ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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Forced electroacoustic oscillations along piezoelectric layer thickness: Applied opportunities

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Keywords: piezoelectric layer, electro-acoustic forced oscillations, surface impact, contactless impact, oscillations control, energy concentration.

Вынужденные электроакустические колебания по толщине пьезоэлектрического слоя. Прикладные возможности

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Ключевые слова: пьезоэлектрический слой, электроакустические вынужденные колебания, поверхностное воздействие, бесконтактное воздействие, управление колебаниями, концентрация энергии.

С целью формулирования постановки задач управления колебаниями пьезоэлемента при помощи поверхностного воздействия электрическим полем, а также для получения способа накопления электрической энергии, рассматривается простая модельная задача вынужденных колебании пьезоэлектрического слоя класса 6mm гексагональной симметрии.

Формулированы задача управления электроакустическими сдвиговыми колебаниями по толщине пьезоэлектрического слоя поверхностным механическим воздействием, а также задача бесконтактного управления этими колебаниями воздействием поверхностного потенциала электрического поля.

Решения задач колебаний представляются в виде разложения искомых функций по собственным модам однородной краевой задачи, а функции, характеризующие поверхностные воздействия, - соответствующими гармониками.

Решены прикладные задачи управления электроакустическими колебаниями в случае, как механического поверхностного воздействия, так и бесконтактного поверхностного воздействия потенциалом электрического поля.

Պիեզոէլեկտրական շերտի հաստությամբ էլեկտրաառաձգական հարկադրական փափանումները. Կիրառման հնարավորություններ

Ավետիսյան Արա Ս., Մկրտչյան Մ.Հ., Ավետիսյան Լ.Վ.

Բանալի բառեր` պիեզոէլեկտրական շերտ, փափանումներ ըստ հաստության, էլեկտրա-ակուստիկ հարկադրական փափանումներ, մակերևութային ազդեցություն, անհպում ազդեցություն, փափանումների ղեկավարում, էներգիայի կուտակում:

Նպատակ ունենալով, էլեկտրական դաշտի մակերևութային ազդեցության միջոցով, պիեզոէլեկտրական շերտի փափանումների անհպում ղեկավարման խնդիրների ձևակերպումը, ինչպես նաև էլեկտրական էներգիայի կուտակման մեթոդ սպանալու համար, դիփարկվում է վեցանկյուն սիմետրիայի 6mm դասի պիեզոէլեկտրական շերտի հարկադրական փափանումների պարզ մոդելային խնդիր:

Ձևակերպված են մակերևութային մեխանիկական ազդեցությամբ պիեզոէլեկտրական շերտում, ըստ հաստության սահքի էլեկտրաակուստիկ փափանումների ղեկավարման խնդիրը, ինչպես նաև էլեկտրական դաշտի մակերևութային անհպում ազդեցությամբ այդ փափանումների ղեկավարման խնդիրը:

Տափանման խնդիրների լուծումները ներկայացվում են համասեռ եզրային խնդրի սեփական ֆունկցիաների շարքերի տեսքով, իսկ մակերևութային ղեկավարող ազդեցությունները բնութագրող ֆունկցիաները ներկայացված են համապատասխան հարմոնիկաներով:

Լուծված են էլեկտրաակուստիկ փափանումների ղեկավարման կիրառական խնդիրներ, ինչպես մակերևութային մեխանիկական ազդեցության, այնպես էլ էլեկտրական դաշտի պոտենցիալով մակերևութային անհպում ազդեցության դեպքերում:

In a view of formulation of the controlling problems of a piezoelectric element oscillation by means of an electric field surface impact, as well as to obtain a method for electric energy harvesting, a simple model problem is considered of forced oscillations of a piezoelectric layer of a 6mm class of hexagonal symmetry.

The problem of controlling electroacoustic shear oscillations along the thickness of the piezoelectric layer by surface mechanical impact, as well as the problem of contactless control is formulated for oscillations caused by impact of the surface potential of an electric field.

The solutions of oscillation problems are represented in the form of an expansion of the sought functions in terms of eigenmodes of a homogeneous boundary value problem, the functions characterizing surface influences are represented by the corresponding harmonics.

The applied problem of control has been solved for electroacoustic oscillations in cases of both mechanical surface impact and non-contact surface impact caused by an electric field potential.

Introduction

In 1880 the Curie brothers discovered the unique property of piezoelectric matter (direct piezoelectric effect). Shortly thereafter in 1881, the inverse effect was also confirmed, specifically, that the substance located between the two electrodes, reacts to an electrical voltage applied to it by changing its shape. The direct piezoelectric effect is currently widely used in high-precision instrumentation, and the inverse piezoelectric effect, for exciting oscillations of mechanical pressures and deformations.

General principles and basic relations of the linear theory of piezoelectrics are well studied. These can be found in famous books [1–5] and etc.

With the development of modern technology, research on the control of related oscillatory and wave processes is encountered more and more [6–9] and etc. But before dealing with process control, it is necessary to explore the controllability of the original physical model [10, 11].

