ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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OPTIMAL CHOICE OF THE TYPE OF THE FINAL CONFIGURATION AT THE LIMITED CONTROL OF GRIPPER MOTION OF THE TWO LINK MANIPULATOR

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Keywords: two link manipulator, limited control, the type of the final configuration

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Оптимальный выбор типа конечной конфигурации при ограниченном управлении перемещением схвата двузвенного манипулятора

Ключевые слова: : двузвенный манипулятор, ограниченное управление, конечный тип конфигурации

Рассматривается задача ограниченного управления перемещением точечного схвата плоского двузвенного манипулятора с прямолинейными звеньями равной длины и со вторым статически уравновешенным звеном. На плоскости обобщенных координат манипулятора построены области, позволяющие по заданным координатам терминального положения манипулятора определить управления, обеспечивающие перемещение манипулятора из начального положения покоя в заданное терминальное положение покоя за конечное время без нарушения ограничений на управления, а также выбрать тип конечной конфигурации, при котором время перемещения схвата минимально. Численными расчетами установлено, что оптимальный выбор типа конечной конфигурации может приводить к значительному уменьшению времени перемещения схвата манипулятора. Дана оценка близости результатов, получаемых с помощью рассматриваемом способе управления и оптимальном по быстродействию управлении.

Ավետիսյան Վ.Վ.

Երկօղակ մանիպուլյատորի վերջնական կոնֆիգուրացիայի տեսակի օպտիմալ ընտրությունը բռնիչի տեղափոխման սահմանափակ ղեկավարման դեպքում

Հիմնաբառեր`երկօղակ մանիպուլյատոր, սահմանափակ ղեկավարում, վերջնական կոնֆիգուրացիայի տեսակ:

Դիտարկվում է ուղղագիծ և հավասար երկարությամբ օղակներով հարթ երկօղակ մանիպուլյատորի կետային բռնիչի տեղաշարժման սահմանափակ ղեկավարման խնդիրը, երբ մանիպուլյատորի երկրորդ օղակը ստատիկորեն հավասարակշռված է: Մանիպուլյատորի ընդհանրացված կոորդինատների հարթության մեջ կառուցված են հասանելի կոնֆիգուրացիաների տիրույթներ , որոնք հնարավորություն են տալիս ըստ մանիպուլյատորի տերմինալ դիրքի կոորդինատների որոշել այն ղեկավարումները, որոնք ապահովում են մանիպուլյատորի տեղափոխումը սկզբնական հանգստի դիրքից տրված տերմինալ հանգստի դիրք վերջավոր ժամանակում՝ առանց ղեկավարումների վրա դրված սահմանափակումների խախտման, ինչպես նաև ընտրել վերջնական կոնֆիգուրացիայի տեսակը, որի դեպքում բռնիչի տեղափոխման ժամանակը նվազագույնն է: Թվային հաշվարկներով հաստատվել է, որ վերջնական կոնֆիգուրացիայի տեսակի օպտիմալ ընտրությունը կարող է բերել մանիպուլյատորի բռնիչի տեղափոխման ժամանակի զգալի նվազեցմանը: Տրված է արդյունքների մոտիկության գնահատականը, որոնք ստացվում են դիտարկվող ղեկավարման և օպտիմալ ըստ արագագործության ղեկավարման դեպքերում:

We consider the limited control problem for the motion of a point gripper of a plane two-link manipulator with linear links of equal length, the second link being statically balanced. Regions are constructed on the plane of the generalized coordinates of the manipulator that allow, based on the specified coordinates of the manipulator's terminal position, to determine the controls that ensures the manipulator moves from the initial rest position to the specified terminal resting position in a finite time without violating the controls constraints, and also to select the type of final configuration in which the movement time is gripper is minimal. It is established by numerical calculations that the optimal choice of the final configuration can significantly reduce the motion time of the manipulqtor gripper. An estimate is given for the proximity of the results obtained using the considered control method and time-optimal control.

