ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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Ghazaryan K., Ghazaryan R., Terzyan S.

Localization of Shear Waves in Inhomogeneous Periodically Stratified Waveguide

Keywords: Localization , shear elastic wave, inhomogeneous material, waveguide.

Локализация сдвиговых волн в слоисто- периодическом неоднородном волноводе

Казарян К., Казарян Р., Терзян С.

Ключевые слова: локализация, сдвиговая упругая волна, неоднородный материал, волновод.

Для определенного класса неоднородных материалов аналитически установлена возможность локализации сдвиговой упругой волны в слоистом волноводе, состоящем из периодически повторяющихся упруго контактирующих неоднородных конечнего числа идентичных слоев. Для неоднородного материала с периодическими несимметричными профилями показано, что вследствии неоднородности имеет место локализация сдвиговых волн на внешних границах волновода Локализация волн значительно усиливается с увеличением числа элементарных ячеек волновода. Для неоднородного материала с симметричными профилями показано, что в этом волноводе локализация сдвиговых волн не имеет место.

Կ. Ղազարյան, Ռ. Ղազարյան, Ս. Թերզյան.

Սահքի ալիքների տեղայնացումը շերտավոր – պարբերական ալիքատարում

Հիմնաբառեր՝ Տեղայնացում, սահքի առաձգական ալիք, անհամասեռ նյութ, ալիքատար:

Անհամասեռ նյութերի որոշակի դասի համար անալիտիկորեն հաստատված է շերտավոր ալիքատարում սահքի առաձգական ալիքի տեղայնացման հնարավորությունը: Շերտավոր ալիքատարը բաղկացած է պարբերաբար կրկնվող, առաձգականորեն հպված վերջավոր թվով նույնական շերտերից: Պարբերական, ոչ սիմետրիկ պրոֆիլներով անհամասեռ նյութի համար ցույց է տրված, որ անհամասեռության հետևանքով ալիքատարի արտաքին եզրերում տեղի ունի սահքի ալիքների տեղայնացում: Ալիքատարի տարրական բջիջների թվի աճին զուգընթաց ալիքների տեղայնացումը զգալիորեն ուժեղանում է: Սիմետրիկ պրոֆիլներով անհամասեռ նյութի համար ցույց է տրված, որ այդ ալիքատարում սահքի ալիքների տեղայնացում տեղի չունի:

For a special class of inhomogeneity materials this analytical study demonstrates localization of shear elastic wave in periodically stratified waveguide, consisting of periodically repeated perfectly bonded inhomogeneous identical finite number unit cells. For inhomogeneous material with periodic non symmetrical profiles is shown that due to inhomogeneity the shear guided waves can be localized at interfaces in the waveguide. The localization of waves significantly increases with the numbers of the waveguide unit cells. For inhomogeneous material with symmetrical profiles is shown that this material does not support localization in waveguide.

Introduction

Functionally graded materials (FGM) are inhomogeneous elastic bodies whose properties vary with space. FGM plays an essential role in the most advanced integrated systems for vibration control and health monitoring. The progress in the characterization, modelling, analysis and principal developments of FGM was reviewed in [1, 2]. In pure elastic FGM materials elastic wave propagation was studied by many authors, and some studies close to presented article topic are worth to be mentioned [3–11]. In piezoelectric medium coupled electro- elastic bulk and surface waves are widely discussed in science periodic, particularly in [12–21]. Elastic and couple electro elastic surface wave prorogation in inhomogeneous materials admitting analytical solutions is considered in [6–10, 13, 15]. In this study, an exact analytical approach and transfer matrix technique are used to investigate localization of shear elastic wave in periodically stratified waveguide of some functionally graded material, consisting of periodically repeated perfectly bonded inhomogeneous identical unit cells.

Statement of the problem.

Let consider shear elastic wave propagation in periodically stratified functionally graded waveguide, consisting of periodically repeated perfectly bonded identical inhomogeneous layers. The material parameters of a inhomogeneous material, the stiffness and the mass density are assumed to be varied in the same proportion in the unit cell as $\mu_n(x) = \mu_0 f_n(x); \rho_n(x) = \rho_o f_n(x)$, where $f_n(x)$ is the inhomogeneity functions which will be specified later, $x \in [(n-1)\,d, nd]\,, n = 1, 2, ..N$, is the number of the unit sells (Fig.1)

