ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 76, №3, 2023 Механика
УДК 539.3; 534.2 **DOI: 10.54503/0002-3051-2023.76.3-35**

ON THE DYNAMIC BEHAVIOUR OF A THREE-LAYERED STRIP IN A NON-CLASSICAL MIXED PLANE DEFORMATION PROBLEM

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Keywords: Three-layered package, asymptotic method, plane deformation, seismology, prediction of earthquakes.

The non-classical dynamic problem of plane deformation of a three-layered isotropic package which is modelling the behaviour of Lithospheric plates and blocks of the Earth's crust is considered. The upper surface of the package is free, and the conditions of full contact are set between the layers. It is assumed that the measurement data were taken from the points of the contact surface between the first and second layers of the package, as data from inclinometers or strainmeters placed between the layers. The solution of the corresponding dynamic equations and correlations of the problem of plane deformation in the theory of elasticity was obtained by the asymptotic method. A numerical analysis has been carried out for a three-layered package by simulating a block of the Earth's crust in Armenia. Monitoring of changes in the stress-strain state of the package according to the data of measuring instruments in time allows tracing the process of preparation of earthquakes and predicting the possibility of their occurrence and their magnitude.

Աղալովյան Լ.Ա., Ղուլղազարյան Լ.Գ., Կապլունով Յու.Դ., Սարգսյան Մ.Զ., Պրիկազչիկով Դ.Ա.
Ոչ դասական խառը հարթ դեֆորմացիայի խնդրում եռաշերտ շերտի դինամիկ վարքի մասին

Հիմնաբառեր Եռաշերտ փաթեթ, ասիմպտոտիկ մեթոդ, հարթ դեֆորմացիա, սեյսմոլոգիա, երկրաշարժերի կանխատեսում:

Դիտարկված է եռաշերտ իզոտրոպ փաթեթի հարթ դեֆորմացիայի ոչ դասական դինամիկ խնդիրը, որը մոդելավորում է Լիտոսֆերային սալերի և երկրակեղևի բլոկների վարքը: Փաթեթի վերին մակերեսը ազատ է, իսկ շերտերի միջև տրված են լրիվ կոնտակտի պայմանները: Ենթադրվում է, որ չափման տվյալները վերցվել են փաթեթի առաջին և երկրորդ շերտերի միջև շփման մակերեսի կետերից՝ որպես տվյալներ շերտերի միջև տեղադրված ինկլինոմետրերից կամ դեֆորմոգրաֆներից: Առաձգականության տեսության հարթ դեֆորմացիայի խնդրի համապատասխան դինամիկ հավասարումների և առընչությունների լուծումը ստացվել է ասիմպտոտիկ մեթոդով: Հայաստանում՝ երկրակեղևի բլոկի մոդելավորմամբ, եռաշերտ փաթեթի համար իրականացվել է թվային վերլուծություն: Փաթեթի լարվածադեֆորմացիոն վիճակի փոփոխությունների մոնիտորինգը ժամանակի ընթացքում ըստ չափիչ սարքերի տվյալների թույլ է տալիս հետևել երկրաշարժերի առաջացման գործընթացին և կանխատեսել դրանց առաջացման հնարավորությունը և մագնիտուդը:

Агаловян Л.А., Гулгазарян Л.Г., Каплунов Ю.Д., Саргсян М.З., Приказчиков Д.А.
О динамическом поведении трехслойной полосы в неклассической смешанной задаче плоской деформации

Ключевые слова: Трехслойный пакет, асимптотический метод, плоская деформация, сейсмология, прогноз землетрясений.

Рассмотрена неклассическая динамическая задача о плоской деформации трехслойного изотропного пакета, моделирующего поведение литосферных плит и блоков земной коры. Верхняя поверхность пакета свободна, а между слоями заданы условия полного контакта. Предполагается, что данные измерений брались с точек поверхности контакта между первым и вторым слоями пакета, как данные инклинометров или деформографов, размещенных между слоями. Асимптотическим методом получено решение соответствующих динамических уравнений и соотношений задачи плоской деформации теории упругости. Проведен численный анализ трехслойного пакета путем моделирования блока земной коры в Армении. Мониторинг изменения напряженно-деформированного состояния пакета по данным измерительных приборов во времени позволяет проследить процесс подготовки землетрясений и прогнозировать возможность их возникновения и их магнитуду.

1. Introduction

According to the modern theory of lithospheric plates, the entire lithosphere is divided into separate blocks by narrow and active zones - deep faults - moving in the plastic layer of the upper mantle relative to each other with a speed of 2-3 cm per year. For the first time the assumption about the horizontal movement of crustal blocks was made by Alfred Wegener in the 1920s [1]. It was the work of A. Wegener formed the basis of the research conducted in the 60s of the last century. It became the foundation for the emergence of the theory of "lithospheric plate tectonics". The main provisions of plate tectonics were formulated in 1967-73 by a group of American geophysicists - W. J. Morgan [2], X. Le Pichon [3], J. Oliver, J. Isaacs, L. Sykes [4]. According to these theories, the Lithosphere is divided into tectonic plates, which have a rigid structure and have different masses, and are placed on the plastic substance of the asthenosphere. They are in an unstable state and are constantly moving. During displacements, the plates constantly collide, overlap one another, and there are joints and zones of separation of the plates. Most of the earthquakes recorded in the world arose as a result of movements of tectonic plates, when there is a sharp displacement of rocks. This can be either collisions with each other, or lowering a thinner plate under a thicker one. Although this shift is usually small, and amounts to only a few centimeters (1-6 cm), mountains located above the epicenter begin to move, which leads to the accumulation of deformations that reach the order of 10^{-4}, and according to the Japanese seismologist Rikitate of the order of $4.7\cdot10^{-5}$ [5], contributes to the release of a huge force of potential energy, as a result of which global destruction occurs - an earthquake. As a result, cracks form on the earth's surface, along the edges of which huge tracts of land begin to shift along with everything that is on it. The focus of an earthquake is a gap, after the formation of which the earth's surface instantly shifts. It should be noted that this gap does not occur immediately. First, the plates collide with each other, as a result of which friction occurs and energy is generated, which gradually begins to accumulate. When the stress reaches its maximum and begins to exceed the force of friction, the rocks rupture, after which the released energy is converted into seismic waves that move at a speed of approximately 8 km/s and cause the earth to shake. An earthquake consists of several stages. The main, most powerful shock is preceded by warning oscillations (foreshocks), and after it, aftershocks begin, subsequent shaking, and the magnitude of the strongest aftershock is 1.2 less than that of the main shock. The period from the onset of foreshocks to the end of aftershocks may well last several years.

