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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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CONTROL OF A SECOND-ORDER ELECTROMECHANICAL SYSTEM UNDER MIXED CONSTRAINTS

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Key words: control of an electromechanical system, limited voltage, limited heat dissipation power

For a system simulating the dynamics of a single-link electromechanical manipulator, the problem of constructing a control voltage and determining the areas of final states is solved, the transition to any point of which from the initial state of rest using the constructed control occurs in a finite time without violating the specified restrictions on the voltage and heat release power in the winding of the rotor of the electric motor.

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Управление электромеханической системой второго порядка при смешанных ограничениях

Ключевые слова: управление электромеханической системой, ограниченное напряжение, ограниченная мощность тепловыделения

Для системы, моделирующей динамику однозвенного электромеханического манипулятора, решается задача построения управляющего напряжения и определения областей конечных состояний, переход в любую точку которых из начального состояния покоя с помощью построенного управления происходит за конечное время без нарушения заданных ограничений на напряжение и мощность тепловыделения в обмотке ротора электродвигателя.

Ավետիսյան Վ.Վ.

Խառը սահմանափակումներով երկրորդ կարգի էլեկտրամեխանիկական համակարգի ղեկավարումը

Բանալի բառեր. էլեկտրամեխանիկական համակարգի ղեկավարում, սահմանափակ լարում, ջերմաանջատման սահմանափակ հզորություն

Մեկ օղակով էլեկտրամեխանիկական մանիպուլյատորի դինամիկան նկարագրող համակարգի համար լուծվում է ղեկավարող լարման և վերջնական վիճակների տիրույթների կառուցման խնդիրը, որոնց յուրաքանչյուր կետ սկզբնական հանգստի վիճակից համակարգի տեղափոխումը կառուցված ղեկավարման օգնությամբ իրականացվում է վերջավոր ժամանակում՝ առանց լարման և էլեկտրաշարժիչի ռոտորի փաթույթի հզորության վրա դրված տրված սահմանափակումների խախտման։

Introduction

An electromechanical system of the second order is considered, which approximately describes the dynamics of a separate arm link of a multilink manipulator, if each link is controlled by a voltage supplied by an independent drive electric motor, and the dynamic mutual influence of various degrees of freedom is sufficiently small [1]. The system under consideration is distinguished by the feature that in the class of bounded controls it is controllable to a state of rest, but is not controllable with respect to an arbitrary state other than rest. For the considered model, in [1-8], the problems of constructing an optimal control that ensures the movement of the system from an arbitrary initial state to a given final state of rest, including under additional restrictions, were studied. For some higher-order systems, in particular, for fourth-order systems with mixed constraints, which are models of mechanical and electromechanical systems containing an electric motor, in [9-11] the problems of constructing a bounded control that bring the system from an arbitrary initial state to terminal state of rest in a finite time. In this paper, we solve the problem of constructing a control for the movement of an electromechanical system with one degree of freedom from the initial state of rest to a given final state with restrictions on the control voltage and on the power of heat generation in the winding of the rotor of the electric motor. The presence of a restriction on the heat release power is the main difference between this problem and the problem considered in [12]. Conditions are obtained that on the phase plane of the system form areas of final states, a controlled transition to any point of which occurs in a finite time without violating the specified restrictions.

1. Calculation model of an electromechanical system and problem statement

Consider an electromechanical system consisting of a electric motor with independent excitation, a gearbox and an absolutely rigid body (inertial load) on its output shaft. Such a system can be interpreted as a model of the simplest manipulator with one degree of freedom. In this case, the inertial load is the arm of the manipulator together with the load fixed in its gripper. The movement of the described electromechanical system is determined by the equations [1]

$$(I+Jn^2)\ddot{\varphi}=n\mu, \qquad Rj+kn\dot{\varphi}=u, \qquad \mu=kj\,. \tag{1.1}$$

