ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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TIME-SUBOPTIMAL CONTROL OF A TWO-LINK MANIPULATOR MOTION

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Key words: two link manipulator, time-suboptimal control

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Субоптимальное по быстродействию управление движением двузвенного манипулятора

Ключевые слова: двузвенный манипулятор, субоптимальное по быстродействию управление

Рассматривается задача построения субоптимальных по быстродействию режимов управления движением плоского двузвенного манипулятора с произвольными геометрическими и инерционными характеристиками. Требуется перевести манипулятор из заданной начальной конфигурации в заданную конечную конфигурацию при условиях, что в начале и в конце процесса система покоится, а модули управляющих обобщенных сил не превышают фиксированных значений. Управления ищутся в классе релейных режимов с минимальным (равным трем) суммарным числом переключений, достаточным для удовлетворения граничных условий. Предложен расчетный алгоритм, позволяющий по начальным и конечным положениям определить количество точек переключения управлений, порядок чередования их знаков, моменты переключения и время приведения манипулятора в требуемое положение покоя.

Ավետիսյան Վ.Վ., Գրիգորյան Շ.Ա.

Երկօղակ մանիպուլյատորի շարժման սուբօպտիմալ ըստ արագագործության ղեկավարումը

Հիմնաբառեր: երկօղակ մանիպուլյատոր, սուբօպտիմալ ըստ արագագործության ղեկավարում

Դիտարկվում է կամայական երկրաչափական և իներցիոն բնութագրիչներով հարթ երկօղակ մանիպուլյատորի շարժման սուբօպտիմալ ըստ արագագործության ղեկավարման ռեժիմների կառուցման խնդիրը: Պահանջվում է մանիպուլյատորը տեղափոխել տրված սկզբնական հանգստի վիճակից տրված վերջնական հանգստի վիճակ ղեկավարումների վրա դրված տրված սահմանափակումների դեպքում: Ղեկավարումները փնտրվում են կտոր առ կտոր հաստատուն ֆունկցիաների դասում՝ նվազագույն (հավասար երեքի) գումարային թվով փոխանցման կետերով, որը բավարար է եզրային պայմաններին բավարարելու համար: Առաջարկված է հաշվարկային ալգորիթմ, որը թույլ է տալիս ըստ սկզբնական և վերջնական դիրքերի որոշել ղեկավարումների փոխանցման կետերի քանակը, դրանց նշանների հերթագայության կարգը, փոխանցման պահերը և տրված հանգստի դիրք մանիպուլյատորի բերման ժամանակը:

We consider the problem of the construction of a time-suboptimal control modes of the motion of a flat two-link manipulator with arbitrary geometric and inertial characteristics. It is required to transfer the manipulator from a given initial configuration to a given final configuration under the conditions that at the beginning and at the end of the process the system is at rest, and the moduli of the generalized control forces do not exceed fixed values. The controls are sought in the class of relay modes with a minimum (equal to three) total number of switches, sufficient to satisfy the boundary conditions. A computational algorithm is proposed that allows, from the initial and final positions, to determine the number of control switching points, the order of alternation of their signs, the switching moments and the time for bringing the manipulator to the required rest position.

Introduction

In practice, two-link manipulators are used both independently and as part of the structures of multi-link manipulation robots for which it is these two links that perform the bulk of the robot's motions when it performs various technological operations. Therefore, the development of effective modes of program control of a two-link robotic manipulator is still an urgent task. When choosing program controlled movements, one should take into account such factors as the time for performing a work operation by a manipulator, energy consumption, various restrictions, ease of implementation, etc. In cases where shortening of the work cycle of the manipulator leads to speed up of the entire technological process, it is advisable to construct programmed motions that are optimal with respect to speed of action. At the same time, an important task from the point of view of practical implementation is the construction of optimal control laws for a two-link manipulator of a simple structure, having the minimum possible number of switchings. In [1,2], optimal and suboptimal control laws were constructed for a two-link manipulator with zero-lag links in the two-point problem of moving a gripper with a load. A significant dependence of the time it takes to bring the gripper to the terminal state on the manipulator configuration type was revealed, and the problem of choosing the optimal configuration type was solved. In [3], a graphic-analytical approach was developed to constructing time-suboptimal open-loop controls that bring a two-link manipulator with arbitrary geometric and lag characteristics from the initial rest configuration to an arbitrary final rest configuration. The publications [4–8] deal with optimization methods for solving the problem of controlling robots, including two-link manipulators, and calculating their design parameters. In [9,10], a parametric optimization method was used to construct a quadratic-functional-suboptimal control of the motion of a plane two-link manipulator taking into account feasible manipulator configurations corresponding to given gripper positions at the beginning and end of the motion. In [11,12], the problems of optimal control of transport movements of an electromechanical two-link manipulator time optimal [11], energy consumption and the functional combined from them [12] are considered. Methods for calculating controls are proposed based on a simplified model that does not take into account the mutual influence of links. Numerical simulation of the dynamics of the complete model under optimal control modes has been carried out. The simulation results have established the practical efficiency of the found modes. In [13], an algorithm was developed for time suboptimal control of a two-link electromechanical manipulator with high positioning accuracy. A combined control is proposed that allows you to bring the manipulator to the desired position with any given accuracy. The works [14-16] are devoted to the construction of time optimal control of the motion of a two-link manipulator with a second balanced link.