Introduction

Two-link robotic manipulators are widely used in various branches of modern industry. They are used both independently and as part of the structures of multi-link manipulation robots for which it is these two links that perform the bulk of the robot's motions when it performs various technological operations. One possible approach to the rational calculation of control modes is their optimization according to some manipulator performance criterion (time of transport operations, energy consumption, etc.). An essential component in the formation of algorithms for controlling the motion of a two-link manipulator is taking into account its design and geometric features. For a plane two-link manipulator, each position of the gripper corresponds to two feasible configurations differing by the sign of the angle between the links. Consequently, the performance of the gripper motion to the terminal position depends both on the type of the final configuration and on the control method that brings the manipulator into this configuration. In [1, 2], optimal and suboptimal control laws were constructed for a two-link manipulator with zero-lag links in the two-point problem of moving a gripper with a load. A significant dependence of the time it takes to bring the gripper to the terminal state on the manipulator configuration type was revealed, and the problem of choosing the optimal configuration type was solved. In [3], a graphic-analytical approach was developed to constructing time-suboptimal open-loop controls that bring a two-link manipulator with arbitrary geometric and lag characteristics from the initial rest configuration to an arbitrary final rest configuration. The publications [4–9] deal with optimization methods for solving the problem of controlling robots, including two-link manipulators, and calculating their design parameters. Models of mechanical and electromechanical plane two-link manipulators with statically balanced second link and with arbitrary lag characteristics are considered in [10–13]. Assuming that the manipulator design allows full clockwise and counterclockwise rotation of the links, it was established that the manipulator can be brought to the same final configuration by various combinations of rotations of the links. For each of the two types of final configurations, the graphic-analytical procedure solved the problem of choosing the directions of rotations of the manipulator links and determining the control method for which a given control criterion (the response speed [10, 11], the energy consumption [13], and a combined functional [12]) attains its minimum value. The optimal type of the final configuration was found by a straightforward calculation. In [14, 15], a parametric optimization method was used to construct a quadratic-functional-suboptimal control of the motion of a plane two-link manipulator taking into account feasible manipulator configurations corresponding to given gripper positions at the beginning and end of the motion. In [16], a mechanical model of a two-link manipulator [3] is considered, the design of which allows only half a revolution of the links in the positive and negative directions. For a given terminal position of the manipulator gripper, the type of final configuration and a control method have been determined, which ensure the movement of the gripper to a given final resting position in a minimum time. In [17, 18], using the generalized method of constructing a limited control [19], explicitly found the controls and the corresponding finite time, at which the two-link manipulator of the initial state of rest is brought to any final state of rest in the working zone without violating the restrictions on the speed.

This article discusses a mechanical model of a two-link manipulator [16]. On the plane of the generalized coordinates of the manipulator, regions are constructed that allow, based on the given coordinates of the terminal position of the manipulator, to determine the limited controls that ensures the movement of the manipulator from the initial resting position to a given terminal rest position in a finite time, as well as to determine the type of final configuration at which the time of movement of the gripper is minimal.

1 Design of the manipulator model

Consider a mechanical two-link system consisting of two absolutely rigid links G_1 and G_2 of the same length joined with a hinge O_2. The link G_1 is attached to a stationary base using the hinge O_1. The hinges are perfect and cylindrical, and their axes are parallel to each other. A gripper is mounted on the end of the second link at point O_3. We will assume that the linear sizes of the gripper are much smaller than the lengths of the links and consider the gripper to be a material point when studying transport motions. The manipulator control under study is accomplished with two independent drives D_1 and D_2. The first link and the base interact via the drive D_1 and D_2 is responsible for the interaction between the links G_1 and G_2 of the manipulator. The control functions in the manipulator model under study are the torques M_1 and M_2 about the axes O_1 and O_2 generated by the drives D_1 and D_2, respectively. The system performs a plane-parallel motion in a horizontal plane perpendicular to the axes of the hinges O_1 and O_2.

The Lagrange equations describing the motion of the system under consideration in the case when the link of the manipulator is statically balanced have the form [3]:

$$(I_1 + m_2 L^2)\ddot{\varphi}_1 = M_1 - M_2, \qquad I_2\ddot{\varphi}_2 = M_2, \tag{1.1}$$

Here we have introduced the following notation: φ_1 is the angle between the axis O_1x and the straight line O_1O_2; φ_2 is the angle between the axis O_1x and the straight line O_2O_3; $L = |O_1O_2| = |O_2O_3|$ is the length of the first and second links; I_1 and I_2 are the moments of inertia of the links G_1 and G_2 about the axes of the hinges O_1 and O_2, respectively; and m_2 is the mass of the link G_2. We assume that the positive sense of the angles φ_1 and φ_2 is counterclockwise from the line O_1x.

The control torques M_1 and M_2 are subject to the constraints

$$|M_1| \le M_1^0, \quad |M_2| \le M_2^0, \tag{1.2}$$

where M_1^0 and M_2^0 are given constants.

2 Statement of the problem

The manipulator control objective is to bring the gripper into a given spatial position by rotating the manipulator links in the positive and negative directions within a half turn. It follows from the geometry of the two-link manipulator that there exists a one-to-one correspondence

$$x = L(\cos\varphi_1 + \cos\varphi_2), \quad y = L(\sin\varphi_1 + \sin\varphi_2). \tag{2.1}$$

between the Cartesian coordinates x, y of the projection of the point O_3 and the generalized coordinates φ_1, φ_2.

However, the angles φ_1, φ_2 are not uniquely determined by the Cartesian coordinates x, y. Let the working area of the manipulator be a semicircle $R = \{(x,y):\ x^2 + y^2 \le 4L^2,\ x > 0\}$. We solve system (2.1) with respect to φ_1, φ_2

$$\varphi_i(x,y) = \operatorname{arctg}\frac{y}{x} + (-1)^i \frac{1}{2}K\delta, \quad \delta = \arccos\left[\frac{x^2+y^2-2L^2}{2L^2}\right], \quad K = \pm 1, \quad i = 1, 2. \tag{2.2}$$

It follows from (2.2) that each gripper position (x, y) inside the manipulator working area R is associated with two configurations of the two-link manipulator that differ in the sign of the angle $\theta = \varphi_2 - \varphi_1$ between the links. The quantity δ in (2.2) is the angle at the vertices O_1 and O_3 of the triangle $O_1O_2O_3$. As follows from (2.2), the values $K = 1$ and $K = -1$ are associated with the configurations for which $\theta > 0$ and $\theta < 0$, respectively; i.e. $K = \mathrm{sign}\theta$. Let us denote them by $\{\varphi_1(x,y),\ \varphi_2(x,y)\}_K$, $K = \pm 1$. In what follows, the arguments (x, y) of the functions $\varphi_i(x, y)$ will be dropped.

