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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 76, №1, 2023 Механика
УДК 539.3 DOI: 10.54503/0002-3051-2023.76.1-56

Propagation of a Hybrid of Heterogeneous Electroacoustic Waves in Composite Piezoelectric Waveguide without Acoustic Contact between Layers

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Keywords: composite waveguide, periodic wave, non-acoustic contact, hybrid of electro-acoustic waves, wave energy localization.

Распространение гибрида разнородных электроакустических волн в Композитном пьезоэлектрическом волноводе без акустического контакта между слоями

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Ключевые слова: составной волновод, периодическая волна, неакустический контакт, гибрид электроакустических волн, локализация волновой энергии.

The problem of propagation of an electroactive unidirectional wave signal of elastic shear (or plane elastic deformation) in an infinite piezoelectric composite waveguide consisting of periodically repeating two-layer cells is considered. In the sagittal plane of one piezo layer in the cell, antiplane electroactive deformation is possible, and in the adjacent layer, electroactive planar deformation is possible. The layers are in a state of non-acoustic contact. The surfaces of the piezoelectric composite waveguide are free from mechanical influences. One of the waveguide surfaces is electrically open, while the other is electrically closed. The propagation of an electroacoustic wave signal occurs due to the penetration of accompanying electrical oscillations through a non-acoustic contact between the piezoelectric layers. There is a multiple transformation of a three-component electroelastic shear wave into a four-component electroelastic wave of plane deformation and vice versa. A hybrid of electroacoustic waves is formed. In the case of a high-frequency wave signal, a hybrid of surface electroacoustic waves of the Rayleigh and Gulyaev-Bluestein types is formed. The distributions of elastic displacements and electric potential along the thickness of the waveguide are determined. The resulting hybrid has the character of a periodic Floquet-Bloch wave. The zones of allowable frequencies and allowed lengths of the hybrid are determined. Rapidly decaying components of the electroacoustic wave are also found.

Ավետիսյան Արա Ս., Խաչատրյան Վազգեն Մ.

Տարասեռ էլեկտրաակուստիկ ալիքների հիբրիդի տարածումը բաղադրյալ պիեզոէլեկտրական ալիքատարում, անհպում շերտերի դեպքում

Բանալի բառեր՝ բաղադրյալ ալիքատար, պարբերական ալիք, անհպում կոնտակտ, էլեկտրաակուստիկ ալիքների հիբրիդ, ալիքային էներգիայի տեղայնացում:

Դիտարկված է առաձգական սահքի (կամ հարթ առաձգական դեֆորմացիայի) էլեկտրաակտիվ, ուղղորդված ալիքային ազդանշանի տարածման խնդիրը անվերջ երկար պիեզոէլեկտրական բաղադրյալ ալիքատարում, որը բաղկացած է պարբերաբար կրկնվող երկշերտ բջիջներից: Բջջի մեկ շերտի սագիտալ հարթությունում հնարավոր է հակահարթ էլեկտրաակտիվ դեֆորմացիա, իսկ հարակից շերտում՝ էլեկտրաակտիվ հարթ դեֆորմացիա։ Միջնաշերտերը գտնվում են անհպում շփման վիճակում։ Պիեզոէլեկտրական բաղադրյալ ալիքատարի մակերևույթները զերծ են մեխանիկական ազդեցություններից: Ալիքատարի մակերևույթներից մեկը էլեկտրականորեն բաց է, իսկ մյուսը էլեկտրականորեն փակ է: Էլեկտրաակուստիկ ալիքի տարածումը տեղի է ունենում շերտից-շերտ էլեկտրական դաշտի ուղեկցող տատանումների ներթափանցման շնորհիվ՝ պիեզոէլեկտրական շերտերի միջև անհպում շփման միջոցով։ Տեղի է ունենում եռաբաղադրիչ սահքի էլեկտրաառաձգական ալիքի բազմակի փոխակերպում, հարթ դեֆորմացիայի քառաբաղադրիչ էլեկտրաառաձգական ալիքի և հակառակը: Ձևավորվում է էլեկտրաակուստիկ ալիքների հիբրիդ։ Բարձր հաճախականության ալիքային ազդանշանի դեպքում ձևավորվում են Ռելեյի և Գուլյաև-Բլյուստեյնի տիպային մակերևութային էլեկտրաակուստիկ ալիքների հիբրիդ։ Որոշվում են առաձգական սահքիի և էլեկտրական դաշտի բաշխումները ալիքատարի հաստությամբ: Ստացված հիբրիդն ունի Ֆլոկե-Բլոխի պարբերական ալիքի բնույթ։ Որոշվում են հիբրիդների թույլատրելի հաճախականությունների և թույլատրելի երկարությունների գոտիները: Գտնվում են նաև էլեկտրաակուստիկ ալիքի արագ մարող բաղադրիչներ։

Рассмотрена задача о распространении электроактивного однонаправленного волнового сигнала упругого сдвига (или плоской упругой деформации) в бесконечном пьезоэлектрическом композитном волноводе, состоящем из периодически повторяющихся двухслойных ячеек. В сагиттальной плоскости одной прослойки в ячейке, возможна анти плоская электро активная деформация, а в соседней прослойке возможна электроактивное плоскостная деформация. Прослойки находятся в состоянии не акустического контакта. Поверхности пьезоэлектрического композитного волновода свободны от механических воздействий. Одна из поверхностей волновода электрически открыта, а другая электрически замкнута.

Распространение сигнала электроакустической волны происходит за счет проникновения сопутствующих колебаний электрического поля, через неакустический контакт между пьезоэлектрическими слоями. Происходит многократное преобразование трехкомпонентной электроупругой сдвиговой волны в четырехкомпонентную электроупругую волну плоской деформации и наоборот. Образуется гибрид электроакустических волн. В случае высокочастотного волнового сигнала формируется гибрид поверхностных электроакустических волн типов Рэлея и Гуляева-Блюстейна. Определены распределения упругих перемещений и электрического потенциала по толщине волновода. Полученный гибрид имеет характер периодической волны Флоке-Блоха. Определены зоны допустимых частот и разрешенных длин гибрида. Обнаружены также быстро затухающие компоненты электроакустической волны.

Introduction.

In **[1]**, and in **[2]** the possibility of localizing of the wave energy of the ***SH*** elastic wave with accompanying oscillations of the electric field on a mechanically free surface of a piezoelectric medium of a certain symmetry, under various boundary conditions is shown. The features of the propagation and localization of the wave energy of a purely shear electroelastic wave are still being studied. In **[3]** the propagation of Bluestein-Gulyaev waves in materials with complicated properties is investigated. The propagation of Bluestein-Gulyaev waves in a prestressed layered piezoelectric structure was considered by **[4].** The propagation of transverse surface waves in a functionally graded substrate carrying a layer of piezoelectric material of the 6mm class of hexagonal symmetry was studied by

[5]. In **[6]** the amplitude-phase interaction during the propagation of an electroelastic monochromatic wave signal in an inhomogeneous piezoelectric with hexagonal symmetry of class 6mm is considered.

Rayleigh-type electroelastic waves have been relatively little studied, although, for plane deformation waves, the localization of wave energy near a mechanically free surface in an isotropic half-space was first discovered by **[7]**. In particular, in **[8]** the propagation of the Rayleigh wave in a rotating initially stressed piezoelectric half-space is considered. In the article **[9]**, the authors proposed an analytical model for studying the propagation of Rayleigh waves in an orthotropic half-space with a piezoelectric layer. Propagation of coupled Rayleigh waves in a piezoelectric layer of a material of the class ***2mm*** of rhombic symmetry over a porous piezo-thermoelastic half-space is studied in the work **[10].**

Under various alternative boundary conditions on mechanically free surfaces of a piezoelectric waveguide, the problem of propagation of high-frequency electroacoustic waves of plane deformation (Rayleigh-type electroacoustic waves) is solved in the work **[11]**.