In order to formulate the formulation of control tasks for oscillations of the piezoelectric layer using surface mechanical impact or contactless action of an electric field, as well as to identify a method of accumulating electrical energy, a simple model problem is considered of forced oscillations of a piezoelectric layer made of a material of 6mm class of hexagonal symmetry

1 One-dimensional shear oscillations across the oscillations of the piezoelectric layer.

In a rectangular Cartesian coordinate system(x, y, z), an elastic layer of a piezoactive material of 6mm class of hexagonal symmetry occupying the region

$\{0 \leq x \leq a;\ -\infty < y < \infty;\ \infty < z < \infty\}$ (Fig. 1).

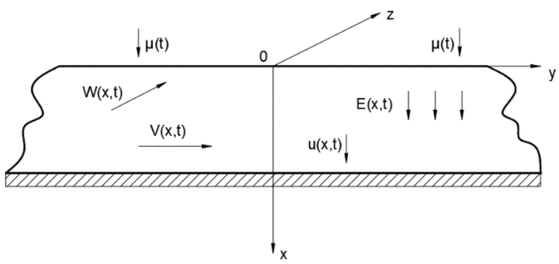


Fig. 1: Physical model of excitation of electroacoustic oscillations along the thickness of the piezoelectric layer and their control

Crystallographic axis of symmetry $\bar{P}_6$ is parallel to the coordinate axis $0z$, and the coordinate plane $x0y$ aligned with the isotropic plane of the material. The equations of propagation of one-dimensional shear waves or transverse oscillations of the layer, in the quasi-static approximation can be specified by the set of equations [1, 2, 4]

$$c_{44}w_{,xx}(x,t) + e_{15}\varphi_{,xx}(x,t) = \rho\ddot{w}(x,t), \quad \varepsilon_{11}\varphi_{,xx}(x,t) = e_{15}w_{,xx}(x,t). \tag{1.1}$$

Here, we used the following notations: function $w(x,t)$ is the elastic displacement along the coordinate z, $E(x,t) = -\varphi_{,x}$ is the component of the electric field along the coordinate x, c_{44} the shear modulus,ρ is the bulk density,ε_{11} dielectric permittivity and e_{15} piezo module of layer material. The expressions for the non-zero components of the mechanical stress and electrical displacement in one-dimensional setting have the form

$$\sigma_{zx}(x,t) = c_{44}w_{,x}(x,t) + e_{15}\varphi_{,x}(x,t), \quad D_x(x,t) = e_{15}w_{,x}(x,t) - \varepsilon_{11}\varphi_{,x}(x,t). \tag{1.2}$$

On planes $x = 0$ and $x = a$ of layer edges, the boundary conditions of surface mechanical impact and clamping are given by

$$w(0,t) = \mu(t), \tag{1.3}$$

$$w(a,t) = 0. \tag{1.4}$$

In the boundary condition (1.3), $\mu(t)$ is an arbitrary function of time corresponding to a nonstationary displacement of the surface. But in problems of electroelasticity, the surface impact can also be set by mechanical force $\sigma_{zx}(0,t)$, or by surface polarization (electrical displacement) $D_x(0,t)$, or by electrical potential $\varphi(0,t)$.

On the planes of layer edges, the boundary conditions for the electric field are also satisfied. Without loss of generality, as applied limiting variants of electrical boundary conditions, we consider the conditions of electrically open and electrically closed surfaces, respectively, on the planes $x = 0$ and $x = a$

$$\left[\varphi_{,x}(x,t) - (e_{15}/\varepsilon_{11}\ ) \cdot w_{,x}(x,t)\right]_{x=0} = 0, \quad \varphi(x,t)|_{x=a} = 0. \tag{1.5}$$

For a complete formulation of the boundary value problem of forced oscillations of elastic shear, it is also necessary to set the initial conditions also. In the case of a quasi-static formulation of the problem, it is natural to set the initial values for the mechanical components $w(x,t)$ and $\dot{w}(x,t)$

$$w(x,0) = f(x), \qquad \dot{w}(x,0) = g(x) \tag{1.6}$$

Initial conditions for accompanying electric field components $\varphi(x,t)$ and $\dot{\varphi}(x,t)$ are determined in accordance with the boundary value problem (1.1) and (1.3)-(1.5). Here, four more types of initial conditions are possible with initial values of the accompanying electric field components

$$w(x,0) = f(x), \qquad \varphi(x,0) = \psi(x) \tag{1.7}$$

$$\dot{w}(x,0) = g(x), \qquad \varphi(x,0) = \psi(x) \tag{1.8}$$

$$w(x,0) = f(x), \qquad \dot{\varphi}(x,0) = \phi(x) \tag{1.9}$$

$$\dot{w}(x,0) = g(x), \qquad \dot{\varphi}(x,0) = \phi(x) \tag{1.10}$$

Reduced boundary value problem (1.1), (1.3), (1.4) and (1.6) is similar to the problem of forced oscillations of a string [10]. When the one of the initial conditions (1.7)-(1.10) is valid instead of conditions (1.6), the solution of boundary value problems does not become more complicated, but the content and applications are expanded.

2 Solution of the boundary value problem of forced oscillations of elastic shear along the thickness of the piezoelectric layer.