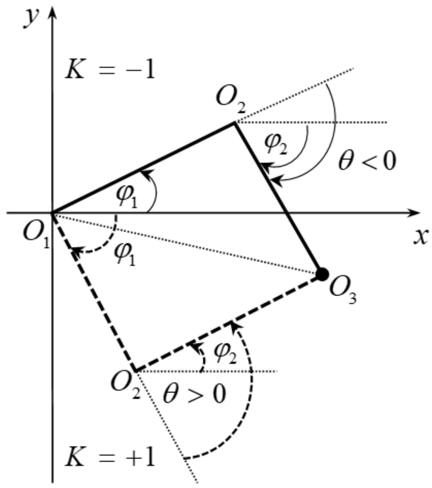

Fig. 1

Thus, on the plane $\Phi = \{\varphi_1,\, \varphi_2 : \ -\pi \le \varphi_1, \varphi_2 \le \pi\}$ of manipulator's generalized coordinates, the points $(\varphi_1,\, \varphi_2)$ and $(\varphi_2,\, \varphi_1)$, corresponding to the configurations $\{\varphi_1,\, \varphi_2\}_{K=1}$ and $\{\varphi_1, \varphi_2\}_{K=-1}$, respectively, are symmetric about the bisector of quadrants I and III, and one has

$$\begin{aligned} (\varphi_1,\, \varphi_2) &\in \{\varphi_1,\, \varphi_2 \in \Phi : \ \ \varphi_2 \ge \varphi_1\} = \Phi(+1), \\ (\varphi_2,\, \varphi_1) &\in \{\varphi_1,\, \varphi_2 \in \Phi : \ \ \varphi_2 \le \varphi_1\} = \Phi(-1). \end{aligned} \tag{2.3}$$

We will consider system (1.2) under the initial conditions

$$\varphi_i(0) = \varphi_i^0, \quad \dot{\varphi}_i(0) = 0, \quad i = 1, 2, \tag{2.4}$$

which are associated, according to (2.1), with the initial gripper rest position

$$\begin{aligned} x(0) &= L(\cos\varphi_1^0 + \cos\varphi_2^0) = x^0, \quad \dot{x}(0) = 0, \\ y(0) &= L(\sin\varphi_1^0 + \sin\varphi_2^0) = y^0, \quad \dot{y}(0) = 0. \end{aligned} \tag{2.5}$$

Assume that we are given distinct initial gripper position (x^0, y^0) (2.5) and final gripper position (x^T, y^T) in the manipulator working area R. Since the position (x^T, y^T) is associated with the two configurations $\{\varphi_1^T, \varphi_2^T\}_K$, $K = \pm 1$, for the terminal conditions for system (1.2) we take the conditions

$$\varphi_i|_K\,(T) = \varphi_i^T|_K\,, \quad \dot{\varphi}_i|_K\,(T) = 0, \quad i = 1, 2, \quad K = \pm 1, \tag{2.6}$$

which are uniquely associated with one and the same gripper rest position

$$\left.\begin{aligned} x(T) &= L(\cos\varphi_1^T|_K + \cos\varphi_2^T|_K) = x^T, \quad \dot{x}(T) = 0, \\ y(T) &= L(\sin\varphi_1^T|_K + \sin\varphi_2^T|_K) = y^T, \quad \dot{y}(T) = 0, \end{aligned}\right. \quad K = \pm 1. \tag{2.7}$$

System (1.2), (1.3) is completely controllable in the class of piecewise continuous functions $M_1(t)$ and $M_2(t)$ [3], therefore, for given edge gripper states (2.5), (2.7), each combination of conditions (2.6) $(K = +1, -1)$ and each feasible controls M_1 and M_2 are associated with some transfer time $T^{(K)}$. From the above it follows that there is a dependency for the travel time

$$T^{(K)} = T\left[\{\varphi_1^T, \varphi_2^T\}|_K, M_1, M_2\right], \quad K = \pm 1. \tag{2.8}$$

Consider the following problem of control of the manipulator gripper motion with allowance for the final configuration type.

Determine the type $K = \pm 1$ of the final configuration (2.6) and the law of change of the controls $M = M_1(t)$ and $M = M_2(t)$, that ensure the minimum of functionality (2.8)

$$T^* = \min_{K=\pm 1} T\left(\{\varphi_1^T, \varphi_2^T\}|_K, M_1, M_2\right) \tag{2.9}$$

when bringing the manipulator gripper from the initial rest state (2.5) into the given rest state (2.8) without violation of restrictions (1.2).

