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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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Controlling Electro-Acoustic Wave Propagation in a Piezoelectric Waveguide with Non-Acoustic Edge Action

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Keywords: electro acoustic wave, wave control, piezoelectric waveguide, non-acoustic action, edge control, control function harmonics.

Управление распространением электроакустической волны в пьезоэлектрическом волноводе с неакустическим краевым воздействием

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Ключевые слова: электроакустическая волна, волновое управление, пьезоэлектрический волновод, неакустическое воздействие, краевое управление, гармоники функции управления.

Рассмотрена задача управления распространением однонаправленной волны упругого сдвига в бесконечном пьезоэлектрическом волноводе на конечном интервале времени. Поверхности пьезоэлектрических волноводов свободны от напряжения и контакта. Нагруженные электроды с переменным потенциалом электрического поля расположены на разном расстоянии от поверхности волновода. Нагруженные электроды вблизи механически свободных поверхностей пьезоэлектрического волновода вызывают эквивалентное электромеханическое воздействие на поверхности волновода.
Сформулирована задача граничного управления распределением компонент волны электроактивной однонаправленной сдвиговой деформации по толщине волновода. Она решается с помощью метода разложения в ряды Фурье, заключающегося в нахождении правильной формы электроакустических волн с использованием соответствующих гармоник поверхностных воздействий.
Так же, были проведены аналитические и численные расчеты для конкретного случая равно нагруженных и равноудаленных от поверхностей волновода электродов.

Էլեկտրաակուստիկ ալիքի տարածման ղեկավարումը պիեզոէլեկտրիկ ալիքատարում ոչ ակուստիկ եզրային ազդեցության միջոցով

Ավետիսյան Արա Ս., Մկրտչյան Մ.Հ., Ավետիսյան Լ.Վ.

Հիմնաբառեր՝ էլեկտրաակուստիկ ալիք, ալիքի ղեկավարում, պիեզոէլեկտրիկ ալիքատար, ոչ ակուստիկ ազդեցություն, եզրային ղեկավարում, ղեկավարման ֆունկցիայի հարմոնիկներ:

Դիտարկվել է անվերջ պիեզոէլեկտրիկ ալիքատարում միակողմանի առաձգական սահքի ալիքի տարածումը սահմանափակ ժամանակի ինտերվալում ղեկավարելու խնդիրը: Պիեզոէլեկտրիկ ալիքատարի մակերևույթները ազատ են լարվածությունից և շփումներից: Էլեկտրական դաշտի տարբեր պոտենցիալներով բեռնված էլեկտրոդները տեղակայված են ալիքատարի մակերևույթներից

տարբեր հեռավորությունների վրա: Պիեզոէլեկտրական ալիքատարի մեխանիկորեն ազատ մակերևույթների մոտ լիցքավորված էլեկտրոդները հանգեցնում են ալիքատարի մակերևույթների վրա համարժեք էլեկտրամեխանիկական ազդեցության:
Ձևակերպված է ալիքատարի հաստությամբ էլեկտրաակտիվ միակողմանի սահքի դեֆորմացիայի ալիքի բաղադրիչների բաշխման եզրային ղեկավարման խնդիրը: Այն լուծվում է օգտագործելով Ֆուրիեի շարքի ներկայացման մեթոդը, որը ներառում է էլեկտրաակուստիկ ալիքների ճիշտ ձևի հայտնաբերում, օգտագործելով մակերևութային ազդեցությունների համապատասխան հարմոնիկները:
Բացի այդ, անալիտիկ և թվային հաշվարկներ են իրականացվել կոնկրետ դեպքի համար, երբ էլեկտրոդները հավասարապես բեռնված են և գտնվում են ալիքատարի մակերևույթներից հավասար հեռավորության վրա:

Considered the problem of controlling unidirectional elastic shear wave propagation in an infinite piezoelectric waveguide over a finite time interval. The piezoelectric waveguide surfaces are free from stress and contact. Loaded electrodes with varying electric field potentials are located at varying distances from the waveguide surfaces. The loaded electrodes near the mechanically free surfaces of the piezoelectric waveguide result in equivalent electromechanical action on the waveguide surfaces.
The boundary control problem for the distribution of electroactive unidirectional shear deformation wave components across the waveguide thickness is formulated. It is solved using the Fourier series expansion method, which involves finding the proper form of electroacoustic waves using corresponding harmonics of surface actions. Additionally, analytical and numerical calculations were conducted for the specific case of electrodes being equally loaded and equidistant from the waveguide surfaces.

Introduction

The widespread use of multi-functional materials in modern electronics has led to the discovery and study of new coupled wave effects, such as electro-magneto-elastic and thermo-elastic, with both plane and antiplane deformation elastic wave properties. The study of wave propagation modes and changes in wave field characteristics is a critical step in solving dynamic problems. The transfer of wave energy, its localization near the waveguide surface, and the distribution of wave characteristics across the piezoelectric waveguide thickness are important aspects of the dynamic process.
In 1968, after confirmation by **Bluestein J.L. [1]**, and in 1969 by **Gulyaev Yu.V. [2]** assumptions about the existence of localization of the wave energy of an electroactive elastic pure shear wave on a mechanically free smooth surface of a piezoelectric medium, under different boundary conditions for the accompanying electric field **Kaganov M.I., Sklovskaya I.L. [3]**.
In **[4] Ingebrigsten K.A.** considered the effect of various electromagnetic boundary conditions on the propagation of surface waves in piezoelectrics, by introducing an electrical "surface impedance". In works **[4, 5] Avetisyan A.S.** is shown that the conditions of conjugation of electric fields on a mechanically free surface of a piezoelectric lead to the appearance of "effective mechanical stresses", which leads to near-surface localization of the component's energy of electro elastic waves.
In the article **Avetisyan A. S., Mkrtchyan M. H. and Avetisyan L. V. [6],** considered a variety of surface non-acoustic influences in problems of surface control of three-component electroacoustic waves in a piezoelectric waveguide.

The possibility of the control problems formulating of the electro-acoustic transverse waves propagating in the piezoelectric half-space by non-acoustic action on its surface is studied **Avetisyan A.S. [7].** It is shown that the non-stationary electric potential on the electric screen or the non-stationary width of the gap between the indicated surfaces leads to a non-acoustic effect on the piezoelectric half-space.

In work **Ilyin V.A., Moiseev E.I. [9],** the seven different problems of boundary control for a stretched string were considered. In work **Ilyin V.A., Moiseev E.I. [10],** to find an unambiguous solution to these problems, it is sometimes necessary to impose additional conditions on the desired functions.

