ՀԱՅԱՍՏԱՆԻԳԻՏՈՒԹՅՈՒՆՆԵՐԻԱԶԳԱՅԻՆԱԿԱԴԵՄԻԱՅԻՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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MULTI-COMPONENT ELECTROACOUSTIC WAVES (MCEAW) IN PIEZO CRYSTALLINE TEXTURES:APPLIED OPPORTUNITIES

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Keywords: electroelasticity, multi-component wave,generalized tensor, electroactive state, piezocrystallinetexture, non-acoustic contact, plane deformation, quasistatic equations.

Ա.Ս. Ավետիսյան

Բազմաբաղադրիչ էլեկտրաակուստիկ ալիքները (MCEAW) պիեզոբյուրեղային կառուցվածքում. Կիրառական հնարավորություններ

Հիմնաբառեր՝ էլեկտրաառաձգականություն, բազմաբաղադրիչ ալիք, ընդհանրացված տենզոր, էլեկտրաակտիվ վիճակ, պիեզոկրիստալային հյուսվածք, ոչ ակուստիկ կոնտակտ, հարթ դեֆորմացիա, քվազիստատիկ հավասարումներ:

Աշխատանքում ցույց է տրվում, որ պիեզոէլեկտրական կառուցվածքի յուրաքանչյուր ընտրված հարթությունում և՛ մեկ բաղադրիչով առաձգական սահքի ալիքը, և՛ երկու բաղադրիչով առաձգական հարթ դեֆորմացիայի ալիքը կարող է ուղեկցվել կամ ընտրված հարթությանը ուղղահայաց էլեկտրական դաշտի բաղադրիչի տատանումներով կամ ընտրված հարթությանը զուգահեռ, հարթ էլեկտրական դաշտի տատանումներով: Ձևավորվում են բազմաբաղադրիչ էլեկտրաառաձգական ալիքների չորս տիպի փաթեթներ: Ստացված են, պիեզոբյուրեղի ընտրված հարթությունում, էլեկտրաառաձգականության երկչափ խնդրի ձևակերպման անհրաժեշտ և բավարար պայմանները, որոնք թույլ են տալիս ձևավորել բազմաբաղադրիչ քվազիստատիկ էլեկտրաառաձգական ալիքի առանձին գրգռում և տարածում:

Ара С. Аветисян

Многокомпонентные электроакустические волны (МЭАВ) в пьезокристаллических текстурах: прикладные возможности

Ключевые слова: электроупругость, многокомпонентная волна, обобщенный тензор, электроактивное состояние, пьезокристаллическая текстура, неакустический контакт, плоская деформация, квазистатические уравнения.

В данной работе показывается, что в каждой сагиттальной плоскости пьезотекстуры как однокомпонентная упругая сдвиговая волна, так и двухкомпонентная упругая плоская деформационная волна могут сопровождаться либо колебаниями поперечной составляющей электрического поля, либо колебаниями плоского электрического поля. Формируются четыре таких пакета многокомпонентных электроупругих волн. Получены необходимые и достаточные условия, позволяющие поставить двумерную задачу электроупругости в вибрируемой сагиттальной плоскости пьезокристалла, где возможно раздельное возбуждение и распространение многокомпонентной квазистатической электроупругой волны.

In this paper is shown, that in each sagittal plane of the piezoelectric texture, both the one-component elastic shear wave and the two-component elastic plane deformation wave can be accompanied by either oscillations of the transverse component of the electric field or oscillations of the plane electric field. Four such packets of multicomponent electroelastic waves are formed.Necessary and sufficient conditions are obtained that allow the formulation of a two-dimensional problem of electroelasticity in a vibrated sagittal plane of a piezocrystal, where separate excitation and propagation of a multicomponent quasistatic electroelastic wave is possible.

Introduction

In many structural diagrams of modern electronic technology, various new crystalline elements, layered composite waveguides, formed from various natural or artificially grown piezoelectric materials with different physical and mechanical properties, are widely used. The piezoelectric or ferroelectric materials have many distinct properties, negative piezoelectric constants and high mechanical flexibility. Piezoelectric or ferroelectric materials are widely used in ***2D*** layered functional heterostructures. These new materials and heterostructures have broad applications in memory, logic, sensing, optical and energy harvesting devices. Piezoelectric crystals are inherently anisotropic structures and the operation of such elements is often based on the emission (or delay) of electroacoustic waves of incomplete component packages.

Separate excitation and propagation of a purely transverse elastic wave (***SH*** type waves) from a plane deformation wave (***P&SV*** type waves) is possible both in an isotropic medium and in all crystal textures of cubic, hexagonal, trigonal, tetragonal and rhombic symmetry **[1]**. A higher anisotropy of the crystal texture leads to the convolution of the components of purely elastic waves and, in fact, the separation of components in elastic bodies is uniquely determined by the structure of the elastic constant tensor.

In contrast to a purely elastic medium, where two types of waves can propagate separately: a two-component wave of the plane stress-strain state (***P&SV*** type wave) and a single-component anti-plane deformation wave (***SH*** type wave), in a piezoelectric medium, four different incomplete sets of multicomponent electroelastic waves can propagate separately. The properties of piezoelectric media are given by the structure of the generalized tensor of electromechanical constants. Therefore, in the case of piezoelectric media, the group of those symmetries and classes of crystals is significantly narrowed, in the sagittal planes of which separate excitation and propagation of the indicated types of electroelastic waves are possible.

The possibility of the separate excitation and propagation of an electroactive elastic plane deformation wave from thenon-electroactive shear elastic wave or separate excitation and propagation of an electroactive shear elastic wave from a non electroactive elastic plane deformation wave, depending on the physical properties of the piezoelectric medium, was studied in the articles [2, 3]. However, the other new cases when separate excitation and propagation of two electroactive wave packets are possiblehave not been considered yet.