The acoustic artificial structures with tunable parameters have attracted much research interest in these days. In **[12]**, the authors presented a tunable composite waveguide based on the piezoelectric phononic crystal shunted by an inductor circuit.

Naturally, various studies have shown the possibilities of various types of localization of wave energy under various boundary conditions near the surfaces of elements of a composite (inhomogeneous) waveguide.

In modern high-precision technologies, composite periodically inhomogeneous waveguides made of piezoelectric crystals are widely used as converters, filters or resonators of electroacoustic wave signals.

For the first time, the presence of frequency cut-off zones in an unidirectional periodic elastic structure was noted in the work **[13].** An overview of the perspectives, current state and future directions of research of wave processes in periodic structures are given in **[14].**

From the mathematical point of view, the spectral theory of transverse vibrations of periodic elastic beams is presented in articles **[15, 16]**. In the work **[17]**, the dispersion relations of SH-waves were obtained and investigated during their propagation in periodic piezoelectric composite layered structures. Papers **[18]** and **[19]** are devoted to the application of the Floquet-Lyapunov theory to the problems of propagation of elastic waves in periodic structures. In the works **[20]**, the authors investigated the spectrum of Floquet-Bloch waves in elastic periodic waveguides.

In the article **[21]** the spectrum of acoustic oscillations generated by interdigital transducers in a plate made from a $LiNbO_3$ piezocrystal with a thickness on the order of the acoustic wavelength is studied. It is shown that, along with zeroth and higher-order modes, this spectrum also contains odd harmonics of the same modes.

Coupled electro-elastic SH waves propagating oblique to the lamination of a one-dimensional piezoelectric periodic structure are considered in the framework of the full system of Maxwell’s electrodynamic equations. The dispersion equation has been obtained and numerical analyses carried out for two kinds of composites both consisting of two different piezoelectric materials **[22, 23]**. Analysis of the role of impedance on the existence of forbidden frequencies are given in the works, where it is also shown that if the impedance of a periodically inhomogeneous 1D structure is constant, then there are no forbidden frequencies in this structure.

In the works listed above, the wave field is uniform, and the character of the initial normal wave does not change during the propagation in a periodically inhomogeneous waveguide.

In **[24]** it is shown that, depending on the crystallographic symmetry of an anisotropic piezoelectric, in its sagittal planes it is possible to excite either an electroelastic wave of pure elastic shear, or an electroelastic wave of plane deformation, accompanied by oscillations of a plane electric field.

In **[25]** the elastic wave propagation properties of phononic crystals (PnCs) composed of an elastic matrix embedded in magnetorheological and electrorheological elastomers are studied. The variations in the band gap characteristics with changes in the electric/magnetic fields are given.

In articles **[26, 27]**, it is shown that the non-acoustic contact between two different piezoelectrics allows the formation of a hybrid of electroactive elastic shear and plane strain waves. In article **[28]** the possibility of propagation of an unidirectional hybrid of electroacoustic waves of elastic shear and plane deformation is showed, in a periodically inhomogeneous composite, the layers of which are made of different piezoelectric materials and are in non-acoustic contact. Two groups of allowed discrete frequencies are revealed. It is shown that if the ratio of the widths of the interlayers and the velocities of elastic waves in them is inversely proportional, then the admissible discrete frequencies are resonant.

We present here in a simple scheme of an inhomogeneous piezoelectric waveguide that allows multiple mutual conversion and co-propagation of localized electroactive normal anti-plane strain waves and plane strain waves under different electrical conditions on the waveguide surfaces.

1. Formulation of the problem

Let us consider the propagation of the electroelastic wave normal signal $F(x,y,t) = f(x,y)\cdot\exp(i\omega t)$, in a periodically longitudinally inhomogeneous layer, which is assigned to an orthogonal coordinate system $0xyz$ (Fig. 1). Composite waveguide layer consists of periodically repeating cells $\Omega(x,y,z) = \Omega_1(x,y,z)\cup\Omega_2(x,y,z)$, from different piezoelectric crystals of rectangular cross section

$$\Omega_{s1}(x,y) \triangleq \left\{x\in[0;a_1],\ y\in[-h;h],\ |z|<\infty\right\},\quad \Omega_{s2}(x,y) \triangleq \left\{x\in[-a_2;0],\ y\in[-h;h]\right\} \qquad (1.1)$$

There is no acoustic contact between adjacent layers and cells.

The crystallographic axes and sagittal crystallographic surfaces of the adjacent layers materials in the cells are referred to the Cartesian coordinate system $0xyz$, so that the multicomponent electroactive waves of anti-planar and planar deformations can exist separately in the adjacent layers of the waveguide.

Without violating the generality of the reasoning, for clarity, we formulate the boundary value problem by choosing specific anisotropies of the materials of the composite waveguide bands.

Let us assume that the material of the strips $\Omega_{n1}(x,y,z)$ belongs to the crystallographic class ***6mm*** of hexagonal symmetry, and the symmetry axis of the piezocrystal $\vec{p}_6$ is parallel to the selected coordinate axis $\vec{p} \parallel 0\vec{z}$. Then, the quasistatic equations for unidirectional waves of electroactive antiplane deformation $\{0;\ 0;\ \mathrm{w}_1(x,y);\ \varphi_1(x,y)\}\cdot\exp(i\omega t)$, with respect to the functions of the elastic shear $\mathrm{w}_1(x,y)$ and the potential of electric field $\varphi_1(x,y)$ in the plane $x0y$ of the Cartesian coordinate system are written as

$$\begin{aligned} &\mathrm{w}_{1,xx}(x,y)+\mathrm{w}_{1,yy}(x,y)=-(\omega^2/C_{1t}^2)\cdot\mathrm{w}_1(x,y) \\ &\varphi_{1,xx}(x,y)+\varphi_{1,yy}(x,y)=(e_{15}^{(1)}/\varepsilon_{11}^{(1)})\cdot\left[\mathrm{w}_{1,xx}(x,y)+\mathrm{w}_{1,yy}(x,y)\right] \end{aligned}, \qquad (1.2)$$

In equations (1.2), $C_{1t}=\sqrt{\tilde{c}_{44}^{(1)}/\rho_1}$ is the velocity of the volumetric electroactive elastic shear wave, $\tilde{c}_{44}^{(1)}=c_{44}^{(1)}\left(1+\chi_1^2\right)$ is the shear rigidity of the material, taking into account the piezoelectric effect, $c_{44}^{(1)}$ is the shear rigidity, $\chi_1^2=\left(e_{15}^{(1)}\right)^2\Big/\left(c_{44}^{(1)}\varepsilon_{11}^{(1)}\right)$ is the electromechanical coupling coefficient, $e_{15}^{(1)}$ is the piezoelectric modulus, $\varepsilon_{11}^{(1)}$ is the relative dielectric constant and ρ_1 is the density of the piezoelectric material.