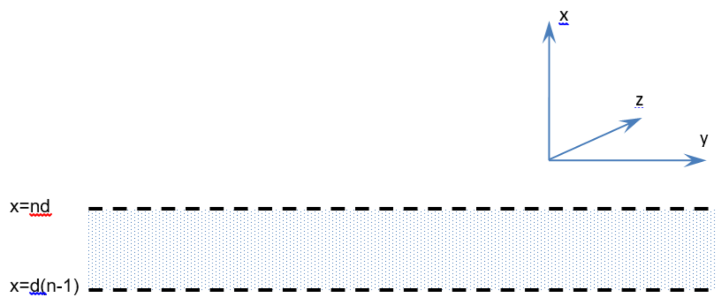

Fig.1 Elementary unit cell of the inhomogeneous elastic waveguide

The elastic displacements and stresses obey to the anti-plane equations of motion and Hooke's law. Choosing the anti-plane deformation in the z-direction and considering a steady SH-wave $u_n\,(x, y, t) = u_{n0}\,(x) \exp\left[i(ky - \omega t)\right]$, where $u_n\,(x, y, t)$ is the displacement in z direction, we come to the following equations $(k, \omega$ are wave number and angular frequency)

$$\begin{aligned} &\partial_x \tau_{0n}(x) + \left(\varrho_0 \omega^2 - \mu_0 k^2\right) f_n(x) u_{0n}(x) = 0 \\ &\tau_{0n}(x) - \mu_0 f_n(x) \partial_x \left(u_{0n}(x)\right) = 0 \end{aligned} \tag{1}$$

For a special class of inhomogeneity functions Eqs. (1) can be converted into differential equations with constant coefficients admitting the exact solutions [10, 16].

$$f_n^{(\pm)}(x) = \left(\cosh\left(a\left(x-(n-1)\,d\right)+d_0\right) + \frac{b}{a}\sinh\left(a\left(x-(n-1)\,d\right)+d_0\right)\right)^{\pm 2} \tag{2}$$

The expression Eqs. (2) is valid also for $b = ib_0$, $a = 0$. When $a = 0$ instead of Eq.(2) we have ($d_0 = 0$)

$$f_n^{(\pm)}(x) = (1 + b(x - d(n-1)))^{\pm 2}; \tag{3}$$

Solutions of Eq. (1), corresponding to the functions $f_n^{(+)}(x)$, can be found as

$$u_{0n}\left(x\right) = \frac{\left(A_{1n}\, e^{ipx} + A_{2n}\, e^{-ipx}\right)}{\sqrt{f_n^{(+)}(x)}}; \quad \tau_{0n}(x) = \mu_0 f_n^{(+)}(x) \partial_x \left(u_n(x)\right) \tag{4}$$

where $p = d^{-1}\sqrt{\theta^2 - \left(kd\right)^2 - \left(ad\right)^2}; \theta = \omega\sqrt{\mu_0^{-1}\rho_0}$; By introducing the column field vector $\bar{U}_n\left(x\right) = \left(u_{0n}(x), \tau_{0n}(x)\right)^T, \bar{A}_n = \left(A_{1n} A_{2n}\right)^T$ the solutions of Eq.(4) can be cast as

$$\bar{U}_n\left(x\right) = \hat{F}_n^{(+)}\left(x\right)\bar{A}_n$$

where

$$\begin{aligned} &\hat{F}_n^{(+)}\left(x\right) = \frac{1}{2\sqrt{f_n^{(+)}(x)}} \times \\ &\times \begin{pmatrix} 2e^{ipx} & 2e^{-ipx} \\ \mu_0\, e^{ipx}\left(2ipf_n^{(+)}(x) - \partial_x f_n^{(+)}(x)\right) & -\mu_0 e^{-ipx}\left(2ipf_n^{(+)}(x) + \partial_x f_n^{(+)}(x)\right) \end{pmatrix} \end{aligned} \tag{5}$$

Solutions of Eqs. (1) corresponding to the functions $f_n^{(-)}(x)$ can be found as

$$\begin{aligned} \tau_{0n}(x) &= \frac{\mu_0\left(A_{1n}e^{ipx} + A_{2n}e^{-ipx}\right)}{\sqrt{f_n^{(-)}(x)}}; \\ u_{0n}(x) &= \frac{\mu_0 k^2 - \varrho_0\omega^2}{f_n^{(-)}(x)} \partial_x \tau_{0n}(x); \end{aligned} \tag{6}$$

and in the matrix form they can cast as

$$\bar{U}_n\left(x\right) = \hat{F}_n^{(-)}\left(x\right) \cdot \bar{A}_n$$

where

$$\hat{F}_n^{(-)}(x) = \sqrt{f_n^{(-)}(x)}\times$$
$$\times \begin{pmatrix} -\frac{e^{ipx}\left(2ipf_n^{(-)}(x)+\partial_x f_n^{(-)}(x)\right)}{2(a^2+p^2)\left(f_n^{(-)}(x)\right)^2} & -\frac{e^{-ipx}\left(-2ipf_n^{(-)}(x)+\partial_x f_n^{(-)}(x)\right)}{2(a^2+p^2)\left(f_n^{(-)}(x)\right)^2} \\ \mu_0 e^{ipx} & \mu_0 e^{-ipx} \end{pmatrix}$$