The ability to formulate the problem of excitation and propagation of incomplete multicomponent electroelastic waves is due to both the anisotropy of the medium and the admissibility of setting a two-dimensional problem in one of the sagittal planes of an anisotropic medium.

As a rule, structural elements used in modern technology are thin-walled and in the formulation of two-dimensional problems, we must take into account possible approaches that allow separate formulations of the problems of electroactive plane deformation and electroactive anti-plane deformation.

The mathematical formulation of the two-dimensional problem of electroelasticity in any of the sagittal planes $0x_\alpha x_\beta$ of the piezoelectric texture requires the observance of several well-known hypotheses:

i) The hypothesis of straight normal.
ii) The hypothesis of the inextensibility of the middle surface of the plate.

iii) The hypothesis about the absence of pressure of the material layers on each other.

Adhering to the physical essence of these hypotheses, in applied problems of thin-walled elastic elements of structures, the two-dimensional problem of the theory of elasticity was mathematically modeled in different ways by many researchers Kirchhoff G. [4], Timoshenko S., Woinowsky-Krieger S. [5], Reissner E. [6], Ambartsumian S.A. [7].In each of these cases, the hypotheses were accepted as additional restrictions based on the nature of the distribution of the mechanical load on the element and the conditions for fixing the end of the elastic element.

The hypothetical approach has also been successfully implemented in the problems of electro-magneto-elasticity of thin plates and shells [8], where, along with hypothetical distributions of mechanical characteristics over the thickness of a thin-walled element, characteristic distributions of the electromagnetic field are also accepted.

Naturally, the need to introduce additional restrictions (hypotheses) also arises when modeling two-dimensional problems on the propagation of electroelastic waves of plane or antiplane deformations in semi-infinite waveguides. In general, in order to fulfill these hypotheses, additional conditions are imposed on the electromechanical characteristics:

a. In the process of deformation, the segment of the straight line normal to the sagittal plane does not change.

b. The characterizing values of the wave-driven process outside the sagittal plane do not change.

c. In the sagittal plane of the piezocrystal, the axial stressandtheaxial polarization (electrical displacement)are absent.

These conditions make it possible to formulate a generalized plane stress-strain state also in a cut perpendicular to the selected axis (material plane of zero thickness) of an infinite waveguide layer. With loosely supported detached ends of the layer, the principles of Saint-Venant [10] and flat sections [9] also work.According to hypotheses, in the case when there are no acting axial stress and axial polarization in the selected planes, the elastic surface is inextensible, and it cannot be transversely polarized. Such modeling also allows formulating an equivalent generalized antiplane stress-strain state.

In this paper, definitions of possible multicomponent waves (package of wave components) are given. The possibility of the separated formulation of two-dimensional problems of electroelasticity in piezoelectric textures is investigated.

As an applied example, it will be shown that non-acoustic contact between the layers of an inhomogeneous waveguide makes it possible to create a wave hybrid in it, when dissimilar electroacoustic fields are created in its layers.

1. Some definitions and basic relations of the electroelastic stress-strain state in homogeneous piezoelectric media

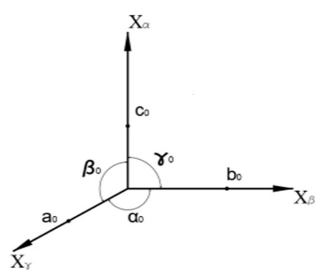
Physicomechanical constants of the homogeneous piezoelectric medium: the elastic stiffness $c_{(ij)(nm)}$, piezoelectric coefficients $e_{j(mn)}$ and dielectric constant ε_{ik}, form a generalized electroelastic tensor of piezoelectric materials of the type $\left(\hat{\gamma}_{jn}\right)_{9\times 9}=\left(\hat{c}_{ij}\right)_{6\times 6}\cup\left(\hat{e}_{mn}\right)_{3\times 6}\cup\left(\hat{\varepsilon}_{ik}\right)_{3\times 3}$ [11, 12]

$$\begin{pmatrix} \left(\hat{c}_{(ij)(nk)}\right)_{6\times6} & \left(\hat{e}_{(ij)m}\right)_{6\times3} \\ \left(\hat{e}_{m(ij)}\right)_{3\times6} & \left(\hat{\varepsilon}_{ik}\right)_{3\times3} \end{pmatrix} \qquad (1.1)$$

In generalized electroelastic tensor of linear electroelasticity of piezoelectric materials (1.1) the notations and known transitions from four-digit indices to two-digit indices $(\alpha\gamma) \rightleftarrows \alpha$ if $\alpha = \gamma$ and $(\alpha\gamma) \rightleftarrows 9-\alpha-\gamma$ if $\alpha \neq \gamma$ are used. It is also assumed that the indices $\{\alpha;\beta;\gamma\} \in \{1;2;3\}$, $\alpha \neq \beta$, $\beta \neq \gamma$ and $\gamma \neq \alpha$ indicated by the Greek letters, are dumb, and summation over them is not carried out.

The conditions permitting separate excitation and propagation of the plane or anti flat stress strain states in the uniform piezoelectric medium of this anisotropy, are imposed on the structure tensor of elastic material stiffness $\left(\hat{c}_{ij}\right)_{6\times6}$, as well as on the corresponding structures of tensors of piezoelectric coefficients $\left(\hat{e}_{nj}\right)$ and dielectric constant of the material $\left(\hat{\varepsilon}_{ik}\right)_{3\times3}$.