The thickness of lithospheric plates is much less than other geometric dimensions. The greatest thickness of the lithospheric plate reaches up to 200km, and the thinnest plate is located in the ocean zone. Its thickness does not exceed 10km, and in some areas this figure is 5km. The length and width of lithospheric plates can reach tens of thousands of kilometers. Therefore, when modeling the problem of the theory of elasticity for studying the stress-strain state of lithospheric plates, the asymptotic method of solving by introducing a small parameter is effective [6].

The issues of generalized plane deformation for anisotropic bodies are considered in [7,8,9]. The dynamic problem of a two-layered plate in the presence of viscous resistance in both layers is considered in the three-dimensional formulation, where the values of the displacement vector components are collected from inclinometers and other measuring instruments at the contact surface between the layers, was solved in Ghulghazaryan et al. (2020) [10]. The dynamic non-classical 3D problem for a layered orthotropic shells with complete contacts between the layers, when the measurement data are taken from the contact surface between certain layers inside the package, was solved in Aghalovyan et al. (2020) [11]. A non-classical boundary value static problem of the theory of elasticity of plane deformation of a three-layer isotropic package was solved in Ghulghazaryan et al (2022) [12].

The corresponding 3D quasi static problem of elasticity theory (the Rikitake problem) for a layered package from isotropic plates was solved in Aghalovyan et al (2022) [13] and dynamic problem of elasticity theory (the Rikitake problem) for a layered package from isotropic plates in [14].

The non-classical dynamic problem of plane deformation of a three-layer isotropic package modelling the behaviour of Lithospheric plates is considered. The upper surface of the package is free and the layers are supposed to be in ideal contact. It is assumed that the measurement data were taken from the points of the contact surface between the first and second layers of the package, as data from inclinometers or strainmeters placed between the layers. Within the current consideration, it is also assumed that the wave length exceeds substantially the thickness, thus providing a natural geometrical small parameter. Explicit asymptotic results for the displacements are obtained, which could be useful for estimates of certain parameters of earthquakes. The problem in particular models the stress-strain state of the territory of Armenia (the area between northern latitudes 39.00-42.00 and eastern longitudes 42.00-47.00) [15].

2. The plane strain-state dynamic problem of elasticity theory for layered package

Consider a static problem of elasticity theory to determine the plane deformation state of a three-layered isotropic package $D = \{(x, y, z),\ (x, z) \in D_0, 0 \le y \le h,\ h = h_1 + h_2 + h_3,\ h << l\}$, in full contact between the layers, where l its characteristic tangential dimension (the smallest of the linear dimensions of the surface D_0), and measurements are taken from the points of the contact surface between the first and second layers of the package, as data from inclinometers placed between these layers.

The formulation of the problem includes: equations of motion

$$\frac{\partial \sigma_{xx}^{(j)}}{\partial x} + \frac{\partial \sigma_{xy}^{(j)}}{\partial y} = \rho^{(j)} \frac{\partial^2 U^{(j)}}{\partial t^2}, \quad j = I, II, III$$
$$\frac{\partial \sigma_{xy}^{(j)}}{\partial x} + \frac{\partial \sigma_{yy}^{(j)}}{\partial y} = \rho^{(j)} \frac{\partial^2 V^{(j)}}{\partial t^2} \tag{1}$$

constitutive relations for an isotropic solid

$$\frac{\partial U^{(j)}}{\partial x} = \beta_{11}^{(j)} \sigma_{xx}^{(j)} + \beta_{12}^{(j)} \sigma_{yy}^{(j)}$$
$$\frac{\partial V^{(j)}}{\partial y} = \beta_{12}^{(j)} \sigma_{xx}^{(j)} + \beta_{11}^{(j)} \sigma_{yy}^{(j)} \tag{2}$$
$$\frac{\partial U^{(j)}}{\partial y} + \frac{\partial V^{(j)}}{\partial x} = a_{66}^{(j)} \sigma_{xy}^{(j)}$$

where $\rho^{(j)}$ is the density, j is the index of the layer. The $\beta_{mn}^{(j)}$ coefficients of elasticity are expressed by formulas:

$$\beta_{11}^{(j)}=\frac{1-\nu^{(j)^2}}{E^{(j)}},\ \beta_{12}^{(j)}=-\frac{\nu^{(j)}(1+\nu^{(j)})}{E^{(j)}},\ a_{66}^{(j)}=\frac{2(1+\nu^{(j)})}{E^{(j)}}=\frac{1}{G^{(j)}} \tag{3}$$

where $E^{(j)}$ is the Young's modulus, $G^{(j)}$ is the shear modulus and $\nu^{(j)}$ is the Poisson's ratio.