The transfer matrix $\hat{M}$ linking field vector values at the surfaces $x = (n-1)d, x = nd$ of the unit cell can now be determined as:

$$\bar{U}_n(nd) = \hat{F}_n^{(\pm)}(nd)\cdot \bar{A}_n, \bar{U}_n((n-1)d) = \hat{F}_n^{(\pm)}((n-1)d)\cdot \bar{A}_n \tag{7}$$

Excluding vector $\bar{A}_n$ in Eq. (5) one has

$$\bar{U}_n(nd) = \hat{M}^{(\pm)}\bar{U}_n((n-1)d); \tag{8}$$

Herein $\hat{M}^{(\pm)} = \hat{F}_n^{(\pm)}(nd)\left(\hat{F}_n^{(\pm)}\right)^{-1}((n-1)d)$ is the unimodal transfer matrix for the inhomogeneous cells corresponding to the functions $f_n^{(\pm)}(x)$.

The condition $\left|\mathrm{Tr}\left(\hat{M}^{(\pm)}\right)\right| > 2$, defines the stopband of frequencies, ranges of eigen frequencies in which waves cannot propagate in the infinite periodic medium consisting of periodically repeated inhomogeneous cells [22]. Let note that elements of matrix $\hat{M}^{(\pm)}$ do not depend of cell number n.

Using the continuity conditions of the field vectors $\bar{U}(x)$ at interfaces $x = nd$ we come to the matrix equations

$$\widehat{U}_{n+1}(x_n) = \hat{M}^{(\pm)}\widehat{U}_n(x_{n-1}) \tag{9}$$

, Repeating this procedure the n-th times the propagator unimodal matrix $\left(\hat{M}^{(\pm)}\right)^n$ can be found. The matrix $\left(\hat{M}^{(\pm)}\right)^n$ links the field vectors at $x = 0$ and $x = nd$ surfaces of the waveguide.

$$\left(\hat{M}^{(\pm)}\right)^n \bar{U}_1(0) = \bar{U}_n(nd), \qquad n = 1, 2, \ldots.N \tag{10}$$

According to Sylvester's matrix polynomial theorem for 2x2 matrices the elements of the n-th power of an unimodal matrix $\hat{M}^{(\pm)}$ can be cast as [23]

$$\left(\hat{M}^{(\pm)}\right)^n = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

and can be simplified using the following matrix identity

$$\begin{aligned} M_{11} &= m_{11}S_{n-1}(\eta) - S_{n-2}(\eta); M_{12} = m_{12}S_{n-1}(\eta) \\ M_{21} &= m_{21}S_{n-1}(\eta); M_{22} = m_{22}S_{n-1}(\eta) - S_{n-2}(\eta) \end{aligned} \tag{11}$$

where $m_{11}, m_{12}, m_{21}, m_{22}$are elements of matrix $\hat{M}^{(\pm)}$

$$\hat{M}^{(\pm)} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},$$

$S_n(\eta)$ are the Chebyshev polynomials of second kind, namely

$$S_n(\eta) = \frac{\sin((n+1)\phi)}{\sin\phi}; \qquad \cos\phi = \eta;$$
$$\eta = \frac{1}{2}\mathrm{Tr}\left(\hat{M}\right) = \frac{1}{2}(m_{11} + m_{22});$$

The first Chebyshev polynomials are

$$S_0(\eta) = 1, \quad S_1(\eta) = 2\eta; \quad S_2(\eta) = 4\eta^2 - 1$$

Subsequent polynomials may be obtained from the recurrence relation of Chebyshev polynomials [?]

$$S_m(\eta) = 2\eta S_{m-1}(\eta) - S_{m-2}(\eta) \tag{12}$$

Consider now a boundary value problem when the waveguide interfaces $x = 0, x = Nd$ and are tractions free

$$\tau_{01}(0) = \tau_{0N}(Nd) = 0 \tag{13}$$

In this case the following matrix equation can be imposed

$$\left(\hat{M}^{(\pm)}\right)^N \begin{pmatrix} u_{01}(0) \\ 0 \end{pmatrix} = \begin{pmatrix} u_{0N}(Nd) \\ 0 \end{pmatrix} \tag{14}$$