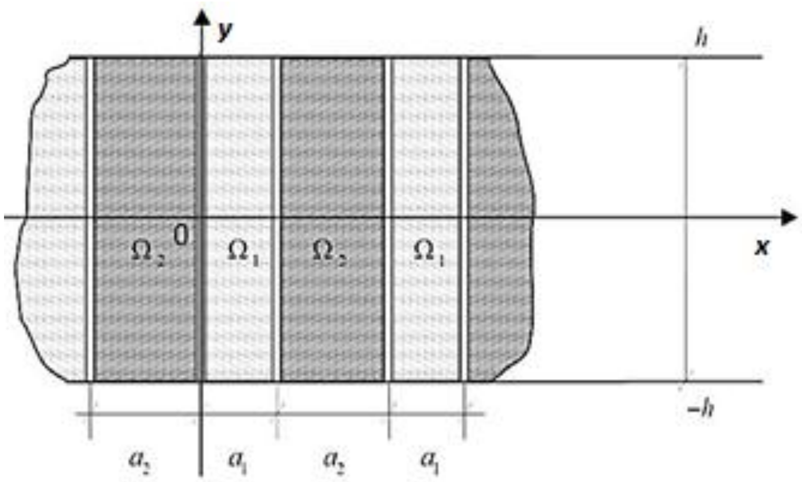

Fig.1 *Periodically longitudinally inhomogeneous composite waveguide, without acoustic contact of piezoelectric interlayers.*

On both surfaces $y=\pm h$ of rectangular sections $\Omega_{n1}(x,y)$ of the waveguide strips free from mechanical loads, the conditions for mechanically free boundaries are written as

$$\left[c_{44}^{(1)} \cdot \mathrm{w}_{1,y}(x,y) + e_{15}^{(1)} \cdot \varphi_{1,y}(x,y)\right]_{y=\pm h} = 0 \tag{1.3}$$

The conditions for the conjugation of the electric field with the external field on the surfaces $y = \pm h$ of rectangular sections $\Omega_{n1}(x,y)$ of strips are written in the form

$$\begin{aligned} &[\varphi_1(x,y) - \varphi_e(x,y)]\big|_{y=\pm h} = 0 \\ &\left[(e_{15}^{(1)}/\varepsilon_{11}^{(1)}) \cdot \mathrm{w}_{1,y}(x,y) - \varphi_{1,y}(x,y)\right]_{y=\pm h} = -(\varepsilon_0/\varepsilon_{11}^{(1)}) \cdot \varphi_{e,y}(x,y)\big|_{y=\pm h} \end{aligned} \tag{1.4}$$

In surface relations (1.4), $\varphi_e(x,y)$ is the amplitude function of the external accompanying electric field.

On the electrically open $y = h$ and electrically closed $y = -h$ surfaces of the piezoelectric rectangular section $\Omega_{n1}(x,y)$, the conditions for the transparency and shielding of the electric field take the form, respectively

$$\left[(e_{15}^{(1)}/\varepsilon_{11}^{(1)}) \cdot \mathrm{w}_{1,y}(x,y) - \varphi_{1,y}(x,y)\right]_{y=h} = 0\,, \tag{1.5}$$

$$\varphi_1(x,y)\big|_{y=-h} = 0 \tag{1.6}$$

Because of the penetration of accompanying electrical vibrations through the vacuum gap, in the rectangular section $\Omega_{n1}(x,y)$, the propagating three component electroelastic shear waveform $\{0;0;\mathrm{w}_1(x,y);\varphi_{1,x}(x,y);\varphi_{1,y}(x,y);0\}\cdot\exp(i\omega t)$, is converted in the rectangular section $\Omega_{n2}(x,y)$, into the four component electroelastic plane deformation wave $\{\mathrm{u}_2(x,y);\ \mathrm{v}_2(x,y);\ 0;\ \varphi_{2,x}(x,y);\ \varphi_{2,y}(x,y);\ 0\}\cdot\exp(i\omega t)$. Similarly, because of the penetration of accompanying electrical vibrations through the vacuum gap, in the rectangular section $\Omega_{n2}(x,y)$, the propagating four-component electroelastic shear waveform is converted in the rectangular section $\Omega_{n1}(x,y)$, into the three-component electroelastic plane deformation wave. Such multiple transformations

$$\begin{aligned} &\{0;0;\mathrm{w}_1(x,y,t);\ \varphi_{1,x}(x,y,t);\varphi_{1,x}(x,y,t);0\} \rightleftarrows \\ &\rightleftarrows \{\mathrm{u}_2(x,y,t);\mathrm{v}_2(x,y,t);0;\ \varphi_{2,x}(x,y,t);\varphi_{2,y}(x,y,t);0\} \end{aligned}$$

of the wave field are possible if adjacent piezoelectric strips of different materials are in non-acoustic contact with each other.

Let the material of the composite bands $\Omega_{n2}(x,y)$ belong to the class $\bar{6}m2$ of hexagonal symmetry and the inversion symmetry axis $\vec{\bar{p}}_6$ of the piezocrystal be aligned with the coordinate axis $0z$. Then in the coordinate plane $0xy$ the quasi-static equations for unidirectional electroactive plane deformation with respect to both amplitude functions of elastic displacements $\mathrm{u}_2(x,y)$, $\mathrm{v}_2(x,y)$ and the potential of the electric field $\varphi_2(x,y)$ will be written in the form [12, 23]

$$\mathrm{u}_{2,xx}(x,y) + \mathrm{u}_{2,yy}(x,y) = -(\omega^2/C_{2l}^2)\cdot\mathrm{u}_2(x,y) \tag{1.7}$$

$$\mathrm{v}_{2,xx}(x,y)+\mathrm{v}_{2,yy}(x,y)=-(\omega^2/C_{2t}^2)\cdot \mathrm{v}_2(x,y) \tag{1.8}$$

$$[\varphi_{2,xx}(x,y)+\varphi_{2,yy}(x,y)]=(e_{11}/\varepsilon_{11})\cdot[\mathrm{u}_{2,xx}(x,y)+\mathrm{u}_{2,yy}(x,y)] \tag{1.9}$$

In equations (1.7) ÷ (1.9), the $c_{11}^{(2)}$, $c_{12}^{(2)}$ and $c_{66}^{(2)}=\left(c_{11}^{(2)}-c_{12}^{(2)}\right)/2$ is the elastic modulus of rigidity and shear, respectively, $e_{11}^{(2)}$ is the piezoelectric modulus, $\varepsilon_{11}^{(2)}$ is the relative dielectric constant, ρ_2 is the density and $\chi_2^{(2)}=\sqrt{e_{11}^{(2)}/(c_{66}^{(2)}\varepsilon_{11}^{(2)})}$ is the coefficient of electromechanical coupling of piezoelectric material. $C_{2l}=\sqrt{C_{2t}^2\cdot(1+\chi_2^2)}=\sqrt{(c_{66}^{(2)}/\rho_2)\cdot(1+\chi_2^2)}$ and $C_{2t}=\sqrt{c_{66}^{(2)}/\rho_2}$ are the velocities of longitudinal and transverse elastic volumetric waves, respectively, without taking into account the piezoelectric properties of the material.

The conditions of mechanically free surfaces $y=\pm h$ of rectangular sections of strips $\Omega_{n2}(x,y)$ in the case of a selected cut of the given piezoelectric will be written in the form

$$\begin{aligned}&\left[c_{12}^{(2)}\cdot \mathrm{u}_{2,x}(x,y)+c_{11}^{(2)}\cdot \mathrm{v}_{2,y}(y)\right]_{y=\pm h}=0\\&\left[\mathrm{u}_{2,y}(x,y)+\mathrm{v}_{2,x}(x,y)+(e_{11}^{(2)}/c_{66}^{(2)})\cdot\varphi_{2,y}(x,y)\right]_{y=\pm h}=0\end{aligned} \tag{1.10}$$

The conditions for the conjugation of the electric field with the external field on the surfaces $y=\pm h$ of rectangular sections of strips $\Omega_{n2}(x,y)$ written in the form

$$\begin{aligned}&\left[(e_{11}^{(2)}/\varepsilon_{11}^{(2)})\cdot[\mathrm{u}_{2,y}(x,y)+\mathrm{v}_{2,x}(x,y)]-\varphi_{2,y}(x,y)+(\varepsilon_0/\varepsilon_{11}^{(2)})\cdot\varphi_{e,y}(x,y)\right]_{y=\pm h}=0,\\&\left[\varphi_2(x,y)-\varphi_e(x,y)\right]_{y=\pm h}=0\end{aligned} \tag{1.11}$$

On the electrically open surface $y=h$ and on the electrically closed surface $y=-h$ of the piezoelectric layer, the conditions for the transparency and screening of the electric field, respectively, take the form

$$\left[(e_{11}^{(2)}/\varepsilon_{11}^{(2)})\cdot[\mathrm{u}_{2,y}(x,y)+\mathrm{v}_{2,x}(x,y)]-\varphi_{2,y}(x,y)\right]_{y=h}=0, \tag{1.12}$$

$$\varphi_2(x,y)\big|_{y=-h}=0 \tag{1.13}$$

Without acoustic contact of the strips of the composite waveguide, on the facial surfaces $x_{-1n}=-a_2\pm n(a_1+a_2)$, $x_{0n}=\pm n(a_1+a_2)$ and $x_{1n}=a_1\pm n(a_1+a_2)$, where $n\in\mathbb{N}^+$ of the interlayers, both the conditions of mechanically free surfaces and the conditions of conjugation of the electric field are satisfied.