The generalized linear tensor of electroelasticity (1.1) for each piezoelectric texture, the material relations of the medium and the basic equations are determined in accordance with the geometric diagram of piezoelectric textures (Fig. 1),according to the rules for installing crystals by to crystal syngonies (table 1) and the rules for choosing crystallographic axes in them (table 2) [11, 12].



$\{X_\alpha, X_\beta, X_\gamma\}$ *–crystallographic axis of the texture,*
$\{\alpha_0, \beta_0, \gamma_0\}$ *- angles in the sagittal planes,*
$\{a_0, b_0, c_0\}$ *- measures of axial unit vectors,*
symmetry axes of the 2nd, 3rd, 4th, 6th orders in the texture,
m - normal to the plane of symmetry.

Fig. 1. *Geometric layout of piezoelectric textures*

These tables describe the order of the axes of symmetry (and/or inversion) and the anisotropy planes of the piezocrystals, the commensurability of the unit vectors and angles of the selected orthogonal system of base coordinates, as well as the order of alignment of the coordinate system with the base axes and planes of piezocrystals. To formulate the problems of electroelasticity in piezoelectric media, it is necessary to combine the coordinate axes and planes with the crystal axes and sagittal planes of the given piezoelectric texture, respectively.

Without loss of generality, let us formulate the problem of linear electroacoustics in one of the sagittal planes of the piezocrystal $x_\alpha 0 x_\beta$, where all components of the electroelastic field depend on the coordinates x_α and x_α, and there are no changes in the electro-mechanical characteristics of the field along the third base coordinate $\partial[*]/\partial x_\gamma \equiv 0$

Table 1. ***Crystal installation rules according to syngonies***

Crystalline textures	***crystallographic axis of the texture***	***angles in the sagittal planes***	***measures of axial unit vectors***
Triclinic	$x_\alpha, x_\beta, x_\gamma \rightleftarrows 1; \bar{1};$	$\alpha_0 \neq \beta_0 \neq \gamma_0 \neq 90^0$	$a_0 \neq b_0 \neq c_0$
Monoclinic	$x_\beta \rightleftarrows 2$ *or* m	$\alpha_0 = \gamma_0 = 90^0 \neq \beta_0$	$a_0 \neq b_0 \neq c_0$
Rhombic	$x_\alpha, x_\beta, x_\gamma \rightleftarrows 2$ *or* m	$\alpha_0 = \beta_0 = \gamma_0 = 90^0$	$a_0 \neq b_0 \neq c_0$
Tetragonal	$x_\alpha, x_\beta \rightleftarrows 2; m$, $x_\gamma \rightleftarrows 4; \bar{4}$	$\alpha_0 = \beta_0 = \gamma_0 = 90^0$	$a_0 = b_0 \neq c_0$
Trigonal and Hexagonal	$x_\alpha, x_\beta \rightleftarrows 2; m$, $x_\gamma \rightleftarrows 3; \bar{3}; 6; \bar{6}$	$\alpha_0 = \beta_0 = 90^0$, $\gamma_0 = 120^0$	$a_0 = b_0 \neq c_0$
Cubic	$x_\alpha, x_\beta, x_\gamma \rightleftarrows 4; \bar{4}; 2$	$\alpha_0 = \beta_0 = \gamma_0 = 90^0$	$a_0 = b_0 = c_0$

In the linear theory of electroelasticity of homogeneous piezoelectric media, the two-dimensionalcomplete system of quasi-static equations written in a crystallographic coordinate system is used[13, 14, 15]

$$\partial\sigma_{ij}(x_\alpha, x_\beta, t)/\partial x_j = \rho\left(\partial^2 u_i(x_\alpha, x_\beta, t)/\partial t^2\right),\ \partial D_n(x_\alpha, x_\beta, t)/\partial x_n = 0, \tag{1.2}$$

where $u_i(x_\alpha, x_\beta, t)$ are the elastic displacement vector components, $\sigma_{ij}(x_\alpha, x_\beta, t)$ are the mechanical stress tensor components, $D_n(x_\alpha, x_\beta, t)$ are the electrical displacement vector components, ρ is the material density and the indices take on the values $j, n \in \{\alpha, \beta\}$ and $i \in \{\alpha, \beta, \gamma\}$.

Generally, the components of the mechanical stress tensor $\sigma_{ij}(x_\alpha, x_\beta, t)$ and the components of the electric displacement vector $D_m(x_\alpha, x_\beta, t)$ in the equations (1.2), on the sagittal planes $x_\alpha 0 x_\beta$ are determined as[13, 14]

$$\sigma_{ij}(x_\alpha, x_\beta, t) = c_{(ij)(nm)}\left(\partial u_n(x_\alpha, x_\beta, t)/\partial x_m\right) - e_{m(ij)}E_m(x_\alpha, x_\beta, t),$$

$$D_k(x_\alpha, x_\beta, t) = e_{k(nm)}\left(\partial u_n(x_\alpha, x_\beta, t)/\partial x_m\right) + \varepsilon_{km}E_m(x_\alpha, x_\beta, t) \tag{1.3}$$

where the mechanical and electric fields are interconnected by the piezoelectric coefficient tensor $(\hat{e}_{j(mn)})$.