In this paper, for a flat two-link manipulator with arbitrary geometric and inertial characteristics, an algorithm for calculating suboptimal program controls is proposed, which allows, based on the initial and final configurations of the manipulator, to determine the number of switching points of control moments, the order of alternation of their signs, the switching moments at which the system moves from the initial rest position to the specified final rest position in a time close to the optimal response time.

1 Formulation of the problem

Consider a two-link manipulator (fig. 1) consisting of two absolutely rigid bodies G_1 and G_2 joined by a hinge O_2. The body G_1 (first link) is joined to a fixed foundation by means of hinge O_1. The hinges are ideal and cylindrical, and their axes are parallel to one another. At the end of the second link (G_2) a reinforced gripper is installed, in which there is a movable object (cargo). We will assume that the linear sizes of the gripper are much smaller than the lengths of the links and consider the gripper to be a material point when studying transport motions. The system performs a plane-parallel motion in a horizontal plane perpendicular to the axes of the hinges O_1 and O_2. The manipulator control under study is accomplished with two independent drives D_1 and D_2. The first link and the base interact via the drive D_1, and D_2 is responsible for the interaction between the links G_1 and G_2 of the manipulator. The control functions in the manipulator model under study are the torques M_1 and M_2 about the axes O_1 and O_2 generated by the drives D_1 and D_2, respectively. The action of other forces is not taken into account.

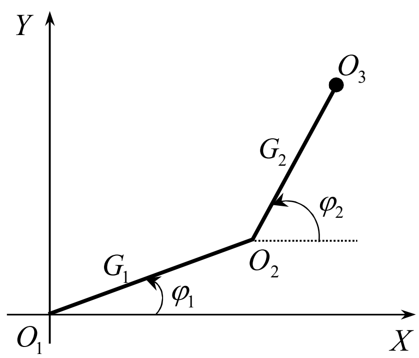

Fig. 1. Calculation model of a two-link manipulator

In the plane we introduce a fixed Cartesian coordinate system O_1XY with origin one the axis of O_1. We denote: φ_1 - angle between the O_1X and the straight line O_1O_2 connecting the hinges; φ_2 - angle between the O_1X axis and the straight line O_2C passing through the axis of the moving hinge O_2 and the center of mass C of G_2; $L_1 = |O_1O_2|$ - distance between the hinge axes; $l = |O_2C|$ - distance from the axis of O_2 to the center of mass of G_2 with the cargo; I_1 and I_2 - moment of inercia of G_1 and G_2 (with the cargo) about the axes of O_1 and O_2, respectively; m - mass of G_2 with the cargo.

The Lagrange equations describing the motion of this system have the form [3]

$$\begin{aligned} &(I_1 + mL_1^2)\ddot{\varphi}_1 + mL_1 l\ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2) + mL_1 l\dot{\varphi}_2^2 \sin(\varphi_1 - \varphi_2) = M_1 - M_2, \\ &I_2\ddot{\varphi}_2 + mL_1 l\,\ddot{\varphi}_1 \cos(\varphi_1 - \varphi_2) - mL_1 l\dot{\varphi}_1^2 \sin(\varphi_1 - \varphi_2) = M_2. \end{aligned} \tag{1.1}$$

We pose the following optimal control problem.