System of equations (1.1) can easily brought to the form

$$\tilde{c}_t^2 \cdot w_{,xx}(x,t) = \ddot{w}(x,t), \quad \varphi_{,xx}(x,t) = (e_{15}/\varepsilon_{11}) \cdot w_{,xx}(x,t), \tag{2.1}$$

where $\tilde{c}_t^2 = c_{44}(1+\chi^2)/\rho$ is the reduced velocity of the electroactive shear wave, $\chi^2 = e_{15}^2/(c_{44}\varepsilon_{11})$ is the electromechanical coupling coefficient of the material. By introducing a transform to move shear

$$w(x,t) = u(x,t) + (1 - x/a\ ) \cdot \mu(t) \tag{2.2}$$

The boundary r conditions (1.3) and (1.4) with regard to function $u(x,t)$ are converted to homogeneous surface conditions

$$u(0,t) = 0, \qquad u(a,t) = 0 \tag{2.3}$$

and the first equation of the system (2.1) takes the form of an inhomogeneous wave equation, with a perturbation $(1 - x/a\ ) \cdot \ddot{\mu}(t)$, at a certain depth of the layer

$$\tilde{c}_t^2 \cdot u_{,xx}(x,t) = \ddot{u}(x,t) + (1 - x/a\ ) \cdot \ddot{\mu}(t), \quad \varphi_{,xx}(x,t) = (e_{15}/\varepsilon_{11}\ ) \cdot u_{,xx}(x,t). \tag{2.4}$$

Homogeneous boundary conditions (2.3) allow to represent the solution of the equation (2.4) in the form of an expansion in eigenfunctions of the homogeneous boundary value problem

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \cdot \sin(\lambda_n x), \tag{2.5}$$

with eigenvalues $\lambda_n = n\pi/a$ and the corresponding harmonics characterizing the dynamics of natural oscillations of the layer $u_n(t)$

$$u(t) = \sum_{n=1}^{\infty} u_n(t), \tag{2.6}$$

where $u_n(t) = \sum\limits_{n=1}^{\infty} [A_{un}\cos(\omega_{un}t) + B_{un}\sin(\omega_{un}t)]$ and $\omega_{un} = \tilde{c}_t\lambda_n$

Factor in inhomogeneous part of the equation (2.4) also is represented as a series of its eigen functions

$$(1 - x/a\ ) \cdot \ddot{\mu}(t) = \sum_{n=1}^{\infty} \ddot{\mu}_n(t) \cdot c_n \cdot \sin(\lambda_n x) \quad \text{where} \quad c_n = 2/n\pi \tag{2.7}$$

with the corresponding harmonics of its acceleration characterizing the surface effect

$$\mu(t) = \sum_{n=1}^{\infty} \mu_n(t) = \sum_{n=1}^{\infty} [A_{\mu n}\cos(\omega_{\mu n}t) + B_{\mu n}\sin(\omega_{\mu n}t)] \tag{2.8}$$

Here,$\omega_{\mu n}$ are the known frequencies of the harmonics of the surface effect, $A_{\mu n}$ and $B_{\mu n}$ are the known amplitudes of harmonics of surface impact. Substituting (2.6) and (2.8) into the first of the equations (2.4), we come to the infinite system of ordinary differential equations

$$\ddot{u}_n(t) + \omega_{un}^2 \cdot u_n(t) = -c_n\omega_{\mu n}^2 \cdot \mu_n(t) \tag{2.9}$$

Here, $\omega_{un} = \tilde{c}_t\lambda_n$ are the eigen frequencies of oscillations of the layer, and $\omega_{\mu n}$ are the surface impact frequency. General solutions to equations (2.9) we find by the method of variation of constants

$$\begin{aligned} u_n(t) &= A_{un}\cos(\omega_{un}t) + B_{un}\sin(\omega_{un}t) + \\ &+ c_n\omega_{\mu n}^2/(\omega_{\mu n}^2 - \omega_{un}^2)\ \cdot [A_{\mu n}\cos(\omega_{\mu n}t) + B_{\mu n}\sin(\omega_{\mu n}t)] \end{aligned} \tag{2.10}$$

where the non-resonant frequency of exposure is defined as

$$\omega_{\mu n} \neq \omega_{un}, \tag{2.11}$$

Unknown amplitudes A_{un} and B_{un}, in solutions (2.10) we find on the basis of the given initial conditions, expanding these conditions in a Fourier series in terms of their eigen forms. In the case when the initial conditions are specified by mechanical characteristics (1.6), these expansions will be written in the form

$$w(x,0) = \sum_{n=1}^{\infty} f_n \cdot \sin(\lambda_n x), \quad \dot{w}(x,0) = \sum_{n=1}^{\infty} g_n \cdot \sin(\lambda_n x) \tag{2.12}$$

Comparing the expressions for the true shear across the layer thickness (2.2) and the expansion of the initial conditions (2.12), to determine unknown amplitudes A_{un} and B_{un} we will have an infinite system of linear algebraic equations in the form

$$\begin{cases} u_n(0) + c_n \cdot \mu_n(0) = f_n \\ \dot{u}_n(0) + c_n \cdot \dot{\mu}_n(0) = g_n \end{cases} \tag{2.13}$$