Let the upper face $y=0$ be traction-free:

$$\sigma_{xy}^{(I)}=0,\ \sigma_{yy}^{(I)}=0 \tag{4}$$

On the contact surface between the layers, the values of the displacements of the points of the contact surface are known, as data from inclinometers or other measuring instruments:

$$U^{(I)}(x,h_1)=U^{(II)}(x,h_1)=U^{+}(x)\exp(i\Omega t),\ \ (U,V) \tag{5}$$

Complete contact between the layers is assumed:

$$\begin{gathered}
\sigma_{xy}^{(I)}(x,h_1)=\sigma_{xy}^{(II)}(x,h_1),\ \ (\sigma_{xy},\sigma_{yy})\\
U^{(I)}(x,h_1)=U^{(II)}(x,h_1),\ \ \ (U,V)\\
\sigma_{xy}^{(II)}(x,h_1+h_2)=\sigma_{xy}^{(III)}(x,h_1+h_2),\ \ (\sigma_{xy},\sigma_{yy})\\
U^{(II)}(x,h_1+h_2)=U^{(III)}(x,h_1+h_2),\ \ \ (U,V)
\end{gathered} \tag{6}$$

Let us introduce the scaling $x=l\xi,\ y=\varepsilon l\zeta=h\zeta, U=lu,\ V=lv$, $\sigma_{mk}^{(j)}=\mu\tilde{\sigma}_{mk}^{(j)}$, the total thickness$h=h_1+h_2+h_3$, μ and ρ are typical values of elastic moduli and density, respectively, and $\varepsilon=h/l$ as a small geometrical parameter.

The solutions transformed equations are sought for in the form:

$$Q_{\alpha\beta}^{(j)}=Q_{mk}^{(j)}(\xi,\zeta)\,exp(i\Omega t),\ \ (\alpha,\beta,\gamma);\ \ m,k=1,2,3; j=I,II,III \tag{7}$$

where $Q_{\alpha\beta}^{(j)}$ denotes any of the stress or displacement components. As a result, we arrive at a singularly perturbed system in respect of $Q_{mk}^{(j)}$ with a small parameter ε.

The stresses and displacements are now represented in asymptotic form as follows (Aghalovyan (2015) [6])

$$\begin{gathered}
\tilde{\sigma}_{11}^{(j)}(\xi,\zeta)=\varepsilon^{-1+s}\tilde{\sigma}_{11}^{(j,s)}(\xi,\zeta)\quad (11,12,22),\quad j=I,II,III,\ \ s=\overline{0,N}\\
u^{(j)}(\xi,\zeta)=\varepsilon^{s}u^{(j,s)}(\xi,\zeta),\quad (U,V)
\end{gathered} \tag{8}$$

The notation $s=\overline{0,N}$ here and below denotes summation along the dummy index s within the region of $0,N$.

By substituting (7)-(8) into the dimensionless forms of the governing equations (1), (2), we arrive at a system taking the form:

$$\begin{gathered}
\frac{\partial\tilde{\sigma}_{11}^{(j,s-1)}}{\partial\xi}+\frac{\partial\tilde{\sigma}_{12}^{(j,s)}}{\partial\zeta}+\tilde{\rho}^{(j)}\Omega_*^2u^{(j,s)}=0,\ \ j=I,II,III\\
\frac{\partial\tilde{\sigma}_{12}^{(j,s-1)}}{\partial\xi}+\frac{\partial\tilde{\sigma}_{22}^{(j,s)}}{\partial\zeta}+\tilde{\rho}^{(j)}\Omega_*^2v^{(j,s)}=0\\
\frac{\partial u^{(j,s-1)}}{\partial\xi}=\tilde{\beta}_{11}^{(j)}\tilde{\sigma}_{11}^{(j,s)}+\tilde{\beta}_{12}^{(j)}\tilde{\sigma}_{22}^{(j,s)},\ \ \frac{\partial v^{(j,s)}}{\partial\zeta}=\tilde{\beta}_{12}^{(j)}\tilde{\sigma}_{11}^{(j,s)}+\tilde{\beta}_{11}^{(j)}\tilde{\sigma}_{22}^{(j,s)}\\
\frac{\partial u^{(j,s)}}{\partial\zeta}+\frac{\partial v^{(j,s-1)}}{\partial\xi}=\tilde{a}_{66}^{(j)}\tilde{\sigma}_{12}^{(j,s)},\ \ \Omega_*^2=\frac{\rho}{\mu}h^2\Omega^2\\
\tilde{\beta}_{mk}^{(j)}=\mu\beta_{mk}^{(j)},\ \tilde{a}_{66}^{(j)}=\mu\, a_{66}^{(j)},\ \tilde{\rho}^{(j)}=\ \rho^{(j)}/\rho
\end{gathered} \tag{9}$$

We use relations (9) to express the stress tensor components in terms of displacement vector components:

$$\tilde{\sigma}_{22}^{(j,s)} = \frac{1}{\tilde{B}_{11}^{(j)}}\frac{\partial v^{(j,s)}}{\partial \zeta} - \frac{1}{\tilde{B}_{12}^{(j)}}\frac{\partial u^{(j,s-1)}}{\partial \xi}, \quad \tilde{\sigma}_{11}^{(j,s)} = -\frac{1}{\tilde{B}_{12}^{(j)}}\frac{\partial v^{(j,s)}}{\partial \zeta} + \frac{1}{\tilde{B}_{11}^{(j)}}\frac{\partial u^{(j,s-1)}}{\partial \xi}$$

$$\tilde{\sigma}_{12}^{(j,s)} = \frac{1}{\tilde{a}_{66}^{(j)}}\frac{\partial u^{(j,s)}}{\partial \zeta} + \frac{1}{\tilde{a}_{66}^{(j)}}\frac{\partial v^{(j,s-1)}}{\partial \xi} \qquad (10)$$

where

$$\tilde{B}_{11}^{(j)} = \frac{\tilde{\beta}_{11}^{(j)2} - \tilde{\beta}_{12}^{(j)2}}{\tilde{\beta}_{11}^{(j)}}, \quad \tilde{B}_{12}^{(j)} = \frac{\tilde{\beta}_{11}^{(j)2} - \tilde{\beta}_{12}^{(j)2}}{\tilde{\beta}_{12}^{(j)}} \quad j = I, II, III$$

