left[[1+\alpha_t^2(k,\omega)+4\chi^2]/2+\sqrt{[1+\alpha_t^2(k,\omega)+4\chi^2]^2/4-\alpha_t^2(k,\omega)}\right]^{1/2} \tag{2.6}$$

are the wave coefficients, in which $\alpha_t^2(k,\omega)=1-(\omega^2\rho/k^2c_{44})$.

From the eigen solutions (2.3) and (2.4) of the boundary value problem (1.1), (2.1) and (2.2) it is obvious that it consists of two groups of linearly independent normal signals.

Propagating with the phase of motion $\vartheta_1(x,t)=(kx-\omega t)$, the first pair of associated characteristics of an electroelastic wave, is represented as

$$\begin{aligned}\mathrm{w}_1(x,y,t)&=A_{1\mathrm{w}}\exp[kq_1(\omega)y]\cdot\exp[i(kx-\omega t)]\\\varphi_1(x,y,t)&=[2i(e_{14}/\varepsilon_{11})q_1(\omega)/(q_1^2(\omega)-1)]\cdot A_{1w}\exp[kq_1(\omega)y]\cdot\exp[i(kx-\omega t)\end{aligned} \tag{2.7}$$

Propagating with the phase of motion $\vartheta_2(x,t)=(kx-\pi/2-\omega t)$, the second pair of associated characteristics of an electroelastic wave, is represented as

$$\begin{aligned}\mathrm{w}_2(x,y,t)&=[2i(e_{14}/c_{44})q_2(\omega)/(q_2^2(\omega)-\alpha_t^2)]A_{2\varphi}\exp[kq_2(\omega)y]\exp[i(kx-\omega t)]\\\varphi_2(x,y,t)&=-A_{2\varphi}\exp[kq_2(\omega)y]\cdot\exp[i(kx-\omega t)\end{aligned}. \tag{2.8}$$

2.2 To solve the initial-boundary value control problem formed by the system of homogeneous electro elasticity equations (1.1), inhomogeneous boundary conditions (1.2), (1.3), as well as the initial state conditions from (1.4)-(1.7) and the final state conditions from (1.8 -(1.11), we reduce the problem to an initial boundary value problem with homogeneous boundary conditions.

Taking advantage of the fact that the surface conditions for elastic shear are already homogeneous, to reduce the boundary value problem we introduce only a new characteristic function of the electric field

$$\psi(y,t)=\varphi(y,t)-\frac{(y-h)^2}{h^2}\phi(t)-\frac{y^2}{2h}\delta(t)\,, \tag{2.9}$$

The system of homogeneous equations (1.1) is reduced to an inhomogeneous system of equations, in the right-hand sides of which surface influences $\phi(t)$ and $\delta(t)$ are appearing

$$\begin{aligned}&\left[\mathrm{w}''(y)-k^2\cdot\mathrm{w}(y)+2i(e_{14}/c_{44})\,k\cdot\psi'(y)\right]\cdot\theta(t)-\mathrm{w}(y)\cdot C_t^{-2}\cdot\ddot{\theta}(t)=\\&=-2i(e_{14}/c_{44})\,k\cdot\left[\left[2(y-h)/h^2\right]\cdot\phi(t)+(y/h)\cdot\delta(t)\right]\end{aligned} \tag{2.10}$$

$$\begin{aligned}&\left[-2i(e_{14}/\varepsilon_{11})\,k\cdot\mathrm{w}'(y)+\psi''(y)-k^2\cdot\psi(y)\right]\cdot\theta(t)=\\&=\left[[k^2(y-h)^2-2]/h^2\right]\cdot\phi(t)+\left[[k^2y^2-2]/2h\right]\cdot\delta(t)\end{aligned} \tag{2.11}$$