Eq. (14) have a non-trivial solution if the following two alternative equations are satisfied

$$m_{21}(\theta) = 0 \tag{15}$$

$$S_{N-1}(\eta(\theta)) = 0 \tag{16}$$

From Eq.(14) besides of these equations it follows also that

$$u_{0N}(Nd) = (m_{11}S_{N-1} - S_{N-2})\, u_{01}(0); \tag{17}$$

Alongside with Eq. (14) one can consider the matrix equation such as

$$\left(\hat{M}^{(\pm)}\right)^n \begin{pmatrix} u_{01}(0) \\ 0 \end{pmatrix} = \begin{pmatrix} u_{0n}(nd) \\ \tau_{0n}(nd) \end{pmatrix} \tag{18}$$

and the relation between field vector values can be found as

$$u_{0n}(nd) = (m_{11}S_{n-1} - S_{n-2})\, u_{01}(0); \qquad n = 1, 2, 3...N \tag{19}$$

The roots of Eqs. (15,16) are curves in the phase plane (θ, kd), each point of which corresponds to a wave travelling in the wave guides.

If $m_{21}(\theta) = 0$ or $m_{12}(\theta) = 0$, then since $\hat{M}^{(\pm)}$ is a unimodular matrix $m_{11}m_{22} - m_{12}m_{21} = 1$ it follows that $m_{11}m_{22} = 1$ and therefore $\eta = \left(\lambda + \lambda^{-1}\right)/2$, where $\lambda = m_{11}(\theta)$ Using recurrent relation Eq.(12) the following new relation can be shown for the Chebyshev polynomials of second kind:

$$P_{n+1} = \lambda S_n(\eta) - S_{n-1}(\eta) = \lambda \left((\lambda + \lambda^{-1})S_{n-1}(\eta) - S_{n-2}(\eta)\right) - S_{n-1}(\eta) = \\ = \lambda\left(\lambda S_{n-1}(\eta) - S_{n-2}(\eta)\right)$$

which can be rewritten as

$$P_{n+1} = \lambda P_n \tag{20}$$

where $P_n = (\lambda S_{n-1}(\eta) - S_{n-2}(\eta))$

Taking into account that $P_1 = \lambda, (S_0(\eta) = 1, S_{-1}(\eta) = 0)$ the following identity can be obtained valid for all integers starting from $n = 1$,

$$P_n = \lambda^n \tag{21}$$

Hence it follows from Eqs.(19,21) that for frequencies $\theta = \theta_0$, where $\theta = \theta_0$ are the roots of the equation $m_{21}(\theta) = 0$, we have

$$u_{0n}(nd) = \lambda^n u_0(0); \qquad n = 1, 2...N \tag{22}$$

Therefore Eq. (22) shows that at frequencies $\theta = \theta_0$, localisation of elastic shear displacements may take place at the top or bottom interfaces of the periodic waveguide if $m_{11}(\theta) \neq m_{22}(\theta)$.

Another possible case is $S_{N-1}(\eta(\theta)) = 0$. This equation has $N-1$ roots in the range $\eta \in (-1, 1)$ which are given by $\eta_m(\theta) = \cos\left(m\pi N^{-1}\right), m = 1, 2...N-1$

Taking into account that in this case $S_{N-2}(\eta_m) = (-1)^m$ one can write

$$u_{0n}(nd) = (-1)^m u_{01}(0); \qquad n = 1, 2......N \tag{23}$$

This means that $N-1$ shear wave normal modes exist where amplitudes of guided waves are distributed along the waveguide width and have the same magnitude at the top and the bottom interfaces.

When the top and bottom faces of the waveguide are clamped

$$u_0(0) = u_0(Nd) = 0$$

Eqs. (15,21) should be replaced by the following equations:

$$m_{12}(\theta) = 0 \tag{24}$$

$$S_{N-1}(\eta(\theta)) = 0$$

$$\tau_{0n}(nd) = \lambda^{-n}\tau_{01}(0)$$

In this case the localization of the elastic shear stresses takes place at the top or bottom interfaces when $m_{12}(\theta_0) = 0$ if $m_{11}(\theta_0) \neq m_{22}(\theta_0)$.

Here are also $N-1$ normal modes distributed along the waveguide width and have the same magnitude of stresses at the top and the bottom interfaces which follows from the following relation:

$$\tau_{0n}(nd) = (-1)^m \tau_{01}(0); \qquad n = 1, 2......N \tag{25}$$

Thus two different families of vibrational modes exist in the inhomogeneous waveguide for both traction free and clamped the top and bottom interfaces . One is a localized mode which exists only when $m_{11}(\theta_0) \neq m_{22}(\theta_0)$, where θ_0 are the roots of $m_{21}(\theta_0) = 0$ or$m_{21}(\theta_0) = 0$. There are also another $N-1$ normal non-localised vibration modes at frequencies defined by $S_{N-1}(\eta(\theta)) = 0$.