Table 2. ***Rules for selecting crystallographic axes in textures***

Crystalline textures	***crystallographic axis***		
	x_α	x_β	x_γ
Triclinic	In a plane perpendicular to the direction [001]		[001]
Monoclinic	[100]	[010]	[001]
Rhombic	[100]	[010]	[001]
Tetragonal	[100]	[010]	[001]
Trigonal and Hexagonal	[100]	[010]	[001]
Cubic	[100]	[010]	[001]

The plane quasi-static electric field is potential $E_m(x_\alpha, x_\beta, t) = -\left(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_m\right)$, where $m \in \{\alpha;\beta\}$ .In problems where elastic waves are accompanied by vibrations of a plane electric field $\{E_\alpha(x_\alpha, x_\beta, t),\ E_\beta(x_\alpha, x_\beta, t),\ 0\}$, the linear material relations (1.3) of piezoelectrics are often represented in the form

$$\sigma_{ij}(x_\alpha, x_\beta, t) = c_{(ij)(nk)}\left(\partial u_n(x_\alpha, x_\beta, t)/\partial x_k\right) + e_{m(ij)}\left(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_m\right),$$

$$D_m(x_\alpha, x_\beta, t) = e_{m(nk)}\left(\partial u_n(x_\alpha, x_\beta, t)/\partial x_k\right) - \varepsilon_{mk}\left(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_k\right). \quad (1.4)$$

Obviously, in two-dimensional quasi-static problem of electroelasticity, both elastic deformations can decompose into the plane $\{u_\alpha(x_\alpha, x_\beta, t),\ u_\beta(x_\alpha, x_\beta, t),\ 0\}$ and antiplane $\{0;\ 0;\ u_\gamma(x_\alpha, x_\beta, t)\}$ components, and their accompanying electric field can be plane $\left\{E_\alpha(x_\alpha, x_\beta, t),\ E_\beta(x_\alpha, x_\beta, t),\ 0\right\}$ or antiplane $\left\{0,\ 0,\ E_\gamma(x_\alpha, x_\beta, t)\right\}$.

Consequently, in piezoelectrics it will be possible to separately excite and propagate an electroelastic wave of a type from four incomplete sets of its components:

i. Four-component electroelastic wave - $\{u_\alpha(x_\alpha, x_\beta, t),\ u_\beta(x_\alpha, x_\beta, t),\ 0,\ E_\alpha(x_\alpha, x_\beta, t),\ E_\beta(x_\alpha, x_\beta, t),\ 0\}$, electractive plane deformation with accompanying oscillations of the plane electric field,

ii. Three-component electroelastic wave - $\{u_\alpha(x_\alpha, x_\beta, t),\ u_\beta(x_\alpha, x_\beta, t),\ 0,\ 0,\ 0,\ E_\gamma(x_\alpha, x_\beta, t)\}$, electractive plane deformation with accompanying oscillations of the anti-plane electric field,

iii. Three-component electroelastic wave - $\{0,\ 0,\ u_\gamma(x_\alpha,x_\beta,t),\ E_\alpha(x_\alpha,x_\beta,t),\ E_\beta(x_\alpha,x_\beta,t),\ 0\}$, electractive anti-plane deformation with accompanying oscillations of the plane electric field,

iv. Two-component electroelastic wave - $\{0,0,u_\gamma(x_\alpha,x_\beta,t),0,0,E_\gamma(x_\alpha,x_\beta,t)\}$, electractive anti-plane deformation with accompanying oscillations of the anti-plane electric field.

From the selected structures of wave packets, it is obvious that in a particular sagittal plane, the first from the fourth, or the second and the third packets are separated from each other.

2. Necessary and sufficient conditions for the separation of electroactive elastic states in a piezoelectric medium

The problems of the possibility for separate excitation and propagation of the waves of electroactive plane deformation and electroactive elastic wave of anti-plane deformation in homogeneous piezoelectric materials, were studied in [2, 3]. In these articles it is shown that in the elastic anisotropic homogeneous media the separation of the plane elastic deformation wave from the anti-plane elastic deformation wave in a selected sagittal plane $x_\alpha 0 x_\beta$ of the crystalline medium is ensured by the absence of the corresponding constants in the structure of the elastic stiffness tensor $(\hat{c}_{(ij)(nk)})_{6\times6}$

$$c_{\alpha(\gamma\alpha)} = c_{\alpha(\beta\gamma)} = c_{\beta(\beta\gamma)} = c_{\beta(\gamma\alpha)} = c_{(\alpha\beta)(\gamma\alpha)} = c_{(\alpha\beta)(\beta\gamma)} \equiv 0 \tag{2.1}$$

In this case, the electroactive wave of plane deformation accompanied by oscillations of the plane electric field in piezoelectrics can be separated from the wave of antiplane elastic deformation, when, along with the conditions (2.1), the conditions for the absence of piezoelectric coefficients in the generalized electroelasticity tensor (1.1) are satisfied

$$e_{\alpha(\gamma\alpha)} = e_{\alpha(\gamma\beta)} = e_{\beta(\gamma\alpha)} = e_{\beta(\gamma\beta)} \equiv 0 \tag{2.2}$$

The electroactive wave of antiplanar elastic deformation accompanied by oscillations of a plane electric field in piezoelectrics can be separated from the wave of plane elastic deformation when, along with the conditions (2.1), the conditions for the absence of other piezoelectric coefficients in the generalized electroelasticity tensor (1.1) are satisfied

$$e_{\alpha(\alpha\alpha)} = e_{\alpha(\beta\beta)} = e_{\beta(\alpha\alpha)} = e_{\beta(\beta\beta)} = e_{\alpha(\alpha\beta)} = e_{\beta(\beta\alpha)} \equiv 0 \tag{2.3}$$

The electroactive wave of plane elastic deformation accompanied by the oscillations of the antiplane electric field $\{u_\alpha(x_\alpha,x_\beta,t),\ u_\beta(x_\alpha,x_\beta,t),\ 0,\ 0,\ 0,\ E_\gamma(x_\alpha,x_\beta,t)\}$ in piezoelectrics can be separated from the wave of plane elastic deformation, when, along with the conditions (2.1), the conditions for the absence of other piezoelectric coefficients in the generalized electroelasticity tensor (1.1) are satisfied

$$e_{\alpha(\alpha\alpha)} = e_{\alpha(\beta\beta)} = e_{\beta(\alpha\alpha)} = e_{\beta(\beta\beta)} = e_{\alpha(\alpha\beta)} = e_{\beta(\beta\alpha)} \equiv 0 \tag{2.4}$$