Solutions of the system of inhomogeneous equations (2.10) and (2.11) with homogeneous surface conditions (2.1) and (2.2) with respect to the newly introduced characteristics of the electromechanical field can be represented in the form of Fourier series with eigenmodes and their corresponding eigen harmonics. Since this homogeneous boundary, value problem has two eigenfunctions $\exp[k_n q_{1n}(\omega)y]$ and $\exp\exp[k_n q_{1n}(\omega)y]$, we write the required characteristics as the sum of two terms

$$\begin{aligned}&\mathrm{w}(y,t)=\mathrm{w}_1(y,t)+\mathrm{w}_2(y,t)=\\&=\sum_{n=0}^{\infty}A_{1\mathrm{w}}\exp[k_n q_{1n}(\omega)y]\cdot\theta_1(t)+\\&+\sum_{n=0}^{\infty}[2i(e_{14}/c_{44})\,q_{2n}/(q_{2n}^2-\alpha_t^2)]\cdot A_{2\varphi n}[k_n q_{2n}(\omega)y]\cdot\theta_2(t)\end{aligned} \tag{2.12}$$

$$\begin{aligned}&\psi(y,t)=\psi_1(y,t)+\psi_2(y,t)=\\&=\sum_{n=0}^{\infty}[2i(e_{14}/\varepsilon_{11})\,q_1/(q_1^2-1)]\cdot A_{1w}\exp[k_n q_{1n}(\omega)y]\cdot\theta_1(t)-,\\&-\sum_{n=0}^{\infty}A_{2\varphi}\exp[k_n q_{2n}(\omega)y]\cdot\theta_2(t)\end{aligned} \tag{2.13}$$

where the wave coefficients $q_1(\omega)$ and $q_2(\omega)$ are defined as in (2.5) and (2.6), in which $\alpha_{tn}^2(k_n,\omega)=1-(\omega_n^2/C_t^2k_n^2)=(n\pi/kh)^2$, $n\in\mathbb{N}$.

In the presented relations for the desired characteristics (2.12) and (2.13), the functions $\theta_1(t)$ and $\theta_2(t)$ are their own harmonics, corresponding to the natural vibration modes $\exp[k_n q_{1n}(\omega)y]$ and $\exp[k_n q_{2n}(\omega)y]$

$$\theta_1(t)=\sum_{n=0}^{\infty}\theta_{1n}(t)=\sum_{n=0}^{\infty}[A_{1\mathrm{w}n}\sin(\omega_{1n}t)+B_{1\mathrm{w}n}\cos(\omega_{1n}t)]\,, \tag{2.14}$$

$$\theta_2(t)=\sum_{n=0}^{\infty}\theta_{2n}(t)=\sum_{n=0}^{\infty}[A_{2\varphi n}\sin(\omega_{2n}t)+B_{2\varphi n}\cos(\omega_{2n}t)] \quad (2.15)$$

For true harmonics of electroelastic vibrations of a piezoelectric medium, we obtain an equation characterizing the sum of its own harmonics and harmonics of the surface action. In accordance with the presented relations for the desired characteristics (2.12), (2.13) and harmonic vibration functions (2.14), we obtain the vibration equation for the first eigen branch of the electroelastic wave in the form

$$\ddot{\theta}_{1n}(t)+C_t^2k_n^2\cdot\left[[4\chi^2q_{1n}^2-(q_{1n}^2-1)^2]/(q_{1n}^2-1)\right]\cdot\theta_{1n}(t)=$$
$$=-\left\{M_{11\phi n}+N_{11\delta n}\cdot\left[M_{12\phi n}/N_{12\delta n}\right]\right\}\cdot\phi_n(t) \quad (2.16)$$

with the Fourier coefficients of the surface actions expansions

$$M_{11\phi n}=\frac{1}{h}\int_0^h -2i(e_{14}/c_{44})k\cdot\left[2(y-h)/h^2\right]\exp(kq_1y)dy,$$

$$N_{11\delta n}=\frac{1}{h}\int_0^h -2i(e_{14}/c_{44})k\cdot(y/h)\exp(kq_1y)dy,$$

$$M_{12\phi n}=\frac{1}{h}\int_0^h\left[[2-k^2(y-h)^2]/h^2\right]\cdot\exp(kq_1y)dy,$$

$$N_{12\delta n}=\frac{1}{h}\int_0^h\left[[k^2y^2-2]/2h\right]\cdot\exp(kq_1y)dy. \quad (2.17)$$