Results and discussions

In this section attention is restricted to some specified inhomogeneity functions. In the first example, the inhomogeneity function are quadratic functions

$f_n^{(+)}(x) = (1 + b(x - d(n-1)))^2$ At the Fig.2 the profiles of quadratic functions $f_n^{(+)}(x)$ in the elementary cells $n = 1, 2, 3, 4$ are presented for different values of inhomogeneity parameter $\beta = bd$, $\beta = 1; \beta = -0.7, \beta = -2$

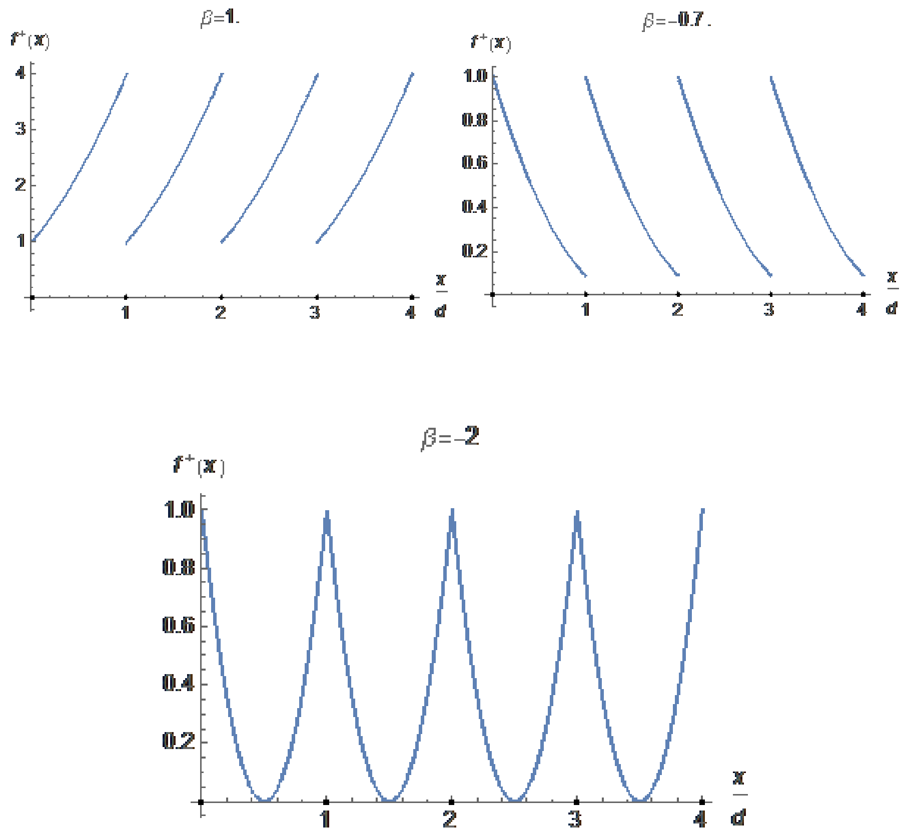

Fig. 2: Profiles of quadratic function $f_n^{(+)}(x)$)

Let note that the inhomogeneous quadratic function when $\beta=-2$ is the symmetric function $f_n^{(+)}\left((n-1)d+x\right)=f_n^{(+)}(nd-x), \qquad x\in(0,d/2\ )$.

For quadratic functions $f_{n0}^{(+)}(x)$ the transfer matrix $\hat{M}^{(+)}$ can be found as

$$\hat{M}^{(+)}=\begin{pmatrix} \frac{Z\cos Z+\beta\sin Z}{Z(1+\beta)} & \frac{d\sin Z}{\mu_0 Z(1+\beta)} \\ \frac{\mu_0}{d}\left(Z\beta^2\cos Z-\left(\beta^2+Z^2(1+\beta)\sin Z\right)\right) & (1+\beta)\cos Z-\frac{\beta}{Z}\sin Z \end{pmatrix} \tag{26}$$

where $Z=\sqrt{\theta^2-(kd)^2}$ is a dimensionless parameter.