Taking into account (2.5), (2.6), (2.8) and (2.10) we can easily obtain the unknown amplitudes

$$\begin{cases} A_{un} + c_n \left[1 + \omega_{\mu n}^2/(\omega_{\mu n}^2 - \omega_{un}^2)\ \right] \cdot A_{\mu n} = f_n \\ B_{un} + c_n \left[1 + \omega_{\mu n}^2/(\omega_{\mu n}^2 - \omega_{un}^2)\ \right] \cdot B_{\mu n} = g_n/\omega_{un} \end{cases} \tag{2.14}$$

Taking into account the transformation (2.2), true shear in layer thickness $w(x,t)$ is defined in the following form

$$w(x,t) = \sum_{n=1}^{\infty} \left[u_n(t) + c_n \cdot \mu_n(t)\right] \cdot \sin\left(\lambda_n x\right) \tag{2.15}$$

where $w_n(t) = u_n(t) + c_n \cdot \mu_n(t)$ are true harmonics of oscillations under surface impact. From the second equation of the system (2.4), taking into account the transformation (2.2), the surface conditions (1.5) and (2.3) on surfaces $x = a$ and $x = 0$, the potential of the accompanying electric field oscillations is determined in the form

$$\varphi(x,t) = (e_{15}/\varepsilon_{11}\ ) \cdot \left[(a-x) \cdot u'(0,t) + u(x,t)\right] \tag{2.16}$$

Using expansions (2.6) and (2.8) the expressions for the electric potential can be written as

$$\varphi(x,t) = (e_{15}/\varepsilon_{11}\ ) \cdot \sum_{n=1}^{\infty} \left[a \cdot \lambda_n c_n + 1\right] \cdot \left[A_{un} \cos(\omega_{un} t) + B_{un} \sin(\omega_{un} t)\right] \cdot \sin(\lambda_n x) \tag{2.17}$$

Taking into account the boundary conditions (2.3), on the clamped surface $x = a$, the electric potential disappears, and its derivative will not be zero

$$\varphi(a,t) = 0, \qquad \varphi'(a,t) = (e_{15}/\varepsilon_{11}\ ) \cdot \left[u'(a,t) - u'(0,t)\right]. \tag{2.18}$$

On the surface of mechanical impact $x = 0$, taking into account the boundary conditions (2.3), the electric potential and its derivative, respectively, take the values

$$\varphi(0,t) = (e_{15}/\varepsilon_{11}\ ) \cdot a \cdot u'(0,t) = (e_{15}/\varepsilon_{11}\ ) \cdot a \cdot \sum_{n=1}^{\infty} \lambda_n \cdot u_n(t), \quad \varphi'(0,t) = 0. \tag{2.19}$$

On the basis of the obtained solutions of the problem of forced oscillations along the thickness of the piezoelectric layer, it is possible to propose new problems: control of electroacoustic oscillations by surface mechanical impact , or contactless impact on electroacoustic oscillations by non-stationary potential of the electric field, accumulation tasks (or absorption) the energy of the electric field.

3 The problem of controlling electroacoustic oscillations of shear along the thickness of a piezoelectric layer

As in the problem of forced oscillations, when solving the control problem, we again use the Fourier series method in the mathematical boundary value problem. This approach reduces the problem of controlling oscillations along the layer thickness by surface impact into an infinite system of the oscillation controlling by means of eigenforms of the harmonics of the surface impact [10].

In the problem of oscillation control, as in the expansion of the impact function (2.8) and so in the solutions (2.10), the amplitudes of harmonics of surface impact $A_{\mu n}$ and $B_{\mu n}$, are unknown ones. These coefficients are determined together with the unknown amplitudes of the harmonics of the eigen vibration modes A_{un} and B_{un}.

3.1 Control of electroacoustic oscillations of shear along the thickness of the piezoelectric layer by surface mechanical impact -1.

Based on the obtained solution of the problem of forced oscillations of a piezo layer in the case of unsteady boundary mechanical loading, we can discuss the control problem requiring to determine mechanical impact function $\mu(t)$ allowing to reach electroelastic state with deflection values

$$w(x, T_0) = R(x), \qquad \dot{w}(x, T_0) = 0, \tag{3.1}$$

at the moment of time T_0. In the ratios (3.1) R and T_0 are given constants. Here, taking into account the expression for the deflection function (2.15), together with an infinite system of linear algebraic equations (2.13), from the conditions of the final state (3.1), one more infinite system of linear algebraic equations can be obtained

$$\begin{cases} u_n(T_0) + c_n \cdot \mu_n(T_0) = r_n \\ \dot{u}_n(T_0) + c_n \cdot \dot{\mu}_n(T_0) = 0 \end{cases} \tag{3.2}$$

In the system of equations (3.2) r_n are the expansion coefficients of the final version of deflection function

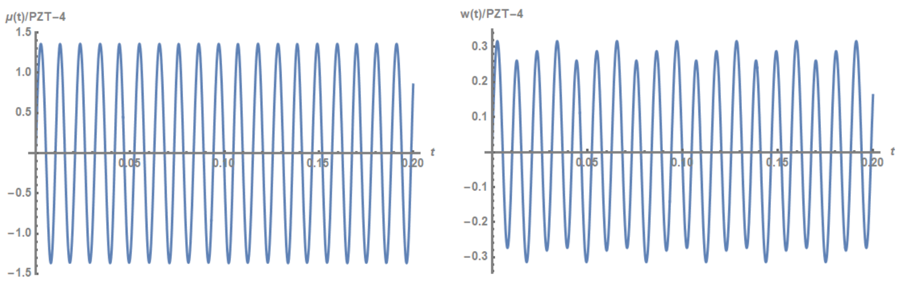
$$w(x, T_0) = \sum_{n=1}^{\infty} r_n \cdot \sin(\lambda_n x) \tag{3.3}$$