In articles **Barseghyan V.R. [11, 12]**, the problem of optimal control of string vibrations and the optimal boundary control of string vibrations with given values of the velocities of the deflection points at intermediate times are examined. The control function for all orthogonal vibration modes in these problems considers the total edge action.

In the last part of the book **Avetisyan A.S. [12]**, the problem of controlling electroacoustic waves in a waveguide is formulated. A method for solving problems of control of electroacoustic waves by non-contact surface action is proposed.

1. Problem statement.

1.1 Formulating and modeling a mathematical boundary value problem.

The control of the propagation of normal electroacoustic shear deformation waves in a piezoelectric waveguide, under external influence by an electric potential is investigated. A piezoelectric layer with a thickness of $2H_0$ is located between electric screens at a distance of $[H_0 + h_+]$ and $[H_0 + h_-]$ from each other respectively. The electric screens are loaded by field potential $\varphi_{0\pm}(t) \cdot \exp(ikx)$ (Figure 1).

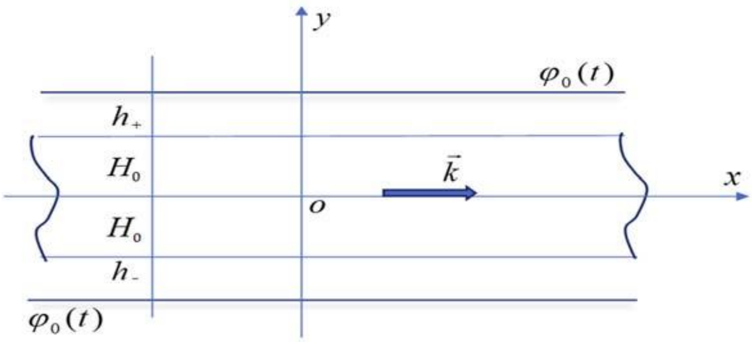

Figure 1. *The scheme of contactless edge control of the distribution of an electro acoustic wave in a piezoelectric waveguide.*

In the investigations of the propagation of electroactive unidirectional elastic shear deformation waves of a type $W(x, y, t) = \mathrm{w}(y, t) \cdot \exp(ikx)$ in a ***6mm*** hexagonal symmetry class piezoelectric layer $y \le |H_0|$, the system of the quasi-static equations of electro elasticity has the form

$$\mathrm{w}''(y,t) - k^2 \cdot \mathrm{w}(y,t) = (1/\tilde{c}_t^2) \cdot \ddot{\mathrm{w}}(y,t)\,,$$

$$\varphi''(y,t) - k^2 \cdot \varphi(y,t) = (e_{15}/\varepsilon_{11}) \cdot \left[\mathrm{w}''(y,t) - k^2 \cdot \mathrm{w}(y,t) \right] \tag{1.1}$$

Here, $\mathrm{w}(y,t)$ is elastic shear distribution function over the waveguide thickness, $\varphi(y,t)$ is distribution function of the accompanying electric field potential over the waveguide thickness, $\tilde{c}_t^2 = (1+\chi^2) \cdot \sqrt{c_{44}/\rho}$ the quadrate of the elastic wave velocity, e_{15} piezoelectric module and ε_{11} coefficient of dielectric and $\chi^2 = e_{15}^2/(c_{44}\varepsilon_{11})$ the coefficient of electromechanically coupling of material.

In the near surface vacuum gaps $y \in [H_0;\ H_0 + h_+]$ and $y \in [-H_0 - h_-;\ -H_0]$, respectively, the accompanying quasi-static electric field vibration equations have the form

$$\varphi''_{\pm e}(y,t) - k^2 \cdot \varphi_{\pm e}(y,t) = 0\,, \tag{1.2}$$

Here $\varphi_{\pm e}(y,t)$ is the accompanying electric field potential distribution function in vacuum gaps respectively.

As follows from equations (1.1) and (1.2), according to the quasi-static theory, an elastic transverse shear wave propagates in the waveguide, with accompanying oscillations of the electric field in it and in vacuum gaps.

Electrical dynamic loads on the electrodes create electric fields in vacuum gaps, forming mechanical stresses and electrical polarization on the surfaces of the piezoelectric waveguide. Taking into account the electromechanical boundary conditions for propagating electroacoustic oscillations of the type $F(x,y,t) = f(y,t) \cdot \exp(ikx)$ on the mechanically free surfaces of the piezoelectric waveguide and the conditions of continuity of the electric field on the electrodes in $y = H_0 + h_+$ and $y = -(H_0 + h_-)$, the solution of equations (1.2), which describe the oscillations of the accompanying electric field in the gaps $y \in [H_0; H_0 + h_+]$ and $y \in [-(H_0 + h_-); -H_0]$ can be written in the following form, respectively

$$\varphi_{+e}(y,t) = -\varphi(H_0) \cdot \frac{sh[k(y - H_0 - h_+)]}{sh(kh_+)} \cdot \theta(t) + \frac{sh[k(y - H_0)]}{sh(kh_+)} \cdot \varphi_{+0}(t) \tag{1.3}$$

$$\varphi_{-e}(y,t) = \varphi(-H_0) \cdot \frac{sh[k(y + H_0 + h_-)]}{sh(kh_-)} \cdot \theta(t) - \frac{sh[k(y + H_0)]}{sh(kh_-)} \cdot \varphi_{-0}(t) \tag{1.4}$$

The obtained values of accompanying electric field oscillations in vacuum gaps (1.3) and (1.4) form mechanical stresses and electrical polarization on the surfaces of the piezoelectric waveguide.

The piezoelectric layer is mechanically balanced when

$$\mathrm{w}'(y,t)\big|_{y=\pm H_0} = \pm(e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\frac{kh}{sh(kh)}\cdot\frac{\varphi_0(t)-ch(kh)\cdot\varphi(H_0)\cdot\theta(t)}{h} \tag{1.5}$$

The piezoelectric waveguide surfaces also have balanced electric polarization

$$\varphi'(y,t)\big|_{y=\pm H_0} = \mp(\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\frac{kh}{sh(kh)}\cdot\frac{\varphi_0(t)-ch(kh)\cdot\varphi(H_0)\cdot\theta(t)}{h} \tag{1.6}$$

It is obvious from (1.5) that in the absence of the piezoelectric effect in the medium $e_{15}\equiv 0$, there will be no mechanical tensions on the surfaces.