For determining the displacement vector components we obtain the equations

$$\frac{\partial^2 u^{(j,s)}}{\partial \zeta^2} + \tilde{a}_{66}^{(j)}\Omega_*^2\tilde{\rho}^{(j)}u^{(j,s)} = -\frac{\partial^2 \mathrm{v}^{(j,s-1)}}{\partial \xi \partial \zeta} - \tilde{a}_{66}^{(j)}\frac{\partial \sigma_{11}^{(j,s-1)}}{\partial \xi}, \quad j = I, II, III$$

$$\frac{\partial^2 v^{(j,s)}}{\partial \zeta^2} + \tilde{B}_{11}^{(j)}\Omega_*^2\tilde{\rho}^{(j)}v^{(j,s)} = \frac{\tilde{\beta}_{12}^{(j)}}{\tilde{\beta}_{11}^{(j)}}\frac{\partial^2 u^{(j,s-1)}}{\partial \xi \partial \zeta} - \tilde{B}_{11}^{(j)}\frac{\partial \sigma_{12}^{(j,s-1)}}{\partial \xi} \qquad (11)$$

whose solutions have the form

$$u^{(j,s)} = C_1^{(j,s)}\sin\chi^{(j,u)}\zeta + C_2^{(j,s)}\cos\chi^{(j,u)}\zeta + \bar{u}^{(j,s)}(\xi,\zeta), \quad j = I, II, III$$

$$v^{(j,s)} = C_3^{(j,s)}\sin\chi^{(j,v)}\zeta + C_4^{(j,s)}\cos\chi^{(j,v)}\zeta + \bar{v}^{(j,s)}(\xi,\zeta) \qquad (12)$$

where $\bar{u}^{(j,s)}(\xi,\zeta)$, $\bar{u}^{(j,s)}(\xi,\zeta)$are particular solutions of equations (11), and

$$\chi^{(j,u)} = \sqrt{\tilde{a}_{66}^{(j)}\tilde{\rho}^{(j)}}\Omega_*, \quad \chi^{(j,v)} = \sqrt{\tilde{B}_{11}^{(j)}\tilde{\rho}^{(j)}}\Omega_*, \qquad j = I, II, III$$

On satisfying the boundary conditions (4) - (6) we obtain independent inhomogeneous algebraic systems with respect to the $C_i^{(j,s)}$ $(i = \overline{1,4};\ j = I, II, III)$. These systems will have finite solutions if

$\cos\chi^{(I,u)}\zeta_1 \neq 0, \quad \zeta_1 = h_1/h, \ (u, v)$

After solving these systems we obtain the solutions:

for the first layer

$$u^{(I,s)} = \frac{(u^{+(s)} - \bar{u}^{(I,s)}(\zeta_1))}{\cos\chi^{(I,u)}\zeta_1}\cos\chi^{(I,u)}\zeta +$$

$$\frac{1}{\chi^{(I,u)}\cos\chi^{(I,u)}\zeta_1}\left(\frac{\partial \bar{u}^{(I,s)}(0)}{\partial \zeta} + \frac{\partial v^{(I,s-1)}(0)}{\partial \xi}\right)\sin\chi^{(I,u)}(\zeta_1 - \zeta) + \bar{u}^{(I,s)}(\zeta) \qquad (13)$$

$$v^{(I,s)} = \frac{(v^{+(s)} - \bar{v}^{(I,s)}(\zeta_1))}{\cos\chi^{(I,v)}\zeta_1}\cos\chi^{(I,v)}\zeta +$$

$$\frac{B_{11}^I}{\chi^{(I,v)}\cos\chi^{(I,v)}\zeta_1}\left(\frac{1}{B_{11}^I}\frac{\partial \bar{v}^{(I,s)}(0)}{\partial \zeta} - \frac{1}{B_{12}^I}\frac{\partial u^{(I,s-1)}(0)}{\partial \xi}\right)\sin\chi^{(I,v)}(\zeta_1 - \zeta) + \bar{v}^{(I,s)}(\zeta)$$

for the second layer

$$u^{(II,s)} = (u^{(I,s)}(\zeta_1) - \bar{u}^{(II,s)}(\zeta_1))\cos\chi^{(II,u)}(\zeta_1 - \zeta) -$$

$$-\frac{\tilde{a}_{66}^{II}}{\chi^{(II,u)}}G_{11}^{(II,s)}\sin\chi^{(II,u)}(\zeta_1 - \zeta) + \bar{u}^{(II,s)}(\zeta) \qquad (14)$$

$$v^{(II,s)} = (v^{(I,s)}(\zeta_1) - \bar{v}^{(II,s)}(\zeta_1))\cos\chi^{(II,v)}(\zeta_1 - \zeta) -$$

$$\frac{\tilde{B}_{11}^{II}}{\chi^{(II,v)}}G_{12}^{(II,s)}\sin\chi^{(II,v)}(\zeta_1 - \zeta) + \bar{v}^{(II,s)}(\zeta)$$

for the third layer

$$u^{(III,s)} = (u^{(II,s)}(\zeta_2) - \bar{u}^{(III,s)}(\zeta_2))\cos\chi^{(III,u)}(\zeta_2 - \zeta) - \frac{\tilde{a}_{66}^{III}}{\chi^{(III,u)}} G_{11}^{(III,s)} \sin\chi^{(III,u)}(\zeta_2 - \zeta) + \bar{u}^{(III,s)}(\zeta) \tag{15}$$

