ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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DISPERSION OF SHEAR SURFACE WAVES IN AN ELASTIC SUBSTRATE IMPERFECTLY BONDED WITH AN ELASTIC LAYER

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Keywords: Surface shear waves, imperfect contact, slip and scattering interfaces, dispersion.

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Дисперсия сдвиговых поверхностных волн в упругой подложке, несовершенно контактирующим с упругим слоем

Ключевые слова: поверхностные поперечные волны, неидеальный контакт, дисперсия.

Исследовано распространение поверхностной поперечной волны для слоистой структуры, состоящей из упругого слоя, лежащего на упругом полупространстве с поверхностью неполного упругого контакта. Рассмотрены две различные модели неполного упругого. Получены дисперсионные уравнения, описывающие зависимость фазовой скорости поверхностной волны от волнового числа. На основе анализа дисперсионных уравнений показано, что несовершенство границы раздела может существенно уменьшить или увеличить фазовую скорость поверхностной волны.

Ղազարյան Կ.Բ., Ղազարյան Ռ.Ա., Թերզյան Ս.Հ.

Առաձգական շերտի հետ թերի կոնտակտ ունեցող առաձգական հարթակում սահքի մակերևութային ալիքների դիսպերսիան

Հիմնաբառեր. մակերևութային լայնական ալիքներ, ոչ իդեալական կոնտակտ, դիսպերսիա:

Առաձգական կիսատարածության վրա գտնվող և թերի կոնտակտ ունեցող առաձգական շերտ բաղադրյալ կառուցվածքի համար ուսումնասիրված է մակերևութային լայնական ալիքի տարածումը: Դիտարկվել են թերի կոնտակտի երկու տարբեր մոդելներ: Ստացվել են դիսպերսիոն հավասարումներ, որոնք նկարագրում են մակերևութային ալիքի ֆազային արագության կախումը ալիքային թվից: Դիսպերսիոն հավասարումների հետազոտումով ցույց է տրվել, որ բաժանման եզրի անկատարությունը կարող է էապես մեծացնել, կամ փոքրացնել մակերևութային ալիքի ֆազային արագությունը:

The surface shear wave is studied in layered bi-material structure consisting from an elastic layer lying on an elastic half-space with an imperfectly bonded interface. The two models of the imperfect “slip” and “scattering” interface are considered. The dispersion equations are obtained describing the surface wave phase speed versus wavenumber. Based on the analysis of dispersion equations it is shown that the interface imperfectness can sufficiently decrease in “slip” case or increase in “scattering” case the surface wave phase speed.

Introduction

The term elastic surface waves is used to denote waves propagating along the interface of elastic media with the energy localized in a band of a width of the order of several wavelengths. The classic results concerning the shear surface waves propagation in layered media were firstly published in [1,2]. Wave propagation through a layered composite material and the formation of surface waves bands is a well-studied topic [3,4,5] and has significance in applied geoscience [6].

A model of an imperfectly bonded interface between two elastic media is proposed in [7] where the displacement discontinuity (slip) is taken to be linearly related to the stress traction which is continuous across the interface. A model of “scattering” imperfectly bonded interface is used in [8] where the stress traction discontinuity is taken to be linearly related to the displacement which is continuous across the interface where one-dimensional time-harmonic waves interact with a finite number of scatterers. The surface shear waves in elastic semi-spaces separated by elastic layer with an imperfectly bonded “slip” interfaces between layer and semi-spaces are considered in [9]. The shear surface waves at the electro-mechanical imperfect interface of two piezoelectric materials is studied in [10,11] . The localized electro-acoustic Rayleigh and Gulyaev-Bluestein waves is studied in [12] It is shown that the choice of materials can increase or decrease the surface electro elastic wave energy localization at the surface of non acoustic interface of bi-material piezoelectric structure. The surface electro-magneto elastic shear waves in a bi-material structure in an external constant magnetic consisting of the bonded piezoelectric and perfectly conducting half-spaces is considered in [13], where the conditions of an existence of surface shear waves localized at the interface between two media were derived.. In [14] the dynamic contact problem is studied concerning propagation and diffraction of shear plane waves in a composite structure consisting of elastic half-space and a layer, weakened by a semi-infinite tunnel crack interface. In [14-15] the dynamic contact problems are studied concerning propagation and diffraction of shear plane waves in a composite structure consisting of elastic half-space and a layer, or consisting of two elastic half-spaces, weakened by a semi-infinite tunnel crack interface. Electroacoustic transverse waves in a piezoelectric half-space via non-acoustic influence on its interface is considered in [16]. The conducting interface near the traction free surface of a piezoelectric half-space changes the character of the near-surface localization of the electroacoustic wave.