Obviously, under non-acoustic influence on the electric surface, the layer will be in equilibrium when the electrodes are equally charged $\varphi_{-0}(t)=\varphi_{+0}(t)$ and are at the same distance $h_- = h_+$ from the mechanically free surfaces of the piezoelectric layer. In this case, the values of the electric potential on the surfaces of the waveguide are equal $\varphi_{+e}(H_0,t)=\varphi_{-e}(-H_0,t)$ and the electric polarization on these surfaces is directed in the opposite direction $\varphi'_{+e}(y,t)\big|_{y=H_0} = -\varphi'_{-e}(y,t)\big|_{y=-H_0}$.

Based on the above, the conditions for conjugating the distribution functions of the electromechanical field on the mechanically free surfaces of the piezoelectric layer will be written in the form

$$\mathrm{w}'(y,t)\big|_{y=\pm H_0} = \pm(e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\varphi'_{\pm e}(y,t)\big|_{y=\pm H_0} \tag{1.7}$$

$$\varphi'(y,t)\big|_{y=\pm H_0} = \mp(\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\varphi'_{\pm e}(y,t)\big|_{y=\pm H_0} \tag{1.8}$$

$$\left[\varphi(y,t)-\varphi_{\pm e}(y,t)\right]_{\mathrm{y}=\pm H_0} = 0 \tag{1.9}$$

Conditions for electric field conjugation on screens with electric potential are written in the form

$$\varphi_{\pm e}(y,t)\big|_{y=\pm(H_0+h_\pm)} = \varphi_{0\pm}(t) \tag{1.10}$$

Formally, the initial conditions of the electroacoustic wave field characteristics at a moment are written in the form

$$\mathrm{w}(y,0)=\xi_0(y);\qquad \varphi(y,0)=\phi_0(y);\qquad \varphi_e(y,0)=\phi_{0e}(y) \tag{1.11}$$

$$\dot{\mathrm{w}}(y,0)=\zeta_0(y);\qquad \dot{\varphi}(y,0)=\psi_0(y);\qquad \dot{\varphi}_e(y,0)=\psi_{0e}(y) \tag{1.12}$$

The final state conditions for the electroacoustic wave field characteristics at a moment are written in the same manner

$$\mathrm{w}(y,T_0)=\tilde{\xi}_0(y);\qquad \varphi(y,T_0)=\tilde{\phi}_0(y);\qquad \varphi_{\pm e}(y,T_0)=\tilde{\phi}_{\pm e}(y) \tag{1.13}$$

$$\dot{\mathrm{w}}(y,T_0)=\tilde{\zeta}_0(y);\qquad \dot{\varphi}(y,T_0)=\tilde{\psi}_0(y);\qquad \dot{\varphi}_{\pm e}(y,T_0)=\tilde{\psi}_{\pm e}(y) \tag{1.14}$$

However, according to the quasi-static problem statement, the accompanying electrical potential vibrations $\Phi(x,y,t)$ and $\Phi_{\pm e}(x,y,t)$ have the same dynamics $\theta(t)$, as electroactive unidirectional elastic deformation waves $W(x,y,t)$

$$\left\{W(x,y,t);\ \Phi(x,y,t);\ \Phi_{\pm e}(x,y,t)\right\} = \left\{\mathrm{w}(y);\ \varphi(y);\ \varphi_{\pm e}(y)\right\}\cdot\exp(ikx)\cdot\theta(t).$$

Therefore, the initial oscillation values of the accompanying electric fields are described by functions $\phi_0(y)$, $\psi_0(y)$, $\phi_{0e}(y)$, $\psi_{0e}(y)$ will be associated with the functions describing the initial state of elastic displacement $\xi_0(y)$, $\tilde{\xi}_0(y)$, $\zeta_0(y)$, $\tilde{\zeta}_0(y)$. The following system of equations describes the connection between them

$$\frac{\xi_0''(y)-k^2\cdot\xi_0(y)}{\xi_0(y)}=\frac{\ddot{\theta}(t)}{\tilde{c}_t^2\theta(t)},\qquad \frac{\phi_0''(y)-k^2\cdot\phi_0(y)}{\phi_0(y)}=(e_{15}/\varepsilon_{11})\cdot\frac{\xi_0''(y)-k^2\cdot\xi_0(y)}{\xi_0(y)},$$

$$\phi_{\pm 0e}''(y)-\phi_{\pm 0e}(y)=0, \tag{1.15}$$

and the boundary conditions

$$\xi_{0,y}(y)\Big|_{y=\pm H_0}=\pm(e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\phi_{\pm 0e,y}(y)\Big|_{y=\pm H_0},$$

$$\phi_{0,y}(y)\Big|_{y=\pm H_0}=\mp(\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\phi_{\pm 0e,y}(y)\Big|_{y=\pm H_0};\qquad [\phi_0(y)-\phi_{\pm e}(y)]_{y=\pm H_0}=0 \tag{1.16}$$

Similarly, a mathematical boundary value problem is formulated for the functions describing the final state of the electroacoustic field $\tilde{\phi}_0(y), \tilde{\psi}_0(y)$, $\tilde{\phi}_{\pm e}(y)$, $\tilde{\psi}_{\pm e}(y)$ and $\tilde{\xi}_0(y)$, $\tilde{\zeta}_0(y)$.

Therefore, in the quasi-static formulation of the wave process control problem, the initial conditions of the electroacoustic wave field characteristics (1.7) and (1.8) in a moment $t=0$, and the final state conditions (1.9) and (1.10) in a moment $t=T_0$, will be written only for the leading component of the process

$$\mathrm{w}(y,0)=\xi_0(y),\qquad \dot{\mathrm{w}}(y,0)=\zeta_0(y) \tag{1.17}$$

$$\mathrm{w}(y,T_0)=\tilde{\xi}_0(y),\qquad \dot{\mathrm{w}}(y,T_0)=\tilde{\zeta}_0(y) \tag{1.18}$$

1.2 The reduced problem of the wave propagation control

The system of the quasi-static equations of electro elasticity has the form

$$\mathrm{w}''(y,t)-k^2\cdot\mathrm{w}(y,t)=(1/\tilde{c}_t^2)\cdot\ddot{\mathrm{w}}(y,t),$$

$$\psi''(y,t)-k^2\cdot\psi(y,t)=0 \tag{1.19}$$

With introduced function $\psi(y,t)=\varphi(y,t)-(e_{15}/\varepsilon_{11})\cdot\mathrm{w}(y,t)$.