Problem 1.1. Find the program laws of change the control moments $M_1 = M_1(t)$, $M_2 = M_2(t)$ that ensure of the system (1.1) from the initial state

$$\varphi_1(0) = \varphi_1^0, \quad \dot{\varphi}_1(0) = 0, \qquad \varphi_2(0) = \varphi_2^0, \quad \dot{\varphi}_2(0) = 0 \tag{1.2}$$

into a given final state

$$\varphi_1(T) = \varphi_1^T, \quad \dot{\varphi}_1(T) = 0, \qquad \varphi_2(T) = \varphi_2^T, \quad \dot{\varphi}_2(T) = 0 \tag{1.3}$$

In a minimum time T, with the condition that the control moments are limited in absolute value by constants:

$$|M_1(t)| \le M_1^0, \quad |M_2(t)| \le M_2^0. \tag{1.4}$$

In (1.1)-(1.4) we go to over to dimensionless variables denoted by primes:

$$t' = (M_1^0 / (mL_1^2))^{1/2} t, \quad l' = l / L_1, \quad I_i' = I_i / (mL_1^2), \quad M_i' = M_i / M_2^0, \quad i = 1, 2. \tag{1.5}$$

If we then omit omit the primes, and also rotate the coordinate system O_1XY through the angle φ_1^0, then relations (1.1)-(1.4) become simpler: in them we have

$$\varphi_1^0 = 0, \quad m = 1, \quad L_1 = 1, \quad M_2^0 = 1. \tag{1.6}$$

2 Solving a linear problem

First consider the special case of the manipulator (1.1) in which the second manipulator link is statically balanced; i.e., one has $l = 0$ in (2.1). Then, taking into account (1.6), the time-optimal control problem takes the form

$$T \to \min \tag{2.1}$$

$$(I_1 + 1)\ddot{\varphi}_1 = M_1 - M_2, \quad I_2 \ddot{\varphi}_2 = M_2, \tag{2.2}$$

$$\begin{aligned} &\varphi_1(0) = 0, \qquad \dot{\varphi}_1(0) = 0, \\ &\varphi_2(0) = \varphi_2^0, \qquad \dot{\varphi}_2(0) = 0, \end{aligned} \tag{2.3}$$

$$\varphi_i(T) = \varphi_i^T, \quad \dot{\varphi}_i(T) = 0, \quad i = 1, 2, \tag{2.4}$$

$$|M_1| \le M_1^0, \quad |M_2| \le 1. \tag{2.5}$$

The solution of problem (2.1)-(2.5) is known [3]. A pair of functions that determine the time optimal control depending on $(\varphi_1^T, \Delta\varphi_2^T)$, $\Delta\varphi_2^T = \varphi_2^T - \varphi_2^0$, can be represented as:

$$M^* = \begin{cases} (M_1^*, M_2^*), & (\varphi_1^T, \Delta\varphi_2^T) \in \Phi_0, \\ (M_1^*, M_2), & (\varphi_1^T, \Delta\varphi_2^T) \in \Phi_1, \\ (M_1, M_2^*), & (\varphi_1^T, \Delta\varphi_2^T) \in \Phi_2. \end{cases} \tag{2.6}$$

Here the areas Φ_i, $i = 0, 1, 2$ are defined as follows:

$$\Phi_0 = \Phi_0^{(0,0)} \cup \Phi_0^{(0,1)} \cup \Phi_0^{(1,0)} \cup \Phi_0^{(1,1)}, \tag{2.7}$$

$$\Phi_0^{(0,0)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi : \Delta\varphi_2^T = B\varphi_1^T,\ \varphi_1^T > 0\right\},$$
$$\Phi_0^{(0,1)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi : \Delta\varphi_2^T = -A\varphi_1^T,\ \varphi_1^T > 0\right\},$$
$$\Phi_0^{(1,0)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi : \Delta\varphi_2^T = -A\varphi_1^T,\ \varphi_1^T < 0\right\},$$
$$\Phi_0^{(1,1)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi : \Delta\varphi_2^T = B\varphi_1^T,\ \varphi_1^T < 0\right\},$$

$$\Phi_1 = \Phi_1^{(0)} \cup \Phi_1^{(1)}, \tag{2.8}$$
$$\Phi_1^{(0)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi :\ -A\varphi_1^T < \Delta\varphi_2^T < B\varphi_1^T,\ \varphi_1^T > 0\right\},$$
$$\Phi_1^{(1)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi :\ B\varphi_1^T < \Delta\varphi_2^T < -A\varphi_1^T,\ \varphi_1^T < 0\right\},$$