In (1.1) φ is the angle of rotation of the arm; I is the moment of inertia together with the driven gear of the reducer; J is the moment of inertia of the rotor of the electric motor together with the driven gear of the gearbox; n is the gear ratio of the drive gearbox; R is the electrical resistance of the rotor winding of the electric motor; μ is the moment of electromagnetic forces generated by the engine; j is the current in the motor rotor circuit; k is a constant (electric motor parameter); u is the input (control) voltage of the electric motor. We will assume that the maximum voltage allowed during engine operation is limited

$$|u|\le U\,. \tag{1.2}$$

The first equation in (1.1) describes the dynamics of the mechanical part of the system, the second equation describes the voltage balance in the motor rotor circuit, if we neglect the phenomenon of self-induction in the rotor winding, the third equation in (1.1) reflects the proportionality of the motor torque and current in its rotor circuit. Neglecting the phenomenon of self-induction when describing the dynamics of the manipulator is possible if the electromagnetic time constant $\tau=L/R$ (L is the inductance of the rotor winding) is much less than the operating time of the manipulator. In practice, this condition is met in most cases. Note that the system of equations (1.1) approximately describes the dynamics of an individual arm link of a multilink manipulator if each link is controlled by an independent drive and the dynamic mutual influence of different degrees of freedom is sufficiently small (see, for example, [1]).

Eliminating the variable μ from (1.1) and in the resulting system with constraint (1.2) passing to dimensionless units (with subsequent omitting primes)

$$t' = \frac{t}{\bar{T}}, \quad u' = \frac{u}{U}, \quad k' = \frac{kn}{U\bar{T}}, \quad R' = \frac{RA}{knU\bar{T}^2}, \quad j' = \sqrt{\frac{knU\bar{T}^5}{A^2}}\, j, \tag{1.3}$$

where $A = I + Jn^2$ and $\bar{T} = \dfrac{nk}{U}$ is the unit of time, we get the following system

$$\ddot{\varphi} = \sqrt{k}\, j, \quad Rj + \sqrt{k}\dot{\varphi} = \frac{u}{\sqrt{k}}, \tag{1.4}$$

$$|u| \le 1. \tag{1.5}$$

In this paper, along with the constraint (1.5), we consider the constraint on the heat release power in the rotor winding of the drive motor. The amount of heat q released in the rotor winding of the electric motor per unit of time (heat dissipation power) is $q = j^2 R$. Substituting j from the second equation (1.1) into the last equality, we obtain (in the initial dimensional variables) the following expression for the amount of heat released in the motor rotor winding $q = \dfrac{(u - kn\dot{\varphi})^2}{R}$. When the drive is continuously loaded, the heat generated in the rotor winding heats up the electric motor. In order to avoid severe overheating and burnout of the motor winding, the heat dissipation power is limited from above $q \le q^0$:

$$q = \frac{(u - kn\dot{\varphi})^2}{R} \le q^0. \tag{1.6}$$

Passing to dimensionless variables (1.3), the value q is transformed according to formula $q' = \dfrac{\bar{T}^3}{A} q$, and the constraint (1.6) (with the strokes omitted) is reduced to the limitation on the angular velocity of the manipulator and on the allowable value of the input voltage:

$$|u - k\dot{\varphi}| \le \sqrt{kRq^0} = \eta. \tag{1.7}$$

Consider the control problem for system (1.4).

Problem. Find the law of change of the control voltage $u(t)$, which provides bringing the manipulator from the initial state of rest

$$\varphi(0) = 0, \quad \dot{\varphi}(0) = 0 \tag{1.8}$$

to a given final state

$$\varphi(T) = \varphi^1, \quad \dot{\varphi}(T) = \dot{\varphi}^1 \tag{1.9}$$

at some point in time $t = T$ under restrictions on the control voltage (1.5) and the heat release power in the motor rotor winding (1.7).

Eliminating the variable j from system (1.4), we obtain the equation

$$R\ddot{\varphi}+k\dot{\varphi}=u \tag{1.10}$$

and passing to new variables in (1.5), (1.7)-(1.10)

$$\mathrm{v}=u-k\varphi, \qquad z_1=R\varphi, \qquad z_2=R\dot{\varphi}, \tag{1.11}$$

equation (1.10) with initial (1.8) and final (1.9) conditions can be written as

$$\dot{z}_1=z_2, \quad \dot{z}_2=\mathrm{v}\,, \tag{1.12}$$

$$z_1(0)=0\,, \quad z_2(0)=0\,, \tag{1.13}$$

$$z_1(T)=z_1^1=R\varphi^1\,, \quad z_2(T)=z_2^1=R\dot{\varphi}^1\,. \tag{1.14}$$

In this case, constraints (1.5) and (1.7) take, respectively, the form

$$\left|\mathrm{v}+\frac{k}{R}z_2\right|\le 1, \tag{1.15}$$

$$|\mathrm{v}|\le\eta\,. \tag{1.16}$$

Thus, the problem formulated above passes to the equivalent problem with mixed constraints (1.12)-(1.16).