On a mechanically free and electrically open surface $y=H_0$, we will have equivalent effects of the first kind

$$\mathrm{w}'(y,t)\Big|_{y=H_0}=(e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\frac{kh}{sh(kh)}\cdot\frac{\varphi_0(t)-ch(kh)\cdot\varphi(H_0)\cdot\theta(t)}{h} \tag{1.20}$$

$$\psi'(y,t)\Big|_{y=H_0}=-(\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\frac{kh}{sh(kh)}\cdot\frac{\varphi_0(t)-ch(kh)\cdot\varphi(H_0)\cdot\theta(t)}{h} \tag{1.21}$$

It is important to note that, in case of ultra-high frequency (the actions of the ultra-short length) actions, for which $2\pi h/\lambda\gg 1$, the mechanical surface tensions and electric surface polarization will have the following forms respectively:

$$\mathrm{w}'(y,t)\big|_{y=H_0} = (2\pi/\lambda_*)\cdot(e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\varphi(H_0)\cdot\theta(t)$$
$$\psi'(y,t)\big|_{y=\pm H_0} = -(2\pi/\lambda_*)\cdot(\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\varphi(H_0)\cdot\theta(t) \tag{1.22}$$

In case of ultralow frequencies of surface actions, for which $2\pi h/\lambda \ll 1$, the mechanical surface tension and electrical surface polarization will have the form, respectively

$$\mathrm{w}'(y,t)\big|_{y=\pm H_0} = (e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\frac{\varphi_0(t)-\varphi(H_0)\cdot\theta(t)}{h}$$
$$\psi'(y,t)\big|_{y=H_0} = (\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\frac{\varphi_0(t)-\varphi(H_0)\cdot\theta(t)}{h} \tag{1.23}$$

In both limiting cases (1.22) and (1.23), surface conditions (1.5) and (1.6) are identically satisfied.

On the middle surface of the waveguide $y=0$, the symmetry of the problem is given by the conditions of the first kind

$$\mathrm{w}(y,t)\big|_{y=0} = 0; \qquad \psi(y,t)\big|_{y=0} = 0 \tag{1.24}$$

Considering relations (1.20), (1.21) and (1.24), the new functions for electroactive elastic shear and the electric potentials are represented in the following form:

$$U(y,t) = \mathrm{w}(y,t) - y\delta_{\mathrm{w}}\cdot\mu(t) \tag{1.25}$$

$$\Psi(y,t) = \psi(y,t) + y\delta_{\psi}(kh)\cdot\mu(t) \tag{1.26}$$

In relations (1.25) and (1.26), a new designation is adopted:

$$\mu(t) = \left[\varphi_0(t) - ch(kh)\cdot\varphi(H_0)\cdot\theta(t)\right]/h \tag{1.27}$$

the total surface non acoustic contact action

$$\delta_{\mathrm{w}}(kh) = (e_{15}/\tilde{c}_{44})(\varepsilon_0/\varepsilon_{11})\cdot\frac{kh}{sh(kh)}, \qquad \delta_{\psi}(kh) = (\varepsilon_0/\tilde{\varepsilon}_{11})\cdot\frac{kh}{sh(kh)} \tag{1.28}$$

are a intensities of the non-acoustic surface actions.

Considering new boundary conditions (1.20), (1.21) and (1.24), as well as the introduced designations (1.25) ÷ (1.28), the boundary value problem (1.1) and (1.7)÷(1.9) is reduced to a system of inhomogeneous differential equations with separated variables

$$\ddot{U}(y,t) - \tilde{c}_t^2 U''(y,t) + \tilde{c}_t^2 k^2\cdot U(y,t) = -y\delta_w\cdot\left[\ddot{\mu}(t) + \tilde{c}_t^2 k^2\mu(t)\right] \tag{1.29}$$

$$\Psi''(y,t) - k^2\Psi(y,t) = -yk^2\delta_{\psi}(kh)\cdot\mu(t) \tag{1.30}$$

with a homogeneous boundary condition

$$U'(y,t)\big|_{y=H_0} = 0, \qquad U(y,t)\big|_{y=0} = 0 \tag{1.31}$$

$$\Psi'(y,t)\big|_{y=H_0} = 0, \qquad \Psi(y,t)\big|_{y=0} = 0 \tag{1.32}$$

The values of the introduced quantities $U(y,t)$, $\Psi(y,t)$ and their velocities $\dot{U}(y,t)$, $\dot{\Psi}(y,t)$, characterizing the initial and final states of the wave process at the moments $t=0$

and $t = T_0$ respectively, are determined from the known relations (1.17) and (1.18) , taking into account both of solution boundary value problem (1.29)÷(1.32) and the introduced notation (1.25)÷(1.28)

$$U(y,0) = \xi_0(y) - y\delta_{\text{w}} \cdot \mu(0)\,, \qquad \dot{U}(y,0) = \zeta_0(y) - y\delta_{\text{w}} \cdot \dot{\mu}(0) \tag{1.33}$$

$$U(y,T_0) = \tilde{\xi}_0(y) - y\delta_{\text{w}} \cdot \mu(T_0)\,, \qquad \dot{U}(y,T_0) = \tilde{\zeta}_0(y) - y\delta_{\text{w}} \cdot \dot{\mu}(T_0) \tag{1.34}$$

Thus, the control problem for unidirectional electroacoustic waves in the piezoelectric layer with a non acoustic action is already can by written as a nonhomogeneous differential equations (1.29) and (1.30), with the homogeneous boundary conditions (1.31) and (1.32), the initial and final states conditions (1.33) and (1.34) with account the relations (1.27), (1.28) and (1.29) with respect to reduced elastic shear and the electric potentials in the coordinate rectangle $Q_T = [0 \le y \le H_0] \times [0 \le t \le T_0]$.

2. Analytical solution of the waveform control problem.

It is necessary to find a unique solution to the initial boundary value problem: a control function $\mu(t)$ in the class $W_2^1[0,T_0]$, that satisfies the smoothness requirements, as well as functions of the required characteristics that satisfy the initial conditions (1.33), final conditions (1.34)

$$\{\xi_0(y,t); \tilde{\xi}_0(y,t)\} \in W_2^1[0,T_0]\,, \tag{2.1}$$

and boundary conditions (1.31) and (1.32), it is necessary for the problem of controlling wave propagation

$$\{\text{w}_0(y,t); \text{w}_0'(y,t)\} \in L_2[-H_0,H_0]\,. \tag{2.2}$$

Representing the solution to the boundary value control problem (1.29)-(1.32) by multiplying the functions of the separating variables allows us to present the characteristics of the wave in the form of Fourier series expansions of the distribution of the wave form over the thickness of the piezoelectric waveguide and the true harmonic of the process in terms of the harmonics of the surface action.