$$\Phi_2 = \Phi_2^{(0)} \cup \Phi_2^{(1)}, \tag{2.9}$$
$$\Phi_2^{(0)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi :\ B\varphi_1^T < \Delta\varphi_2^T,\ \varphi_1^T > 0\ \ u\ \ -A\varphi_1^T < \varphi_2,\ \varphi_1^T < 0\right\},$$
$$\Phi_2^{(1)} = \left\{(\varphi_1^T, \Delta\varphi_2^T) \in \Phi :\ \Delta\varphi_2^T < -A\varphi_1^T,\ \varphi_1^T > 0\ \ u\ \ \Delta\varphi_2^T < B\varphi_1^T,\ \varphi_1^T < 0\right\},$$

where

$$A = (I_1 + 1)(M_1^0 + 1)^{-1} I_2^{-1}, \quad B = (I_1 + 1)(M_1^0 - 1)^{-1} I_2^{-1},$$

and

$$\Phi = \left\{\varphi_1, \varphi_2 :\ -2\pi \le \varphi_1, \varphi_2 \le 2\pi\right\} \tag{2.10}$$

- the region of change of generalized coordinates (angles) of a two-link manipulator.

According to (2.6), the components of the optimal control M^* are defined as follows.

If

$$(\varphi_1^T, \Delta\varphi_2^T) \in \Phi_1 = \bigcup_{\alpha=0}^{1} \Phi_1^{(\alpha)}, \tag{2.11}$$

then M_1^* - control with one switching, and M_2 - control with two switching

$$M_1^* = (-1)^\alpha M_1^0\, \mathrm{s}ign\left[(t_0^{(1)} - t)\left|(I_1 + 1)\varphi_1^T + I_2\Delta\varphi_2^T\right|\right],\quad t_0^{(1)} = T_1^*/2, \tag{2.12}$$
$$\alpha = 0,1,$$

$$M_2 = \begin{cases} (-1)^\beta, & t \in \left[0, t_2^{(2)}\right) \cup \left[t_3^{(2)}, T_1^*\right], \\ (-1)^{\beta+1}, & t \in \left[t_2^{(2)}, t_3^{(2)}\right), \end{cases}$$
$$t_2^{(2)} = (-1)^\beta I_2 \Delta\varphi_2^T / T_1^* + T_1^*/4, \tag{2.13}$$
$$t_3^{(2)} = (-1)^\beta I_2 \Delta\varphi_2^T / T_1^* + 3T_1^*/4, \quad \beta = 0,1.$$

If

$$(\varphi_1^T, \Delta\varphi_2^T) \in \Phi_2 = \bigcup_{\alpha=0}^{1} \Phi_2^{(\alpha)} , \tag{2.14}$$

then M_1 - control with two switching, and M_2^* - control with one switching

$$M_1 = \begin{cases} (-1)^{\beta} M_1^0, & t \in \left[0, t_2^{(1)}\right) \cup \left[t_3^{(1)}, T_2^*\right], \\ (-1)^{\beta+1} M_1^0, & t \in \left[t_2^{(1)}, t_3^{(1)}\right), \end{cases}$$

$$t_2^{(1)} = (-1)^{\beta} \left[(I_1+1)\varphi_1^T + I_2\Delta\varphi_2^T\right] / (M_1^0 T_2^*) + T_2^* / 4, \tag{2.15}$$

$$t_3^{(1)} = (-1)^{\beta} \left[(I_1+1)\varphi_1^T + I_2\Delta\varphi_2^T\right] / (M_1^0 T_2^*) + 3T_2^* / 4, \quad \beta = 0,1,$$

$$M_2^* = (-1)^{\alpha}\, \mathrm{s}ign\left[(t_0^{(2)} - t)\left|I_2\Delta\varphi_2^T\right|\right], \quad t_0^{(2)} = T_2^* / 2, \quad \alpha = 0,1. \tag{2.16}$$

If

$$(\varphi_1^T, \varphi_2^T) \in \Phi_0 = \left(\bigcup_{\alpha=0}^{1} \Phi_0^{(0,\alpha)}\right) \cup \left(\bigcup_{\alpha=0}^{1} \Phi_0^{(\alpha,1)}\right), \tag{2.17}$$

then M_1^*, M_2^* - controls with one switching

$$M_1^* = (-1)^{\alpha} M_1^0\, \mathrm{s}ign\left[(t_1^{(1)} - t)\left|(I_1+1)\varphi_1^T + I_2\Delta\varphi_2^T\right|\right], \quad \alpha = 0,1,$$