2. Construction of the law of change of the control voltage, taking into account the restrictions

The law of change of the control voltage $\mathrm{v}(t)$, which ensures the transition of the system (1.12) from the initial state of rest (1.13) to the final state (1.14) at time T without taking into account the restrictions (1.15), (1.16) is constructed using the Kalman method described in [9] and has the following view [12]:

$$\mathrm{v}(t)=\left(-\frac{12}{T^3}t+\frac{6}{T^2}\right)z_1^1+\left(\frac{6}{T^2}t-\frac{2}{T}\right)z_2^1\,. \tag{2.1}$$

Integrating equations (1.12) with initial conditions (1.13) under control (2.1), we find

$$z_2=\left(-\frac{6}{T^3}t^2+\frac{6}{T^2}t\right)z_1^1+\left(\frac{3}{T^2}t^2-\frac{2}{T}t\right)z_2^1\,. \tag{2.2}$$

Taking into account (2.1), (2.2), constraints (1.15), (1.16) can be written in the form

$$\begin{aligned}&\left|h_1(t,T)z_1^1+h_2(t,T)z_2^1\right|\le 1,\\ &\left|f_1(t,T)z_1^1+f_2(t,T)z_2^1\right|\le\eta,\end{aligned}\qquad t\in[0,T], \tag{2.3}$$

Where

$$h_1(t,T)=-\frac{6k}{RT^3}t^2+\left(\frac{6k}{RT^2}-\frac{12}{T^3}\right)t+\frac{6}{T^2}\,,$$

$$h_2(t,T)=\frac{3k}{RT^2}t^2+\left(\frac{6}{T^2}-\frac{2k}{RT}\right)t-\frac{2}{T}, \tag{2.4}$$

$$f_1(t,T)=-\frac{12}{T^3}t+\frac{6}{T^2}, \quad f_2(t,T)=\frac{6}{T^2}t-\frac{2}{T}.$$

Let us turn to the analysis of constraints (2.3). To satisfy these restrictions, it suffices to require that the following inequalities hold:

$$\begin{aligned} &|h_1(t,T)||z_1^1|+|h_2(t,T)||z_2^1|\le 1, \\ &|f_1(t,T)||z_1^1|+|f_2(t,T)||z_2^1|\le \eta. \end{aligned} \qquad t\in[0,T]. \tag{2.5}$$

Inequalities (2.5) must hold for all $t\in[0,T]$. First, let us estimate from above the maximum (with respect to $t\in[0,T]$) values of the quantities $|h_i(t,T)|$, $|f_i(t,T)|$, $i=1,2$, appearing in (2.5). From (2.4) it follows that the quadratic function $h_1(t,T)$ with respect to t reaches its maximum value at the point $t_1^*=\frac{T}{2}-\frac{R}{k}$. There are the following two cases. If $t_1^*=\frac{T}{2}-\frac{R}{k}\le 0$, i.e. $0<T\le\frac{2R}{k}$, then the function $h_1(t,T)$ monotonically decreases on $[0,T]$, taking the maximum and minimum values of $h_1(0,T)=\frac{6}{T^2}$, $h_1(T,T)=-\frac{6}{T^2}$ at the ends of this interval, respectively. Hence,

$$\max_{t\in[0,T]}|h_1(t,T)|=\frac{6}{T^2}, \quad 0<T\le\frac{2R}{k}. \tag{2.6}$$

If $0\le t_1^*=\frac{T}{2}-\frac{R}{k}<\infty$, i.e. $\frac{2R}{k}\le T<\infty$, then on interval $[0,T]$ the maximum value of function $h_1(t,T)$ is $h_1(t_1^*,T)=\frac{3k}{2RT}+\frac{6R}{kT^3}$, and the minimum is $h_1(T,T)=-\frac{6}{T^2}$. Thus $|h_1(t_1^*,T)|>|h_1(T,T)|$, when $T\in\left[\frac{2R}{k},\infty\right)$ and $|h_1(t_1^*,T)|=|h_1(T,T)|$, when $T=T'=\frac{2R}{k}$. Hence,

$$\max_{t\in[0,T]}|h_1(t,T)|=\frac{3k}{2RT}+\frac{6R}{kT^3}, \quad \frac{2R}{k}\le T<\infty. \tag{2.7}$$