1.Statement and solution of the problem

In Cartesian coordinate system (x_1, x_2, x_3) we consider the layered structure consisting from an elastic semi infinite substrate $\left(|x_1| < \infty, x_2 \in (-\infty, 0), |x_3| < \infty\right)$ imperfectly bonded with an elastic layer $\left(|x_1| < \infty, x_2 \in (0, h), |x_3| < \infty\right)$.

The anti- plane equations of motion and material relations are given by

$$\frac{\partial \sigma_{13}^{(s)}}{\partial x_1} + \frac{\partial \sigma_{23}^{(s)}}{\partial x_2} = \rho^{(s)} \frac{\partial^2 U_3^{(s)}}{\partial t^2};$$
$$\sigma_{13}^{(s)} = G^{(s)} \frac{\partial U_3^{(s)}}{\partial x_1}; \sigma_{23}^{(s)} = G^{(s)} \frac{\partial U_3^{(s)}}{\partial x_2} \tag{1}$$

Here $U_3^{(s)}$ are elastic displacements, $\sigma_{13}^{(s)}, \sigma_{23}^{(s)}$ are the shear stresses, $\rho^{(s)}$ are the mass densities, $G^{(s)}$ are the shear elastic modulus, respectively. The indexes $s = 1; 2$ stand for the layer and substrate, respectively.

The model of the imperfect “slip –scattering” interface between elastic layer and substrate will be used [7,8]. According to this model the traction and displacement are not continuous across the interface and the following contact conditions are valid at interface $x_2 = 0$

$$\sigma_{23}^{(1)}\left(x_1,0,t\right)-\sigma_{23}^{(2)}\left(x_1,0,t\right)=\frac{g}{2}\left(U_3^{(1)}\left(x_1,0,t\right)+U_3^{(2)}\left(x_1,0,t\right)\right)$$

$$U_3^{(1)}\left(x_1,0,t\right)-U_3^{(2)}\left(x_1,0,t\right)=\frac{f}{2}\left(\sigma_{23}^{(1)}\left(x_1,0,t\right)+\sigma_{23}^{(2)}\left(x_1,0,t\right)\right) \quad (2)$$

$$g \ge 0, \quad f \ge 0,$$

When $g = 0$, we have the model of the “slip” interface at $x_2 = 0$ where the shear stresses are continuous but the displacements have a jump [7] . When $f = 0$ we have the model of the “scattering” interface where the displacements are continuous but the shear stresses have a jump [8].

At the layer upper surface $x_2 = h$ we consider the traction free interface condition

$$\sigma_{23}^{(1)}\left(x_1,h,t\right)=0 \quad (3)$$

or the clamped interface condition

$$U_3^{(1)}(x_1,h,t)=0 \quad (4)$$

In the semi-space the displacement decaying to zero at infinite distance from contact interface $x_2 = 0$

$$U_3^{(1)}(x_1,x_2,t)\to 0, x_2 \to -\infty \quad (5)$$

We consider harmonic wave travelling along the x_1 direction, $U_3^{(s)}\left(x_1,x_2,t\right)=U^{(s)}(x_2)\exp\left[i\left(kx_1-\omega t\right)\right]$, where ω is the wave angular frequency, k is the wave number.

Since the interface conditions at $x_2 = 0$ are imposed on functions $U^{(s)}\left(x_2\right),\sigma_{23}^{(s)}(x_2)$ it is convenient to introduce the following column vectors

$$\mathbf{U}^{(s)}(x_2)=\left(U^{(s)}\left(x_2\right),\sigma_{23}^{(s)}(x_2)\right)^T \quad (6)$$

In the matrix form the solutions of (1) in the layer can be cast as

$$\mathbf{U}^{(1)}(x_2)=\mathbf{F}^{(1)}\left(x_2\right)\cdot\mathbf{C},\ \mathbf{C}=\left(C_1,C_2\right)^T, \quad (7)$$

$$\mathbf{F}^{(1)}\left(x_2\right)=\begin{pmatrix}\exp\left(ir_1x_2\right) & \exp\left(-ir_1x_2\right)\\ iG_1r_1\exp\left(ir_1x_2\right) & -iG_1r_1\exp\left(-ir_1x_2\right)\end{pmatrix} \quad (8)$$

In substrate the solutions can be cast as

$$\mathbf{U}^{(2)}\left(x_2\right)=A\begin{pmatrix}\exp\left(r_2x_2\right)\\ r_2G_2\exp\left(r_2x_2\right)\end{pmatrix} \quad (9)$$

In (7,9) A, C_1, C_2 are constants, $v_s = \sqrt{G_s/\rho_s}$ denote the shear wave speeds in the bi- material structure

$$r_1 = \sqrt{\frac{\omega^2}{v_1^2} - k^2}, \ \ r_2 = \sqrt{k^2 - \frac{\omega^2}{v_2^2}}; \tag{10}$$

The condition $\omega < kv_2$ provides decaying of the surface wave from the interface $x_2 = 0$.