Considering homogeneous surface conditions (1.31) and (1.32), the generated wave with the corresponding harmonic for inhomogeneous equations (1.29) and (1.30) is obtained in the following form:

$$U(y,t) = \sum_{m=0}^{\infty} U_m(y) \cdot \theta_{0m}(t) = \sum_{m=0}^{\infty} A_m \sin(\alpha_{\text{w}m} y) \cdot \left[A_{0m} \sin(\omega_{0m} t) + B_{0m} \cos(\omega_{0m} t) \right] \tag{2.3}$$

$$\Psi(y,t) = \sum_{m=0}^{\infty} \Psi_m(y) \cdot \theta_{0m}(t) = \sum_{m=0}^{\infty} B_m \sin(k_m y) \cdot \left[A_{0m} \sin(\omega_{0m} t) + B_{0m} \cos(\omega_{0m} t) \right] \tag{2.4}$$

Here $\omega_{0m} = \tilde{c}_t k_m \cdot \sqrt{1 + (m\lambda_m / 4H_0)^2}$ are the frequencies of the eigen harmonics, and $\lambda_m = 2\pi / k_m$ is the length «***m***» forms of the wave.

Expanding all the factors in equation (1.29) and using the Fourier series over the thickness of the piezoelectric waveguide for the natural waveforms, the inhomogeneity leads to the appearance of new modes of vibration, which are the combined influence of the harmonic functions of the natural modes of vibration and the surface action function.

$$\begin{aligned}&\left[\ddot{\theta}_m(t)-C_m\cdot\delta_{wm}\cdot(1/\tilde{c}_t^2)\,\ddot{\mu}_m(t)\right]+\omega_{\theta m}^2\cdot\left[\theta(t)+C_m\cdot\delta_{wm}\cdot(1/\tilde{c}_t^2)\cdot\mu_m(t)\right]=\\&=-C_m\cdot\delta_w\cdot(1/\tilde{c}_t^2)\left[\omega_{\mu m}^2-\omega_{\theta m}^2\right]\cdot\mu_m(t)\end{aligned} \tag{2.5}$$

Here the Fourier coefficients $C_m=(1/H_0)\cdot\int_0^{H_0}y\cdot\sin[\alpha_m y]\cdot\mathrm{d}y$ are determined from right-hand side of the equation (1.29).

From equations (2.3) is also follows, that the true oscillation is formed as a sum of two oscillations with different frequencies. It is important that the frequency characteristics $\omega_{\theta m}=\tilde{c}_t k_m\cdot\sqrt{1+(m\lambda_m/4H_0)^2}$ and $\omega_{\varphi m}=\tilde{c}_t\cdot k_m$, as well as the coefficients $H_m(hk)=C_m\cdot\delta_w\cdot(1/\tilde{c}_t^2)\cdot(\omega_{\mu m}^2-\omega_{\theta m}^2)$ characterizing the intensity of the surface impact, are correspond to the eigenmodes of oscillation in the propagating wave. Hence, it is obvious that the frequency of the zero harmonic $\omega_{\theta 0}=\tilde{c}_t\cdot k_m$, will be resonant $\omega_{\varphi m}=\tilde{c}_t\cdot k_m=\omega_{\theta 0}$.

Therefore, this equation represents the relationship between natural vibration harmonics $\theta_m(t)$ and external surface action harmonics $\varphi_{0m}(t)$ for $m\geq 1$.

Equation (2.3) can be written as an infinite system of linear differential equations, representing both the true vibration harmonics and the harmonics of the surface action function:

$$\ddot{f}_m(\omega_{\theta m}t)+\omega_{\theta m}^2\cdot f_m(\omega_{\theta m}t)=H_m(hk_m)\cdot(\omega_{\theta m}^2-\omega_{\mu m}^2)\cdot\mu_m(\omega_{\mu m}t) \tag{2.6}$$

In equation (2.5), the true (total) vibration harmonic for the layer oscillation is represented in the form

$$f(t)=\sum_{m=1}^{\infty}f_m(t)=\sum_{m=1}^{\infty}\left[\theta_m(\omega_{\theta m}t)+H_m(hk)\cdot\mu_m(\omega_{\mu m}t)\right] \tag{2.7}$$

The dynamics function corresponding to the natural modes of oscillation of the layer $\theta(t)$ is determined from equations (2.3)

$$\theta(t)=\sum_{m=1}^{\infty}\theta_m(t)=\sum_{m=1}^{\infty}A_{\theta m}\cos(\omega_{\theta m}t)+B_{\theta m}\sin(\omega_{\theta m}t) \tag{2.8}$$

The surface action $\mu(t)$ of non-acoustic contact can be represented in a Fourier series, with indeterminate coefficients $A_{\mu m}$, $B_{\mu m}$ and new frequencies $\omega_{\mu m}$

$$\mu(t)=\sum_{m=1}^{\infty}\mu_m(t)=\sum_{m=1}^{\infty}\left[A_{\mu m}\cos(\omega_{\mu m}t)+B_{\mu m}\sin(\omega_{\mu m}t)\right] \tag{2.9}$$

The surface non-acoustic contact action function also decomposes into harmonics $\varphi_{0m}(t)=A_{\varphi m}\cos(\omega_{\varphi m}t)+B_{\varphi m}\sin(\omega_{\varphi m}t)$ corresponding to the eigenmodes of the layer vibration

$$\varphi_0(t)=\sum_{m=1}^{\infty}\varphi_{0m}(t)=\sum_{m=1}^{\infty}\left[A_{\varphi m}\cos(\omega_{\varphi m}t)+B_{\varphi m}\sin(\omega_{\varphi m}t)\right] \tag{2.10}$$

On the other hand, the total surface action is determined by (1.27), and therefore

$$\begin{gathered}\mu_m(t)=A_{\varphi m}\cos(\omega_{\varphi m}t)+B_{\varphi m}\sin(\omega_{\varphi m}t)-\\ -[ch(hk_m)/h]\cdot\varphi_m(H_0)\cdot\left[A_{\theta m}\cos(\omega_{\theta m}t)+B_{\theta m}\sin(\omega_{\theta m}t)\right]\end{gathered} \tag{2.11}$$

In the time interval $\left[0\le t\le T_0\right]$, the general solution for the m^{th} harmonic orthogonal, from the infinite system of equations (2.8), is obtained by the method of constant variations, in the form of the addition of the harmonics of the intrinsic and forced formation of forms

$$\begin{gathered}f_m(t)=A_{fm}\cdot\cos(\omega_{\theta m}t)+B_{fm}\cdot\sin(\omega_{\theta m}t)+\\ +H_m(hk_m)\cdot\left[A_{\mu m}\cos(\omega_{\mu m}t)+B_{\mu m}\sin(\omega_{\mu m}t)\right]\end{gathered} \tag{2.12}$$

The coefficients A_{fm}, B_{fm}, $A_{\mu m}$ and $B_{\mu m}$, like the frequencies $\omega_{\theta m}$ and $\omega_{\mu m}$, as well the coefficient of the intensity $H(hk_m)$ are determined by the physical constants of the medium and the geometric dimensions of the piezoelectric layer and vacuum gaps.