The electroactive wave of the antiplane elastic deformation accompanied by the oscillations of the antiplane electric field $\{0,\ 0,\ u_\gamma(x_\alpha,x_\beta,t),\ 0,\ 0,\ E_\gamma(x_\alpha,x_\beta,t)\}$ in piezoelectrics can be separated from the wave of plane elastic deformation, when, along with conditions (2.1), the conditions for the absence of other piezoelectric coefficients in the generalized electroelasticity tensor (1.1) are satisfied

$$e_{\gamma(\alpha\alpha)} = e_{\gamma(\beta\beta)} = e_{\gamma(\alpha\beta)} \equiv 0 \tag{2.5}$$

For separate excitation and propagation of plane or antiplane electroactive stress-strain states, the above pairs of conditions (2.1) and (2.2), or (2.1) and (2.3), or (2.1) and (2.4), or (2.1) and (2.5) as constraints on the anisotropy of the medium are necessary but not sufficient.

From material relations (1.4) and the conditions (2.1) and (2.2) corresponding to them, taking into account the form of the generalized electroelasticity tensor (1.1), it follows that, in the formulation of the two-dimensional problem of electroelasticity, side by side with nonzero stresses characteristic of the plane stress state $\sigma_{\alpha\alpha}(x_\alpha, x_\beta, t)$ $\sigma_{\beta\beta}(x_\alpha, x_\beta, t)$ and $\sigma_{\alpha\beta}(x_\alpha, x_\beta, t)$, there also arises an axial mechanical stress $\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t)$ .As well as, along with the nonzero components of the electric displacement of the plane electric field $D_\alpha(x_\alpha, x_\beta, t)$ and $D_\beta(x_\alpha, x_\beta, t)$, the third component $D_\gamma(x_\alpha, x_\beta, t)$ of the electric displacement vector can arise.

The presence of non-zero axial components of the mechanical stress and the vector of electrical displacement along the axis $0x_\gamma$, in the general case will violate the formulation of the two-dimensional problem of electroelasticity in the material sagittal plane $x_\alpha 0 x_\beta$ .Therefore, to fulfill the accepted hypotheses, additional conditions are imposed on the electromechanical characteristics: the absence of axial mechanical stress $\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t)$ and the axial component of the electric polarization (electric displacement) $D_\gamma(x_\alpha, x_\beta, t)$ perpendicular to the sagittal plane of the piezoelectric crystal.

In all piezoelectric crystals for which conditions (2.1) ÷ (2.5) are satisfied, the dielectric constant tensors $(\hat{\varepsilon}_{ik})_{3\times3}$ are diagonal. Therefore, the third component of the electric displacement is represented only by the elastic elongations $(\partial u_\alpha / \partial x_\alpha)$, $(\partial u_\beta / \partial x_\beta)$ and shift $(\partial u_\alpha / \partial x_\beta) + (\partial u_\beta / \partial x_\alpha)$ in the sagittal plane.

It is known that in any basic plane $x_\alpha 0 x_\beta$ the elastic stiffnesses $c_{\gamma\alpha} \neq 0$ and $c_{\gamma\beta} \neq 0$, as well as the elastic compliance coefficients $s_{\gamma\alpha} = (-1)^{\alpha+\gamma} \cdot \Delta c_{\alpha\gamma} / \Delta^c$ and $s_{\gamma\beta} = (-1)^{\beta+\gamma} \cdot \Delta c_{\beta\gamma} / \Delta^c$ cannot be zeros. Therefore, the existence of a non-zero axial stress $\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t)$ can lead to the axial tensions (compressions) $r_{\gamma\gamma}(x_\alpha, x_\beta, t)$ in the direction of the axis $0x_\gamma$, violating the planar deformed state.

Taking into account the above statements, from the material rations of axial mechanical stress and electrical displacement

$$\sigma_{\gamma\gamma}(x_\alpha,x_\beta,t)=c_{\gamma\alpha}\frac{\partial u_\alpha(x_\alpha,x_\beta,t)}{\partial x_\alpha}+c_{\gamma\beta}\frac{\partial u_\beta(x_\alpha,x_\beta,t)}{\partial x_\beta}+c_{\gamma(\gamma\beta)}\frac{\partial u_\gamma(x_\alpha,x_\beta,t)}{\partial x_\beta}-$$
$$-e_{\alpha(\gamma\gamma)}E_\alpha(x_\alpha,x_\beta,t)-e_{\beta(\gamma\gamma)}E_\beta(x_\alpha,x_\beta,t)-e_{\gamma(\gamma\gamma)}E_\gamma(x_\alpha,x_\beta,t)+ \tag{2.6}$$
$$+c_{\gamma(\alpha\gamma)}\frac{\partial u_\gamma(x_\alpha,x_\beta,t)}{\partial x_\alpha}+c_{\gamma(\alpha\beta)}\left[\frac{\partial u_\alpha(x_\alpha,x_\beta,t)}{\partial x_\beta}+\frac{\partial u_\beta(x_\alpha,x_\beta,t)}{\partial x_\alpha}\right],$$