The vibration equation for the second eigen branch of the electroelastic wave in the form

$$\ddot{\theta}_{2n}(t)-C_t^2(\alpha_t^2-1)k_n^2\cdot\theta_{2n}(t)=-\left[M_{21\phi n}\cdot(N_{22\delta n}/M_{22\phi n})+N_{21\delta n}\right]\cdot\delta_n(t) \quad (2.18)$$

$$M_{21\phi n}=\frac{1}{h}\int_0^h C_t^2k_n[(q_{2n}^2-\alpha_t^2)/q_{2n}]\cdot\left[2(y-h)/h^2\right]\exp(kq_{2n}y)dy,$$

$$N_{21\delta n}=\frac{1}{h}\int_0^h C_t^2k_n[(q_{2n}^2-\alpha_t^2)/q_{2n}]\cdot(y/h)\exp(kq_{2n}y)dy,$$

$$M_{22\phi n}=\frac{1}{h}\int_0^h\left[[2-k^2(y-h)^2]/h^2\right]\cdot\exp(kq_{2n}y)dy,$$

$$N_{22\delta n}=\frac{1}{h}\int_0^h\left[[k^2y^2-2]/2h\right]\cdot\exp(kq_{2n}y)dy. \quad (2.19)$$

In the presented equations of oscillations of electroelastic wave characteristics (2.15) and (2.16) there are harmonics $\phi_n(t)$ and $\delta_n(t)$ surface influences corresponding to the eigenmodes $\exp[k_nq_{1n}(\omega)y]$ and $\exp[k_nq_{2n}(\omega)y]$

$$\phi(t)=\sum_{n=0}^{\infty}\phi_n(t)=\sum_{n=0}^{\infty}[A_{\phi n}\sin(\omega_{\phi n}t)+B_{\phi n}\cos(\omega_{\phi n}t)], \tag{2.20}$$

$$\delta(t)=\sum_{n=0}^{\infty}\delta_n(t)=\sum_{n=0}^{\infty}[A_{\delta n}\sin(\omega_{\delta n}t)+B_{\delta n}\cos(\omega_{\delta n}t)] \tag{2.21}$$

For the true harmonics of oscillations $f_{\phi n}(t)=\theta_{1n}(t)-(\omega_{1\phi n}^2/\omega_{1\theta n}^2)\cdot\phi_n(t)$, of the first eigen branch of the electroelastic wave (2.7), an equation is obtained that characterizes the sum of eigen harmonics $\theta_{1n}(t)$ and harmonics of surface action $\phi_n(t)$

$$\ddot{f}_{\phi n}(t)+\omega_{1\theta n}^2\cdot f_{\phi n}(t)=-(\omega_{\phi n}^2/\omega_{1\theta n}^2)\cdot\ddot{\phi}_n(t), \tag{2.22}$$

with the corresponding frequencies $\omega_{1\theta n}^2=C_t^2k_n^2\cdot\left[[4\chi^2q_{1n}^2-(q_{1n}^2-1)^2]/(q_{1n}^2-1)\right]$ and $\omega_{\phi n}^2=M_{11\phi n}+M_{12\phi n}\cdot\left[N_{11\delta n}/N_{12\delta n}\right]$.

In the characteristic equation (2.22), we use the Fourier expansions of the eigen harmonics (2.14) and harmonics of surface effects (2.20).

For the true harmonics of the medium oscillations the solution of equation (2.22) is obtained in the following form

$$\begin{aligned}f_{\phi n}(t)=A_{1wn}\cdot\cos(\omega_{1\theta n}t)+B_{1wn}\cdot\sin(\omega_{1\theta m}t)-\\-(\omega_{\phi n}^2/\omega_{1\theta n}^2)\cdot\left[A_{\phi n}\cos(\omega_{\phi n}t)+B_{\phi n}\sin(\omega_{\phi n}t)\right].\end{aligned} \tag{2.23}$$

From the solution (2.23) for the true harmonics of vibrations of the first pair of coupled characteristics of an electroelastic wave, it follows that the surface action $\phi_n(t)$ changes only the harmonic of the first natural vibration mode $\exp[k_nq_{1n}(\omega)y]$.