The values of the unknown amplitudes of the corresponding harmonics A_{un}, B_{un}, $A_{\mu n}]$ and $B_{\mu n}$, we can obtain from the system of equations (2.13) and (3.2)

$$\begin{cases} A_{un} + c_n(1+\gamma_{\omega n}) \cdot A_{\mu n} = f_n \\ B_{un} + c_n(1+\gamma_{\omega n}) \cdot B_{\mu n} = g_n/\omega_{un} \\ \cos(\omega_{un}T_0) \cdot A_{un} + \sin(\omega_{un}T_0) \cdot B_{un} + \\ + c_n(1+\gamma_{\omega n}) \cdot \cos(\omega_{\mu n}T_0) \cdot A_{\mu n} + c_n(1+\gamma_{\omega n}) \cdot \sin(\omega_{\mu n}T_0) \cdot B_{\mu n} = r_n \\ \sin(\omega_{un}T_0) \cdot A_{un} - \cos(\omega_{un}T_0) \cdot B_{un} + \\ + c_n(1+\gamma_{\omega n}) \cdot \sin(\omega_{\mu n}T_0) \cdot A_{\mu n} - c_n(1+\gamma_{\omega n}) \cdot \cos(\omega_{\mu n}T_0) \cdot B_{\mu n} = 0 \end{cases} \tag{3.4}$$

In the infinite system (3.4) the following notations are used for dimensionless frequency characteristics $\delta_{\omega n} = \omega_{\mu n}/\omega_{un}$ and $\gamma_{\omega n} = \delta_{\omega n}^2/(\delta_{\omega n}^2 - 1)$.

From the conditions of the existence of nontrivial solutions of a system of algebraic linear equations, we can determine the time interval $t \in [0; T_0]$ during which the surface impact $\mu(t) = \sum_{n=1}^{\infty} \mu_n(t)$ leads to the excited state of the piezoelectric layer from (1.6) state to state (3.1). Calculating the main determinant $Det\|d_{ij}(\omega_{un}; \omega_{\mu n}; T_0)\|_{4\times 4} = c_n^2 \neq 0$ of the system (3.4), we convince the possibility of controlling the shear oscillations along the thickness of the piezoelectric layer.

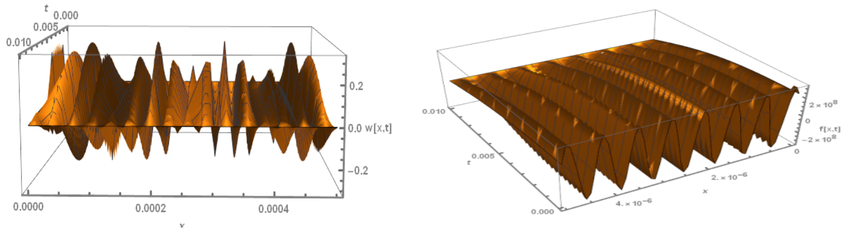


(a) Surface mechanical impact function $\mu(t)$ for controlling electro-acoustic oscillations over the thickness of the piezoelectric layer

(b) The behavior of time depending function of elastic shear of electroacoustic oscillations of a piezoelectric layer $w(t)$

Fig. 2

Finding the amplitudes of the corresponding harmonics A_{un}, B_{un}, $A_{\mu n}$ and $B_{\mu n}$ from the system of equations (3.4), we define the function of the surface effect according to (2.8) and the surface of the electroacoustic shear and the potential of the accompanying electric field according to (2.15) and (2.17) , accordingly.



(a) Elastic shear change surface $w(x,t)$ in the process of controlling electro-acoustic oscillations of the piezoelectric layer in the case of surface mechanical action $\mu(t)$

(b) Surface of change of electric potential $\varphi(x,t)$ in the process of controlling electro-acoustic oscillations of the piezoelectric layer in the case of surface mechanical action $\mu(t)$

Fig. 3

In a particular case, when the initial and final states of the piezoelectric layer descriptions are known

$$w(x,0) = F(x) = \sum_{n=1}^{\infty} f_n \cdot \sin(\lambda_n x), \quad \dot{w}(x,0) = G(x) = \sum_{n=1}^{\infty} g_n \cdot \sin(\lambda_n x) \tag{3.5}$$

$$w(x,T_0) = R(x) = \sum_{n=1}^{\infty} r_n \cdot \sin(\lambda_n x), \quad \dot{w}(x,T_0) = 0, \tag{3.6}$$

for a period of time $t \in [0;\ 1.0sec]$ the control function $\mu(t)$ is constructed, as a mechanical surface effect (Fig.2.a). A function $w(t)$ of the temporal behavior of the elastic shear of electroacoustic oscillations of the piezoelectric layer has been constructed also (Fig.2.b). Wave surfaces of elastic shear and electric potential along the thickness of a piezolayer with a thickness $a = 10^{-3}m$ are shown in the figures 3a and 3b ,respectively.

