

**HYBRID OF SURFACE SHEAR WAVES AT THE CONTACT  
INTERFACE BETWEEN PIEZOELECTRIC AND ELECTRICALLY  
CONDUCTIVE HALF-SPACES**

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**Keywords:** piezoelectric, perfect conductor, surface shear wave, electro active wave, magneto active wave, wave dispersion, Alfvén velocity.

**Гибрид поверхностных волн сдвига на границе раздела  
пьезоэлектрического и электропроводящего полупространств**

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**Ключевые слова:** пьезоэлектрик, идеальный проводник, поверхностная волна сдвига, электро активная волна, магнито активная волна, дисперсия волн, скорость Альфена.

**Аннотация:** Рассматривается совместное распространение электро-магнито активных, поверхностных волн упругого сдвига в двухслойном композите. Композит состоит из пьезоэлектрического и идеально проводящего полупространств, и размещен в параллельном поверхности раздела полупространств внешнем постоянном магнитном поле. Сформулирована математическая краевая задача. Анализ полученного дисперсионного уравнения позволяет определить условия существования локализованных поперечных волн на границе раздела двух сред и характер локализаций магнитоупругих и электроупругих волн. Показано, что волновой процесс в композите можно оптимизировать, изменяя напряженность магнитного поля. Наличие параллельного магнитного поля может привести к устранению существующего гибрида поверхностных электроактивных и магнитоактивных упругих поперечных волн или может привести к их возникновению.

**Սահքի մակերևութային ալիքների հիբրիդ՝ պիեզոէլեկտրական և էլեկտրական  
հաղորդիչ կիսափարածոյությունների ամրակցման միջերեսին**

**Ավերիսյան Արա Ս., Գևորգյան Ա.Վ., Ավերիսյան Լևոն Վ.**

**Տիմնարաներ՝** պիեզոէլեկտրիկ, իդեալական հաղորդիչ, սահքի մակերևութային ալիք, էլեկտրաակտիվ ալիք, մագնիսաակտիվ ալիք, ալիքի ցրում, Ալֆենի արագոյություն:

Դիտարկվում է երկշերտ կոմպոզիտում էլեկտրամագնիսական ակտիվ մակերևութային առաձգական սահքի ալիքների համաբերական փարածումը: Կոմպոզիտը բաղկացած է պիեզոէլեկտրական և կապարյալ հաղորդիչ կիսափարածոյութից և փեղակալված է կիսափարածոյութունների միջերեսին զուգահեռ արտաքին հասարակ մագնիսական դաշտում: Ձևակերպված է մաթեմատիկական եզրային արժեքի խնդիր: Մտածված դիսպերսիայի հավասարման վերլուծությունը հնարավորություն է տալիս որոշելու երկու միջավայրերի միջերեսի մոտ փեղանագված սահքի ալիքների առկայության պայմանները, ինչպես նաև մագնիսաառաձգական և էլեկտրաառաձգական սահքի ալիքների փեղայնացման բնույթը: Յուրյ է փրկված, որ կոմպոզիտում ալիքային գործընթացը

կարելի է օպտիմալացնել՝ փոխելով արտաքին մագնիսական դաշտի լարվածությունը: Զուգահեռ մագնիսական դաշտի առկայությունը կարող է հանգեցնել մակերևույթի էլեկտրակրիվ և մագնիսաակրիվ առաձգական կարող ալիքների առկա հիբրիդի վերացմանը կամ կարող է հանգեցնել դրանց առաջացման:

Զուգահեռ մագնիսական դաշտի առկայությունը կարող է հանգեցնել մակերևույթի էլեկտրակրիվ և մագնիսաակրիվ առաձգական սահքի ալիքների հիբրիդի առաջացման, կամ կարող է հանգեցնել առկա հիբրիդի վերացմանը:

**Abstract:** The propagation of electro-magnetically active, surface elastic shear waves in a bi-material composite is considered. The composite consists of the contacting piezoelectric and perfectly conducting half-spaces. There is an external constant magnetic field parallel to the interface of the half-spaces. A mathematical boundary value contact problem is formulated. An analysis of the derived dispersion equation provides the conditions for the existence of surface shear waves localized at the interface between two media. The effects and nature of the localizations of magnetoelastic and electroelastic waves are discussed. It is shown that the wave process in the composite can be optimized by changing the magnetic field intensity. It is shown also that the magnetic field can eliminate the existing hybrid of surface electroactive and magnetically active elastic shear waves, or can lead to their origin.