The interface conditions (2) in the matrix form can be cast as

$$\mathbf{U}^{(1)}(0) = \mathbf{S}\mathbf{U}^{(2)}(0) \tag{11}$$

$$\mathbf{S} = \begin{pmatrix} \dfrac{fg+4}{4-fg} & \dfrac{4f}{4-fg} \\ \dfrac{4g}{4-fg} & \dfrac{fg+4}{4-fg} \end{pmatrix}$$

Using the procedure of transfer matrix approach [9] we obtain the following relation linking the values of the vector $\mathbf{U}^{(1)}(x_2)$ at the layer interfaces $x_2 = 0, x_2 = h$ via transfer matrix

$$\mathbf{U}^{(1)}(h) = \mathbf{T}\mathbf{U}^{(1)}(0) \tag{12}$$

$$\mathbf{T} = \begin{pmatrix} \cos(hr_1) & (G_1 r_1)^{-1}\sin(hr_1) \\ -G_1 r_1 \sin(hr_1) & \cos(hr_1) \end{pmatrix}; \tag{13}$$

In the case of the traction free surface at $x_2 = h$ we have

$$\mathbf{U}^{(1)}(h) = (U_0, 0)^T \tag{14}$$

At the clamped interface $x_2 = h$ we have

$$\mathbf{U}^{(1)}(h) = (0, \sigma_0)^T \tag{15}$$

In (14,15) U_0, σ_0 are constants.

Using (5) and (6) we come to the following homogeneous set of equations with respect to constants A, σ_0 for the “clamped” case, or A, U_0 for “traction free” case

$$\mathbf{U}^{(1)}(h) - \mathbf{TS}U^{(2)}(0) = 0 \tag{16}$$

Equating the determinant of these sets of equations (16) to zero, we obtain the following dispersion equations:

Traction free interface at $x_2 = h$

$$K\sqrt{\beta^2\eta^2 - 1}\tan\left(K\sqrt{\beta^2\eta^2 - 1}\right) = \frac{\sqrt{1-\eta^2}K(\theta\xi + 4) + 4\xi}{\gamma\left(\theta\xi + 4\sqrt{1-\eta^2}\theta K + 4\right)}, \tag{17}$$

Clamped interface at $x_2 = h$

$$K\sqrt{\beta^2\eta^2-1}\cot\left(K\sqrt{\beta^2\eta^2-1}\right)=-\frac{\sqrt{1-\eta^2}K(\theta\xi+4)+4\xi}{\gamma\left(\theta\xi+4\sqrt{1-\eta^2}\theta K+4\right)} \quad (18)$$

In (17, 18) the following dimensionless notations are used

$$\beta=v_2/v_1,\eta=\omega/kv_2,\quad \gamma=G_1/G_2,K=kh,\theta=fG_2/h,\xi=gh/G_2; \quad (19)$$

The equations (17,18) may have real solutions corresponding to the surface wave if only

$$\beta^{-1}<\eta<1,\quad v_1<\omega k^{-1}<v_2 \quad (20)$$

Dispersion equations (18,19) reveal that the phase velocity of the surface wave is a function of the wave number $\eta(K)$, and these equations have the finite numbers of solutions (modes). For the given value of K the number of the modes do not depend of interface complaints θ,ξ and is defined by the formula

$$n=\mathrm{Floor}\left(\frac{K\sqrt{\beta^2-1}}{\pi}\right)+1$$

The function $\mathrm{Floor}(x)$ gives the greatest integer less than or equal to x.

Traditionally, the first mode of the dispersion equations solutions is assumed as Love waves.

2.Numerical Results

The curves on Fig1., Fig 2. are plotted and the numerical data of Tabl.1 are calculated for substrate and layer materials corresponding to shear modulus ratio $\gamma=G_1/G_2=0.5$, speeds ratio $\beta=v_2/v_1=2$.