The boundary control problem of wave propagation decompound into an infinite number of boundary control problems of orthogonal waveforms with the corresponding harmonics of the boundary action.

It is necessary to take into account that equation (2.12) is characterizes the dynamics of m - number form in the process of wave formation with the eigenvalue $k_m=\sqrt{(\omega_{\theta m}^2/\tilde{c}_t^2)-(m\pi/2H_0)^2}$ and the frequency $\omega_{\theta m}=\tilde{c}_t k_m\cdot\sqrt{1+(m\lambda_m/4H_0)^2}$.

In the control process of the wave propagation, the boundary action $\varphi_0(t)$, with its action harmonics $\varphi_{0m}(t)$, is transmitted by the frequency $\omega_{\varphi m}(k_m)=\tilde{c}_t\cdot k_m$.

The current harmonic of wave formation $\theta(t)$, with its harmonics $\theta_m(t)$ is represented by the expansion of frequencies $\omega_{\theta m}(k_m)=\tilde{c}_t k_m\cdot\sqrt{1+(m\lambda_m/4H_0)^2}$.

The initial values of the deflection and it vibration speed functions $\xi_0(y)$ and $\tilde{\xi}_0(y)$, as well as the final values of its functions $\zeta_T(y)$ and $\tilde{\zeta}_T(y)$, we expand into the Fourier series, to satisfy the initial conditions (1.17) and the final conditions (1.8)

$$\xi_0(y)=\sum_{m=1}^{\infty}\gamma_m\cdot W_{0m}(y),\qquad \tilde{\xi}_0(y)=\sum_{m=1}^{\infty}\tilde{\gamma}_m\cdot W_{0m}(y) \tag{2.13}$$

$$\zeta_T(y)=\sum_{m=1}^{\infty}\delta_m\cdot W_{0m}(y),\qquad \tilde{\zeta}_T(y)=\sum_{n=1}^{\infty}\tilde{\delta}_n\cdot W_{0m}(y) \tag{2.14}$$

Comparing the obtained relations for the initial and terminal conditions (2.13) and (2.14) with the relations (2.7) and (2.9), we obtain the matching conditions at the beginning and at the end of the control process

$$\theta_m(0) + H(hk_m)\cdot\mu_m(0) = \gamma_m, \tag{2.15}$$
$$\dot{\theta}_m(0) + H(hk_m)\cdot\dot{\mu}_m(0) = \tilde{\gamma}_m, \tag{2.16}$$
$$\theta_m(T_0) + H(hk_m)\cdot\mu_m(T_0) = \delta_m, \tag{2.17}$$
$$\dot{\theta}_m(T_0) + H(hk_m)\cdot\dot{\mu}_m(T_0) = \tilde{\delta}_m. \tag{2.18}$$

Taking into account the representation of the deflection function $H_m(hk) = C_m\cdot\delta_w\cdot(1/\tilde{c}_t^2)\cdot(\omega_{\mu m}^2 - \omega_{\theta m}^2)$, expansions of the initial and final conditions for the elastic shear (2.8) and (2.9), and its velocities, the system of equations (2.15) ÷ (2.18) are written as an infinite system of linear algebraic equations in relatively to undefined amplitudes $A_{\varphi m}$, $B_{\varphi m}$, $A_{\theta m}$ and $B_{\theta m}$

$$\begin{aligned}
&A_{\theta m} + H_m(hk_m)\cdot A_{\mu m} = \gamma_m \\
&B_{\theta m} + (\omega_{\mu m}/\omega_{\theta m})\cdot H_m(hk_m)\cdot B_{\mu m} = \tilde{\gamma}_m/\omega_{\theta m} \\
&A_{\theta m}\cdot\cos(\omega_{\theta m}T_0) + B_{\theta m}\cdot\sin(\omega_{\theta m}T_0) + A_{\mu m}H_m(hk_m)\cdot\cos(\omega_{\mu m}T_0) + \\
&+B_{\mu m}H_m(hk_m)\cdot\sin(\omega_{\mu m}T_0) = \delta_m \\
&A_{\theta m}\cdot\sin(\omega_{\theta m}T_0) - B_{\theta m}\cdot\cos(\omega_{\theta m}T_0) + \\
&+(\omega_{\mu m}/\omega_{\theta m})\cdot H_m(hk_m)\cdot\left[A_{\mu m}\sin(\omega_{\mu m}T_0) - B_{\mu m}\cos(\omega_{\mu m}T_0)\right] = -\tilde{\delta}_m/\omega_{\theta m}
\end{aligned} \tag{2.19}$$

By finding four unknown constant coefficients $A_{\varphi m}$, $B_{\varphi m}$, $A_{\theta m}$ and $B_{\theta m}$, it is easy to construct the boundary control function $\varphi_0(t)$ according to (1.27) and the elastic shear distribution function $\mathrm{w}(y,t)$ in the electroacoustic wave, taking into account (1.25) and (2.3), in the coordinate rectangle $\left[-H_0 \le y \le H_0\right]\times\left[0 \le t \le T_0\right]$

$$\mathrm{w}(y,t) = \sum_{m=0}^{m_*} W_{0m}(y)\cdot f_m(t) = \sum_{m=1}^{m_*} W_{0m}(y)\cdot\left[\theta_m(\omega_{\theta m}t) + H_m(hk)\cdot\mu_m(\omega_{\mu m}t)\right] \tag{2.20}$$

$$\begin{aligned}
\varphi_0(t) = &\sum_{m=1}^{\infty}\left[\frac{sh(hk_m)}{hk_m}\cdot\frac{W'_{0m}(H_0)}{(e_{15}/c_{44})(\varepsilon_0/\varepsilon_{11})} + ch(hk_m)\cdot\Phi_{0m}(H_0)\right]\cdot\theta_m(\omega_{\theta m}t) + \\
&+\sum_{m=1}^{\infty}\frac{sh(hk_m)}{hk_m}\cdot\frac{W'_{0m}(H_0)\cdot H_m(hk)}{(e_{15}/c_{44})(\varepsilon_0/\varepsilon_{11})}\cdot\mu_m(\omega_{\mu m}t)
\end{aligned} \tag{2.21}$$

The required time of the edge action is defined as

$$T_0 = \max_{m\in[0;m_*]}\left\{\,2\pi/\min\{\omega_{\theta m}\};\ 2\pi/\min\{\omega_{\varphi m}\}\right\} \tag{2.22}$$

3. Analysis of numerical results for the surface control of electroacoustic wave propagation in the boundary value problem.