3 Construction of bounded controls

In (1.1), (1.2), (2.4) - (2.7) we pass to the dimensionless variables

$$\begin{aligned} &t' = \left(M_2^0/(m_2L^2)\right)^{1/2} t, \quad I'_i = I_i/(m_2L^2), \quad M'_i = M_i/M_2^0, \\ &x' = x/L, \quad y' = y/L, \quad \varphi'_i = \varphi_i - \varphi_i^0, \quad \varphi'^{0,T}_i = \varphi_i^{0,T} - \varphi_i^0, \quad i = 1, 2. \end{aligned} \tag{3.1}$$

If now we omit the primes, then relations (1.1), (1.2), (2.4) are simplified $\varphi_{1,2}^0 = 0$, $m_2 = 1$, $L = 1$, $M_2^0 = 1$, and system (1.1), the constraints (1.2), and the boundary conditions (2.4), (2.6) acquire the form

$$\begin{aligned} (I_1 + 1)\ddot{\varphi}_1 &= M_1 - M_2, \\ I_2\ddot{\varphi}_2 &= M_2, \end{aligned} \tag{3.2}$$

$$|M_1| \le M_1^0, \quad |M_2| \le 1, \tag{3.3}$$

$$\varphi_i(0) = 0, \quad \dot{\varphi}_i(0) = 0, \quad i = 1, 2, \tag{3.4}$$

$$\varphi_i|_K(T) = \varphi_i^T|_K, \quad \dot{\varphi}_i|_K(T) = 0, \quad i = 1, 2; \quad K = \pm 1. \tag{3.5}$$

First, consider the problem of constructing a limited control $M = (M_1, M_2)$ of the system (3.2) - (3.5) without taking into account the type of the final configuration. When solving this problem under the boundary conditions (3.5), we omit the parameter K.

The change of variables

$$q_1 = (I_1 + 1)\varphi_1, \qquad q_2 = I_2\varphi_2 \tag{3.6}$$

reduces system (3.2)-(3.5) to the form

$$\ddot{q}_1 = M_1 - M_2, \qquad \ddot{q}_2 = M_2, \tag{3.7}$$

$$|M_1| \le M_1^0, \qquad |M_2| \le 1, \tag{3.8}$$

$$q_1(0) = q_1^0 = 0, \quad \dot{q}_1(0) = 0, \quad q_2(0) = q_2^0 = 0, \quad \dot{q}_2(0) = 0 \tag{3.9}$$

$$q_1(T) = q_1^T = (I_1 + 1)\varphi_1^T, \quad \dot{q}_1(T) = 0, \quad q_2(T) = q_2^T = I_2\varphi_2^T, \quad \dot{q}_2(T) = 0. \tag{3.10}$$

The phase vector of system (3.7) is formed by the variables q_1, $\dot{q}_1$, q_2, $\dot{q}_2$. We represent the system of equations (3.7) in the vector form

$$\dot{x} = Ax + BM, \tag{3.11}$$

$$x = (x_1, x_2, x_3, x_4)^{\mathrm{T}} = (q_1, \dot{q}_1, q_2, \dot{q}_2)^{\mathrm{T}}, \quad M = (M_1, M_2)^{\mathrm{T}}$$

with constant matrices A and B having dimensions 4x4 and 4x2, and a fundamental matrix $\Omega(t)$, respectively

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Omega(t) = \begin{pmatrix} 1 & t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & t \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.12}$$

We write the initial (3.9) and final (3.10) conditions in the form

$$x(0) = 0, \tag{3.13}$$

$$x(T) = x^1 = (x_1^1, 0, x_3^1, 0)^{\mathrm{T}}. \tag{3.14}$$

We will use the well-known approach to constructing the control [19]. Following the indicated approach, we seek the control solving the problem without taking into account constraints (3.8) in the form

$$M(t) = Q^{\mathrm{T}}(t)C, \quad Q(t) = \Omega^{-1}(t)B \tag{3.15}$$

Here $C = (C_1, C_2, C_3, C_4)^{\mathrm{T}}$ is the constant vekctor determined from the system of the following linear algebraic equation

$$R(T) \cdot C = \Phi^{-1}(T)x^1, \quad R(T) = \int_0^T Q(t)Q^{\mathrm{T}}(t)dt. \tag{3.16}$$

Since the system (3.11), (3.12) is completely controllable, then the matrix $R(T)$ is nondegenerate [19] and therefore (3.16) has a unique solution

$$C = R^{-1}(T)\, \Omega^{-1}(T)\, x^1, \quad x^1 = (x_1^1, 0, x_3^1, 0)^{\mathrm{T}}. \tag{3.17}$$

After calculating the matrices $Q^{\mathrm{T}}(t), \Omega^{-1}(T), R^{-1}(T), C$, taking into account (3.12), the components of the sought vector control (3.15) can be represented in the form

$$M_1(t) = g(t,T)x_1^1 + g(t,T)x_3^1, \quad M_2(t) = g(t,T)x_3^1, \quad g(t,T) = -12T^{-3}t + 6T^{-2}. \tag{3.18}$$

Solution $x(t)$ of system (3.11) - (3.13) under control $M(t)$ with components (3.18) for any $T > 0$ satisfies the boundary condition (3.14). In this case, however, the components of the constructed control do not necessarily satisfy the imposed constraints (3.8). In order to take these constraints into account, let us estimate the control modules M_1 and M_2

$$|M_1(t)| \leq |g(t,T)| \left|x_1^1 + x_3^1\right|, \quad |M_2| \leq \left|x_3^1\right| |g(t,T)|. \tag{3.19}$$