This impact is determined through the expansion coefficients of the surface impact (2.17) and the frequency of impacts created by them $\omega_{\phi n}^2=M_{11\phi n}+M_{12\phi n}\cdot\left[N_{11\phi n}/N_{12\phi n}\right]$.

For the true harmonics of oscillations $f_{\delta n}(t)=\theta_{2n}(t)-(\omega_{\delta n}^2/\omega_{2\theta n}^2)\cdot\delta_n(t)$, of the second eigen branch of the electroelastic wave (2.8), an equation is obtained that characterizes the sum of eigen harmonics $\theta_{2n}(t)$ and harmonics of surface action $\delta_n(t)$

$$\ddot{f}_{\delta n}(t)-\omega_{2\theta n}^2\cdot f_{\delta n}(t)=-(\omega_{\delta n}^2/\omega_{2\theta n}^2)\cdot\ddot{\delta}_n(t), \tag{2.24}$$

with the corresponding frequencies $\omega_{\delta n}^2=M_{21\phi n}+M_{22\phi n}\cdot(N_{21\delta n}/N_{22\delta n})$ and $\omega_{2\theta n}^2=(\alpha_t^2-1)\cdot C_t^2k_n^2$.

In the characteristic equation (2.24), we use the Fourier expansions of the eigen harmonics (2.15) and harmonics of surface effects (2.21).

For the true harmonics of the medium oscillations the solution of equation (2.24) is obtained in the following form

$$f_{\delta n}(t) = A_{2\varphi n} \cdot \cos(\omega_{2\theta n} t) + B_{2\varphi n} \cdot \sin(\omega_{2\theta m} t) - \\ -(\omega_{\delta n}^2 / \omega_{2\theta n}^2) \cdot \left[A_{\delta n} \cos(\omega_{\delta n} t) + B_{\delta n} \sin(\omega_{\delta n} t) \right]. \tag{2.25}$$

From the solution (2.25) for the true harmonics of vibrations of the first pair of coupled characteristics of an electroelastic wave, it follows that the surface action $\delta_n(t)$ changes only the harmonic of the first natural vibration mode $\exp[k_n q_{2n}(\omega) y]$.

This impact is determined through the expansion coefficients of the surface impact (2.17) and the frequency of impacts created by them $\omega_{\delta n}^2 = M_{21\phi n} + M_{22\phi n} \cdot \left[N_{21\delta n} / N_{22\delta n} \right]$.

Both in the above expansions and in the resulting solutions (2.20), (2.21) there are coefficients $A_{1\text{w}n}$; $B_{1\text{w}n}$; $A_{2\varphi n}$; $B_{2\varphi n}$ and $A_{\phi n}$; $B_{\phi n}$; $A_{\delta n}$; $B_{\delta n}$ undetermined constants. In the obtained solutions, they separated in pairs according to the wave's eigenmodes

$$\text{w}(y,t) = \text{w}_1(y,t) + \text{w}_2(y,t) = \\ = \sum_{n=0}^{\infty} \exp[k_n q_{1n}(\omega) y] \cdot \begin{bmatrix} A_{1\text{w}n} \cdot \cos(\omega_{1\theta n} t) + B_{1\text{w}n} \cdot \sin(\omega_{1\theta m} t) - \\ -(\omega_{\phi n}^2 / \omega_{1\theta n}^2) \cdot \left[A_{\phi n} \cos(\omega_{\phi n} t) + B_{\phi n} \sin(\omega_{\phi n} t) \right] \end{bmatrix} + \\ + \sum_{n=0}^{\infty} \beta_{\text{w}n}(\omega) \cdot \exp[k_n q_{2n}(\omega) y] \cdot \begin{bmatrix} A_{2\varphi n} \cdot \cos(\omega_{2\theta n} t) + B_{2\varphi n} \cdot \sin(\omega_{2\theta m} t) - \\ -(\omega_{\delta n}^2 / \omega_{2\theta n}^2) \cdot \left[A_{\delta n} \cos(\omega_{\delta n} t) + B_{\delta n} \sin(\omega_{\delta n} t) \right] \end{bmatrix} \tag{2.26}$$