From Eqs. (26) it follows that when $\beta=-2$ then $m_{11}(\theta)=m_{22}(\theta)$ and therefore localization does not take place. For inhomogeneous quadratic function when $\beta\neq-2$ localization of guided wave amplitudes takes place at eigen frequencies determining from equations $m_{21}(\theta_0)=0$or $m_{12}(\theta_0)=0$. The eigen frequencies of the waveguide with clamped interfaces determines from equation

$$\sin Z=0; \qquad \theta_0=\sqrt{(kd)^2+\pi^2m^2}. \tag{27}$$

At these frequencies the localization coefficient $|\lambda|=|1+\beta|$ is a monotonically decreasing function in interval $\beta\in(-\infty,-1)$ and a monotonically increasing function in interval $\beta\in(-1,\infty)$.

In the interval $\beta\in(-2,0)$ we have $|\lambda|<1$, outside of this interval $|\lambda|>1$.

The eigen frequencies of waveguide with traction free interfaces determines from equation

$$\beta^2\cos Z-\left(\beta^2+Z^2(1+\beta)\sin Z\right)=0 \tag{28}$$

The graphs of localization coefficients for traction free waveguide are presented on the Fig.3 .

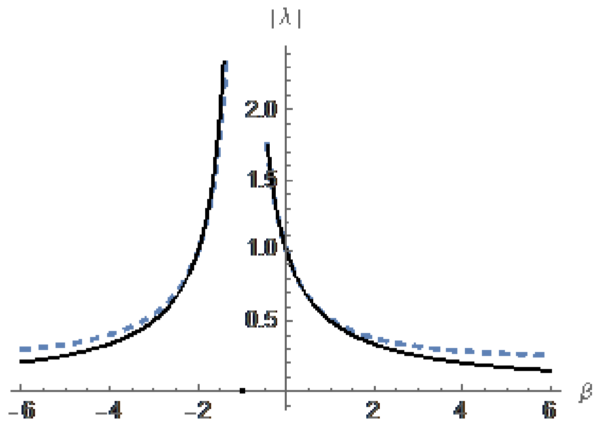

Fig. 3: Localization coefficient of shear stresse wave amplitudes for quadratic function.

As it follows from data of Fig.3 the localization coefficient weakly depends from eigen frequencies θ_0.

Contrary to the above case for waveguide with traction free interfaces In the interval $\beta \in (-2, 0)$, $|\lambda| > 1$, outside of this interval $|\lambda| < 1$.

When $|\lambda| < 1$, in the traction free or clamped waveguide with quadratic periodic inhomogeneity one can state that the amplitudes of guided waves attenuate from the waveguide bottom interface to the top interface with increasing of cell numbers. When $|\lambda| > 1$ the amplitudes of guided waves attenuate from the waveguide top interface to the bottom interface. In the second example, the inhomogeneity functions are inverse quadratic functions .

$$f_{n0}^{(-)}(x) = (1 + b(x - d(n-1)))^{-2}$$

In the Fig.4 the profiles of the inverse quadratic functions $f_n^{(-)}(x)$ in the elementary cells $n = 1, 2, 3, 4$ are presented for different values of inhomogeneity parameter $\beta = bd$, $\beta = 1$; $\beta = -0.8$, $\beta = -2$