On all the facial surfaces $x_{-1n}=-a_2\pm n(a_1+a_2)$, $x_{0n}=\pm n(a_1+a_2)$ and $x_{1n}=a_1\pm n(a_1+a_2)$, where $n\in\mathbb{N}^+$ of the interlayers, the conditions of mechanically free surfaces are written in the following form

$$c_{44}^{(1)}\cdot \mathrm{w}_{1,x}(x,y)+e_{15}^{(1)}\cdot\varphi_{1,x}(x,y)=0, \tag{1.14}$$

$$c_{11}^{(2)}\cdot \mathrm{u}_{2,x}(x,y)+c_{12}^{(2)}\cdot \mathrm{v}_{2,y}(x,y)=0, \tag{1.15}$$

$$\mathrm{u}_{2,y}(x,y)+\mathrm{v}_{2,x}(x,y)+(e_{11}^{(2)}/c_{66}^{(2)})\cdot\varphi_{2,y}(x,y)=0 \tag{1.16}$$

The wave signal propagates along the inhomogeneous waveguide by means of the penetration of accompanying electrical oscillations through vacuum gaps on the same surfaces. On all these facial surfaces, the conditions for the conjugation of the electric field are as follows

$$\varphi_1(x,y)=\varphi_2(x,y) \tag{1.17}$$

$$e_{15}^{(1)}\cdot\mathrm{w}_{1,x}(x,y)+\varepsilon_{11}^{(1)}\cdot\varphi_{1,x}(x,y)=e_{11}^{(2)}\cdot\mathrm{u}_{2,x}(x,y)-e_{11}^{(2)}\cdot\mathrm{v}_{2,y}(x,y)+\varepsilon_{11}^{(2)}\cdot\varphi_{2,x}(x,y) \tag{1.18}$$

Systems of equations (1.2) and (1.7)÷(1.9) together with boundary conditions (1.5) and (1.6) on the surfaces of the inhomogeneous waveguide $y=\pm h$, as well as with boundary conditions (1.14)÷(1.18) at the ends $x_{-1n}=-a_2\pm n(a_1+a_2)$, $x_{0n}=\pm n(a_1+a_2)$ and $x_{1n}=a_1\pm n(a_1+a_2)$, constitute the complete mathematical boundary value problem for studying the propagation of the hybrid of four-component and three-component electroactive elastic waves of plane and antiplane deformations.

The general solutions of equations (1.2) and (1.7)÷(1.9) satisfying the boundary conditions (1.5) and (1.6) on the surfaces of an inhomogeneous waveguide characterize the distribution of the wave field (intensity of wave quantities) over the thickness of the waveguide.

The solutions satisfying the boundary conditions (1.14)÷(1.18) at the inner ends of the interlayers of the composite waveguide correspond to the filtration mode (the admissible frequencies of waves propagation).

2. Solution of the mathematical boundary value problem

Based on the structural periodicity of the inhomogeneous waveguide, it is natural to study the propagation of the electroelastic wave signal according to the Floquet-Lyapunov theory. The periodicity of the structural inhomogeneity of the composite waveguide makes it possible to construct the solution to the formulated boundary value problem for the unit periodic composite cell $\Omega_0(x,y)=\Omega_{01}(x,y)\cup\Omega_{02}(x,y)$, taking into account the Floquet conditions on the facial surfaces of the composite.

2.1. Formation of a hybrid of multicomponent electroacoustic waveforms over the thickness of the interlayers of the unit cell.

Propagating along the infinite periodically inhomogeneous waveguide, the Normal wave signal induces a three-component and a four-component waveform of $f_{mn}(x,y)=Y_m(y)\cdot X_m(x)$ type, in each layer, respectively

$$X_m(x)=\sum_{n=1}^{\infty}\left[C_n\cos(k_{mn}x)+D_n\sin(k_{mn}x)\right],$$

$$Y_m(y)=\sum_{n=1}^{\infty}\left[A_n\cos(\alpha_n k_{mn}y)+B_n\sin(\alpha_n k_{mn}y)\right]. \tag{2.1}$$

In wave functions $f_{mn}(x,y)$, the multiplier $Y_m(y)$ characterizes the shape of the wave component along the thickness of the waveguide, $X_m(x)$ characterizes the shape of the propagation of the wave component, $m\in\{1;2\}$ is the numbering of the wave numbers in the layers $\Omega_{1n}(x,y)$ and $\Omega_{2n}(x,y)$, respectively.

Induced in the interlayers $\Omega_{1n}(x,y)$, the three-component normal electroelastic shear waveforms, as solutions of the system of equations (1.2) are written in the form

$$\mathrm{w}_1(x,y)=\sum_{n=1}^{\infty}\left[A_{\mathrm{wn}}\cos(\beta_{1n}k_{1n}y)+B_{\mathrm{wn}}\sin(\beta_{1n}k_{1n}y)\right]\left[C_{1n}\cos(k_{1n}x)+D_{1n}\sin(k_{1n}x)\right], \tag{2.2}$$

$$\varphi_1(x,y)=\sum_{n=1}^{\infty}\left\{\begin{gathered}\left[A_{\varphi n}\cos(k_{1n}y)+B_{\varphi n}\sin(k_{1n}y)\right]+\\ +(e_{15}^{(1)}/\varepsilon_{11}^{(1)})\begin{bmatrix}A_{\mathrm{wn}}\cos(\beta_{1n}k_{1n}y)+\\ +B_{\mathrm{wn}}\sin(\beta_{1n}k_{1n}y)\end{bmatrix}\end{gathered}\right\}\left[C_{1n}\cos(k_{1n}x)+D_{1n}\sin(k_{1n}x)\right], \tag{2.3}$$

where the wave number $k_{1n}(\omega)$ for each form will be defined from dispersion equation

$$\beta_{1n}(\omega)\left[\beta_{1n}(\omega)\cdot\frac{\tan[2\beta_{1n}(\omega)\cdot k_{1n}(\omega)h]}{\tan[2k_{1n}(\omega)h]}-\chi_1^2\right]=0 \tag{2.4}$$

Solutions (2.2) ÷ (2.4) involve the wave coefficient $\beta_{1n}(\omega)=\sqrt{[\omega^2C_{1t}^{-2}]/k_{1n}^2(\omega)-1}$ of shear oscillations for swift waves of antiplane deformation, for the phase velocities of the forms of which $\omega/k_{1n}(\omega)\geq C_{1t}$. Generally, swift waves correspond to long waveforms (low oscillation frequencies), for which $\lambda_{1n}(\omega)\cong h$ or $k_{1n}(\omega)\cdot h\cong 1$.

In the case of the high-frequency wave signal, in the piezoelectric rectangle $\Omega_{n1}(x,y)$, the short electroactive elastic transverse single-mode wave propagates in the piezoelectric rectangle, for which $\lambda_1(\omega)\ll h$ or $k_1(\omega)\cdot h\gg 1$. Then, electroactive shear waveforms can be damped deep into the piezoelectric rectangle $\Omega_{n1}(x,y)$.