## Introduction

In [1], 1911 Love A. E. H. showed that, unlike plane strain waves, localization of the wave energy of pure elastic shear is possible in the near-surface zone of an elastic half-space at the junction with a layer of softer material. In 1924, Lord Stoneley R. [2], investigated the possibility of the existence of waves similar to Rayleigh waves and Love waves, which propagate along the inner joints of the layers of the earth's crust. In his work, he also investigated the circumstances under which a Love-type wave can exist if a layer of constant thickness is bounded on both sides by very thick layers of various materials.

In [3], 1968 Bleustein J.L. and in [4], 1969 Gulyaev Yu.V., are showed the possibility of localizing the energy of an electroelastic shear wave near a mechanically free surface of a piezoelectric medium of certain symmetry, under different boundary conditions for the accompanying electric field. After these primary sources, soviet and foreign scientists solved a number of problems on the propagation of electroacoustic waves in piezoelectric crystals and in electrically conductive media, a review of which was given in books [5, 6] already in the eighties. The features of the localization of the wave energy of a purely shear electroelastic wave of the Gulyaev- Bleustein type, in multifunctional media, or in layered composites are still being studied. There are a thousand works where the regularities of the propagation of electroelastic shear waves in composite structures, or in media with complicated properties are investigated.

In modern electronics, inhomogeneous composite (in particular layered, piecewise homogeneous) waveguides are widely used as converters, filters or resonators of electro-magneto-acoustic high-frequency wave signals with the use of piezoelectric crystals [7–9], etc.

Qualitatively different interests are represented by the cases when the electroacoustic signal overcomes a transversely inhomogeneous layered structure [10], and when the

electroacoustic signal flows along the interface of homogeneous layers of the structure [11]. In these works, the formation of a hybrid of electroactive waves of elastic shear and elastic plane deformation in the case of non-acoustic contact between layers of piezoelectric materials is investigated.

Naturally, a hybrid of an electroactive elastic shear wave and a magneto active elastic shear wave in a layered composite of a piezoelectric and an electrically conductive material (multifunctional composite material) can provide a peculiar interest. In this case, different surface conditions of mechanical fields at the interface between layers can allow different hybridizations of elastic shear waves.

The proposed work studies the formation and propagation of a hybrid of electroactive and magneto active surface elastic shear waves at the interface between contacting piezoelectric and perfect conducting half-spaces, in an external constant magnetic field parallel to the interface of half-spaces.

## 1 Formulation of the mathematical boundary value contact problem

The composite medium of piezoelectric and electro conductive half-spaces is referred to a rectangular Cartesian coordinate system  $0x_1x_2x_3$ . The coordinate plane  $0x_3x_1$  is the contact interface between half-spaces, the coordinate plane  $0x_1x_2$  is coincides with the isotropy plane of a 6mm piezoelectric medium with hexagonal symmetry. The coordinate axis  $0x_3$  is parallel to the polarization axis of the piezoelectric crystal  $\bar{p}_6$ , and the coordinate axis  $0x_2$  is directed into the piezoelectric half-space (Fig. 1).