Since the linear function $g(t,T)$ (3.17) decreases monotonically on the interval $[0,T]$, taking maximum $g(0,T) = 6T^{-2}$ and minimum $g(T,T) = -6T^{-2}$ values at the ends of this interval, respectively, then inequalities (3.19) can be rewritten as

$$|M_1| \leq 6\left|x_1^1 + x_3^1\right| T^{-2}, \quad |M_2| \leq 6\left|x_3^1\right| T^{-2}. \tag{3.20}$$

If now the required transition time T of the system (3.11), (3.12) from state (3.13) to the state (3.14) is chosen from the relation

$$T = \max(T_1, T_2), \tag{3.21}$$

where

$$T_1 = \sqrt{6\left|x_1^1 + x_3^1\right|(M_1^0)^{-1}}, \quad T_2 = \sqrt{6\left|x_3^1\right|} \tag{3.22}$$

is the roots of the following equations $6\left|x_1^1 + x_3^1\right|T^{-2} = M_1^0$, $6\left|x_3^1\right|T^{-2} = 1$ respectively, then constraints (3.7) will be satisfied for all $t \in [0, T]$.

From (3.21), (3.22) we obtain

$$T = \begin{cases} T_1 = \sqrt{6\left|x_1^1 + x_3^1\right|(M_1^0)^{-1}}, & \text{if } M_1^0\left|x_3^1\right| < \left|x_1^1 + x_3^1\right|, \quad (a) \\ T_2 = \sqrt{6\left|x_3^1\right|}, & \text{if } M_1^0\left|x_3^1\right| > \left|x_1^1 + x_3^1\right|, \quad (b) \\ T_1 = T_2, & \text{if } M_1^0\left|x_3^1\right| = \left|x_1^1 + x_3^1\right|. \quad (c) \end{cases} \tag{3.23}$$

After the time of motion (3.23) is determined for a given final state x^1 (3.14), the control functions $M_1(t)$ and, $M_2(t)$ can be calculated at each moment using formulas (3.18).

In the initial variables (3.6), (3.11), relations (3.18), (3.23) take the form

$$\begin{aligned} M_1(t) &= (-12T^3t + 6T^2)(I_1 + 1)\varphi_1^T + (-12T^3t + 6T^2)I_2\varphi_2^T, \\ M_2(t) &= (-12T^3t + 6T^2)I_2\varphi_2^T, \end{aligned} \tag{3.24}$$

where

$$T = \begin{cases} T_1 = \sqrt{6\left|(I_1+1)\varphi_1^T + I_2\varphi_2^T\right|(M_1^0)^{-1}}, & \text{if } (\varphi_1^T, \varphi_2^T) \in \Phi_1, \quad (a) \\ T_2 = \sqrt{6\left|I_2\varphi_2^T\right|}, & \text{if } (\varphi_1^T, \varphi_2^T) \in \Phi_2, \quad (b) \\ T_1 = T_2, & \text{if } (\varphi_1^T, \varphi_2^T) \in \Phi' \cup \Phi''. \quad (c) \end{cases} \tag{3.25}$$

On the plane of finite configurations $(\varphi_1^T, \varphi_2^T)$, the regions Φ_i, $i = 1, 2$ and Φ', Φ'', appearing in formulas (3.25) are determined as follows:

$$\Phi_1 = \left\{(\varphi_1^T, \varphi_2^T) \in \Phi : \begin{array}{c} \left\{-A\varphi_1^T < \varphi_2^T < B\varphi_1^T,\ \varphi_1^T \geq 0\right\} \cup \\ \cup \left\{B\varphi_1^T < \varphi_2^T < -A\varphi_1^T,\ \varphi_1^T \leq 0\right\} \end{array}\right\}, \tag{3.26}$$

$$\Phi_2 = \left\{(\varphi_1^T, \varphi_2^T) \in \Phi : \begin{array}{c} \left\{B\varphi_1^T < \varphi_2^T,\ \varphi_1^T \geq 0\right\} \cup \left\{-A\varphi_1^T < \varphi_2^T,\ \varphi_1^T \leq 0\right\} \cup \\ \cup \left\{\varphi_2^T < -A\varphi_1^T,\ \varphi_1^T \geq 0\right\} \cup \left\{\varphi_2^T < B\varphi_1^T,\ \varphi_1^T \leq 0\right\} \end{array}\right\}, \tag{3.27}$$

$$\Phi' = \left\{(\varphi_1^T, \varphi_2^T) \in \Phi : \ \varphi_2^T = -A\varphi_1^T\right\}, \Phi'' = \left\{(\varphi_1^T, \varphi_2^T) \in \Phi : \ \varphi_2^T = B\varphi_1^T\right\}, \tag{3.28}$$

where

$$A = (I_1 + 1)(M_1^0 + 1)^{-1}I_2^{-1}, \quad B = (I_1 + 1)(M_1^0 - 1)^{-1}I_2^{-1}. \tag{3.29}$$

Thus, according to the given values φ_1^T, φ_2^T, the time of the process (3.25) is first determined, and then the desired controls (3.24), at which the required displacement of the manipulator is carried out without violating the constraints (3.3).