Modes		$\xi=0,\theta=0$ Perfect		$\xi=7,\theta=0$ Scattering		$\xi=7,\theta=7$ Slip+Scattering	
		Free	Clamped	Free	Clamped	Free	Clamped
1	K	0	0.8	0.7	1.7	0.57	1.3
2	K	1.8	2.7	2.5	3.5	2.2	3.0
3	K	3.7	4.6	4.1	5.0	4.3	49
4	K	5.5	6.3	6.6	7.8	6.7	7.2

Table1. Data for the thresholds of wave number $K=kh$ corresponding to normalized phase speed $\eta=1$, for the first four modes in the cases of different imperfect interfaces

In the Table 1. the thresholds $K=kh$ of the lowest and subsequent three higher modes are presented which are correspond to phase speed $\eta=1$; The different imperfect interfaces are considered. The results of the case $\xi=0,\theta=7$ are not presented since they coincide with the perfect contact case $\xi=0,\theta=0$.

On the Fig.1., Fig2. the first lowest modes of the normalized phase speed $\eta = \omega/kv_1$ are given as a function of the dimensionless wave number $K = kd$. Red curve corresponds to the classic Love wave perfect contact case $\xi = 0, \theta = 0$, green curve to "scattering" interface $\xi = 7, \theta = 0$, blue curve to "slip" interface $\xi = 0, \theta = 7$, black curve to "slip-scattering" interface $\xi = 7, \theta = 7$ **(see online version for colors).**

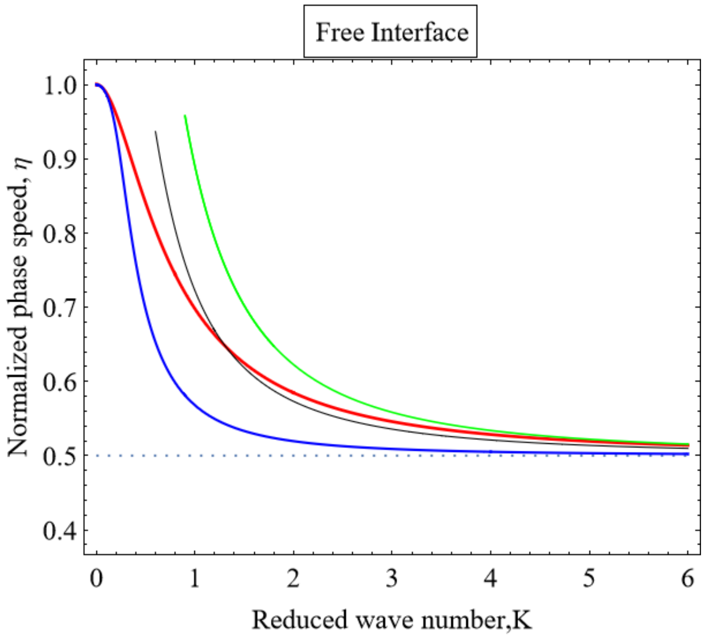

Fig1. The dispersion curve of the shear surface wave first mode: the layer upper interface is traction free

3.Results and Discussion

As it follows from the analysis of the first mode of dispersion equations the slip interface essentially decreasing the values of phase speed. For scattering interface we have the contrary effect of increasing of phase speed. This influence is more notable in bands of the long waves . When the upper interface is traction free we have the maximal deviation of the phase speed up to 30 % which takes place at $K \sim 1.5$. In the case the of clamped interface this same effect takes place at $K \sim 2.7$. For short waves these deviations are small. The case of the "slip-scattering" interface is very interesting one since in this case the dispersion curves are close to the dispersion curves of perfect contact case of Love waves. In combined "slip-scattering" interface the effects caused by "slip" and "scattering" interfaces are practically compensate each other.

It is necessary to mention that imperfect interfaces do not change the number of surface wave modes.

From the data of the Table 1. we can conclude that the imperfect interfaces increasing the thresholds of the wavenumbers in the first and the highest modes.