$$v^{(III,s)} = (v^{(II,s)}(\zeta_2) - \bar{v}^{(III,s)}(\zeta_2))\cos\chi^{(III,v)}(\zeta_2 - \zeta) - \frac{\tilde{B}_{11}^{III}}{\chi^{(III,v)}} G_{12}^{(III,s)} \sin\chi^{(III,v)}(\zeta_2 - \zeta) + \bar{v}^{(III,s)}(\zeta)$$

where

$$G_{11}^{(II,s)} = \frac{1}{\tilde{a}_{66}^{I}}\left(\frac{\partial u^{(I,s)}(\zeta_1)}{\partial\zeta} + \frac{\partial v^{(I,s-1)}(\zeta_1)}{\partial\xi}\right) - \frac{1}{\tilde{a}_{66}^{II}}\left(\frac{\partial \bar{u}^{(II,s)}(\zeta_1)}{\partial\zeta} + \frac{\partial v^{(II,s-1)}(\zeta_1)}{\partial\xi}\right)$$

$$G_{12}^{(II,s)} = \frac{1}{\tilde{B}_{11}^{I}}\frac{\partial v^{(I,s)}(\zeta_1)}{\partial\zeta} - \frac{1}{\tilde{B}_{12}^{I}}\frac{\partial u^{(I,s-1)}(\zeta_1)}{\partial\xi} - \frac{1}{\tilde{B}_{11}^{II}}\frac{\partial \bar{v}^{(II,s)}(\zeta_1)}{\partial\zeta} + \frac{1}{\tilde{B}_{12}^{II}}\frac{\partial u^{(II,s-1)}(\zeta_1)}{\partial\xi} \tag{16}$$

$$G_{11}^{(III,s)} = \frac{1}{\tilde{a}_{66}^{II}}\left(\frac{\partial u^{(II,s)}(\zeta_2)}{\partial\zeta} + \frac{\partial v^{(II,s-1)}(\zeta_2)}{\partial\xi}\right) - \frac{1}{\tilde{a}_{66}^{III}}\left(\frac{\partial \bar{u}^{(III,s)}(\zeta_2)}{\partial\zeta} + \frac{\partial v^{(III,s-1)}(\zeta_2)}{\partial\xi}\right)$$

$$G_{12}^{(III,s)} = \frac{1}{\tilde{B}_{11}^{II}}\frac{\partial v^{(II,s)}(\zeta_2)}{\partial\zeta} - \frac{1}{\tilde{B}_{12}^{II}}\frac{\partial u^{(II,s-1)}(\zeta_2)}{\partial\xi} - \frac{1}{\tilde{B}_{11}^{III}}\frac{\partial \bar{v}^{(III,s)}(\zeta_2)}{\partial\zeta} + \frac{1}{\tilde{B}_{12}^{III}}\frac{\partial u^{(III,s-1)}(\zeta_2)}{\partial\xi}$$

$\zeta_2 = (h_1 + h_2)/h,\ u^{+(0)} = \frac{U^+}{l},\ \ u^{+(s)} = 0, s > 0\ (u, v).$

3. Numerical calculation

For the three-layer orthotropic package what modeling Lithospheric plates calculations are made for the s = 0 and s=1 approximation. Then for the components of the displacement vector in dimensional form we obtain:

for the first layer

$$U^{(I)} = l\,u^{(I,0)} + h\,u^{(I,1)},\ \ V^{(I)} = l\,v^{(I,0)} + h\,v^{(I,1)}$$

$$u^{(I,0)} = \frac{u^+(\xi)}{\cos\chi^{(I,u)}\zeta_1}\cos\chi^{(I,u)}\zeta, \quad v^{(I,0)} = \frac{v^+(\xi)}{\cos\chi^{(I,v)}\zeta_1}\cos\chi^{(I,v)}\zeta$$

$$u^{(I,1)} = \frac{-\bar{u}^{(I,1)}(\zeta_1))}{\cos\chi^{(I,u)}\zeta_1}\cos\chi^{(I,u)}\zeta + \frac{1}{\chi^{(I,u)}}\left(\frac{\partial\bar{u}^{(I,1)}(0)}{\partial\zeta} + \frac{\partial v^{(I,0)}(0)}{\partial\xi}\right)\sin\chi^{(I,u)}(\zeta_1 - \zeta) + \bar{u}^{(I,1)}(\zeta) \tag{17}$$

$$v^{(I,1)} = \frac{-\bar{v}^{(I,1)}(\zeta_1)}{\cos\chi^{(I,v)}\zeta_1}\cos\chi^{(I,v)}\zeta + \frac{1}{\chi^{(I,v)}}\left(\frac{1}{B_{11}^{I}}\frac{\partial\bar{v}^{(I,1)}(0)}{\partial\zeta} - \frac{1}{B_{12}^{I}}\frac{\partial u^{(I,0)}(0)}{\partial\xi}\right)\sin\chi^{(I,v)}(\zeta_1 - \zeta) + \bar{v}^{(I,1)}(\zeta)$$

$$\bar{u}^{(I,1)}(\zeta) = \frac{A_u^I}{\chi^{(I,v)^2} - \chi^{(I,u)^2}}\sin\chi^{(I,v)}\zeta, \quad \bar{v}^{(I,1)}(\zeta) = \frac{B_u^I}{\chi^{(I,v)^2} - \chi^{(I,u)^2}}\sin\chi^{(I,u)}\zeta$$

$$A_u^I = \frac{\chi^{(I,v)}}{\cos\chi^{(I,v)}\zeta_1}\left(\frac{a_{66}^I}{B_{12}^I} - 1\right)\frac{\partial v^+(\xi)}{\partial\xi}, \quad B_u^I = \frac{\chi^{(I,u)}}{\cos\chi^{(I,u)}\zeta_1}\left(\frac{B_{11}^I}{a_{66}^I} - \frac{\beta_{12}^I}{\beta_{11}^I}\right)\frac{\partial u^+(\xi)}{\partial\xi}$$