The high frequency components of the electroelastic wave are represented as

$$\mathrm{w}_1(x,y)=\begin{bmatrix}A_{\mathrm{w}}\sinh[\alpha_1(\omega)\cdot k_1(\omega)y]+\\ +B_{\mathrm{w}}\cosh[\alpha_1(\omega)\cdot k_1(\omega)y]\end{bmatrix}\cdot\left[C_1\cos(k_1x)+D_1\sin(k_1x)\right], \tag{2.5}$$

$$\varphi_1(x,y)=\left\{\begin{gathered}\left[A_{\varphi}\sinh[k_1(\omega)y]+B_{\varphi}\cosh[k_1(\omega)y]\right]+\\ +(e_{15}^{(1)}/\varepsilon_{11}^{(1)})\begin{bmatrix}A_{\mathrm{w}}\sinh[\alpha_1(\omega)\cdot k_1(\omega)y]+\\ +B_{\mathrm{w}}\cosh[\alpha_1(\omega)k_1(\omega)y]\end{bmatrix}\end{gathered}\right\}\cdot\left[C_1\cos(k_1(\omega)x)+D_1\sin(k_1(\omega)x)\right] \tag{2.6}$$

and the wave number in the interlayer is determined from the dispersion equation

$$\alpha_1(\omega)\cdot\left[\alpha_1(\omega)\cdot\frac{\tanh[2\alpha_1(\omega)\cdot k_1(\omega)h]}{\tanh[2k_1(\omega)h]}-\chi_1^2\right]=0 \tag{2.7}$$

Solutions (2.5) and (2.6), as well as the dispersion equation include the wave coefficient $\alpha_1(\omega)=\sqrt{1-[\omega^2 C_{1t}^{-2}]/k_1^2(\omega)}$ of shear oscillations for slow waves of antiplane deformation, for the phase velocities of the forms of which $\omega/k_1(\omega)<C_{1t}$.

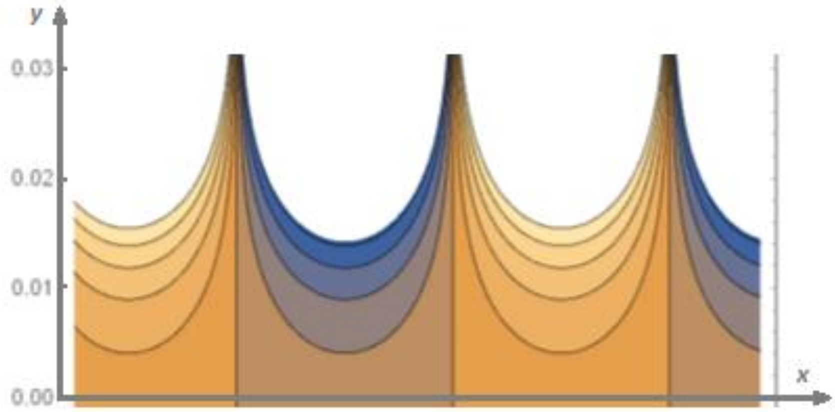

Fig.2 *The shear displacement distributions along the thickness of the interlayer* $\Omega_{n1}(x,y)$ *, in the case of different electrical surface conditions on the waveguide surfaces*

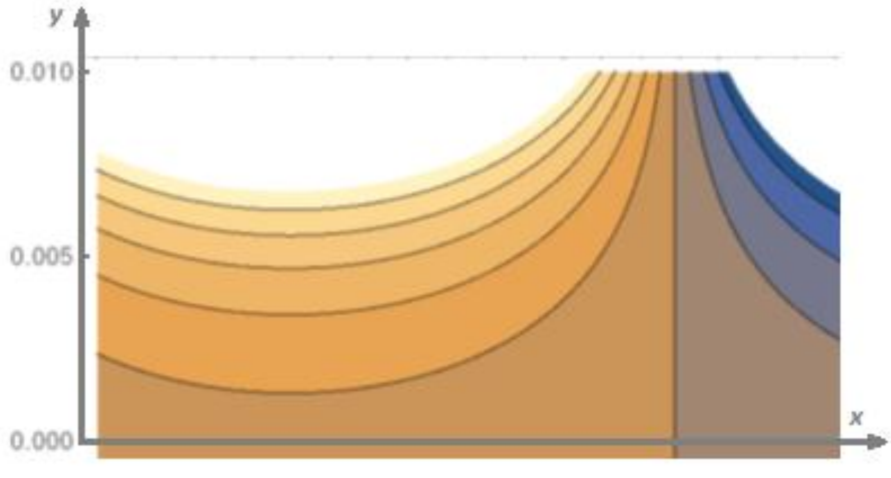

Fig.3 *The electric field potential distributions along the thickness of the interlayer* $\Omega_{n1}(x,y)$ *, in the case of different electrical surface conditions on the waveguide surfaces*

From the dispersion equation (2.7), it follows that in the piezoelectric waveguide with different surface conditions, the Gulyaev-Bluestein-type waves become highly dispersive.

In the case of identical surface conditions of the electric field (1.4), at the oscillation frequency $\omega \gg C_{1t} h^{-1} \sqrt{1-\chi_1^2 [\varepsilon_0 / (\varepsilon_{11}^{(1)} + \varepsilon_0)]}$, the same localization of the wave energy of high-frequency electroacoustic waves of the Gulyaev-Bluestein type occurs near mechanically free surfaces (Fig. 2).

In the case of different surface conditions on the electric field (1.5) and (1.6), near the surfaces $y = h$ and $y = -h$ the wave energy localization of the Gulyaev-Bluestein type electroelastic waves occurs in different ways (Fig. 3). Near the electrically open surface $y = h$, there will not be a localization of wave energy. Near the electrically closed surface $y = -h$, at high frequencies $\omega \gg C_{1t} h^{-1} \sqrt{1-\chi_1^4}$, there will be a localization of the energy of electroacoustic waves.

Due to the conjugation of the electric fields (1.17) and (1.18) on the mechanically free end surfaces of adjacent interlayers, the electroelastic wave of plane deformation arises in the second interlayer $\Omega_{n2}(x, y)$ [11, 25]. The four-component normal electroelastic wave of plane deformation induced in the interlayer $\Omega_{n2}(x, y)$, as solutions of the system of equations (1.7)÷(1.9) with surface conditions (1.10), (1.12), (1.13), can be represented as

$$\mathrm{u}_2(x, y) = \sum_{n=1}^{\infty} \left[A_{un} \cos(\beta_{2l} k_{2n} y) + B_{un} \sin(\beta_{2l} k_{2n} y) \right] \left[C_{2n} \cos(k_{2n} x) + D_{2n} \sin(k_{2n} x) \right] \qquad (2.8)$$

$$\mathrm{v}_2(x, y) = \sum_{n=1}^{\infty} \left[A_{vn} \cos(\beta_{2t} k_{2n} y) + B_{vn} \sin(\beta_{2t} k_{2n} y) \right] \left[C_{2n} \cos(k_{2n} x) + D_{2n} \sin(k_{2n} x) \right] \qquad (2.9)$$

$$\varphi_2(x, y) = \sum_{n=1}^{\infty} \left\{ \begin{array}{l} \left[A_{\phi n} \cos(k_{2n} y) + B_{\phi n} \sin(k_{2n} y) \right] + \\ +(e_{11}^{(2)} / \varepsilon_{11}^{(2)}) \times \begin{bmatrix} A_{un} \cos(\beta_{un} k_{2n} y) + \\ +B_{un} \sin(\beta_{un} k_{2n} y) \end{bmatrix} \end{array} \right\} \left[C_{2n} \cos(k_{2n} x) + D_{2n} \sin(k_{2n} x) \right] \qquad (2.10)$$

Solutions (2.8) ÷ (2.10) include $\beta_{2l}(\omega) = \sqrt{[\omega^2 C_{2l}^{-2}] / k_{2n}^2(\omega) - 1}$ and $\beta_{2t}(\omega) = \sqrt{[\omega^2 C_{2t}^{-2}] / k_{2n}^2(\omega) - 1}$ wave coefficients for longitudinal and shear oscillations of plane deformation swift waves, for the phase velocities of the forms of which $\omega / k_{2n}(\omega) \geq \max\{C_{2l}; C_{2t}\}$.