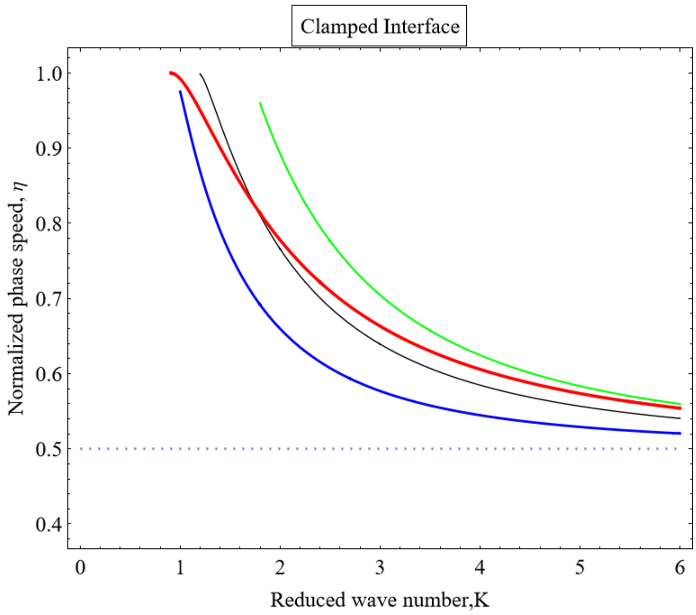

Fig.2. The dispersion curve of the shear surface wave first mode: the layer upper interface is clamped

References

1. Love AEH. Some Problems of Geodynamics. UK: Cambridge University Press; 1911. pp. 89-104 and 149-152
2. R. Stoneley, Elastic waves at the surface of separation of two solids, Proceeding of the Royal Society, v, 106, issue 738, (1924). 416-428.
3. Brehovskikh L.M, Godin O.A, Acoustics of layered media, 1989, p.268, Nauka, Moskva, (In Russian),
4. M. V. Belubekyan, Surface waves in elastic medium. In: Problems of solid deformable body mechanics, Institute of Mechanics of NAS Armenia, Yerevan, 1997, p. 79–100 (in Russian).
5. Mikhasev, G., Erbaş, B., & Eremeyev, V. A. (2023). Anti-plane shear waves in an elastic strip rigidly attached to an elastic half-space. International Journal of Engineering Science, 184, 103809. doi.org/10.1016/j.ijengsci.2022.103809
6. Aki K., Richards P. G., Quantitative Seismology, 2nd edition,University Science Books. Sausalito, CA (2002).
7. M. Schoenberg, Elastic wave behaviour across linear slip interfaces. The Journal of the Acoustical Society of America 68.5 (1980): 1516-1521. doi.org/10.1121/1.385077
8. Martin, P. A. (2014). N masses on an infinite string and related one-dimensional scattering problems. Wave Motion, 51(2), 296-307. doi.org/10.1016/j.wavemoti.2013.08.005
9. Ghazaryan, K.B., Mozharovsky, V.V., Sarkisyan,S.V. and Ohanyan,S.K. (2021) 'Shear surface wave propagation in stratified media with slip interfaces', Int. Journal of

Materials and Structural Integrity, Vol. 14, Nos. 2/3/4, pp.120–126. DOI: 10.1504/IJMSI.2021.10050843

10. Fan, H., Yang, J., & Xu, L. (2006). Piezoelectric waves near an imperfectly bonded interface between two half-spaces. Applied Physics Letters, 88(20), 203509. doi.org/10.1063/1.2206702
11. Otero, J. A., Rodríguez-Ramos, R., Bravo-Castillero, J., & Monsivais, G. (2012). Interfacial waves between two piezoelectric half-spaces with electro-mechanical imperfect interface. Philosophical magazine letters, 92(10), 534-540. doi.org/10.1080/09500839.2012.698758
12. Avetisyan A.S., Jilavyan H.S., Hybrid of Rayleigh and Gulyaev-Bluestein electro-acoustic waves near the inner surface of a layered pezoelectric composite, , Proceedings of NAS RA, (2020) ,Mechanics, vol. 73, ,2, pp. 10-30, http://doi.org/10.33018/73.2.2
13. Avetisyan . S., Gevorgyan, A. V., , Avetisyan, L. V. Hybrid of surface shear waves at the contact interface between piezoelectric and electrically conductive half-spaces. Proceedings of NAS RA, Mechanics, 2021,74,3,53-61, .(In Russian), doi.org/10.33018/74.3.4
14. Agayan K.L, Diffraction of shear flat waves on a semi-infinite crack in a compound elastic half-space.. Proceedings of NAS RA, Mechanics, 2020,73(2), 22-34.(In Russian), doi.org/10.33018/73.2.2
15. Агаян К.Л., Григорян Э.Х., Гулян К.Л. Diffraction of a shear plane wave in a composite elastic space with a semi-infinite crack parallel to the line of inhomogeneity. МТТ , № 2, 2013, с.106-113. .(In Russian)
16. Avetisyan, A. S. On formulating problems of contactless surface control of electroacoustic wave propagation. *Acoustical Physics*, (2022).,*68*(3), 227-234. doi.org/10.1134/S1063771022030022

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