4 Determining the optimal type of the manipulator's final configuration

Let us assume that the manipulator is characterized by the following dimensional parameters appearing in (1.1), (1.2)

$$L = 1\,\mathrm{m}, m_2 = 4\,\mathrm{kg}, I_1 = I_2 = (10/3)\,\mathrm{kg}\cdot\mathrm{m}^2, M_1^0 = 1\ \mathrm{N}\cdot\mathrm{m}, M_2^0 = 1\ \mathrm{N}\cdot\mathrm{m}, \qquad (4.1)$$

which correspond to the manipulator, the links of which are the same homogeneous rods.

After passing to dimensionless parameters according to (3.1), we obtain from (4.1) that

$$L = 1, m_2 = 1, I_1 = I_2 = 1/3, M_1^0 = 1, M_2^0 = 1. \qquad (4.2)$$

Note that the problem solving procedure does not substantially change for other geometric and physical manipulator parameters.

Let us proceed to finding the minimum with respect to the parameter K in (2.9). Let the end position of the gripper $(x^T, y^T) \in R = \left\{(x,y):\ x^2+y^2 \le 4,\ x > 0\right\}$ be fixed. Then from formula (2.2) we find two terminal points $(\varphi_1^T,\, \varphi_2^T) \in \Phi(+1)$ and $(\varphi_2^T,\, \varphi_1^T) \in \Phi(-1)$ (2.3), which, according to (2.2), are associated with the manipulator configurations $\left\{\varphi_1^T,\, \varphi_2^T\right\}_{K=+1}$ and $\left\{\varphi_1^T,\, \varphi_2^T\right\}_{K=-1}$, respectively. In both cases, we write down the constructed controls, in which the superscripts $^{(+)}$ and $^{(-)}$ will correspond to the values $K = +1$ and $K = -1$, respectively.

If $K = +1$, then according to (2.3) $(\varphi_1^T, \varphi_2^T) \in \Phi(+1)$. Then (3.24) takes the form

$$\begin{aligned} M_1{}^{(+)}(t) &= (-\frac{12}{{T^{(+)}}^3}t + \frac{6}{{T^{(+)}}^2})(I_1+1)\varphi_1^T + (-\frac{12}{{T^{(+)}}^3}t + \frac{6}{{T^{(+)}}^2})I_2\varphi_2^T, \\ M_2{}^{(+)}(t) &= (-\frac{12}{{T^{(+)}}^3}t + \frac{6}{{T^{(+)}}^2})I_2\varphi_2^T, \end{aligned} \qquad (4.3)$$

where

$$T^{(+)}(\varphi_1^T, \varphi_2^T) = \begin{cases} T_1{}^{(+)} = \sqrt{6\left|(I_1+1)\varphi_1^T + I_2\varphi_2^T\right|}, & (a) \\ \text{if } (\varphi_1^T, \varphi_2^T) \in \Psi_1(+1) \cup \left(\bigcup\limits_{i=3}^{6} \Psi_i(+1)\right), & \\ T_2{}^{(+)} = \sqrt{6\left|I_2\varphi_2^T\right|},\ \text{ if } (\varphi_1^T, \varphi_2^T) \in \Psi_2(+1), & (b) \\ T_1{}^{(+)} = T_2{}^{(+)},\ \text{ if } (\varphi_1^T, \varphi_2^T) \in \Phi(+1) \cap (\Phi' \cup \Phi''). & (c) \end{cases} \qquad (4.4)$$

The following notation is introduced in formula (4.4):

$$\begin{aligned} \Psi_1(+1) &= \left\{\varphi_1^T, \varphi_2^T \in \Phi(+1):\ \ \varphi_1^T \le \varphi_2^T,\ \ \varphi_1^T \ge 0\right\}, \\ \Psi_2(+1) &= \left\{\varphi_1^T, \varphi_2^T \in \Phi(+1):\ \ -A\varphi_1^T \le \varphi_2^T,\ \ \varphi_1^T \le 0\right\}, \\ \Psi_3(+1) &= \left\{\varphi_1^T, \varphi_2^T \in \Phi(+1):\ \ -\varphi_1^T \le \varphi_2^T \le -A\varphi_1^T,\ \ \varphi_1^T \le 0\right\}, \\ \Psi_4(+1) &= \left\{\varphi_1^T, \varphi_2^T \in \Phi(+1):\ \ -A^{-1}\varphi_1^T \le \varphi_2^T \le -\varphi_1^T,\ \ \varphi_1^T \le 0\right\}, \\ \Psi_5(+1) &= \left\{\varphi_1^T, \varphi_2^T \in \Phi(+1):\ \ 0 \le \varphi_2^T \le -A^{-1}\varphi_1^T,\ \ \varphi_1^T \le 0\right\}, \\ \Psi_6(+1) &= \left\{\varphi_1^T, \varphi_2^T \in \Phi(+1):\ \ \varphi_1^T \le \varphi_2^T \le 0,\ \ \varphi_1^T \le 0\right\}, \end{aligned} \qquad (4.5)$$

where, taking into account (3.29), (4.2), $A = 2$. At the same time $\left(\bigcup\limits_{i=1}^{6} \Psi_i(+1)\right) = \Phi(+1)$.