Generally, swift waves correspond to long waveforms (low oscillation frequencies), for which $\lambda_{2n}(\omega) \cong h$ or $k_{2n}(\omega) \cdot h \cong 1$.

Satisfying the surface conditions (1.10), (1.12), and (1.13), for determining the wave number, we obtain the dispersion equations of the generated waveforms

$$\sin(2k_2 h) \cdot \sin(2\beta_{2t} k_2 h) \cdot \sin(2\beta_{2l} k_2 h) = 0 \qquad (2.11)$$

$$\beta_{2t}^2 \beta_{2l}^2 = \chi_{12}^2 \beta_{2t} \cdot \left(2\theta_{12}^* - \beta_{2l} \beta_{2t} \right) \cdot \sin(2k_2 h) \cdot ctg(2\beta_{2l} k_2 h) + (1 + \chi_2^2) \cdot (\theta_{12}^*)^2 \qquad (2.12)$$

Dispersion equations (2.11) and (2.12) include the wave coefficients $\beta_{2l}(\omega, k_{2n})$ and $\beta_{2t}(\omega, k_{2n})$ for longitudinal and shear oscillations of plane deformation swift waves. And here we also have the dimensionless material anisotropy parameters $\theta_{12} = c_{12}^{(2)} / c_{11}^{(2)}$ and $\theta_{12} = c_{12}^{(2)} / \tilde{c}_{11}^{(2)}$ with the given piezoelectric effect of material, with the stiffness coefficient $\tilde{c}_{11}^{(2)} = c_{11}^{(2)} \cdot (1+\chi_2^2)$ and $\chi_{12}^2 = \chi_2^2 / (1+\chi_2^2)$, $\chi_2^2 = (e_{11}^{(2)})^2 / (c_{66}^{(2)} \varepsilon_{11}^{(2)})$ - the electromechanical coupling coefficient for the second piezoelectric.

Dispersion equation (2.11) describes the formation of eigenmodes of bulk oscillations of the electric potential, longitudinal and transverse displacements, which do not decay over the thickness of the waveguide. Their wave numbers are defined as $k_{2n}^{(e)} = n\pi/2h$, $k_{2n}^{(u)}(\omega) = \sqrt{(\omega/C_{2l})^2 - (n\pi/2h)^2}$ and $k_{2n}^{(v)}(\omega) = \sqrt{(\omega/C_{2t})^2 - (n\pi/2h)^2}$, respectively, where $n \in \mathbb{N}^+$.

Dispersion equation (2.12) describes coupled electroacoustic oscillations of plane strain.

In the case of high-frequency (short-wavelength), when $\omega / k_{2n}(\omega) \le \sqrt{c_{66}^{(2)} / \rho_2}$ the slow wave signal is converted into an equation of the form

$$\alpha_{2t}^2 \alpha_{2l}^2 = \chi_{12}^2 \alpha_{2t} \cdot \left(2\theta_{12}^* + \alpha_{2l}\alpha_{2t}\right) \cdot \sinh(2k_2 h) \cdot \coth(2\alpha_{2l} k_2 h) + (1+\chi_2^2) \cdot (\theta_{12}^*)^2 \tag{2.13}$$

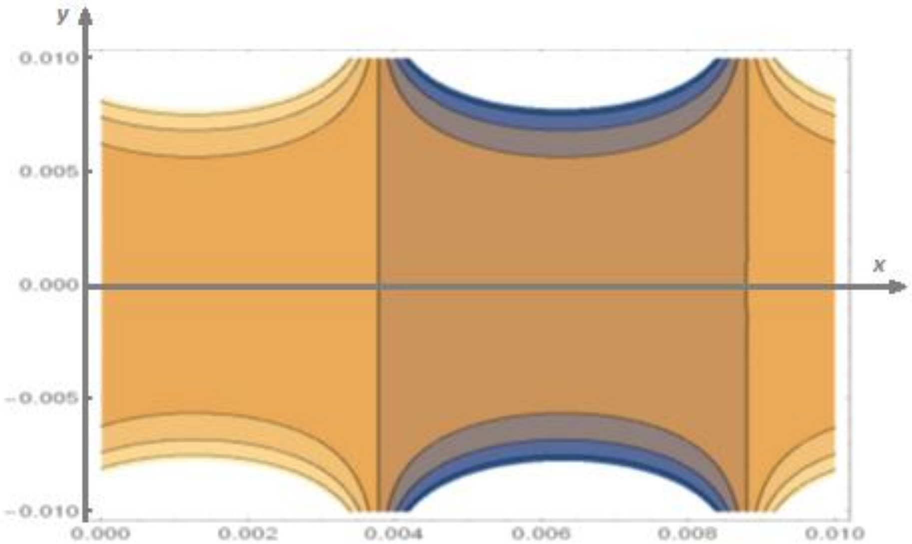

Fig.4 *The planar and shear displacement distributions along the thickness of the interlayer* $\Omega_{2n}(x, y)$ *, in the case of different electrical surface conditions on the waveguide surfaces*

In this case, the four-component normal electroelastic wave of plane deformation induced in the interlayer, as solutions of the system of equations (1.7)÷(1.9) with surface conditions (1.10), (1.12), (1.13), is represented as

$$\mathrm{u}_2(x,y)=\left[A_u^*\cosh(\alpha_{2l}k_2y)+B_u^*\sinh(\alpha_{2l}k_2y)\right]\left[C_2\cosh(k_2x)+D_2\sinh(k_2x)\right] \tag{2.14}$$

$$\mathrm{v}_2(x,y)=\left[A_v^*\cosh(\alpha_{2t}k_2y)+B_v^*\sinh(\alpha_{2t}k_2y)\right]\left[C_2^*\cosh(k_2x)+D_2^*\sinh(k_2x)\right] \tag{2.15}$$

$$\varphi_2(x,y)=\left\{\begin{matrix}\left[A_\phi^*\cosh(k_2y)+B_\phi^*\sinh(k_2y)\right]+\\ +(e_{11}^{(2)}/\varepsilon_{11}^{(2)})\begin{bmatrix}A_u^*\cosh(\alpha_{2l}k_2y)+\\ +B_u^*\sinh(\alpha_{2t}k_2y)\end{bmatrix}\end{matrix}\right\}\left[C_2\cosh(k_2x)+D_2\sinh(k_2x)\right] \tag{2.16}$$

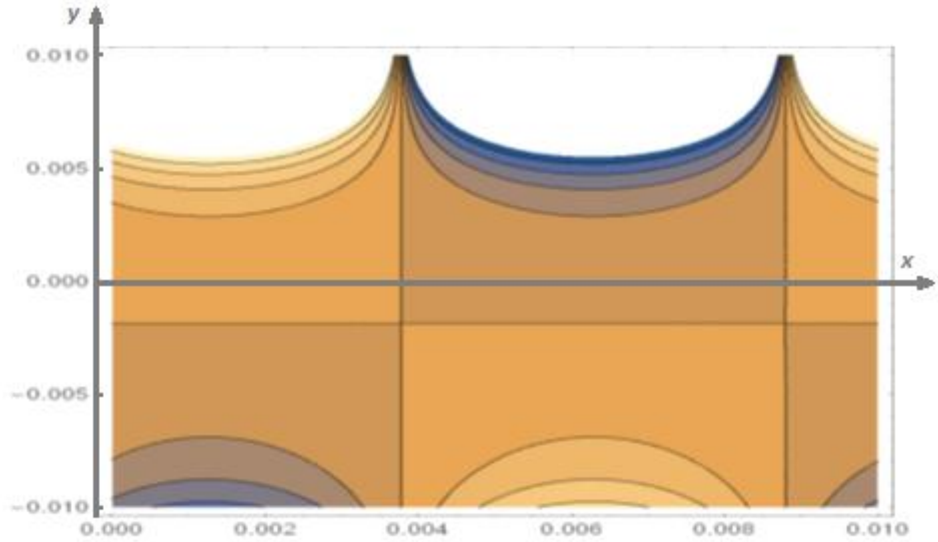

Fig.5 *The electric field potential distributions along the thickness of the interlayer* $\Omega_{2n}(x,y)$ *, in the case of different electrical surface conditions on the waveguide surfaces*

Solutions (2.14) ÷ (2.16) include $\alpha_{2l}(\omega)=\sqrt{1-[\omega^2C_{2l}^{-2}]/k_2^2(\omega)}$ and $\alpha_{2t}(\omega)=\sqrt{1-[\omega^2C_{2t}^{-2}]/k_2^2(\omega)}$ wave coefficients for longitudinal and shear oscillations of plane deformation slow waves, for the phase velocities of the forms of which $\omega/k_2(\omega)\geq\min\{C_{2l};C_{2t}\}$.