$$\psi(y,t) = \psi_1(y,t) + \psi_2(y,t) = \\ = \sum_{n=0}^{\infty} \beta_{\varphi n}(\omega) \cdot \exp[k_n q_{1n}(\omega) y] \cdot \begin{bmatrix} A_{1\text{w}n} \cdot \cos(\omega_{1\theta n} t) + B_{1\text{w}n} \cdot \sin(\omega_{1\theta m} t) - \\ -(\omega_{\phi n}^2 / \omega_{1\theta n}^2) \cdot \left[A_{\phi n} \cos(\omega_{\phi n} t) + B_{\phi n} \sin(\omega_{\phi n} t) \right] \end{bmatrix}, \\ - \sum_{n=0}^{\infty} \exp[k_n q_{2n}(\omega) y] \cdot \begin{bmatrix} A_{2\varphi n} \cdot \cos(\omega_{2\theta n} t) + B_{2\varphi n} \cdot \sin(\omega_{2\theta m} t) - \\ -(\omega_{\delta n}^2 / \omega_{2\theta n}^2) \cdot \left[A_{\delta n} \cos(\omega_{\delta n} t) + B_{\delta n} \sin(\omega_{\delta n} t) \right] \end{bmatrix} \tag{2.27}$$

In the solution representations (2.28) and (2.27), as well as in (2.12) and (2.13), $\beta_{\text{w}n}(\omega) = [2i(e_{14}/c_{44}) q_{2n}(\omega) / (q_{2n}^2(\omega) - \alpha_t^2(\omega))]$ and $\beta_{\varphi n}(\omega) = [2i(e_{14}/\varepsilon_{11}) q_{1n}(\omega) / (q_{1n}^2(\omega) - 1)]$ are the amplitude coefficients of the corresponding accompanying characteristics of the wave field.

The uncertain coefficients $\{A_{1\text{w}n};\ B_{1\text{w}n}\}$, with $\{A_{\phi n};\ B_{\phi n}\}$ and $\{A_{\delta n};\ B_{\delta n}\}$, with $\{A_{2\varphi n};\ B_{2\varphi n}\}$ are determined from the data defining the initial and final states in the waveguide.

Taking into account the structure of solutions (2.12) and (2.13), the initial and final states of the wave process, are also represented in the form of the corresponding Fourier series

$$\xi_0(y)=\sum_{n=1}^{\infty}\left[\gamma_{\phi n}\cdot\exp[k_n q_{1n}(\omega)y]+\gamma_{\delta n}\cdot\exp[k_n q_{2n}(\omega)y]\right]$$
$$\tilde{\xi}_0(y)=\sum_{n=1}^{\infty}\left[\tilde{\gamma}_{\phi n}\cdot\exp[k_n q_{1n}(\omega)y]+\tilde{\gamma}_{\delta n}\cdot\exp[k_n q_{2n}(\omega)y]\right], \tag{2.28}$$

$$\zeta_T(y)=\sum_{n=1}^{\infty}\left[\eta_{\phi n}\cdot\exp[k_n q_{1n}(\omega)y]+\eta_{\delta n}\cdot\exp[k_n q_{2n}(\omega)y]\right]$$
$$\tilde{\zeta}_T(y)=\sum_{n=1}^{\infty}\left[\tilde{\eta}_{\phi n}\cdot\exp[k_n q_{1n}(\omega)y]+\tilde{\eta}_{\delta n}\cdot\exp[k_n q_{2n}(\omega)y]\right], \tag{2.29}$$