$$M_2^* = (-1)^{\beta}\, \mathrm{s}ign\left[(t_1^{(2)} - t)\left|I_2\Delta\varphi_2^T\right|\right], \qquad \beta = 0,1, \tag{2.18}$$

$$t_1^{(1)} = t_1^{(2)} = T_1^* / 2 = T_2^* / 2,$$

In (2.11)-(2.16) T_1^* and T_2^* are defined using the following formulas

$$T_1^* = 2\left(\left|(I_1+1)\varphi_1^T + I_2\Delta\varphi_2^T\right| / M_1^0\right)^{1/2}, \quad T_2^* = 2\left(\left|I_2\Delta\varphi_2^T\right|\right)^{1/2} . \tag{2.19}$$

Formulas (2.6)-(2.19), depending on whether point $(\varphi_1^T, \Delta\varphi_2^T)$ belongs to a particular area $\Phi_i(\varphi_1^T, \Delta\varphi_2^T)$, $i = 1,2,3$, allow us to determine both the type of control mode and the switching moments $t_i^{(j)} = t_i^{(j)}(\varphi_1^T, \Delta\varphi_2^T)$, $i = 1,2,3$; $j = 1,2$ and time $T^* = T^*(\varphi_1^T, \Delta\varphi_2^T)$, under which the system (2.2) moves from the initial state of rest (2.3) to the given state of rest (2.4) in the minimum time. As follows from formulas (2.11)–(2.16), in case $l = 0$, the pairs of modes (2.12), (2.13) and (2.15), (2.16) for $\alpha = 0, \beta = 0$ and $\alpha = 0, \beta = 1$ are equivalent in terms of time optimal and, therefore, we can restrict ourselves to considering one of them. However, in case $l \neq 0$, generally speaking, one should not expect these pairs to be equivalent.

3 Algorithm for Solving a Nonlinear Problem

To solve Problem 1.1 in the case $l \neq 0$, the following algorithm is proposed.

3.1. Since there are eight boundary conditions (1.2), (1.3) for the fourth-order controlled system (1.1), then in order to fulfill these conditions, in the general case, it is required that the control law contain at least four free parameters, including the non-fixed time T of the end of the process. From the maximum principle of Pontryagin it follows that the control functions will be relay functions and that they are maximum (with respect to the modulus). In accordance with the linear case, we set piecewise constant control laws with four free parameters (the switching moments t_1, t_2, t_3 and the process time T) in one of the following forms:

$$\begin{aligned}
&(1)\quad M_1 = M_1^0 u_\alpha(t),\ \ M_2 = v_\beta(t),\\
&(2)\quad M_1 = M_1^0 v_\beta(t),\ \ M_2 = u_\alpha(t),
\end{aligned} \tag{3.1}$$

$$u_\alpha(t) = \begin{cases} (-1)^\alpha, & t \in [0, t_1), \\ (-1)^{\alpha+1}, & t \in [t_1, T], \end{cases} \qquad v_\beta(t) = \begin{cases} (-1)^\beta, & t \in [0, t_2) \cup [t_3, T], \\ (-1)^{\beta+1}, & t \in [t_2, t_3), \end{cases}$$

$$0 \le t_1 \le T, \qquad 0 \le t_2 \le t_3 \le T, \qquad \alpha, \beta = 0, 1.$$

According to (3.1), both moments M_1 and M_2 take the maximum possible (with respect to the modulus) values and have not more than three switching in total. The function u_α corresponds to one switching at the moment t_1, and the function v_β corresponds to two switchings at the moments t_2 and t_3. Formulas (3.1) describe eight different methods of control, differing from each other by the number of the moment that has one switching (cases (1) and (2)), as well as by combinations of values $\alpha, \beta = 0, 1$ that determine the order of alternation of signs of the control moment with two switching.

Fixing one of these eight modes, we can look for parameters t_1, t_2, t_3, T so that solution $\varphi_i(t_1, t_2, t_3; t)$, $i = 1, 2$ of the system (1.1) with initial conditions (1.2) would satisfy conditions (1.3) at the end of the motion. We get a system of four equalities

$$\begin{aligned}
&\varphi_1(t_1, t_2, t_3, T) = \varphi_1^T, \quad \varphi_2(t_1, t_2, t_3, T) = \varphi_2^T, \qquad (a)\\
&\dot\varphi_1(t_1, t_2, t_3, T) = 0, \qquad \dot\varphi_2(t_1, t_2, t_3, T) = 0 \qquad\ \ (b)
\end{aligned} \tag{3.2}$$

for the sought parameters t_1, t_2, t_3, T.