If the piezoelectric effect in the second piezoelectric is zero $\chi_2^2=0$, from (2.13) we obtain the dispersion equation for Rayleigh waves in the isotropic medium $\alpha_{2t}^2\alpha_{2l}^2=\theta_{12}^2$.

In the case of different surface electric conditions (1.12) and (1.13) on the surfaces of the waveguide the distributions of both planar and shear displacements, as well as of potential of the electric field over the thickness of the interlayer $\Omega_{2n}(x,y)$ are shown in Fig. 4 and Fig. 5.

2.2. Propagation of waveforms localized near the surfaces of the waveguide in the periodically longitudinally inhomogeneous composite layer.

Taking into account the nature of the periodicity of the structure of the inhomogeneous waveguide, to determine the patterns of propagation of forms of electroelastic waves localized near the surfaces of the waveguide (2.5) and (2.6), as well as (2.14) ÷ (2.16) we will use the Floquet-Lyapunov theory for periodic structures. Non-acoustic contact on the face surfaces of the periodic structure $x=-a_2$ and $x=a_1$, simplifies the corresponding surface conditions according to the Floquet-Lyapunov theory. It makes it possible to use the conditions of the mechanically free surface on the frontal surfaces of visible interlayers (1.14) ÷ (1.16)

$$\begin{aligned}&\left[c_{11}^{(2)}\cdot \mathrm{u}_{2,x}(x,y)+c_{12}^{(2)}\cdot \mathrm{v}_{2,y}(x,y)\right]_{x=-a_2}=0\\&\left[c_{11}^{(2)}\cdot \mathrm{u}_{2,x}(x,y)+c_{12}^{(2)}\cdot \mathrm{v}_{2,y}(x,y)\right]_{x=0}=0\end{aligned}, \tag{2.17}$$

$$\begin{aligned}&\left[\mathrm{u}_{2,y}(x,y)+\mathrm{v}_{2,x}(x,y)+(e_{11}^{(2)}/c_{66}^{(2)})\cdot\varphi_{2,y}(x,y)\right]_{x=-a_2}=0\\&\left[\mathrm{u}_{2,y}(x,y)+\mathrm{v}_{2,x}(x,y)+(e_{11}^{(2)}/c_{66}^{(2)})\cdot\varphi_{2,y}(x,y)\right]_{x=0}=0\end{aligned} \tag{2.18}$$

$$\begin{aligned}&\left[c_{44}^{(1)}\cdot \mathrm{w}_{1,x}(x,y)+e_{15}^{(1)}\cdot\varphi_{1,x}(x,y)\right]_{x=0}=0\\&\left[c_{44}^{(1)}\cdot \mathrm{w}_{1,x}(x,y)+e_{15}^{(1)}\cdot\varphi_{1,x}(x,y)\right]_{x=a_1}=0\end{aligned}, \tag{2.19}$$

On the face surfaces of the composite waveguide, the conjugation conditions for the electric field are satisfied. On face surface $x=0$ these conditions can be written as

$$\varphi_1(x,y)\big|_{x=0}=\varphi_2(x,y)\big|_{x=0} \tag{2.20}$$

$$\begin{aligned}&\left[e_{15}^{(1)}\cdot \mathrm{w}_{1,x}(x,y)-\varepsilon_{11}^{(1)}\cdot\varphi_{1,x}(x,y)\right]_{x=0}=\\&=\left[e_{11}^{(2)}\cdot \mathrm{u}_{2,x}(x;y)-e_{11}^{(2)}\cdot \mathrm{v}_{2,y}(x,y)-\varepsilon_{11}^{(2)}\cdot\varphi_{2,x}(x,y)\right]_{x=0}\end{aligned} \tag{2.21}$$

On the face surfaces $x=-a_2$ and $x=a_1$, according to the Floquet-Lyapunov theory, the periodicity of the longitudinal inhomogeneity of the composite waveguide allows the conjugation conditions for the electric field to be written as

$$\varphi_1(x,y,t)\big|_{x=a_1}=\mu\cdot\varphi_2(x,y,t)\big|_{x=-a_2} \tag{2.22}$$

$$\begin{aligned}&\left[e_{15}^{(1)}\cdot \mathrm{w}_{1,x}(x,y)-\varepsilon_{11}^{(1)}\cdot\varphi_{1,x}(x,y)\right]_{x=a_1}=\\&=\mu\cdot\left[e_{11}^{(2)}\cdot \mathrm{u}_{2,x}(x;y)-e_{11}^{(2)}\cdot \mathrm{v}_{2,y}(x,y)-\varepsilon_{11}^{(2)}\cdot\varphi_{2,x}(x,y)\right]_{x=-a_2}\end{aligned} \tag{2.23}$$

In the boundary conditions (2.22) and (2.23), the multiplier $\mu=\exp[iL\cdot k(\omega)]$ is Floquet periodicity coefficient and $L=a_1+a_2$ is the linear periodicity parameter, $k(\omega)=2\pi/\lambda(\omega)$ is the Floquet wave number (the wave number of the generated wave), corresponding to the resolvable wavelengths $\lambda(\omega)$ in the layered waveguide.

Substituting the solutions (2.5), (2.6) and (2.14) ÷ (2.16) into the boundary conditions of the mechanically free surfaces (2.17) ÷ (2.19), as well as into the conditions of the electric fields conjugation (2.20) and (2.23) on the face surfaces of the piezoelectric layers, we obtain the algebraic system of linear equations with respect to amplitude functions. From the condition for the existence of nonzero amplitude functions, we obtain the dispersion equation for frequency filtering for the hybrid of electroelastic waves

$$k(\omega)=\frac{1}{a_1+a_2}\times$$
$$\times\mathrm{Arccos}\left[\frac{k_1^2(\omega)\cdot\cos^2[k_1(\omega)a_1]+k_2^2(\omega)\cdot\left[4\cos[k_2(\omega)a_2]-\cos[k_1(\omega)a_1]\right]^2}{2k_1(\omega)k_2(\omega)\cdot\cos[k_1(\omega)a_1]\cdot\left[4\cos[k_2(\omega)a_2]-\cos[k_1(\omega)a_1]\right]}\right] \tag{2.24}$$

If we take into account that the hybrid components in the interlayers of the inhomogeneous waveguide are Gulyaev-Bluestein and Rayleigh-type waves, for which $k_1(\omega)=k_{GB}(\omega)=\omega/C_{GB}$ and $k_2(\omega)=k_R(\omega)=\omega/C_R$, respectively, then the dispersion equation for frequency filtering can be written as

$$k(\omega)=\frac{1}{a_1+a_2}\times$$
$$\times\mathrm{Arccos}\left[\frac{\left(\omega/C_{GB}\right)^2\cdot\cos^2[\omega a_1/C_{GB}]+\left(\omega/C_R\right)^2\cdot\left[4\cos[\omega a_2/C_R]-\cos[\omega a_1/C_{GB}]\right]^2}{2\left(\omega/C_{GB}\right)\cdot\left(\omega/C_R\right)\cdot\cos[\omega a_1/C_{GB}]\cdot\left[4\cos[\omega a_2/C_R]-\cos[\omega a_1/C_{GB}]\right]}\right] \tag{2.25}$$

The allowable wavelengths in the Floquet-type hybrid wave are determined from the equation

$$\lambda(\omega)=$$
$$=\frac{2\pi(a_1+a_2)}{\mathrm{Arccos}\left[\frac{\left(\omega/C_{GB}\right)^2\cdot\cos^2[\omega a_1/C_{GB}]+\left(\omega/C_R\right)^2\cdot\left[4\cos[\omega a_2/C_R]-\cos[\omega a_1/C_{GB}]\right]^2}{2\left(\omega/C_{GB}\right)\cdot\left(\omega/C_R\right)\cdot\cos[\omega a_1/C_{GB}]\cdot\left[4\cos[\omega a_2/C_R]-\cos[\omega a_1/C_{GB}]\right]}\right]} \tag{2.26}$$

The filtration equation, with different combinations of the selected pairs of piezoelectric materials, the zones of permissible frequencies for localized and non-localized electroelastic waves propagating along the composite waveguide are determined. The filtration equation also gives bands of forbidden frequencies, at which the composite waveguide of certain piezoelectrics and linear dimensions does not allow the propagation of localized electroelastic waves, or waves in general.