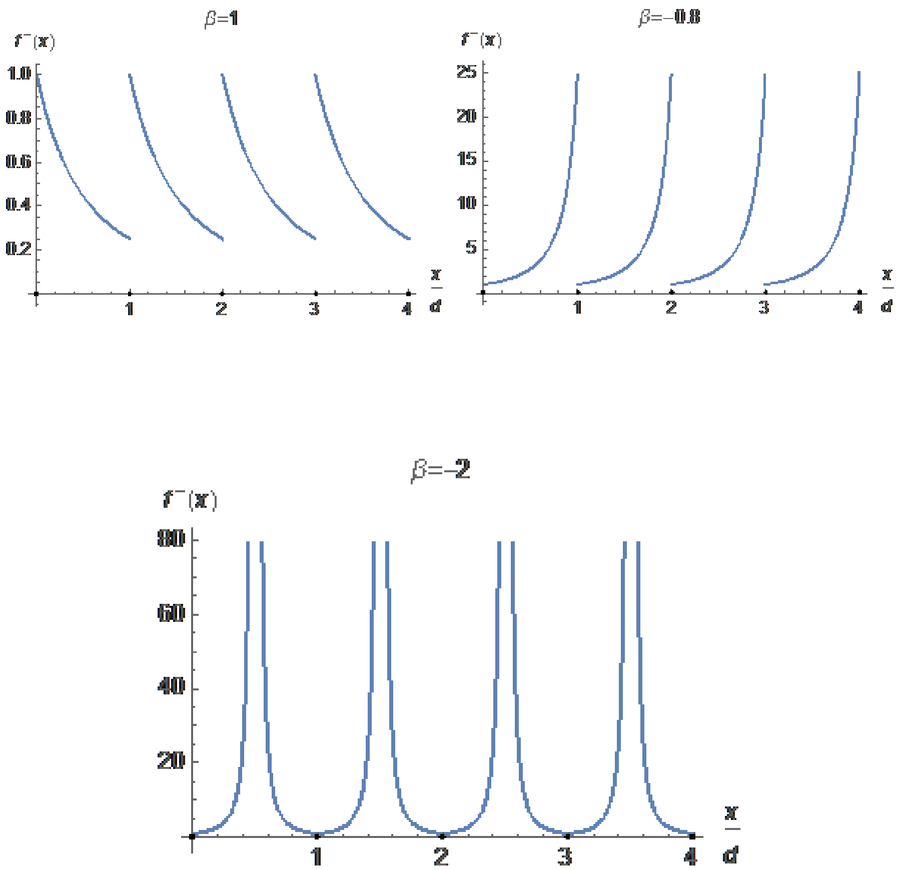

Fig. 5: Profiles of inverse quadratic function $f_n^{(-)}(x)$)

Corresponding to the inverse quadratic function $f_{n0}^{(-)}(x)$ the transfer matrix $\hat{M}_{(-)}$

can be found as

$$\hat{M}^- = \begin{pmatrix} (1+\beta)\cos Z - \frac{\beta}{Z}\sin Z & \frac{d}{\mu_0 Z^3}\left(Z\beta^2\cos Z - \left(\beta^2 + Z^2(1+\beta)\sin Z\right)\right) \\ \frac{\mu_0 Z \sin Z}{d(1+\beta)} & \frac{Z\cos Z + \beta\sin Z}{Z(1+\beta)} \end{pmatrix} \tag{29}$$

Juxtaposition of the matrix $\hat{M}^-$ with matrix $\hat{M}^+$ leads to the conclusion that contrary to quadratic function case Eq.(27) determines eigen frequencies for waveguide with traction free interfaces and Eq.(28) determines eigen frequencies for waveguide with clamped interfaces.

Therefore we can state in the case of inverse quadratic function inhomogeneity the results concerning localization effects for clamped /free traction waveguide coincides with results of traction free /clamped waveguide with quadratic function inhomogeneity.

Conclusion

Based on an exact analytical approach and transfer matrix technique a localization of shear elastic wave is established in waveguide consisting of periodically repeated perfectly bonded inhomogeneous identical of finite numbers unit cells. For a special class of inhomogeneity functions admitting exact solutions, relationships are established between elastic displacements of the top and bottom interfaces of the waveguide when these interfaces are traction free. When they are clamped a relationship is established between tangential stresses on these interfaces. It is shown the localization of guided waves are take place in the traction free or clamped waveguide with quadratic and inverse quadratic non symmetrical periodic inhomogeneity. In the case of inverse quadratic function inhomogeneity the results concerning localization effects for clamped /free traction waveguide coincides with results of traction free /clamped waveguide with quadratic function inhomogeneity. The localization of waves significantly increases with the numbers of the waveguide unit cells. It is shown also that the waveguide with inhomogeneous cells of which are symmetrical periodic quadratic and inverse functions do not support wave localization.

References

[1] R.M. Mahamood, E.T. Akinlabi, M. Shukla, S. Pityana, Functionally Graded Material: An Overview, Proceedings of the World Congress on Engineering, London, UK, 2012, vol. 3, pp. 35-41. http://hdl.handle.net/10204/6548

[2] V. Birman, L.W. Byrd, Modeling and Analysis of Functionally Graded Materials and Structures, Applied Mechanics Reviews, vol.60, 2007, pp. 195-215.

[3] S.N. Bhattacharya, Exact solution of SH wave equation in transversely isotropic inhomogeneous elastic media, Pure and Appl. Geophysics, vol. 93, 1972, pp. 19-75

[4] М.В. Белубекян, Об условиях существования волн Лява с неоднородным слоем, Изв. АН Армении. Механика, Т. 44, №3, 1991 с. 7-10,

[5] М.В. Белубекян, А.Р. Мухсихачоян, Сдвиговая поверхностная волна в слабонеоднородных упругих телах, Акуст. Ж., Т.42, №2, 1996, с.179-182,

[6] З. Даноян, Л. Атоян, С. Саакян, Н. Даноян, Волны типа Лява в слоистой структуре, состоящей из двух однородных полупространств с неоднородными слоями, соединённых однородным слоем, Известия НАН Армении, Механика, Т.67,3, 2014, с. 49-55.

[7] K.B. Ghazaryan, D.G. Piliposyan, Love Waves in a Structure with an Inhomogeneous layer, NAS RA, v. 111, , 2011, pp. 138-147.