$$D_\gamma(x_\alpha,x_\beta,t)=e_{\gamma(\alpha\alpha)}\frac{\partial u_\alpha(x_\alpha,x_\beta,t)}{\partial x_\alpha}+e_{\gamma(\beta\beta)}\frac{\partial u_\beta(x_\alpha,x_\beta,t)}{\partial x_\beta}+$$
$$+e_{\gamma(\gamma\beta)}\frac{\partial u_\gamma(x_\alpha,x_\beta,t)}{\partial x_\beta}+e_{\gamma(\alpha\gamma)}\frac{\partial u_\gamma(x_\alpha,x_\beta,t)}{\partial x_\alpha}+ \tag{2.7}$$
$$+e_{\gamma(\alpha\beta)}\left[\frac{\partial u_\alpha(x_\alpha,x_\beta,t)}{\partial x_\beta}+\frac{\partial u_\beta(x_\alpha,x_\beta,t)}{\partial x_\alpha}\right]+\varepsilon_{\gamma\gamma}E_\gamma(x_\alpha,x_\beta,t)$$

it follows that in piezoelectric crystals, the formulation of theelectractive plane deformation with accompanying oscillations of the plane electric field $\{u_\alpha(x_\alpha,x_\beta,t),\ u_\beta(x_\alpha,x_\beta,t),\ 0,\ E_\alpha(x_\alpha,x_\beta,t),\ E_\beta(x_\alpha,x_\beta,t),\ 0\}$, in the sagittal plane $x_\alpha 0 x_\beta$ is possible, when

$$\begin{cases} c_{\gamma\alpha}(\partial u_\alpha/\partial x_\alpha)+c_{\gamma\beta}(\partial u_\beta/\partial x_\beta)=-c_{\gamma(\alpha\beta)}\left[(\partial u_\alpha/\partial x_\beta)+(\partial u_\beta/\partial x_\alpha)\right]- \\ -e_{\alpha(\gamma\gamma)}(\partial\varphi/\partial x_\alpha)-e_{\beta(\gamma\gamma)}(\partial\varphi/\partial x_\beta) \\ e_{\gamma(\alpha\alpha)}(\partial u_\alpha/\partial x_\alpha)+e_{\gamma(\beta\beta)}(\partial u_\beta/\partial x_\beta)=-e_{\gamma(\alpha\beta)}\left[(\partial u_\alpha/\partial x_\beta)+(\partial u_\beta/\partial x_\alpha)\right] \end{cases} \tag{2.8}$$

The system of linear equations (2.6) and -(2.7) has nontrivial (arbitrary) solutions with respect to elastic elongations (or compressions) $(\partial u_\alpha/\partial x_\alpha)$ and $(\partial u_\beta/\partial x_\beta)$

$$\begin{cases} \dfrac{\partial u_\alpha}{\partial x_\alpha}=\dfrac{\Delta_\alpha}{\Delta}\left[\dfrac{\partial u_\alpha}{\partial x_\beta}+\dfrac{\partial u_\beta}{\partial x_\alpha}\right]+\dfrac{e_{\gamma(\beta\beta)}}{\Delta}\left[e_{\alpha(\gamma\gamma)}(\partial\varphi/\partial x_\alpha)+e_{\beta(\gamma\gamma)}(\partial\varphi/\partial x_\beta)\right] \\ \dfrac{\partial u_\beta}{\partial x_\beta}=\dfrac{\Delta_\beta}{\Delta}\left[\dfrac{\partial u_\alpha}{\partial x_\beta}+\dfrac{\partial u_\beta}{\partial x_\alpha}\right]+\dfrac{e_{\gamma(\alpha\alpha)}}{\Delta}\left[e_{\alpha(\gamma\gamma)}(\partial\varphi/\partial x_\alpha)+e_{\beta(\gamma\gamma)}(\partial\varphi/\partial x_\beta)\right] \end{cases} \tag{2.9}$$

In relations(2.6), (2.7) and (2.8) it is taken into account, that the accompanying plane quasi-static electric field is potential $E_m(x_\alpha,x_\beta,t)=-\partial\varphi(x_\alpha,x_\beta,t)/\partial x_m$, where $m\in\{\alpha;\beta\}$, as well as the descriptions $\Delta=c_{\gamma\beta}e_{\gamma(\alpha\alpha)}-c_{\gamma\alpha}e_{\gamma(\beta\beta)}$, $\Delta_\alpha=c_{\gamma(\alpha\beta)}e_{\gamma(\beta\beta)}-c_{\gamma\beta}e_{\gamma(\alpha\beta)}$, and $\Delta_\beta=c_{\gamma(\alpha\beta)}e_{\gamma(\alpha\alpha)}-c_{\gamma\alpha}e_{\gamma(\alpha\beta)}$ are taken.

Statement-1. In the chosen sagittal plane $x_\alpha 0 x_\beta$ of the piezoelectric crystal, a four-component electroacoustic wave $\{u_\alpha(x_\alpha,x_\beta,t), u_\beta(x_\alpha,x_\beta,t), 0, E_\alpha(x_\alpha,x_\beta,t), E_\beta(x_\alpha,x_\beta,t), 0\}$ is possible, if the elements of the generalized electroelastic tensor (1.1) of the medium satisfy conditions (2.1), (2.2) and the additional functional representation (2.8), according to which the elastic tensions $\partial u_\alpha(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\partial u_\beta(x_\alpha,x_\beta,t)/\partial x_\beta$ in the sagittal plane are expressed in terms of elastic shear $\partial u_\alpha(x_\alpha,x_\beta,t)/\partial x_\beta$ and $\partial u_\beta(x_\alpha,x_\beta,t)/\partial x_\alpha$, as well as the plane electric field components $\partial\varphi(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\partial\varphi(x_\alpha,x_\beta,t)/\partial x_\beta$.
From relations (2.6) and (2.7) it also follows, that the formulation of the electractive anti plane deformation with accompanying oscillations of anti plane electric field $\{0,\ 0,\ u_\gamma(x_\alpha,x_\beta,t),\ 0,\ 0,\ E_\gamma(x_\alpha,x_\beta,t)\}$, in the sagittal plane $x_\alpha 0 x_\beta$ of piezoelectric crystals is possible, when

$$\begin{cases} c_{\gamma(\gamma\beta)}(\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\beta) + c_{\gamma(\alpha\gamma)}(\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\alpha) = e_{\gamma(\gamma\gamma)}E_\gamma(x_\alpha,x_\beta,t) \\ e_{\gamma(\gamma\beta)}(\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\beta) + e_{\gamma(\alpha\gamma)}(\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\alpha) = -\varepsilon_{\gamma\gamma}E_\gamma(x_\alpha,x_\beta,t) \end{cases} \tag{2.10}$$