Consequently, by the proper choice of materials and linear dimensions of the interlayers, it is possible to achieve optimal transfer of wave energy from one interlayer to another, or vice versa. Thus, the inhomogeneous waveguide can become a kind of electromechanical filter or resonator.

Let us consider a particular case when, in the interlayers, the velocities of localized electroacoustic Gulyaev-Bluestein and Rayleigh waves are equal to $C_{GB}=2.574\times10^3\ m/\sec$ and $C_R=2.752\times10^3\ m/\sec$, respectively. And the electromechanical coupling coefficients of these materials are respectively $\chi_1^2 = 0.9013$ and $\chi_2^2 = 0.1021$.

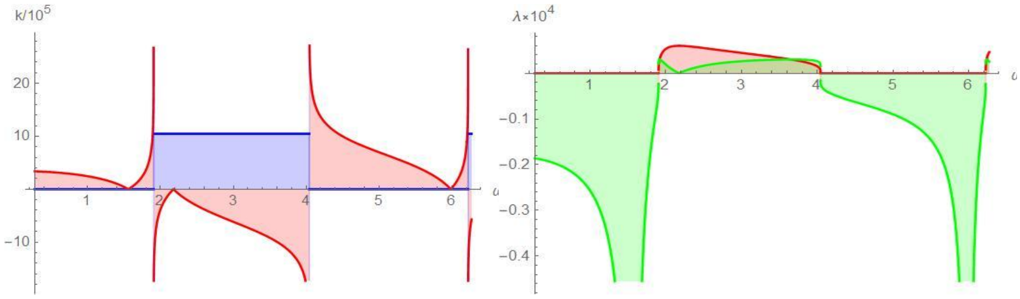

Fig. 6. *Permissible frequencies and wavelengths of the Floquet type in the case of piezo layer thicknesses* $a_1 = 1\times10^{-6}\,m$ *and* $a_2 = 2\times10^{-6}\,m$.

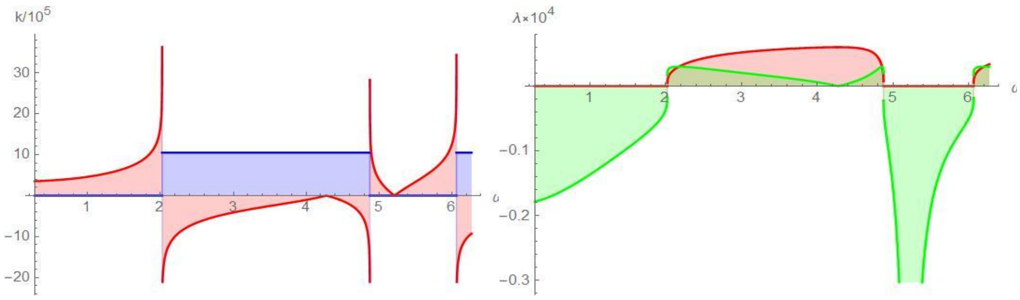

Fig. 7. *Permissible frequencies and wavelengths of the Floquet type in the case of piezo layer thicknesses* $a_1 = 2\times10^{-6}\,m$ *and* $a_2 = 1\times10^{-6}\,m$.

In different cases of choosing the thicknesses of the piezoelectric interlayers, when the thickness of the waveguide is $h = 1\times10^{-5}\,m$, from the relations (2.25) and (2.26) for the allowable frequencies and wavelengths of the Floquet type waves are given in Figures 6, 7, 8. From all the above graphs, it is clear that at relatively low frequencies, when $0 < \omega \le 1\times10^9\ Hz$, there can be rapidly decaying electroacoustic wave signals, since in this case $0 < \mathrm{Im}[k(\omega)] \le 5\times10^5\ m$.

In all the above cases, when the allowable frequency of wave hybrid propagation changes in the segment $2\times10^9\ Hz < \omega < 2\times10^9\ Hz$ (Fig. 6), or in the segment $2\times10^9\ Hz < \omega < 5\times10^9\ Hz$ (Fig. 7), or in the rendition $4\times10^9\ Hz < \omega < 4{,}43\times10^9\ Hz$ (Fig. 8), a hybrid of localized normal waves propagates with wavelengths

$0 < \mathrm{Re}[\lambda(\omega)] \le 0.06\times10^{-4}\,m$, $0 < \mathrm{Re}[\lambda(\omega)] \le 0.08\times10^{-4}\,m$, or $0 < \mathrm{Re}[\lambda(\omega)] \le 0.04\times10^{-4}\,m$, respectively, or a rapidly decaying wave signal, for which $0 < \mathrm{Im}[\lambda(\omega)] \le 0.038\times10^{-4}\,m$.

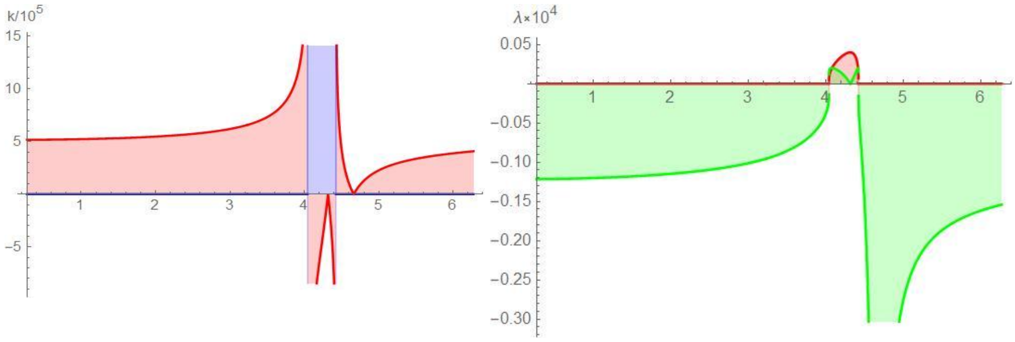

Fig. 8. *Permissible frequencies and wavelengths of the Floquet type in the case of piezo layer thicknesses* $a_1 = 1\times10^{-6}\,m$ *and* $a_2 = 1\times10^{-6}\,m$.

Conclusion.

An electroacoustic wave signal propagating in a transversely periodically inhomogeneous piezoelectric waveguide is converted into a hybrid of electroactive waves of plane and antiplane deformations, when the inhomogeneous periodic cell in the waveguide consists of different piezoelectrics that are in non-acoustic contact. In the case of propagation of a high-frequency wave signal, it forms a hybrid of electroactive waves of the Gulyaev-Bluestein and Rayleigh types localized near the outer surfaces of the waveguide. The zones of permissible frequencies for the propagation of the formed hybrid of waves of the Floquet type are determined. The zones of the corresponding lengths of the propagating wave are also determined. Rapidly decaying localized waves were also found in the allowable frequency zones.

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Received 25.02.2023