3.2 Control of electroacoustic oscillations of shear across the thickness of the piezoelectric layer by surface mechanical impact - 2.

The most interesting applied control problem will be obtained if there is need to determine the function of the mechanical surface impact $\mu(t)$ so, that over time $t \in [0; T_0]$ the electric field potential accompanying mechanical oscillations will be equal to a given value $\Phi(x, T_0)$. In this case, instead of the final conditions (2.1) we need to consider the state

$$\varphi(x,T_0) = \Phi_0(x) = \sum_{n=1}^{\infty} \phi_n \cdot \sin(\lambda_n x), \quad \dot{w}(x,T_0) = 0, \tag{3.7}$$

Taking into account that the electric field potential accompanying mechanical oscillations have the form (1.17), the determined infinite system of equations can be written as we obtain in the form

$$
\begin{cases}
A_{un} + c_n(1+\gamma_{\omega n}) \cdot A_{\mu n} = f_n \\
B_{un} + c_n(1+\gamma_{\omega n}) \cdot B_{\mu n} = g_n/\omega_{un} \\
\cos(\omega_{un}T_0) \cdot A_{un} + \sin(\omega_{un}T_0) \cdot B_{un} + c_n\gamma_{\omega n} \cdot \cos(\omega_{\mu n}T_0) \cdot A_{\mu n} + \\
+ c_n\gamma_{\omega n} \cdot \sin(\omega_{\mu n}T_0) \cdot B_{\mu n} = (\varepsilon_{11}/e_{15}\ )\,(\phi_n/(a \cdot \lambda_n c_n + 1)\ ) \\
\sin(\omega_{un}T_0) \cdot A_{un} - \cos(\omega_{un}T_0) \cdot B_{un} + \\
+ c_n(1+\gamma_{\omega n}) \cdot \sin(\omega_{\mu n}T_0) \cdot A_{\mu n} - c_n(1+\gamma_{\omega n}) \cdot \cos(\omega_{\mu n}T_0) \cdot B_{\mu n} = 0
\end{cases}
\tag{3.8}
$$

Naturally, as in the system (3.4), in an infinite system (3.8 the same notations for dimensionless frequency characteristics are used $\delta_{\omega n} = \omega_{\mu n}/\omega_{un}$ and $\gamma_{\omega n} = \delta_{\omega n}^2/(\delta_{\omega n}^2 - 1)$.

Finding the amplitudes of the corresponding harmonics A_{un}, B_{un}, $A_{\mu n}$ and $B_{\mu n}$ from the system of equations (2.8), we define the function of the surface effect according to (2.8) and the surface of the electroacoustic shear and the potential of the accompanying electric field according to (1.15) and (1.17,) respectively.

In the particular case when the numerical parameters of the initial and final states of the piezoelectric layer are known (2.5) and (2.7), for a period of time $t \in [0;\ 1.0sec]$ the control function is built $\mu(t)$, as a mechanical surface effect (Fig.4.a). The behavior of time depending function of the elastic shear of electroacoustic oscillations of the piezoelectric layer has also been constructed $w(t)$ (Fig.4.b) changing from initial condition (2.5) to final condition (2.7).

4 Contactless control of electroacoustic oscillations of the piezoelectric layer shear by surface action of an electric field.

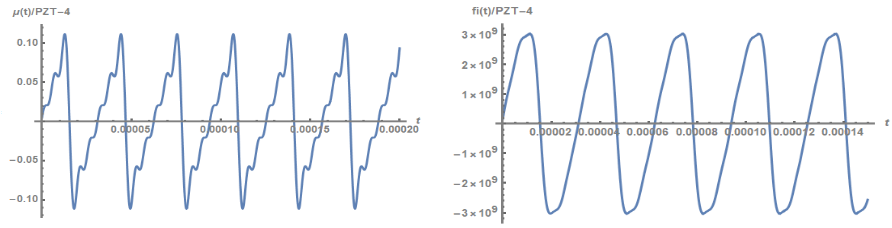
The more interesting case will be the option of contactless surface action, when on the traction free surface of the piezoelectric layer $x = 0$ a non-stationary electric field acts

$$\varphi(0,t) = \varphi_0(t), \tag{4.1}$$

and the second surface $x = a$, as before is clamped

$$w(a,t) = 0. \tag{4.2}$$

In condition (4.1), $\varphi_0(t) \in \mathbb{C}_2$ the times function corresponding to the unsteady potential of the electric field.