In accordance with the cases considered, we have

$$\max_{t\in[0,T]}|h_1(t,T)| = \bar{h}_1(T) = \begin{cases} \dfrac{6}{T^2}, & 0 < T \le \dfrac{2R}{k} = T', \\ \dfrac{3k}{2RT} + \dfrac{6R}{kT^3}, & T' \le T < \infty. \end{cases} \tag{2.8}$$

Further, from (2.4) it follows that the quadratic function $h_2(t,T)$ with respect to t reaches its maximum value at the point $t_2^* = \dfrac{T}{3} - \dfrac{R}{k}$. If $t_2^* = \dfrac{T}{3} - \dfrac{R}{k} \le 0$, i.e. $0 < T \le \dfrac{3R}{k}$, then the function $h_2(t,T)$ monotonically increases on $[0,T]$, taking the minimum and maximum values of $h_2(0,T) = -\dfrac{2}{T}$, $h_2(T,T) = \dfrac{k}{R} + \dfrac{4}{T}$ at the ends of this interval, respectively. Therefore

$$\max_{t\in[0,T]}|h_2(t,T)| = \frac{k}{R} + \frac{4}{T}, \quad 0 < T \le \frac{3R}{k} = T''. \tag{2.9}$$

If $0 \le t_2^* = \dfrac{T}{3} - \dfrac{R}{k}$, i.e. $\dfrac{3R}{k} = T'' \le T < \infty$, then the function $h_2(t,T)$ monotonically decreases on the interval $[0,t_2^*]$ and monotonically increases on the interval $[t_2^*,T]$. At the same time $h_2(0,T) = -\dfrac{2}{T}$, $h_2(t_2^*,T) = -\dfrac{3R}{kT^2} - \dfrac{k}{3R} < 0$, $h_2(T,T) = \dfrac{k}{R} + \dfrac{4}{T} > 0$. Inequality $|h_2(t_2^*,T)| \ge |h_2(T,T)|$ performed when $T \in [0,T''']$, $T''' = \dfrac{3mR}{k}$, $0 < m = \sqrt{\dfrac{3}{2}} - 1 < 1$. But, since $[0,T'''] \cap [T'',\infty] = \varnothing$, the last inequality is not satisfied on the interval $[T'',\infty]$. The reverse inequality holds for $T \in [\dfrac{3mR}{k},\infty)$, i.e. at $T \in [\dfrac{3mR}{k},\infty) \cap [\dfrac{3R}{k},\infty) = [\dfrac{3R}{k},\infty)$. Hence,

$$\max_{t\in[0,T]}|h_2(t,T)| = |h_2(T,T)| = \frac{k}{R} + \frac{4}{T}, \quad T'' < T < \infty. \tag{2.10}$$

Taking into account (2.9) and (2.10), we obtain

$$\max_{t\in[0,T]}|h_2(t,T)| = \frac{k}{R} + \frac{4}{T}, \quad 0 \le T < \infty. \tag{2.11}$$

Functions $f_1(t,T), f_2(t,T)$ (2.4) with respect to t are linear and increasing, therefore, on the interval $[0,T]$

$$\max_{t\in[0,T]}|f_1(t,T)| = \max_{t\in[0,T]}\left|-\frac{12}{T^3}t+\frac{6}{T^2}\right| = \frac{6}{T^2}, \tag{2.12}$$

$$\max_{t\in[0,T]}|f_2(t,T)| = \max_{t\in[0,T]}\left|\frac{6}{T^2}t-\frac{2}{T}\right| = \frac{4}{T}. \tag{2.13}$$

Substituting estimates (2.8), (2.11)-(2.13) into (2.5) we obtain

$$\begin{aligned}
&|h_1(t,T)||z_1^1|+|h_2(t,T)||z_2^1| \le \bar{h}_1(T)|z_1^1|+\left(\frac{k}{R}+\frac{4}{T}\right)|z_2^1| \le 1,\\
&|f_1(t,T)||z_1^1|+|f_2(t,T)||z_2^1| \le \frac{6}{T^2}|z_1^1|+\frac{4}{T}|z_2^1| \le \eta,
\end{aligned} \tag{2.14}$$

where the function $\bar{h}_1(T)$ is given by formula (2.8).