Since the dielectric constant $\varepsilon_{\gamma\gamma}$ is a positive definite quantity, withthe additional condition on the coefficients of the generalized electromechanical tensor $c_{\gamma(\alpha\gamma)}\cdot e_{\gamma(\gamma\beta)} - c_{\gamma(\gamma\beta)}\cdot e_{\gamma(\alpha\gamma)} \neq 0$, the system of linear equations (2.9) always has nontrivial solutions with respect to elastic shears $\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\beta$

$$\begin{cases} (\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\alpha) = \dfrac{c_{\gamma(\gamma\beta)}\varepsilon_{\gamma\gamma}\left((e_{\gamma(\gamma\gamma)}e_{\gamma(\alpha\beta)})/(c_{\gamma(\gamma\beta)}\varepsilon_{\gamma\gamma})+1\right)}{c_{\gamma(\alpha\gamma)}\cdot e_{\gamma(\gamma\beta)} - c_{\gamma(\gamma\beta)}\cdot e_{\gamma(\alpha\gamma)}}\cdot E_\gamma(x_\alpha,x_\beta,t) \\ (\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\beta) = \dfrac{c_{\gamma(\gamma\alpha)}\varepsilon_{\gamma\gamma}\left((e_{\gamma(\gamma\gamma)}e_{\gamma(\alpha\beta)})/(c_{\gamma(\gamma\alpha)}\varepsilon_{\gamma\gamma})+1\right)}{c_{\gamma(\alpha\gamma)}\cdot e_{\gamma(\gamma\beta)} - c_{\gamma(\gamma\beta)}\cdot e_{\gamma(\alpha\gamma)}}\cdot E_\gamma(x_\alpha,x_\beta,t) \end{cases} \tag{2.11}$$

Statement-2. In the chosen sagittal plane $x_\alpha 0 x_\beta$ of the piezoelectric crystal, a four-component electroacoustic wave $\{0,\ 0,\ u_\gamma(x_\alpha,x_\beta,t),\ 0,\ 0,\ E_\gamma(x_\alpha,x_\beta,t)\}$ is possible, if the elements of the generalized electroelastic tensor (1.1) of the medium satisfy conditions (2.1), (2.5) and the additional functional representation (2.10), according to which, the shifts $\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\beta$ in an elastic shear wave are expressed in terms of the axial component of the accompanying electric field $E_\gamma(x_\alpha,x_\beta,t)$.
Similarly, from relations (2.6) and (2.7)it also follows, that for the formulation of the two-dimensional problem of an electractive plane deformation with accompanying oscillations of the anti-plane electric field $\{u_\alpha(x_\alpha,x_\beta,t),\ u_\beta(x_\alpha,x_\beta,t),\ 0,\ 0,\ 0,\ E_\gamma(x_\alpha,x_\beta,t)\}$, in the sagittal plane $x_\alpha 0 x_\beta$, the additional conditions are obtained

$$
\begin{cases}
(\partial u_\alpha(x_\alpha,x_\beta,t)/\partial x_\alpha) = \dfrac{\Delta_\alpha}{\Delta}\left[(\partial u_\alpha/\partial x_\beta)+(\partial u_\beta/\partial x_\alpha)\right]+ \\
+\dfrac{\left((e_{\alpha(\gamma\gamma)}e_{\gamma(\beta\beta)})/(c_{\gamma\beta}\varepsilon_{\gamma\gamma})+1\right)\cdot c_{\gamma\beta}\varepsilon_{\gamma\gamma}}{\Delta}\cdot E_\gamma(x_\alpha,x_\beta,t) \\
(\partial u_\beta(x_\alpha,x_\beta,t)/\partial x_\beta) = \dfrac{\Delta_\beta}{\Delta}\left[(\partial u_\alpha/\partial x_\beta)+(\partial u_\beta/\partial x_\alpha)\right]+ \\
+\dfrac{\left((e_{\beta(\gamma\gamma)}e_{\gamma(\alpha\alpha)})/(c_{\gamma\alpha}\varepsilon_{\gamma\gamma})+1\right)\cdot c_{\gamma\alpha}\varepsilon_{\gamma\gamma}}{\Delta}\cdot E_\gamma(x_\alpha,x_\beta,t)
\end{cases}
\tag{2.12}
$$

The non trivial presentations (2.11) for axial elongations (or compression) $(\partial u_\alpha(x_\alpha,x_\beta,t)/\partial x_\alpha)$ and $(\partial u_\beta(x_\alpha,x_\beta,t)/\partial x_\beta)$, in this case are possible with the additional condition on the coefficients of the generalized electro elastic tensor $c_{\gamma(\alpha\gamma)}\cdot e_{\gamma(\gamma\beta)} - c_{\gamma(\gamma\beta)}\cdot e_{\gamma(\alpha\gamma)} \neq 0$.