(a) Surface electric impact function $\mu(t)$ for controlling electro-acoustic oscillations of shear along the thickness of the piezoelectric layer

(b) Function of the behavior of the electric potential of electroacoustic oscillations of the piezoelectric layer as a function of time $\varphi(t)$

Fig. 4

On the layer edge planes, it is necessary to satisfy the two more boundary conditions for the electromechanical values of the field. Without loss of generality, as applied versions of electromechanical boundary conditions, we consider the conditions of a traction free surface on $x = 0$ and an electrically closed surface on $x = a$, respectively

$$[w_{,x}(x,t) + (e_{15}/c_{44}) \cdot \varphi_{,x}(x,t)]_{x=0} = 0, \quad \varphi(a,t) = 0 \tag{4.3}$$

For a complete statement of the control problem, it is also necessary to set the initial and final conditions. Initial and final conditions for the components of the accompanying electric field $\varphi(x,t)$ and $\dot{\varphi}(x,t)$ are determined in accordance with the boundary value problem (1.1) and (4.1) - (4.3). Here, one more types of initial conditions is possible with the initial values of the components of the electroactive vibration

$$w(x,0) = f(x), \quad \dot{\varphi}(x,0) = \phi(x) \tag{4.4}$$

In this case, we will consider the conditions of such a final state when the potential of the electric field accompanying mechanical oscillations is equal to a given value $\Phi(x,T_0)$

$$\varphi(x,T_0) = \tilde{\psi}(x), \quad \dot{w}(x,T_0) = 0. \tag{4.5}$$

To solve the control problem, we introduce the transformation

$$\varphi(x,t) = \Phi(x,t) + (1 - x/a\ ) \cdot \varphi_0(t) \tag{4.6}$$

For the introduced function $\Phi(x,t)$, condition (4.1) and the second condition from (4.3) are converted to homogeneous surface conditions

$$\Phi(0,t) = 0, \qquad \Phi(a,t) = 0 \tag{4.7}$$

The first equation of the system (1.1) takes the form of an inhomogeneous wave equation, with a perturbation $(1 - x/a\ ) \cdot \ddot{\mu}(t)$ at a certain depth of the layer

$$\tilde{C}_t^2 \cdot \Phi_{,xx}(x,t) = \ddot{\Phi}(x,t) + (1 - x/a\ ) \cdot \ddot{\varphi}_0(t), \tag{4.8}$$

The second equation confirms the synchronism of the shift function $w(x,t)$ and the new introduced function $\Phi(x,t)$

$$w_{,xx}(x,t) = (\varepsilon_{11}/e_{15}) \cdot \Phi_{,xx}(x,t) \tag{4.9}$$

The first surface condition from (4.3) converted to form

$$[w_{,x}(x,t) + (e_{15}/c_{44}) \ \cdot \Phi_{,x}(x,t)]_{x=0} = (e_{15}/ac_{44}) \ \cdot \varphi_0(t) \tag{4.10}$$

Homogeneous boundary conditions (4.7) allow to represent the equation (4.8) solution in the form of an expansion in eigenfunctions of the homogeneous boundary value problem

$$\Phi(x,t) = \sum_{n=1}^{\infty} \Phi_n(t) \cdot \sin(\lambda_n x) \tag{4.11}$$

with eigenvalues $\lambda_n = n\pi/a$ and with the corresponding harmonics characterizing the dynamics of the process $\phi_n(t)$

$$\Phi(t) = \sum_{n=1}^{\infty} \Phi_n(t). \tag{4.12}$$

Factor in the inhomogeneous part of the equation (4.8) also is represented as a series of its eigenfunctions

$$(1 - x/a\ ) \cdot \ddot{\varphi}_0(t) = \sum_{n=1}^{\infty} \ddot{\varphi}_{0n}(t) \cdot c_n \cdot \sin(\lambda_n x) \quad \text{where} \quad c_n = 2/n\pi \tag{4.13}$$

and with the corresponding harmonics of its acceleration characterizing the surface effect

$$\varphi_0(t) = \sum_{n=1}^{\infty} \varphi_{0n}(t) = \sum_{n=1}^{\infty} [A_{\varphi n} \cos(\omega_{\varphi n} t) + B_{\varphi n} \sin(\omega_{\varphi n} t)] \tag{4.14}$$

Here $\omega_{\varphi n}$ are unknown frequencies, $A_{\varphi n}$ and $B_{\varphi n}$ unknown amplitudes of harmonics of surface action. Substituting decompositions (4.12) and (4.14) into the equation (4.8), we come to the solution of the infinite system of ordinary differential equations

$$\ddot{\Phi}_n(t) + \omega_{wn}^2 \cdot \Phi_n(t) = -c_n \omega_{\varphi n}^2 \cdot \varphi_{0n}(t) \tag{4.15}$$

Here, $\omega_{wn} = \tilde{C}_t \lambda_n$ natural frequency of oscillations of the layer, and $\omega_{\varphi n}$ surface action frequency.

General solutions of equations (4.15) we find by the method of variation of constants

$$\begin{aligned} \Phi_n(t) = A_{wn} \cos(\omega_{wn} t) + B_{wn} \sin(\omega_{wn} t) - \\ - c_n \cdot \omega_{\varphi n}^2 / (\omega_{wn}^2 - \omega_{\varphi n}^2\ ) \cdot [A_{\varphi n} \cos(\omega_{\varphi n} t) + B_{\varphi n} \sin(\omega_{\varphi n} t)] \end{aligned} \tag{4.16}$$

where the non-resonant frequency of exposure is defined as

$$\omega_{\varphi n} \neq \omega_{wn} \tag{4.17}$$

Taking into account the transformation (4.6), electric field potential across the layer thickness $\varphi(x,t)$ is defined as follows

$$\varphi(x,t) = \sum_{n=1}^{\infty} [\Phi_n(t) + c_n \cdot \varphi_{0n}(t)] \cdot \sin(\lambda_n x) \tag{4.18}$$

where $\varphi_n(t) = \Phi_n(t) + c_n \cdot \varphi_{0n}(t)$ true harmonic of oscillations under surface action. From the equation (4.9), taking into account the transformation (4.6) and surface conditions (4.7) on surfaces $x = a$ and $x = 0$, electroactive elastic shear is defined as

$$w(x,t) = (e_{15}/c_{44}) \cdot [(a - x) \cdot \Phi'(0,t) + \Phi(x,t)] \tag{4.19}$$