for the second layer

$$u^{(II,0)} = u^+(\xi)\cos\chi^{(II,u)}(\zeta_1 - \zeta) + \frac{a_{66}^{II}}{\chi^{(II,u)}}\frac{\chi^{(I,u)}}{a_{66}^{I}}u^+(\xi)\ tg\chi^{(I,u)}\zeta_1\ \sin\chi^{(II,u)}(\zeta_1 - \zeta)$$

$$v^{(II,0)} = v^+(\xi)\cos\chi^{(II,v)}(\zeta_1 - \zeta) + \frac{B_{11}^{II}}{\chi^{(II,v)}}\frac{\chi^{(I,v)}}{B_{11}^{I}}v^+(\xi)\ tg\chi^{(I,v)}\zeta_1\ \sin\chi^{(II,v)}(\zeta_1 - \zeta)$$

$$u^{(II,1)} = \left(u^{(I,1)}(\zeta_1) - \bar{u}^{(II,1)}(\zeta_1)\right)\cos\chi^{(II,u)}(\zeta_1 - \zeta) -$$
$$- \frac{a_{66}^{II}}{\chi^{(II,u)}} G_{11}^{(II,1)} \sin\chi^{(II,u)}(\zeta_1 - \zeta) + \bar{u}^{(II,1)}(\zeta) \qquad (18)$$
$$v^{(II,1)} = (v^{(I,1)}(\zeta_1) - \bar{v}^{(II,1)}(\zeta_1))\cos\chi^{(II,v)}(\zeta_1 - \zeta) - \frac{B_{11}^{II}}{\chi^{(II,v)}} G_{12}^{(II,1)} \sin\chi^{(II,v)}(\zeta_1 - \zeta) + \bar{v}^{(II,1)}(\zeta)$$
$$\bar{u}^{(II,1)}(\zeta) = A_u^{II}\sin\chi^{(II,v)}(\zeta_1 - \zeta) + B_u^{II}\cos\chi^{(II,v)}(\zeta_1 - \zeta)$$
$$\bar{v}^{(II,1)}(\zeta) = A_v^{II}\sin\chi^{(II,u)}(\zeta_1 - \zeta) + B_v^{II}\cos\chi^{(II,u)}(\zeta_1 - \zeta)$$
$$A_u^{II} = \frac{\frac{\partial v^+(\xi)}{\partial \xi}\left(\frac{a_{66}^{II}}{B_{12}^{II}} - 1\right)\chi^{(II,v)}}{{\chi^{(II,u)}}^2 - {\chi^{(II,v)}}^2}, \quad B_u^{II} = \frac{\frac{\partial v^+(\xi)}{\partial \xi}\left(1 - \frac{a_{66}^{II}}{B_{12}^{II}}\right)\frac{B_{11}^{II}}{B_{11}^{I}}\chi^{(I,v)} tg\chi^{(I,v)}\zeta_1}{{\chi^{(II,u)}}^2 - {\chi^{(II,v)}}^2}$$
$$A_v^{II} = \frac{\left(\frac{\beta_{12}^{II}}{\beta_{11}^{II}} - \frac{B_{11}^{II}}{a_{66}^{II}}\right)\frac{\partial u^+(\xi)}{\partial \xi}\chi^{(II,u)}}{{\chi^{(II,v)}}^2 - {\chi^{(II,u)}}^2}, \quad B_v^{II} = \frac{\left(\frac{B_{11}^{II}}{a_{66}^{II}} - \frac{\beta_{12}^{II}}{\beta_{11}^{II}}\right)\frac{a_{66}^{II}}{a_{66}^{I}}\frac{\partial u^+(\xi)}{\partial \xi}\chi^{(I,u)} tg\chi^{(I,u)}\zeta_1}{{\chi^{(II,v)}}^2 - {\chi^{(II,u)}}^2}$$
$$G_{11}^{(II,1)} = \frac{1}{a_{66}^{I}}\left(\frac{\partial u^{(I,1)}(\zeta_1)}{\partial \zeta} + \frac{\partial v^{(I,0)}(\zeta_1)}{\partial \xi}\right) - \frac{1}{a_{66}^{II}}\left(\frac{\partial \bar{u}^{(II,1)}(\zeta_1)}{\partial \zeta} + \frac{\partial v^{(II,0)}(\zeta_1)}{\partial \xi}\right)$$
$$G_{12}^{(II,1)} = \frac{1}{B_{11}^{I}}\frac{\partial v^{(I,1)}(\zeta_1)}{\partial \zeta} - \frac{1}{B_{12}^{I}}\frac{\partial u^{(I,0)}(\zeta_1)}{\partial \xi} - \frac{1}{B_{11}^{II}}\frac{\partial \bar{v}^{(II,1)}(\zeta_1)}{\partial \zeta} + \frac{1}{B_{12}^{II}}\frac{\partial u^{(II,0)}(\zeta_1)}{\partial \xi}$$