However, for given terminal values φ_1^T, φ_2^T, the existence of a solution to system (3.2) depends (as in the linear case) on the choice of the type of control regime (3.1). Therefore, it is first necessary to construct regions $\Phi_i(l)$, $i = 0, 1, 2$, and then, depending on whether point $(\varphi_1^T, \varphi_2^T)$ belongs to one or another region $\Phi_i(l)$, $i = 0, 1, 2$, determine the type of control mode (3.1) in which system (1.1), (1.2) is brought to this terminal position with zero velocities.

Let's move on to constructing the region $\Phi_0(l)$. Consider the following semi-reverse method. By analogy with case $l = 0$, controls under which system (1.1) with initial conditions (1.2) is reduced to the terminal point (1.3) lying on the boundary $\Phi_0(l)$ will be sought in the class of controls (3.1) with one switching

$$M_1 = M_1^0 u_\alpha(t),\quad M_2 = \mu_\beta(t),$$
$$u_\alpha(t) = (-1)^\alpha sign(t_1 - t),\quad \mu_\beta(t) = (-1)^\beta sign(t_2 - t), \tag{3.3}$$
$$0 \le t_1 \le T,\quad 0 \le t_2 \le T,\quad \alpha, \beta = 0,1.$$

Fix one of the four modes (3.3) and one of the parameters t_1, t_2 (for example, t_2). Then we calculate parameters t_1 and T from the last two conditions of the following system

$$\varphi_1(t_1,t_2,T) = \varphi_1^T,\quad \varphi_2(t_1,t_2,T) = \varphi_2^T,\quad \dot\varphi_1(t_1,t_2,T) = 0,\quad \dot\varphi_2(t_1,t_2,T) = 0. \tag{3.4}$$

This procedure is reduced to finding the root (by the method of half division of the segment) of a function of one variable t_1, the parameter T is easily found in the process of numerical integration of equations (1.1) as the moment of the first vanishing of one of the angular velocities ($\dot\varphi_1$ or $\dot\varphi_2$). When searching for the root t_1, the value $t_1 = t_1^{(1)}$ calculated by formulas (2.18), (2.19) of section 2 is used as the initial value. After that, from the first two conditions (3.4) we determine the boundary values φ_1^T, φ_2^T that correspond to the obtained set of parameters t_1, t_2, T. By sampling (with a certain step) parameter t_1, one can construct regions of finite configurations that are achievable for given initial conditions and control type (3.3). If the described procedure is carried out for all types of control with one switching (3.3), then lines $\Phi_0^{(\alpha,0)}(l)$, $\Phi_0^{(\alpha,1)}(l)$ will stand out on plane φ_1^T, φ_2^T , defining region $\Phi_0(l)$:

$$\Phi_0(l) = \left(\bigcup_{\alpha=0}^{1} \Phi_0^{(0,\alpha)}(l)\right) \cup \left(\bigcup_{\alpha=0}^{1} \Phi_0^{(\alpha,1)}(l)\right). \tag{3.5}$$

Since under controls (3.3) the solutions of system (1.1), (1.2) continuously depend on the parameter l, then $\Phi_0^{(0,\alpha)}(l)\Big|_{\alpha=0,1} \to \Phi_0^{(0,\alpha)}\Big|_{\alpha=0,1}$, $\Phi_0^{(1,\alpha)}(l)\Big|_{\alpha=0,1} \to \Phi_0^{(1,\alpha)}\Big|_{\alpha=0,1}$, as $l \to 0$.

3.2. By analogy with the linear case, taking into account (2.10), outside region $\Phi_0(l)$, in the region $\Phi \setminus \Phi_0(l)$:

$$\Phi_1(l) \cup \Phi_2(l) = \left(\bigcup_{\alpha=0}^{1} \Phi_1^{(\alpha)}(l)\right) \cup \left(\bigcup_{\alpha=0}^{1} \Phi_2^{(\alpha)}(l)\right) \tag{3.6}$$

we will consider control modes with three switching times (3.1)(1),(2):
in the regions

$$\Phi_1^\alpha(l), \alpha = 0,1 \quad \text{- control mode (3.1)(1),} \tag{3.7}$$

i.e. control M_1 with one switching at time t_1 and control M_2 with two switching at times t_2 and t_3, and in the regions

$$\Phi_2^{\alpha}(l), \alpha = 0,1 \quad \text{- control mode (3.1)(2),} \tag{3.8}$$

i.e. control M_1 with two switching at times t_2 and t_3, and control M_2 with one switching at time t_1. Thus, depending on which region (3.5) or (3.6) the point $(\varphi_1^T, \varphi_2^T)$ lies in, from (3.7) or (3.8), respectively, it can be defined the type of control mode (3.1)((1) or (2)), which can be used to transfer the manipulator to (1.1), (1.2) to a given final state (1.3).