Using expansions (4.6), (4.18) and (4.19) the expression for the electroactive elastic shear can be written in the form of the expansion

$$w(x,t) = (e_{15}/c_{44}) \cdot \sum_{n=1}^{\infty} [a\lambda_n c_n + 1] \cdot \Phi_n(t) \cdot \sin(\lambda_n x) \tag{4.20}$$

Taking into account the boundary conditions (4.4), on the clamped surface $x = a$, the electric potential disappears, and its derivative is not zero

$$w(a,t) = 0, \qquad w'(a,t) = (e_{15}/c_{44}) \cdot [\Phi'(a,t) - \Phi'(0,t)]\,. \tag{4.21}$$

On the surface of mechanical action $x = 0$, taking into account the boundary conditions (3.3), the electric potential and its derivative, respectively, take the values

$$w(0,t) = (e_{15}/c_{44}) \cdot a \cdot \Phi'(0,t) = (e_{15}/c_{44}) \cdot a \cdot \sum_{n=1}^{\infty} \lambda_n \cdot \Phi_n(t), \quad w'(0,t) = 0. \tag{4.22}$$

Presenting the initial and final conditions (4.5) in the form of Fourier series in eigenforms $\sin(\lambda_n x)$

$$w(x,0) = \sum_{n=1}^{\infty} f_n \cdot \sin(\lambda_n x), \quad \dot{\varphi}(x,0) = \sum_{n=1}^{\infty} \phi_n \cdot \sin(\lambda_n x) \tag{4.23}$$

$$\varphi(x,T_0) = \sum_{n=1}^{\infty} \tilde{\psi}_n \cdot \sin(\lambda_n x). \quad \dot{w}(x,T_0) = 0 \tag{4.24}$$

The values of the unknown amplitudes of the corresponding harmonics A_{wn}, B_{wn}, $A_{\varphi n}$ and $B_{\varphi n}$,

$$\begin{cases} A_{wn} - c_n\gamma_{\omega n} \cdot A_{\varphi n} = (c_{44}/e_{15}) \cdot f_n/(a\lambda_n c_n + 1) \\ B_{wn} + c_n \cdot \delta_\omega (1 - \gamma_{\omega n}) \cdot B_{\varphi n} = \phi_n/\omega_{wn} \\ \cos(\omega_{wn}t) \cdot A_{wn} + \sin(\omega_{wn}t) \cdot B_{wn} + \\ + c_n (1 - \gamma_{\omega n}) \cdot \cos(\omega_{\varphi n}t) \cdot A_{\varphi n} + c_n (1 - \gamma_{\omega n}) \cdot \sin(\omega_{\varphi n}t) \cdot B_{\varphi n} = \tilde{\psi}_n \\ \sin(\omega_{wn}T_0) \cdot A_{wn} - \cos(\omega_{wn}T_0) \cdot B_{wn} - \\ - c_n\delta_{\omega n}\gamma_{\omega n} \cdot \sin(\omega_{\varphi n}T_0) \cdot A_{\varphi n} + c_n\delta_{\omega n}\gamma_{\omega n} \cdot \cos(\omega_{\varphi n}T_0) \cdot B_{\varphi n} = 0 \end{cases} \tag{4.25}$$

we find from the system of linear algebraic equations, which obtained from (4.23) and (4.24).

Into an endless system (4.25) the notations are used for dimensionless frequency characteristics $\delta_{\omega n} = \omega_{\varphi n}/\omega_{wn}$ and $\gamma_{\omega n} = \delta_{\omega n}^2/(1-\delta_{\omega n}^2)$

Finding the amplitudes of the corresponding harmonics A_{un}, B_{un}, $A_{\mu n}$ and $B_{\mu n}$ from the system of equations (4.25), we define the function of the surface effect according to (4.14) and electroacoustic shear surfaces and potential of the accompanying electric field according to (4.20) and (4.18) accordingly.

Conclusion

The problem is established of controlling shear oscillations along the thickness of the piezoelectric layer by surface mechanical impact, as well as the problem of contactless control of electroacoustic oscillations of shear along the thickness of the piezoelectric layer by the potential of the electric field.

The solution of the oscillation equation is represented in the form of an expansion in terms of eigenfunctions of a homogeneous boundary value problem, with the corresponding harmonics and their accelerations characterizing the surface impact.

An interesting applied problem of controlling electroacoustic oscillations has been solved, when it is required to determine the function of mechanical surface impact so that throughout the finite period of time the potential of the electric field accompanying mechanical oscillations will be equal to a given value.

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Dedicated to the memory of our Teacher: To Professor Mels Vagharshak Belubekyan

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