for the third layer

$$u^{(III,0)} = u^{(II,0)}(\zeta_2)\cos\chi^{(III,u)}(\zeta_2 - \zeta) + \frac{a_{66}^{III}}{\chi^{(III,u)}a_{66}^{II}}\frac{\partial u^{(II,0)}(\zeta_2)}{\partial \zeta} \sin\chi^{(III,u)}(\zeta_2 - \zeta)$$
$$v^{(III,0)} = v^{(II,0)}(\zeta_2)\cos\chi^{(III,v)}(\zeta_2 - \zeta) + \frac{B_{11}^{III}}{\chi^{(III,v)}B_{11}^{II}}\frac{\partial v^{(II,0)}(\zeta_2)}{\partial \zeta} \sin\chi^{(III,v)}(\zeta_2 - \zeta)$$
$$u^{(III,1)} = \left(u^{(II,1)}(\zeta_2) - \bar{u}^{(III,1)}(\zeta_2)\right)\cos\chi^{(III,u)}(\zeta_2 - \zeta) -$$
$$- \frac{a_{66}^{III}}{\chi^{(III,u)}} G_{11}^{(III,1)} \sin\chi^{(III,u)}(\zeta_2 - \zeta) + \bar{u}^{(III,1)}(\zeta) \qquad (19)$$
$$v^{(III,1)} = (v^{(II,1)}(\zeta_2) - \bar{v}^{(III,1)}(\zeta_2))\cos\chi^{(III,v)}(\zeta_2 - \zeta) - \frac{B_{11}^{III}}{\chi^{(III,v)}} G_{12}^{(III,1)} \sin\chi^{(III,v)}(\zeta_2 - \zeta) + \bar{v}^{(III,1)}(\zeta)$$
$$\bar{u}^{(III,1)}(\zeta) = A_u^{III}\sin\chi^{(III,v)}(\zeta_2 - \zeta) + B_u^{III}\cos\chi^{(III,v)}(\zeta_2 - \zeta)$$
$$\bar{v}^{(III,1)}(\zeta) = A_v^{III}\sin\chi^{(III,u)}(\zeta_2 - \zeta) + B_v^{III}\cos\chi^{(III,u)}(\zeta_2 - \zeta)$$
$$A_u^{III} = \frac{\left(\frac{a_{66}^{III}}{B_{12}^{III}} - 1\right)\chi^{(III,v)}\frac{\partial v^{(II,0)}(\xi_2)}{\partial \xi}}{{\chi^{(III,u)}}^2 - {\chi^{(III,v)}}^2}, \quad B_u^{III} = \frac{\left(\frac{a_{66}^{III}}{B_{12}^{III}} - 1\right)\frac{B_{11}^{III}}{B_{11}^{II}}\frac{\partial^2 v^{(II,0)}(\xi_2)}{\partial \xi \partial \zeta}}{{\chi^{(III,u)}}^2 - {\chi^{(III,v)}}^2}$$
$$A_v^{III} = \frac{\left(\frac{\beta_{12}^{III}}{\beta_{11}^{III}} - \frac{B_{11}^{III}}{a_{66}^{III}}\right)\chi^{(III,u)}\frac{\partial u^{(II,0)}(\xi_2)}{\partial \xi}}{{\chi^{(III,v)}}^2 - {\chi^{(III,u)}}^2}, \quad B_v^{III} = \frac{\left(\frac{\beta_{12}^{III}}{\beta_{11}^{III}} - \frac{B_{11}^{III}}{a_{66}^{III}}\right)\frac{a_{66}^{III}}{a_{66}^{II}}\frac{\partial^2 u^{(II,0)}(\xi_2)}{\partial \xi \partial \zeta}}{{\chi^{(III,v)}}^2 - {\chi^{(III,u)}}^2}$$

$$G_{11}^{(III,1)} = \frac{1}{a_{66}^{II}}\left(\frac{\partial u^{(II,1)}(\zeta_2)}{\partial \zeta} + \frac{\partial v^{(II,0)}(\zeta_2)}{\partial \xi}\right) - \frac{1}{a_{66}^{III}}\left(\frac{\partial \bar{u}^{(III,1)}(\zeta_2)}{\partial \zeta} + \frac{\partial v^{(III,0)}(\zeta_2)}{\partial \xi}\right)$$

$$G_{12}^{(III,1)} = \frac{1}{B_{11}^{II}}\frac{\partial v^{(II,1)}(\zeta_2)}{\partial \zeta} - \frac{1}{B_{12}^{II}}\frac{\partial u^{(II,0)}(\zeta_2)}{\partial \xi} - \frac{1}{B_{11}^{III}}\frac{\partial \bar{v}^{(III,1)}(\zeta_2)}{\partial \zeta} + \frac{1}{B_{12}^{III}}\frac{\partial u^{(III,0)}(\zeta_2)}{\partial \xi}$$

For calculations, a three-layer isotropic package with the following geometrical and mechanical parameters of the layers given in Table 1 is used. The initial data of the layer materials are taken from the works [15].

Three cases of values obtained with the help of measuring instruments at different points in time were taken for analysis.

Table 1. The geometrical and mechanical parameters of the layers of the package ($l = 150$ ***)***

j	Layers	$E^{(j)}$ Young's modulus, Pa	$G^{(j)}$ shear modulus, Pa	$\nu^{(j)}$ Poisson's ratio	$\rho^{(j)}$ density, kg/m^3	Layer thickness, km
I	Sedimentary	55*10^9	23.2*10^9	0.184	2050	5
II	Granite	74.83*10^9	30.82*10^9	0.21	2610	10
III	Basalt	75.11*10^9	29.22*10^9	0.27	2910	20

First case: $u^+ = 0.05 + 0.02\xi,\ v^+ = 0.06 + 0.01\xi$.

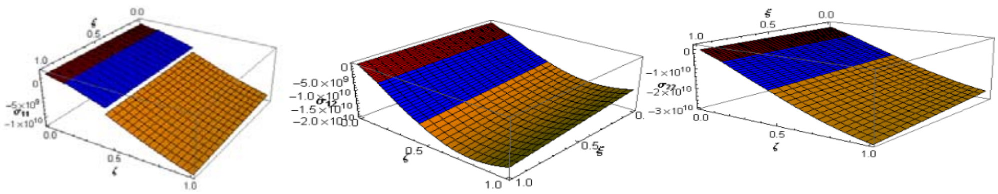

Fig. 1 - Stress tensor components for the package: first case.