The set of two inequalities (2.14) represents sufficient conditions for the solvability of the posed control problem (1.12)-(1.16). These conditions, linking the final state and the process time can be considered as sufficient controllability of the system from the initial state of rest to the given final state (z_1^1, z_2^1) in time T.

Right relations (2.14) will be considered in the case of equality, written in the form

$$H(T,|z_1^1|,|z_2^1|) = 1, \qquad 0 < T < \infty, \tag{2.15}$$

$$F(T,|z_1^1|,|z_2^1|) = \eta, \qquad 0 < T < \infty, \tag{2.16}$$

where

$$H(T,|z_1^1|,|z_2^1|) = \begin{cases} \dfrac{6}{T^2}|z_1^1|+\left(\dfrac{k}{R}+\dfrac{4}{T}\right)|z_2^1|, & 0 < T \le T', \\ \left(\dfrac{3k}{2RT}+\dfrac{6R}{kT^3}\right)|z_1^1|+\left(\dfrac{k}{R}+\dfrac{4}{T}\right)|z_2^1|, & T' \le T < \infty. \end{cases} \tag{2.17}$$

$$F(T,|z_1^1|,|z_2^1|) = \frac{6}{T^2}|z_1^1|+\frac{4}{T}|z_2^1|, \qquad 0 < T < \infty. \tag{2.18}$$

Since the left-hand side of equality (2.17) is positive, it follows that $|z_2^1| < \dfrac{R}{k}$. Functions (2.17) and (2.18) are continuous with respect to T and decrease monotonically from ∞ to 0 as changes T from 0 to ∞. Therefore, for any pair of final states $(z_1^1, z_2^1) \in Z$, where

$$Z = \left\{(z_1^1, z_2^1): \quad |z_1^1| < \infty, \ \ |z_2^1| < \frac{R}{k}\right\}, \tag{2.19}$$

inequalities (2.5) will hold for all $t \in [0,T]$, if the end time of the process T is chosen from conditions (2.15), (2.16).

If we denote by T_1 and T_2 the solutions of equations (2.15) and (2.16), respectively, then constraints (2.3) are not violated for any t from the interval $[0,T^*]$, where

$$T^* = \max(T_1,T_2). \tag{2.20}$$

Consider two regions of change of the pair $(z_1^1, z_2^1) \in Z$ (fig. 1):

$$Z_- = \left\{ (z_1^1, z_2^1) \in Z: \ \left|z_2^1\right| \ge \frac{R}{3k} - \frac{k}{2R}\left|z_1^1\right| \right\}, \tag{2.21}$$

$$Z_+ = \left\{ (z_1^1, z_2^1) \in Z: \ \left|z_2^1\right| \le \frac{R}{3k} - \frac{k}{2R}\left|z_1^1\right| \right\}. \tag{2.22}$$

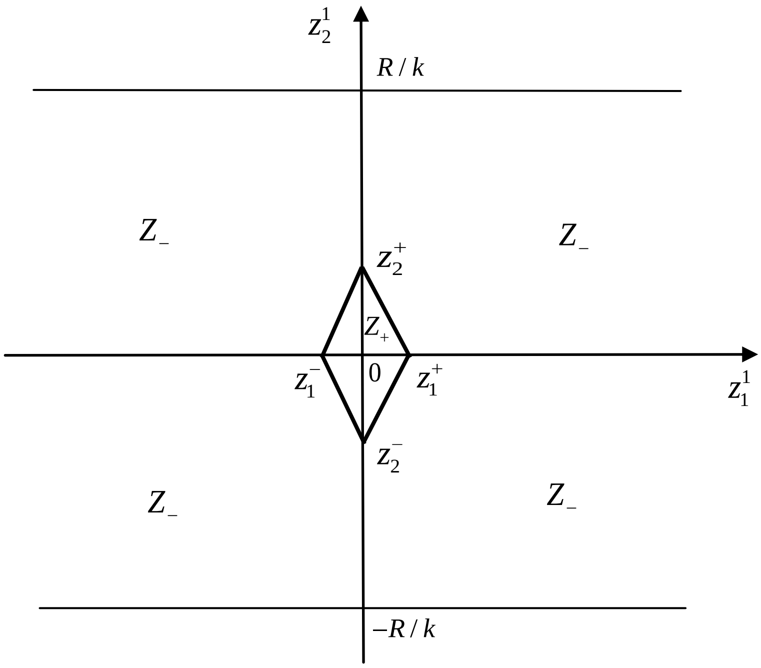

Fig. 1. Regions of final states (z_1^1, z_2^1), where the system can be brought from the initial state of rest $(0,0)$.