[8] G.A. Maugin, Elastic surface waves with transverse horizontal polarization, Adv. Appl. Mech., vol. 23, 1983, pp. 373–434. DOI:10.1016/S0065-2156(08)70246-1

[9] D. Hasanyan, G. Piliposian, A. Kamalyan, M.Karakhanyan, Some dynamic problems for elastic materials with functional inhomogeneities anti-plane deformations. 2003, Continuum Mech. Thermodynamics. 15, 519–527., https://dx.doi.org/10.1007/s00161-003-0130-8

[10] B.Collet, M.Destrade, G.Maugin, Bleustein–Gulyaev waves in some functionally graded materials, European Journal of Mechanics A/Solids, 25 (2006) p.695–706.

[11] Mei, C., Li, L., Tang, H., Han, X., Wang, X., & Hu, Y. Broadening band gaps of shear horizontal waves of metamaterials via graded hierarchical architectures. (2021) Composite Structures, 271, p.114118. https://doi.org/10.1016/j.compstruct.2021.114118

[12] Han, X., & Liu, G. R. (2003). Elastic waves in a functionally graded piezoelectric cylinder. Smart Materials and Structures, 12(6), 962, https://doi.org/10.1088/0964-1726/12/6/014

[13] А.С. Аветисян, Поверхностные электроупругие волны Лява в случае неоднород-ного пьезоэлектрического слоя, Изв. АН Арм.ССР, Механика, 1987, №1, с. 24-29

[14] Liu, J., X. S. Cao, and Z. K. Wang. "Propagation of Love waves in a smart functionally graded piezoelectric composite structure."Smart materials and structures 16, no. 1 (2006), https://doi.org/10.1088/0964-1726/16/1/002

[15] А.С. Аветисян, А.А. Камалян, О распространении электроупругого сигнала в неоднородном пьезодиэлектрическом слое класса 6мм, Докл.НАН Армении, 114, 2, 2014, с. 108- 115,

[16] А. Камалян, О распространении и поведении электроупругой волны в пьезоди-электрике в зависимости от неоднородности, Изв. НАН Армении, Механика, 66, 4, 2014, с. 38-48 .

[17] A.Avetisyan, A. Kamalyan, A. Hunanyan, Features of localization of wave energy at rough surfaces of piezodielectric waveguide, Mechanics. Proceedings of National Academy of Sciences of Armenia, 2017,70 (1), 40-63. DOI:10.33018/70.1.3

[18] Cao, X., Jin, F., Jeon, I., & Lu, T. J. (2009). Propagation of Love waves in a functionally graded piezoelectric material (FGPM) layered composite system. International Journal of Solids and Structures, 46(22-23), 4123-4132. http://dx.doi.org/10.1016/j.ijsolstr.2009.08.005

[19] Qian, Z. H., Jin, F., Lu, T., & Kishimoto, K. (2009). Transverse surface waves in a layered structure with a functionally graded piezoelectric substrate and a hard dielectric layer. Ultrasonics, 49(3), 293-297. https://doi.org/10.1016/j.ultras.2008.10.004

[20] A.Avetisyan, (2019). The Mixed Boundary Conditions Problem of Layered Composites with Meta-Surfaces in Electro Elasticity. In Wave Dynamics, Mechanics and Physics of Microstructured Metamaterials (pp. 73-96). Springer, Cham.

[21] A.Avetisyan, V. Khachatryan Propagation of hybrid electro elastic waves in transversally inhomogeneous periodic piezoelectric structure. Proceedings of NAS of Armenia. Mechanics 2020,73 (1), 1-17,http://doi.org/10.33018/73.1.1

[22] A. Srivastava, Metamaterial properties of periodic laminates, Journal of the mechanics and physics of solids, 96 (2016) 252–263. doi:https://doi.org/10.1016/j.jmps.2016.07.018.A.

[23] 23. A. Tovar, L. W. Casperson, Generalized Sylvester theorems for periodic applications in matrix optics, J. Opt. Soc. Am. A 12 (3) (1995) 578–590, https://doi.org/10.1364/JOSAA.12.000578

Сведения об авторах:

Казарян Карен Багратович - *главный научный сотрудник ин-та Механики НАН Армении. Тел.: +37499 227395, Email - ghkarren@gmail.com,*

Казарян Рафаэль Аракелович - *научный сотрудник ин-та Механики НАН Армении. Тел.: +37499 396344, Email- rafaelghazaryan52@gmail.com,*

Терзян Саркис Арутюнович - *научный сотрудник ин-та Механики НАН Армении. Тел.: +37499 340432, Email - sat_and_21@yahoo.com.*

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