Suppose $(\varphi_1^T, \varphi_2^T) \in \Phi_1^{(0)}(l)$. This point according to (3.7) corresponds to the case (1): $\alpha = 0, \beta = 0$ or $\alpha = 0, \beta = 1$ in (3.1). Let's fix the first of them and solve system (3.2) with respect to the parameters t_1, t_2, t_3, T. Let us carry out multiple integration of system (1.1) with "zeroing" in three parameters t_1, t_2, t_3. First, two (for example, t_2, t_3) of the four parameters t_1, t_2, t_3, T are set, and from the conditions of velocities vanishing (3.2)(a), the remaining parameters (t_1, T) are calculated (this procedure is described in detail above, when solving the system (3.4)). Then, trying with some step the parameters t_2, t_3, the previous procedure is repeated many times until the boundary conditions in (3.2)(b) are satisfied. Finding parameters t_2, t_3 again reduces to finding first the root of the first equation (3.2)(a) with respect to variable t_2 and then the second equation (3.2)(a) with respect to variable t_3. As a result, the parameters t_1, t_2, t_3, T, at which the control mode (3.1)(1)($\alpha = 0, \beta = 0$) realizes the movement of the system (1.1), (1.2) to the state (1.3) are determined.

4 Calculation results

Let's assume that the manipulator is characterized by the following dimensional parameters appearing in (1.1), (1.4):

$$\begin{gathered} L_1 = 1\,\text{m},\ l = 0.2\,\text{m},\ m_2 = 10\,\text{kg},\ I_1 = I_2 = (10/3)\,\text{kg}\cdot\text{m}^2, \\ M_1^0 = 2\,\text{N}\cdot\text{m},\ M_2^0 = 1\,\text{N}\cdot\text{m}. \end{gathered} \tag{4.1}$$

which correspond to the manipulator, the links of which are the same uniform rods.

After passing to dimensionless parameters, according to (1.5), we obtain from (4.1)

$$L_1 = 1,\ l = 0.2,\ m = 1,\ I_1 = I_2 = 1/3,\ M_1^0 = 2,\ M_2^0 = 1. \tag{4.2}$$

At values (4.2) boundary $\Phi_0(l)$ was constructed to the semi-inverse numerical technique described in the section 3.1 (fig. 2). For both calculation options $\varphi_1^0 = \varphi_2^0 = 0$. On the axes of fig. 2 the final values of the angles $\varphi_1 = \varphi_1^T, \varphi_2 = \varphi_2^T$ are plotted. The bold line in fig. 2 shows the border $\Phi_0(l)$ ($l = 0, 0.2$) between regions $\Phi_1(l)$ and $\Phi_2(l)$.

We take the initial and final conditions in the form

$$\varphi_1^0 = \varphi_2^0 = 0\,,\quad \varphi_1^T = 57°17' \approx 1\,\text{rad} \quad \varphi_2^T = 114°34' \approx 2\,\text{rad}\,. \tag{4.3}$$

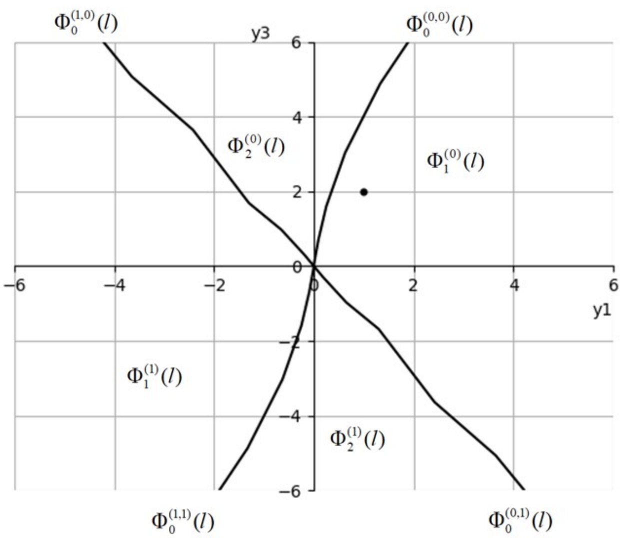

Fig. 2. Diagram on the plane of generalized coordinates $y_1 = \varphi_1$, $y_3 = \varphi_2$ for determining the type of modes in the case $l = 0.2$.