Statement-3. In the chosen sagittal plane $x_\alpha 0 x_\beta$ of the piezoelectric crystal, a four-component electroacoustic wave $\{u_\alpha(x_\alpha,x_\beta,t),\ u_\beta(x_\alpha,x_\beta,t),\ 0,\ 0,\ 0,\ E_\gamma(x_\alpha,x_\beta,t)\}$ is possible, if the elements of the generalized electroelastic tensor (1.1) of the medium satisfy conditions (2.1), (2.4) and the additional functional representation (2.11), according to which the elastic tensions $\partial u_\alpha(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\partial u_\beta(x_\alpha,x_\beta,t)/\partial x_\beta$ in the sagittal plane are expressed in terms of elastic shear $\partial u_\alpha(x_\alpha,x_\beta,t)/\partial x_\beta$ and $\partial u_\beta(x_\alpha,x_\beta,t)/\partial x_\alpha$, as well as the shear electric field $E_\gamma(x_\alpha,x_\beta,t)$.

In the case of the electractive anti-plane deformation with accompanying oscillations of the plane electric field $\{0,\ 0,\ u_\gamma(x_\alpha,x_\beta,t),\ \partial\varphi(x_\alpha,x_\beta,t)/x_\alpha,\ \partial\varphi(x_\alpha,x_\beta,t)/x_\beta,\ 0\}$, in the sagittal plane $x_\alpha 0 x_\beta$, the additional conditions imposed on the structure of the electromechanical generalized tensor are obtained similar to $c_{\gamma(\alpha\gamma)}\cdot e_{\gamma(\gamma\beta)} - c_{\gamma(\gamma\beta)}\cdot e_{\gamma(\alpha\gamma)} \neq 0$. Under these condition on the coefficients of the generalized electromechanical tensor (1.1), we obtain the additional non trivial presentations with respect to elastic shears $\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\alpha$ and $\partial u_\gamma(x_\alpha,x_\beta,t)/\partial x_\beta$

$$
\begin{aligned}
(\partial u_\gamma / \partial x_\alpha) &= \frac{e_{\alpha(\gamma\gamma)}}{c_{\gamma(\gamma\beta)}\,(e_{\gamma(\alpha\gamma)}/e_{\gamma(\gamma\beta)}) - c_{\gamma(\alpha\gamma)}}(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha) + \\
&+ \frac{e_{\beta(\gamma\gamma)}}{c_{\gamma(\gamma\beta)}\,(e_{\gamma(\alpha\gamma)}/e_{\gamma(\gamma\beta)}) - c_{\gamma(\alpha\gamma)}}(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta) \\
(\partial u_\gamma / \partial x_\beta) &= \frac{e_{\beta(\gamma\gamma)}}{c_{\gamma(\gamma\alpha)}\,(e_{\gamma(\beta\gamma)}/e_{\gamma(\gamma\alpha)}) - c_{\gamma(\beta\gamma)}}(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta) + \\
&+ \frac{e_{\alpha(\gamma\gamma)}}{c_{\gamma(\gamma\alpha)}\,(e_{\gamma(\beta\gamma)}/e_{\gamma(\gamma\alpha)}) - c_{\gamma(\beta\gamma)}}(\partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha)
\end{aligned}
\tag{2.13}
$$

Statement-4. In the chosen sagittal plane $x_\alpha 0 x_\beta$ of the piezoelectric crystal, a four-component electroacoustic wave $\{0, 0, u_\gamma(x_\alpha, x_\beta, t), \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha, \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta, 0\}$ is possible, if the elements of the generalized electroelastic tensor (1.1) of the medium satisfy conditions (2.1), (2.3) and the additional functional representation (2.12), according to which, the shifts $\partial u_\gamma(x_\alpha, x_\beta, t)/\partial x_\alpha$ and $\partial u_\gamma(x_\alpha, x_\beta, t)/\partial x_\beta$ in the elastic shear wave are expressed in terms of the axial component of the accompanying plane electric field $\partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha$ and $\partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta$.

Along with the necessary conditions for separate excitation and propagation of electroactive multicomponent waves (2.1) and one of (2.2)÷(2.5), additional conditions (2.8), (2.10), (2.11), (2.12) are obtained, which are sufficient for the formulation of the two-dimensional problem of electroelasticity. In each case of studying the electroelastic two-dimensional problem, it is necessary along with the material relations of the medium to take into account additional representations of elastic elongations and shears (2.8), (2.10), (2.11) and (2.12), respectively.

The necessary conditions for the coefficients of the generalized electroelasticity tensor, as well as additional sufficient representations of elastic elongations and shifts in the other two sagittal planes, can be obtained, without repeating all the calculations by simply rotating the coordinate indices $\{\alpha,\ \beta,\ \gamma\} \to \{\gamma,\ \alpha,\ \beta\} \to \{\beta,\ \gamma,\ \alpha\}$.

3. Conclusion

The necessary conditions imposed on the coefficients of the generalized electroelasticity tensor, as well as the additional relations between elastic displacements and electric field components are formulated that allow the formulation of the two-dimensional problem of linear electroelasticity.

In two-dimensional electroelasticity of anisotropic piezoelectrics, both purely planar and purely antiplanar, as well as mixed (planar elastic and antiplanar electric, or antiplanar elastic and planar electric) multicomponent electroelastic fields are formed.

Taking into account the obtained additional relations between the elastic displacements and electric field components, the constitutive equations for the nonzero components of the mechanical stress tensor and the electric displacement vector, as well as the quasi-static electroelasticity equations for each piezoelectric texture, are derived, respectively.

A catalog of possible multicomponent electroelastic fields in all sagittal planes of all piezoelectric textures has been compiled.

On the example of relatively simple separately existing electroelastic multicomponent fields, an example of their practical application in composite waveguides is given.

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