Bold lines in fig. 1 shows the border between the regions Z_- and Z_+, which is a rhombus centered at the point $(0,0)$. The vertices of the rhombus are located at the points $z_1^\pm = \left(\pm\frac{2R^2}{3k^2}, 0 \right)$, $z_2^\pm = \left(0, \pm\frac{R}{3k} \right)$.

In accordance with (2.15), (2.17), the intervals $(0,T']$ and $[T',\infty)$ correspond to the regions (2.21) and (2.22).

Then, if $(z_1^1, z_2^1) \in Z_-$, then the desired $T_1 \in (0, T']$ is found from the equation

$$H(T, |z_1^1|, |z_2^1|) = \frac{6}{T^2}|z_1^1| + \left(\frac{k}{R} + \frac{4}{T}\right)|z_2^1| = 1$$

and is determined by the formula

$$T_1^{(-)} = \frac{2|z_2^1| + \sqrt{4|z_2^1|^2 + 6\left(1 - \frac{k}{R}|z_2^1|\right)|z_1^1|}}{1 - \frac{k}{R}|z_2^1|}. \tag{2.23}$$

And if $(z_1^1, z_2^1) \in Z_+$, then the desired $T_1 \in [T', \infty)$ is determined from the equation

$$H(T, |z_1^1|, |z_2^1|) = \left(\frac{3k}{2RT} + \frac{6R}{kT^3}\right)|z_1^1| + \left(\frac{k}{R} + \frac{4}{T}\right)|z_2^1| = 1,$$

which is reduced to the cubic equation with negative discriminant

$$aT^3 + bT^2 + d = 0, \qquad a = \frac{2k}{R}\left(1 - \frac{k}{R}|z_2^1|\right) > 0,$$

$$b = -\left(\frac{3k}{R}|z_1^1| + 8|z_2^1|\right)\frac{k}{R} < 0, \qquad d = -12|z_1^1| < 0.$$

This equation has one real positive root, which is determined using Cardano's formula

$$T_1^{(+)} = \sqrt[3]{-\frac{\alpha}{2} + \sqrt{Q}} + \sqrt[3]{-\frac{\alpha}{2} - \sqrt{Q}} - \frac{b}{3a}, \quad Q = \left(\frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{3}\right)^3 > 0, \tag{2.24}$$

$$\alpha = \frac{2b^3}{27a^3} + \frac{d}{a} = -\frac{\left(\frac{3k}{R}|z_1^1| + 8|z_2^1|\right)^3}{108\left(1 - \frac{k}{R}|z_2^1|\right)^3} - \frac{6|z_1^1|}{\frac{k}{R}\left(1 - \frac{k}{R}|z_2^1|\right)},$$

$$\beta = -\frac{b^2}{3a^2} = -\frac{\left(\frac{3k}{R}|z_1^1| + 8|z_2^1|\right)^2}{12\left(1 - \frac{k}{R}|z_2^1|\right)^2}, \qquad \frac{b}{3a} = -\frac{\frac{3k}{R}|z_1^1| + 8|z_2^1|}{6\left(1 - \frac{k}{R}|z_2^1|\right)},$$

The solution of equation (2.16) for any pair $(z_1^1, z_2^1) \in Z$ (2.19) has the form

$$T_2 = \frac{2|z_2^1| + \sqrt{4|z_2^1|^2 + 6\eta|z_1^1|}}{\eta}. \tag{2.25}$$

Taking into account (2.23)-(2.25), from (2.20) we obtain

$$T^* = \begin{cases} \max\left(T_1^{(-)}, T_2\right), \ if \ \ (z_1^1, z_2^1) \in Z_-, \\ \max\left(T_1^{(+)}, T_2\right), \ if \ \ (z_1^1, z_2^1) \in Z_+. \end{cases} \tag{2.26}$$