Considering the problem's symmetry, the solution and numerical calculations are performed in an area $\{|x|<\infty;\ 0\le y\le H_0;\ 0\le t\le T_0\}$, for a layer with a thickness of

$2H_0 = 2\times10^{-3} m$, for a piezoelectric material ***PZT – 4.*** This material is from class ***6mm*** of hexagonal symmetry and its physical mechanical constants are given in **Table 1**. Electrosed screens are located at a distance $h = 2.5\times10^{-4} m$ from the mechanically free surfaces of the piezoelectric layer.

It is worth paying attention to the fact that the intensity of the non-acoustic contact action $H_m(hk) = \delta_w(k_m h)\cdot D_m$, (transmission coefficient of the surface action) depends on the relative length $hk_m = 2\pi h/\lambda_m$ of the propagating wave signal modes. It is obvious from relations (2.5) and (2.12) that this dependence is true for all eigenforms of the wave. It is natural, that the non-acoustic contact action intensity for the materials with a smaller electromechanical coefficient is small. For the piezoelectric material ***PZT-4***, with an electromechanical coefficient $\chi^2 = 0.9409$ the action intensity coefficient for non-acoustic contact is small $5\ \% < H_m(hk) < 16\ \%$, for wavelengths $0 < 2\pi h/\lambda_m < 3$.

Table 1. Shear modules, densities and velocities of shear waves in piezoelectric crystal

	Shear module of the material $c_{44}\ (n/m^2)$	Density of the material $\rho\ (kg/m^3)$	Piezoelectric module $e_{15}(C/m^2)$	Dielectric permeability $\varepsilon_{11}/\varepsilon_0\ (F/m)$	coefficient of *EM* coupling χ^2	(SH) wave Velocity $\tilde{C}_t\ (m/\sec)$
PZT-4	2.56×10^{10}	7.5×10^3	12.7	6.45	0.9409	2.574×10^3

The structural symmetry of the problem allows us to write the reduced inhomogeneous equation (1.23) in the form

$$U_{,yy}(y,t) - k^2\cdot U(y,t) - (\omega^2/\tilde{c}_t^2)\cdot U(y,t) = y\delta_w\cdot(1/\tilde{c}_t^2)\cdot\ddot{\mu}(t) + k^2 y\delta_w\cdot\mu(t) \qquad (3.1)$$

With homogeneous boundary conditions

$$U(y,t)\big|_{y=0} = 0, \qquad U_{,y}(y,t)\big|_{y=H_0} = 0 \qquad (3.2)$$

Where $\delta_w = (e_{15}\varepsilon_0)/(hc_{44}\varepsilon_{11})\cdot(kh)/\sinh(kh)$ and $\mu(t) = \left[\varphi_0(t) - \varphi(H_0,t)\cdot\cosh[kh]\right]$.

For the true harmonics of oscillations $f_m(t) = \theta_m(t) - C_m\cdot\delta_w\cdot(1/\tilde{c}_t^2)\,\mu_m(t)$, as well as for the harmonics of the surface action function $\mu_m(t)$, an infinite system of linear differential equations is obtained

$$\ddot{f}_m(\omega_{\theta m}t) + \omega_{\theta m}^2\cdot f_m(\omega_{\theta m}t) = -C_m\cdot\delta_{wm}\cdot(1/\tilde{c}_t^2)\left(\omega_{\mu m}^2 - \omega_{\theta m}^2\right)\cdot\mu_m(\omega_{\mu m}t) \qquad (3.3)$$

It should be noted, that the coefficient $C_m\delta_{wm}\cdot(1/\tilde{c}_t^2)\cdot(\omega_{\mu m}^2 - \omega_{\theta m}^2)$ on the right-hand side of equation (3.3) indicates the intensity of the non-acoustic surface action on the propagation of an electro-elastic wave. As we can see, this coefficient is different for different waveforms

$k_m = m\pi/2H_0$ and for different vibration frequencies $\omega_{\theta m} = \tilde{C}_{1t}\sqrt{(\pi m)/(2H_0) + k_m^2}$ and $\omega_{\mu m} = \tilde{C}_{1t} k_m$.

Let's consider a low-frequency steady-state electro-elastic shear wave $W_{In}(x, y, 0)$, with a frequency $\omega_l = 5\times10^5\,Hz$ and a wavelength $\lambda_l = 0.001\,m$, that propagates along the waveguide. Initial state conditions in the control problem are written in the form

$$W_{In}(x,y,0) = \left[\cos\left[\frac{2\pi y}{\lambda_l}\cdot\sqrt{\frac{\lambda_l^2\omega_l^2}{(2\pi)^2\cdot\tilde{C}_{1t}^2} - 1}\right] + \sin\left[\frac{2\pi y}{\lambda_l}\cdot\sqrt{\frac{\lambda_l^2\omega_l^2}{(2\pi)^2\,\tilde{C}_{1t}^2} - 1}\right]\right]\times \\ \times\left[\cos\left(\lambda_l x\right) + \sin\left(\lambda_l x\right)\right] \tag{3.4}$$

$$\dot{W}_{In}(x, y, 0) = 0 \tag{3.5}$$

It is required to find such a surface action $\mu(t)$ that will bring the process into a state with an elastic displacement distribution $W_{Fi}(x, y)$, with a frequency $\omega_h = 2\times10^6\,Hz$ and a wavelength $\lambda_{sh} = 10^{-5}\ m$.

The final state conditions in the control problem are written in the form

$$W_{Fi}(x,y,T_0) = \left[\cosh\left[\frac{2\pi y}{\lambda_{sh}}\cdot\sqrt{1 - \frac{\lambda_{sh}^2\cdot\omega_h^2}{(2\pi)^2\cdot\tilde{C}_{1t}^2}}\right] + \sinh\left[\frac{2\pi y}{\lambda_{sh}}\cdot\sqrt{1 - \frac{\lambda_{sh}^2\cdot\omega_h^2}{(2\pi)^2\cdot\tilde{C}_{1t}^2}}\right]\right]\times \\ \times\left[\cos\left(\lambda_{sh}\cdot x\right) + \sin\left(\lambda_{sh}\cdot x\right)\right] \tag{3.6}$$

$$\dot{W}_{Fi}(x, y, T_0) = 0 \tag{3.7}$$

The initial values of the deflection and its vibration speed functions (3.4) and (3.5), as well as the final values of the deflection and its vibration speed functions (3.6) and (3.7), we expand into the Fourier series, to satisfy the initial conditions (1.13) and the final conditions (1.4)

$$W_{In}(y,0) = \sum_{m=1}^{\infty}\gamma_m\cdot\sin(\alpha_m y), \qquad \dot{W}_{In}(y,0) = 0 \tag{3.8}$$

$$W_{Fi}(y,T_0) = \sum_{m=1}^{\infty}\delta_m\cdot\sin(\alpha_m y), \qquad \dot{W}_{Fi}(y,T_0) = 0 \tag{3.9}$$