Since the domains $\Phi(+1)$ and $\Phi(-1)(2.3)$ are symmetric to each other about the straight line $\varphi_2^T = \varphi_1^T$, we conclude that the change of variables $\varphi_1^T \to \varphi_2^T$, $\varphi_2^T \to \varphi_1^T$ transforms the domains

$\Psi_i(+1)$, $i=1,...,6$, into the respective domains $\Psi_i(-1)$, $i=1,...,6$, symmetric about the straight line $\varphi_2^T=\varphi_1^T$.

Therefore, if $K=-1$, then the point $(\varphi_2^T,\varphi_1^T)\in\Phi(-1)$. Then from (3.24) we obtain

$$\begin{aligned} M_1{}^{(-)}(t) &= (-\frac{12}{{T^{(-)}}^3}t+\frac{6}{{T^{(-)}}^2})(I_1+1)\varphi_2^T+(-\frac{12}{{T^{(-)}}^3}t+\frac{6}{{T^{(-)}}^2})I_2\varphi_1^T,\\ M_2{}^{(-)}(t) &= (-\frac{12}{{T^{(-)}}^3}t+\frac{6}{{T^{(-)}}^2})I_2\varphi_1^T, \end{aligned} \tag{4.6}$$

where

$$T^{(-)}(\varphi_2^T,\varphi_1^T)=\begin{cases} T_1{}^{(-)}=\sqrt{6\left|(I_1+1)\varphi_2^T+I_2\varphi_1^T\right|}, & (a)\\ \text{if } (\varphi_2^T,\varphi_1^T)\in\left(\bigcup\limits_{i=1}^{4}\Psi_i(-1)\right)\cup\Psi_6(-1), & \\ T_2{}^{(-)}=\sqrt{6\left|I_2\varphi_1^T\right|},\quad \text{if } (\varphi_2^T,\varphi_1^T)\in\Psi_5(-1), & (b)\\ T_1{}^{(-)}=T_2{}^{(-)},\ \text{if } (\varphi_2^T,\varphi_1^T)\in\Phi(-1)\cap(\Phi'\cup\Phi''). & (c) \end{cases} \tag{4.7}$$

where

$$\begin{aligned} \Psi_1(-1) &= \left\{\varphi_1^T,\varphi_2^T\in\Phi(-1):\ \ 0\le\varphi_2^T\le\varphi_1^T\right\},\\ \Psi_2(-1) &= \left\{\varphi_1^T,\varphi_2^T\in\Phi(-1):\ \ -A^{-1}\varphi_1^T\le\varphi_2^T\le0,\ \ \varphi_1^T\ge0\right\},\\ \Psi_3(-1) &= \left\{\varphi_1^T,\varphi_2^T\in\Phi(-1):\ \ -\varphi_1^T\le\varphi_2^T\le-A^{-1}\varphi_1^T,\ \ \varphi_1^T\ge0\right\},\\ \Psi_4(-1) &= \left\{\varphi_1^T,\varphi_2^T\in\Phi(-1):\ \ -A\varphi_1^T\le\varphi_2^T\le-\varphi_1^T,\ \ \varphi_1^T\ge0\right\},\\ \Psi_5(-1) &= \left\{\varphi_1^T,\varphi_2^T\in\Phi(-1):\ \ \varphi_2^T\le-A\varphi_1^T,\ \ \varphi_1^T\ge0\right\},\quad A=2,\\ \Psi_6(-1) &= \left\{\varphi_1^T,\varphi_2^T\in\Phi(-1):\ \ \varphi_2^T\le\varphi_1^T,\ \ \varphi_1^T\le0\right\}. \end{aligned} \tag{4.8}$$

Thus, calculating the minimum in (2.9), taking into account (4.5), (4.6) and (4.7), (4.8), reduces to choosing the smallest of two times:

$$T^*=\min\left[T^{(+)}(\varphi_1^T,\varphi_2^T)\Big|_{(\varphi_1^T,\varphi_2^T)\in\Psi_i(+1)},\ T^{(-)}(\varphi_2^T,\varphi_1^T)\Big|_{(\varphi_2^T,\varphi_1^T)\in\Psi_i(-1)},\ 1\le i\le6\right]. \tag{4.9}$$

Calculating the minimum in (4.9) for the numerical values of (4.2), as a result, we obtain

$$T^*=\begin{cases} T_1{}^{(+)}(\varphi_1^T,\varphi_2^T),\ \text{if } (\varphi_1^T,\varphi_2^T)\in\Psi_1(+1)\cup\Psi_3(+1), & (1)\\ T_2{}^{(+)}(\varphi_1^T,\varphi_2^T),\ \text{if } (\varphi_1^T,\varphi_2^T)\in\Psi_2(+1), & (2)\\ T_1{}^{(-)}(\varphi_2^T,\varphi_1^T),\ \text{if } (\varphi_2^T,\varphi_1^T)\in\Psi_4(-1)\cup\Psi_6(-1), & (3)\\ T_2{}^{(-)}(\varphi_2^T,\varphi_1^T),\ \text{if } (\varphi_2^T,\varphi_1^T)\in\Psi_5(-1), & (4)\\ T_1{}^{(+)}(\varphi_1^T,\varphi_2^T)=T_1{}^{(-)}(\varphi_2^T,\varphi_1^T),\ \text{if } \varphi_2^T=\varphi_1^T. & (5) \end{cases} \tag{4.10}$$