Thus, the desired control $u(t)$ is constructed according to the following sequence: 1) for a given final state $(z_1^1, z_2^1) \in Z$ (2.19) we determine the time of motion T^* (2.26); 2) fixing any $T \geq T^*$, we find the auxiliary control function $\mathrm{v}(t)$ and the phase variable $z_2(t)$ using explicit formulas (2.1), (2.2); 3) passing to the original variables by formulas (1.3), (1.11) we find the control $u(t)$:

$$\begin{gathered} u = At^2 + Bt + C, \quad A = -\frac{3k}{R^2T^2}\left(\frac{2}{T}\varphi^1 - \dot{\varphi}^1\right), \\ B = -\left[\frac{2}{RT^2}\left(\frac{6}{T} - \frac{3k}{R}\right)\varphi^1 + \frac{2}{RT}\left(\frac{k}{R} - \frac{3}{T}\right)\dot{\varphi}^1\right], \quad C = \frac{2}{RT}\left(\frac{3}{T}\varphi^1 - \dot{\varphi}^1\right). \end{gathered} \tag{2.27}$$

3. Results of numerical calculations

Let us give a numerical example of the implementation of the proposed control construction algorithm. Let us assume that the manipulator is characterized by the following dimensional parameters appearing in (1.1), (1.2), (1.6)[1,4]:

$$\begin{gathered} I = 5.9\,\mathrm{kg}\cdot\mathrm{m}^2, \quad J = 2.45\cdot 10^{-4}\mathrm{kg}\cdot\mathrm{m}^2, \quad n = 163, \quad R = 3.6\,\mathrm{Ohm}, \\ k = 0.233\,\mathrm{N}\cdot\mathrm{m/A}, \quad U = 110\mathrm{V}, \quad j^0 = 2\,\mathrm{A}. \end{gathered} \tag{3.1}$$

After passing to dimensionless parameters according to (1.3), for the system parameters (1.7), (1.10) from (3.1) we obtain the following values:

$$R \approx 0.09, \quad k \approx 1, \quad \eta \approx 0.07. \tag{3.2}$$

For values (3.2), according to the method described in Section 2, using the transition formula (1.11), on the phase plane of the final states of system (1.12), regions (2.19), (2.21), (2.22) were constructed:

$$Z = \left\{(z_1^1, z_2^1): \quad \left|z_1^1\right| < \infty, \ \ \left|z_2^1\right| < 0.09\right\}, \tag{3.3}$$

$$Z_- = \left\{(z_1^1, z_2^1) \in Z: \quad \left|z_2^1\right| \geq 0.03 - 5.6\left|z_1^1\right|\right\}, \tag{3.4}$$

$$Z_+ = \left\{(z_1^1, z_2^1) \in Z: \quad \left|z_2^1\right| \leq 0.03 - 5.6\left|z_1^1\right|\right\}. \tag{3.5}$$

Let us choose the final state of the system (1.12) from the region (3.3) as follows:

$$(z_1^1, z_2^1) = (0.27;\ 0.08). \tag{3.6}$$

Point (3.6) belongs to the region Z_-. Therefore, when using control (2.27), the time of moving the system (1.12) from the initial state of rest (1.13) to state (3.3) is determined by formula (2.26) and is equal

$$T^* = T_2 = 7.62\,. \tag{3.7}$$

In the initial dimensional variables (1.3), (1.11), point (3.6) corresponds to the final state of the manipulator $(\varphi^1, \dot{\varphi}^1) = (3\,\mathrm{rad};\ 2.56\,\mathrm{rad/s})$ and time (3.7) to $T^* = 2.63s$.

Let us compare the solution with the solution obtained in [4] of the time-optimal control problem for system (1.10) with constraints (1.5), (1.7) and boundary conditions (1.8), (1.9). We restrict ourselves to the case of zero final velocity $z_2^1 = 0$. Let us choose the final state at the point $(z_1^1, z_2^1) = (0.18; 0) \in Z_-$, which in the dimensional variables (1.3), (1.11) corresponds to the final state of rest of the manipulator $(\varphi^1, \dot{\varphi}^1) = (2\,\mathrm{rad};\ 0)$. For comparison, we note that the time-optimal movement from the initial zero state to this final state of rest is $T^* = 3.28(1.13s)$ [4]. Thus, the proposed control method brings the system to a given state in a time not much different from the optimal one.

4. Conclusion

By applying the generalized Kalman scheme for constructing control, extended to the case of restrictions on the control voltage and on the heat release power in the rotor winding of the electric motor of an electromechanical system with one degree of freedom, the control law is explicitly found, and conditions are obtained that form certain regions of final states on the phase planes of the system, the movement to each point of which from the initial state of rest with the help of the found control occurs in a finite time without violating the considered restrictions.

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