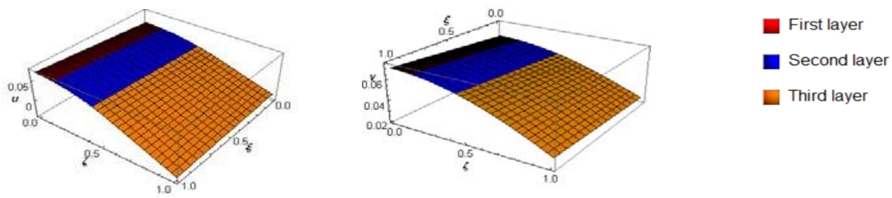

Fig. 2 - Displacement vector components for the package: first case.

Second case: $u^{+} = 0.05 + 0.04\xi,\ v^{+} = 0.06 + 0.03\xi$.

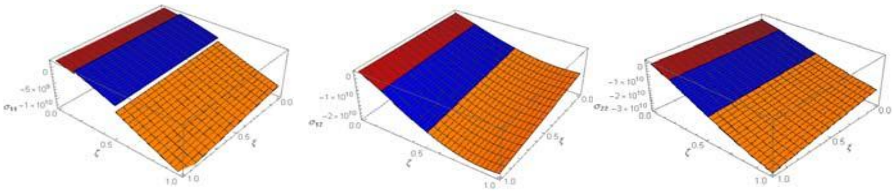

Fig. 3 - Stress tensor components for the package: second case.

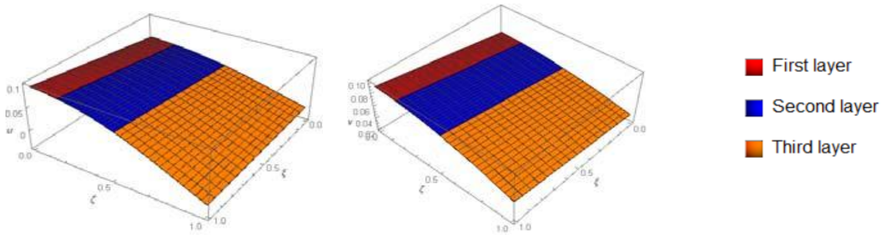

Fig. 4 - Displacement vector components for the package: second case.

Third case: $u^{+} = 0.05 + 0.06\xi,\ v^{+} = 0.06 + 0.05\xi$.

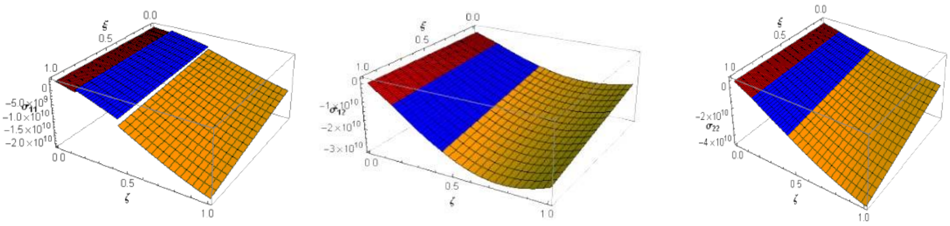

Fig. 5 - Stress tensor components for the package: third case.

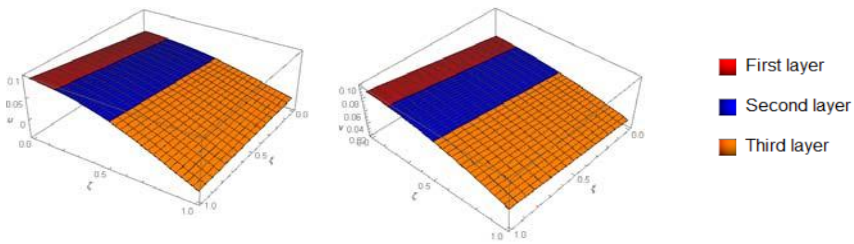

Fig. 6 - Displacement vector components for the package: third case.

4. Conclusion

The Rikitake plane dynamic problem for a three-layer isotropic package of strip is solved using data of the displacements of contact surface points between the first and second layers. The procedure for determining the amplitude of forced oscillations in the foreshock stage for a three-layer package is given. Having the solution to the external problem, it is possible to monitor the change of the stress-strain states over time, in accordance with regularly carried out measurements. Judging by the numerical calculations given above, it can be said that if the data of the displacement amplitudes of the contact points between the layers, obtained with the help of measuring devices, increase along the length of the package with time, then the values of the amplitudes of vibrations when moving away from the surface into the inside of the package increase significantly.

It is possible to detect separation between some layers - when tangential stress becomes greater than the admissible value.

The found solution let to calculate the accumulated potential energy of deformation W by formula:

$$W = \frac{1}{2}\int_V \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{zz}\varepsilon_{zz} + \sigma_{xy}\varepsilon_{xy} + \sigma_{xz}\varepsilon_{xz} + \sigma_{yz}\varepsilon_{yz}\right) dv \quad (20)$$

where ε_{ij} - components of deformations tensor.

Having the value of W corresponding to the time $t = t_*$ of the primary measurement, it is possible to carry out the monitoring of its change in time $t > t_*$, in accordance with new measurements and fix the time when W, reaches the critical value, using the relationship between W and the magnitude M of the expected earthquake [16,17].

$$\lg W = 11{,}8 + 1{,}5M \quad (21)$$

Having the value W, according to the formula (21) it is possible to predict the magnitude M of the expected earthquake.

Acknowledgements

The work was supported by the Science Committee of RA, in the frames of the research project № 21T-2C075.

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Received 17.08.2023