Comparing the representations of the initial and final states (2.28) and (2.29) with solutions (2.26) and (2.27), in the form of two independent systems of linear algebraic equations for uncertain amplitudes we obtain the matching conditions at the initial and final states of the control process

$$\begin{aligned}
&A_{1\mathrm{w}n}-(\omega_{\phi n}^2/\omega_{1\theta n}^2)\cdot A_{\phi n}=\gamma_{\phi n}\\
&[(\omega_{1\theta n})/(\omega_{\phi n})]\cdot B_{1\mathrm{w}n}-(\omega_{\phi n}^2/\omega_{1\theta n}^2)\cdot B_{\phi n}=\tilde{\gamma}_{\phi n}/\omega_{\phi n}\\
&A_{1\mathrm{w}n}\cdot\cos(\omega_{1\theta n}T_0)+B_{1\mathrm{w}n}\cdot\sin(\omega_{1\theta m}T_0)-\\
&-(\omega_{\phi n}^2/\omega_{1\theta n}^2)\cdot\left[A_{\phi n}\cos(\omega_{\phi n}T_0)+B_{\phi n}\sin(\omega_{\phi n}T_0)\right]=\eta_{\phi n}\\
&[(\omega_{1\theta n})/(\omega_{\phi n})]\left[-A_{1\mathrm{w}n}\cdot\sin(\omega_{1\theta n}T_0)+B_{1\mathrm{w}n}\cdot\cos(\omega_{1\theta n}T_0)\right]-\\
&-(\omega_{\phi n}^2/\omega_{1\theta n}^2)\cdot\left[-A_{\phi n}\sin(\omega_{\phi n}T_0)+B_{\phi n}\cos(\omega_{\phi n}T_0)\right]=-\tilde{\eta}_{\phi n}/(\omega_{\phi n})
\end{aligned} \tag{2.30}$$

$$\begin{aligned}
&A_{2\varphi n}-(\omega_{\delta n}^2/\omega_{2\theta n}^2)\cdot A_{\delta n}=\gamma_{\delta n}\\
&[(\omega_{2\theta n})/(\omega_{\delta n})]\cdot B_{2\varphi n}-(\omega_{\delta n}^2/\omega_{2\theta n}^2)\cdot B_{\delta n}=\tilde{\gamma}_{\delta n}/\omega_{\delta n}\\
&A_{2\varphi n}\cdot\cos(\omega_{2\theta n}T_0)+B_{2\varphi n}\cdot\sin(\omega_{2\theta m}T_0)-\\
&-(\omega_{\delta n}^2/\omega_{2\theta n}^2)\cdot\left[A_{\delta n}\cos(\omega_{\delta n}T_0)+B_{\delta n}\sin(\omega_{\delta n}T_0)\right]=\eta_{\delta n}\\
&[(\omega_{2\theta n})/(\omega_{\delta n})]\left[-A_{2\varphi n}\cdot\sin(\omega_{2\theta n}T_0)+B_{2\kappa n}\cdot\cos(\omega_{2\theta n}T_0)\right]-\\
&-(\omega_{\delta n}^2/\omega_{2\theta n}^2)\cdot\left[-A_{\delta n}\sin(\omega_{\delta n}T_0)+B_{\delta n}\cos(\omega_{\delta n}T_0)\right]=-\tilde{\eta}_{\delta n}/(\omega_{\delta n})
\end{aligned} \tag{2.31}$$

From the conditions for the existence of nontrivial solutions of systems (2.30) and (2.31), in the form of two multitudes of discrete times $\mathfrak{T}_{\phi n}=\{T_{0\phi n}\}$ and $\mathfrak{T}_{\delta m}=\{T_{0\delta m}\}$ of surface influences $\phi_n(t)$ and $\delta_m(t)$, are determined, respectively.