Taking into account (3.8) and (3.9), from the system of equations (2.20) we find four groups of unknown coefficients $A_{\varphi m}$, $B_{\varphi m}$, $A_{\theta m}$ and $B_{\theta m}$. The non-acoustic surface action function $\mu(t)$ can be represented as an expansion with already defined coefficients

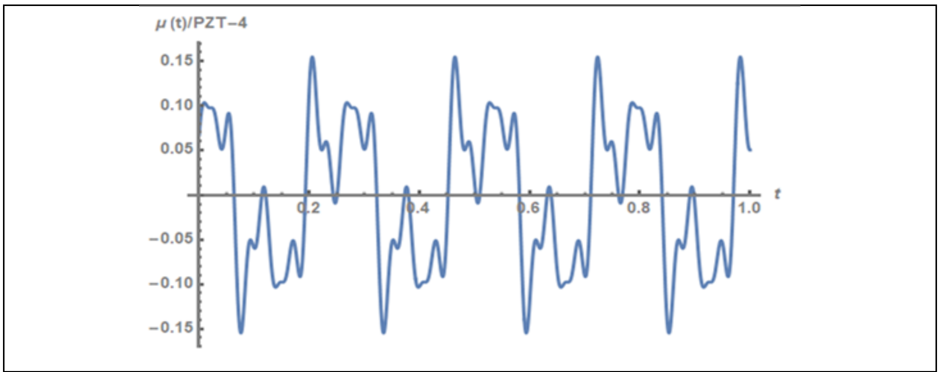

Figure 2. *The graph of the function of surface action* $\mu(t)$*, when m=10.*

$$\mu(t)=\sum_{m=1}^{\infty}\left[A_{\varphi m}\cos(\omega_{\varphi m}t)+B_{\varphi m}\sin(\omega_{\varphi m}t)\right]- \\ -\Phi(H_0)\cdot\sum_{m=1}^{\infty}\cosh(k_m h)\cdot\left[A_{\theta m}\cos(\omega_{\theta m}t)+B_{\theta m}\sin(\omega_{\theta m}t)\right] \tag{3.10}$$

The graph of the non-acoustic surface action function $\mu(t)$ when $m=10$, is given in Figure 2.

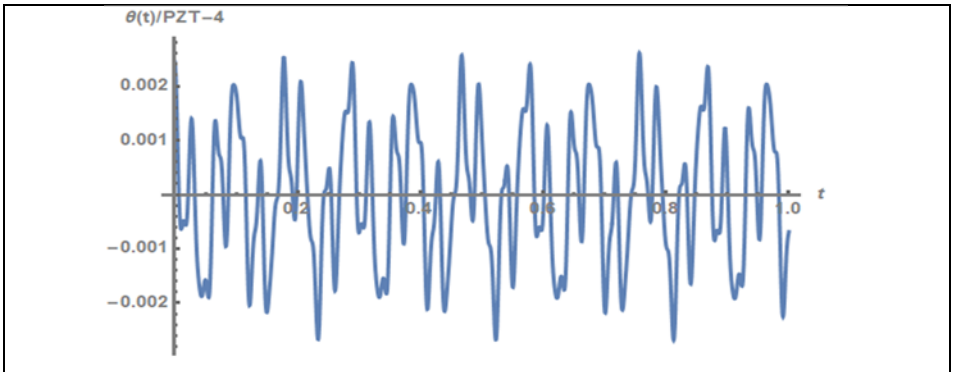

Figure 3. *The graph of the function of natural vibrations* $\theta(t)$*, when m=10.*

The graph of the function $\theta(t)$ of the corresponding to the eigenmodes of the layer vibration of dynamics is given in Figure 3, in case when $m=10$.

The function of the true oscillations $f(t)$ can be represented as

$$f(t)=\sum_{m=1}^{\infty}\begin{pmatrix} A_{\theta m}\cdot\cos(\omega_{\theta m}t)+B_{\theta m}\cdot\sin(\omega_{\theta m}t)+ \\ +C_m\cdot\delta_{wm}\cdot(1/\tilde{c}_t^2)\cdot\left(\omega_{\mu m}^2-\omega_{\theta m}^2\right)\cdot\left[A_{\mu m}\sin(\omega_{\mu m}t)+B_{\mu m}\cos(\omega_{\mu m}t)\right]\end{pmatrix} \tag{3.11}$$

The figures of these functions are obtained in the form (Figures 2÷4)

Conclusions

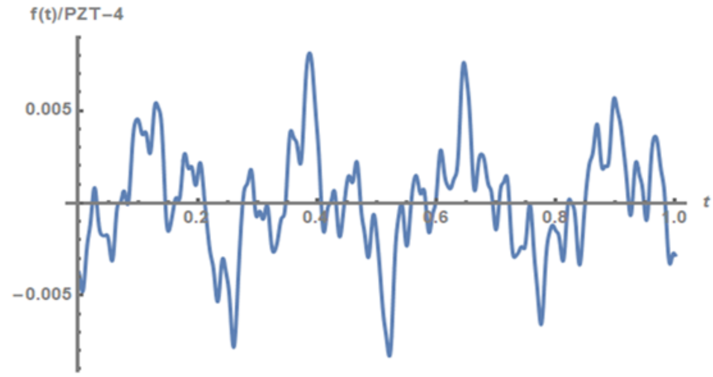

Figure 4 *The graph of the function of true oscillations* $f(t)$ *, when m=10.*

The problem of controlling wave formation and propagation of electroactive shear waves in a piezoelectric waveguide has been solved by applying an electric potential to the mechanically free surfaces of the layer without contact.

In the quasi-static formulation of the control problem for the formation and propagation of an electroelastic wave, the initial and final conditions are set only for elastic shear. The initial and final functions for the accompanying components are derived from the basic equations.

The boundary value problem is simplified into an infinite system of control problems for shaping and propagating eigenforms of electroactive shear waves by expanding the functions of two variables into Fourier series and using harmonics of surface action.

By solving the infinite system of differential equations, we obtain the function of surface action and the true harmonics of wave vibration, which brings the wave surface from a known initial state to a given final state.

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