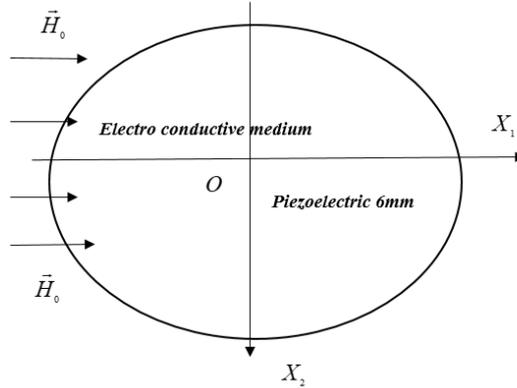


Fig. 1: Located in an external magnetic field  $\tilde{\mathbf{H}}_0(H_{01}, 0, 0)$  a composite space consisting of a piezoelectric medium and an perfectly conducting medium

Constant initial magnetic field  $\tilde{\mathbf{H}}_0(H_{01}, 0, 0)$  is directed along the axis  $0x_1$ . The magnetoelasticity equations for a perfectly conducting isotropic medium, in the general

three-dimensional case, have the form [12]

$$G_2 \nabla^2 \vec{u}^{(2)} + \frac{\mu^{(2)}}{4\pi} \left\{ \vec{\nabla} \times \left[ \vec{\nabla} \times \vec{u}^{(2)} \times \tilde{\mathbf{H}}_0 \right] \right\} \times \tilde{\mathbf{H}}_0 = \rho^{(2)} \frac{\partial^2 \vec{u}^{(2)}}{\partial t^2} \quad (1.1)$$

The coupled equations of elasticity and the equations of electrodynamics for a dielectric piezoelectric medium and material relations have the form [5, 6]

$$\frac{\partial \hat{\sigma}_{ij}^{(1)}}{\partial x_j} = \rho^{(1)} \frac{\partial^2 u_i^{(1)}}{\partial t^2}, \quad (1.2)$$

$$\frac{\partial^2 E_i^{(1)}}{\partial x_k^2} - \frac{\partial^2 E_i^{(1)}}{\partial x_i \partial x_k} = \frac{\mu^{(1)}}{c_0^2} \cdot \frac{\partial^2 D_i^{(1)}}{\partial t^2}, \quad \frac{\partial D_i^{(1)}}{\partial x_i} = 0. \quad (1.3)$$

$$\sigma_{ij}^{(1)} = \hat{C}_{ijkl} u_{kl}^{(1)} - e_{mij} E_m^{(1)}, \quad D_i^{(1)} = e_{ikl} u_{kl}^{(1)} + \varepsilon_{ij} E_j^{(1)}. \quad (1.4)$$

In equations (1.1) ÷ (1.4), the indices  $i, j, k, l, m \in \{1, 2, 3\}$ ,  $G_2$  is the shear elastic modulus of the conducting medium,  $\rho^{(1)}$  and  $\rho^{(2)}$  are the bulk densities of materials,  $\vec{u}^{(n)} = \{u_i^{(n)}(x_1, x_2, x_3, t)\}$  are the vectors of elastic displacements,  $\mu^{(1)}$  and  $\mu^{(2)}$  are the magnetic fields permeability in piezoelectric and electrically conductive media, respectively.  $\hat{C}_{ijkl}$  is a tensor of elastic constants,  $\hat{e}_{mij}$  is a tensor of piezoelectric modules,  $\hat{\varepsilon}_{ik}$  is a tensor of dielectric constant of piezoelectric, and  $\hat{u}_{ij} = \partial u_i / \partial x_j + \partial u_j / \partial x_i$  is the tensor of linear deformation of piezoelectric medium.

Hereinafter, the permeability of the magnetic field of the piezoelectric will be considered  $\mu^{(1)} = 1$ .

The superscripts (1) and (2) denote the quantities corresponding to the piezoelectric half-space and perfect conducting half-space, respectively.

Since the electroactive elastic shear wave is possible in the sagittal plane  $0x_1x_2$  of a piezoelectric material, the displacement field can be represented by two-dimensional equations in the plane  $0x_1x_2$  in the half-spaces  $n \in \{1, 2\}$ .