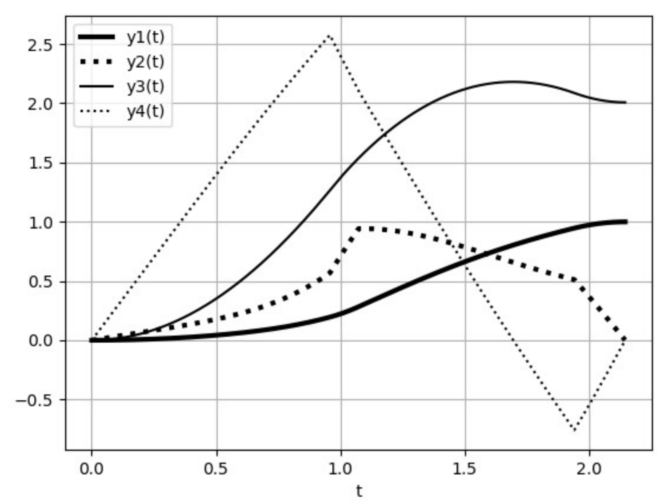

Fig.3. Dependence of generalized coordinates $y_1 = \varphi_1$, $y_3 = \varphi_2$, $y_2 = \dot{\varphi}_1$, $y_4 = \dot{\varphi}_2$ on time in the case $l = 0.2$.

Values of parameters (4.2), (4.3) correspond to fig. 2, from which we find that in the case under consideration $(\varphi_1^T, \varphi_2^T) \in \Phi_1^{(0)}$. Therefore, the control mode (3.1) (1) from (3.7) should be used: control M_1 with one switching t_1 and control M_2 with two switching t_2, t_3. Calculations of the values of parameters t_1, t_2, t_3, T from system (3.2) at finite values (4.2) according to the algorithm described in the section 3.2 gave the following results:

$$t_1 = 1.073 \text{ (2.401s)}, \; t_2 = 0.957 \text{ (2.141s)}, \; t_3 = 1.940 \text{ (4.341s)}, \; T = 2.146 \text{ (4.802s)}. \quad (4.4)$$

At values of (4.4), the fulfillment of equations (3.2), (4.3) is ensured with an accuracy of 0.001 for both angles and both velocities. In parentheses in (4.4), (4.5) the dimensional values of times are given using the transition formulas (1.5).

In the case $l = 0$, for finite values (4.3), parameters t_1, t_2, t_3, T are determined using the analytical solutions of the section 2 (formulas (2.12), (2.13), (2.19)):

$$t_1 = t_1^{(1)} = 1, \quad t_2 = t_2^{(2)} = 0.833, \quad t_3 = t_3^{(2)} = 1.833, \quad T = T^* = 2. \quad (4.5)$$

Note that the first three values from (4.5) were used as starting values when searching for the parameters (4.3) from the system (3.2) by the algorithm of the section 3.2.

The motion of a two-link manipulator was numerically simulated with controls (3.1)(1). Equations (1.1) were integrated under initial conditions $\varphi_i^0 = \dot{\varphi}_i^0 = 0$, $i = 1,2$ and control mode (3.1)(1) ($\alpha = 0, \beta = 0$) with parameters (4.4). The results of the simulation are shown in fig. 3. The bold solid line and the solid line, respectively, show the dependences of angles and on time, and the bold dashed line and the dashed line show the dependences of angular velocities and on time.

Comparison of calculated results (4.4), (4.5) shows that moving the manipulator to the required rest position in control modes (3.1) is accomplished in time close to optimal calculated for the linear case with optimal control modes of the section 2. Therefore, the constructed control modes can be regarded as time suboptimal.

Conclusion

The described algorithm allows constructing software relay controls that are suboptimal in terms of speed with the minimum possible number of switching, sufficient for the transition of a two-link manipulator from the initial state of rest to any terminal state of rest in the working region of the manipulator. The results of numerical simulation of the dynamics of a two-link manipulator with constructed control modes have established the acceptability of the proposed calculation methodology. It can be used to calculate suboptimal program motions of manipulating robots.

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