where $\Psi_i(\pm1)$, $i=1,...,6$ determined from (4.5), (4.8). Let us provide a numerical example. For the manipulator with the dimensionless parameters in (4.2), we take the gripper initial and terminal coordinates in the form

$$x^0=2,\ y^0=0,\quad x^T=1,\ y^T=-0,5. \tag{4.11}$$

The values of the coordinates of the gripper initial and terminal states (4.11) are associated with the initial configuration $\varphi_1^0=\varphi_2^0=0$ and with the two terminal

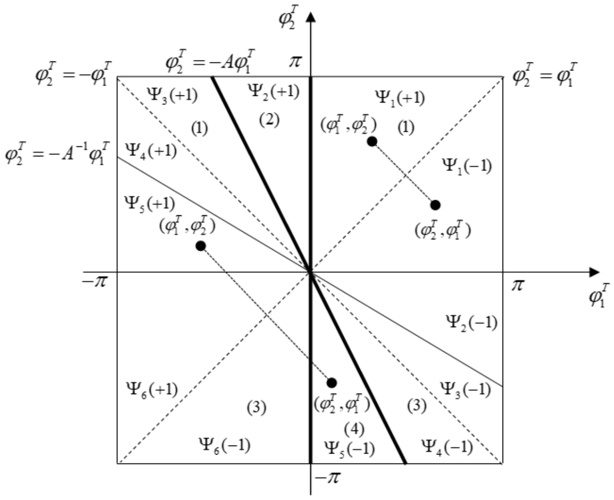

Fig. 2

configurations (2.2), wich are in turn associated with the two points, symmetric about the straight line $\varphi_2^T = \varphi_1^T$, on the angular manipulator plane (the angles are given in radian and angle degree measures):

$$\{\varphi_1^T, \varphi_2^T\}_{K=1}: \quad (\varphi_1^T, \varphi_2^T) = (-1.699\,\mathrm{rad},\ 0.514\,\mathrm{rad}) = (-97^0 37',\ 29^0 46'), \tag{4.12}$$

$$\{\varphi_1^T, \varphi_2^T\}_{K=-1}: \quad (\varphi_2^T, \varphi_1^T) = (0.514\,\mathrm{rad},\ -1.699\,\mathrm{rad}) = (29^0 46',\ -97^0 37'). \tag{4.13}$$

In the case under consideration, the point $(\varphi_1^T, \varphi_2^T) \in \Psi_5(+1)$, and the point $(\varphi_2^T, \varphi_1^T) \in \Psi_5(-1)$(fig. 2). Consequently, according to formula (4.10) (4), the minimum travel time of the gripper is achieved at point (4.13), which corresponds to the final configuration corresponding to the value $K = -1$. In this case, one should use the controls $M_1^{(-)}, M_2^{(-)}$(4.6), in which the travel time is determined by the formula (4.7)(b): $T^* = T_2{}^{(-)} = \sqrt{6\left|I_2\varphi_1^T\right|}$ and equal $T^* = 1,84\,(2.92\,\mathrm{s})$. The figure in parentheses give dimensional values of these times with the use of the conversion formulas (3.1). For comparison, note that if we choose the terminal configuration (4.12) ($K = +1$), then the gripper transfer time turns equal $T_1{}^{(+)}(\varphi_1^T, \varphi_2^T) = \sqrt{6\left|(I_1+1)\varphi_1^T + I_2\varphi_2^T\right|}$(4.4) (a) and out to be much worse; namely, $T_1{}^{(+)} = 3.54\,(5.59\,\mathrm{s})$.

The constructed controls are not time optimal, but simple enough for calculation and practical implementation. It is established by formulas (3.25) - (3.28) that

1) the domains Φ_1 and Φ_2 up to notation, coincide with the domains constructed in a similar time-optimal problem [16],

2) between the time of optimal movement T^0[16] and the time (3.25) there is the following ratio: $T^0/T^* \approx 0.816$. For comparison, we present the results of calculating the time-optimal movement T^0 for the end positions of the gripper (4.12) and (4.13). With the final configuration $K = -1$ the time-optimal is equal $T^0 = 1.5\,(2.37\,\mathrm{s})$ and with $K = +1$ is equal $T^0 = 2.89\,(4.57\,\mathrm{s})$.

Conclusions

On the configuration plane of a two-link manipulator with a second statically balanced link, regions are constructed that allow, based on a given terminal position of the manipulator, to determine the laws of controls change, which brings the manipulator from the initial resting configuration to the terminal resting configuration in a finite time without violating controls constraints, and also to choice the type of final configuration for which the time of movement of the gripper is minimal. It has been established by numerical calculations that the optimal choice of the type of the final configuration can lead to a significant decrease in the travel time. An estimate is given for the proximity of the results obtained using the considered control method and time-optimal control.

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