The control time T_* of the wave process, during which the combined action of surface influences leads the process from the initial state (2.28) to the final state (2.29), defined as

$$T_*=\min_{n,m\in\mathbb{N}}\left[T_0\in\{T_{0\phi n}\}\cap\{T_{0\delta m}\}\right]. \tag{2.32}$$

For certain amplitude coefficients from (2.30) and (2.31), the wave characteristics of the current process will be determined according to relations (2.26) and (2.27)

$$\begin{aligned} \mathrm{w}(x,y,t) = \sum_{n=0}^{\infty} & \left\{ \begin{aligned} & \exp[k_n(q_{1n}(\omega)y + ix)] \times \\ & \times \left[\begin{aligned} & A_{1\mathrm{w}n} \cdot \cos(\omega_{1\theta n} t) + B_{1\mathrm{w}n} \cdot \sin(\omega_{1\theta m} t) - \\ & -(\omega_{\phi n}^2 / \omega_{1\theta n}^2) \cdot \left[A_{\phi n} \cos(\omega_{\phi n} t) + B_{\phi n} \sin(\omega_{\phi n} t) \right] \end{aligned} \right] \end{aligned} \right\} + \\ + \sum_{n=0}^{\infty} & \left\{ \begin{aligned} & \beta_{\mathrm{w}n}(\omega) \cdot \exp[k_n(q_{2n}(\omega)y + ix + i\pi/2k_n)] \times \\ & \times \left[\begin{aligned} & A_{2\varphi n} \cdot \cos(\omega_{2\theta n} t) + B_{2\varphi n} \cdot \sin(\omega_{2\theta m} t) - \\ & -(\omega_{\delta n}^2 / \omega_{2\theta n}^2) \cdot \left[A_{\delta n} \cos(\omega_{\delta n} t) + B_{\delta n} \sin(\omega_{\delta n} t) \right] \end{aligned} \right] \end{aligned} \right\} \end{aligned}, \qquad (2.33)$$

$$\begin{aligned} \psi(x,y,t) = \sum_{n=0}^{\infty} & \left\{ \begin{aligned} & \beta_{\varphi n}(\omega) \cdot \exp[k_n(q_{1n}(\omega)y + ix - i\pi/2k_n)] \times \\ & \times \left[\begin{aligned} & A_{1\mathrm{w}n} \cdot \cos(\omega_{1\theta n} t) + B_{1\mathrm{w}n} \cdot \sin(\omega_{1\theta m} t) - \\ & -(\omega_{\phi n}^2 / \omega_{1\theta n}^2) \cdot \left[A_{\phi n} \cos(\omega_{\phi n} t) + B_{\phi n} \sin(\omega_{\phi n} t) \right] \end{aligned} \right] \end{aligned} \right\} \\ - \sum_{n=0}^{\infty} & \left\{ \begin{aligned} & \exp[k_n(q_{2n}(\omega)y + ix)] \times \\ & \times \left[\begin{aligned} & A_{2\varphi n} \cdot \cos(\omega_{2\theta n} t) + B_{2\varphi n} \cdot \sin(\omega_{2\theta m} t) - \\ & -(\omega_{\delta n}^2 / \omega_{2\theta n}^2) \cdot \left[A_{\delta n} \cos(\omega_{\delta n} t) + B_{\delta n} \sin(\omega_{\delta n} t) \right] \end{aligned} \right] \end{aligned} \right\} \end{aligned}, \qquad (2.34)$$

Conclusion

In the process of surface non-contact control of the propagation of an electroactive elastic shear wave in an infinite piezoelectric waveguide of cubic symmetry, the equations and mixed surface conditions in the mathematical boundary value problem are not separated. Electromechanical interaction occurs in low frequency mode (acoustic frequencies). The initial and final states are specified by two pairs of conditions for the characteristics of the electroelastic shear wave. The other two pairs of initial and final state conditions describing accompanying oscillations are derived from the basic equations.

The initial boundary value problem is transformed into an infinite system of control problems for the formation and propagation of eigenforms of electroactive transverse waves by expanding the desired characteristics and functions of surface action into Fourier series.

In the case of the short-wave approximation, by solving an infinite system of differential equations, we obtain surface action functions and true harmonics of wave oscillations, which correspond to the wave surface that brings the process from a known initial state to a given final state.

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