Elastic displacements field of this type is permissible if, in the solutions of equations (1.2) and (1.3), we put  $E_3^{(1)}(x_1, x_2, t) \equiv 0$  taking into account the material relations (1.4).

Then, an electric field is potential also in a piezoelectric medium  $\vec{E}^{(1)}(x_1, x_2, t) = -grad\varphi^{(1)}(x_1, x_2, t)$ . For piezoelectric half-space under consideration, we obtain the following system of equations

$$\begin{aligned} c_1^2 \nabla^2 u_3^{(1)}(x_1, x_2, t) &= \frac{\partial^2 u_3^{(1)}(x_1, x_2, t)}{\partial t^2} \\ \nabla^2 \left[ u_3^{(1)}(x_1, x_2, t) - (e_{15}/\varepsilon_{11}) \cdot \varphi^{(1)}(x_1, x_2, t) \right] &= 0 \end{aligned} \quad (1.5)$$

Where,  $c_1 = \sqrt{G_1/\rho^{(1)}}$  is the speed of the volume shear wave,  $G_1 = C_{44}(1 + \chi^2)$  is the piezoelectrically stiffened elastic shear modulus,  $\chi = \sqrt{e_{15}^2/(\varepsilon_{11}C_{44})}$  is the coefficient of electromechanical coupling in piezoelectric medium.

For a magneto active elastic shear wave travelling in perfect conducting half-space,

from equation (1.1) we obtain

$$(c_2^2 + a_2^2) \frac{\partial^2 u_3^{(2)}(x_1, x_2, t)}{\partial x_1^2} + c_2^2 \frac{\partial^2 u_3^{(2)}(x_1, x_2, t)}{\partial x_2^2} = \frac{\partial^2 u_3^{(2)}(x_1, x_2, t)}{\partial t^2}. \quad (1.6)$$

Here  $c_2 = \sqrt{G_2/\rho^{(2)}}$  is the velocity of the bulk shear wave and  $a_2 = H_0/\sqrt{4\pi\rho^{(2)}}$  is the speed of Alfvén in a conducting medium.

Since the accompanying magnetic field is neglected in a piezoelectric medium, and the accompanying electric field is neglected in a perfect conducting media, the electroelasticity equations (1.5) in the piezoelectric media and the equation (1.6) of magnetoelasticity in the perfect conducting media can be obtained in a rather simplified form.

Accordingly, they will be small of the order  $\approx 10^{-5}$ .

The boundary conditions can be also simplified. Taking into account that the corresponding component of the Maxwell stress tensor is identically zero at the interface,  $x_2 = 0$  we obtain

$$C_{44} \frac{\partial u_3^{(1)}(x_1, x_2, t)}{\partial x_2} + e_{15} \frac{\partial \varphi^{(1)}(x_1, x_2, t)}{\partial x_2} = G_2 \frac{\partial u_3^{(2)}(x_1, x_2, t)}{\partial x_2}, \quad (1.7)$$

$$u_3^{(1)}(x_1, x_2, t) = u_3^{(2)}(x_1, x_2, t), \quad \varphi^{(1)}(x_1, x_2, t) = 0. \quad (1.8)$$

The condition (1.7) is obtained from the condition of continuity of mechanical stresses. The conditions (1.8) are obtained from the condition continuity of elastic displacements and the tangential component of the electric field at the interface of the half-planes.

Since we consider the surface waves, the conditions of disturbances attenuation for damping of disturbances over the depth of half-spaces must be satisfied also

$$\lim_{x_2 \rightarrow +\infty} u_3^{(1)}(x_1, x_2, t) = 0, \quad \lim_{x_2 \rightarrow +\infty} \varphi^{(1)}(x_1, x_2, t) = 0, \quad \lim_{x_2 \rightarrow -\infty} u_3^{(2)}(x_1, x_2, t) = 0. \quad (1.9)$$

## 2 Formation of the hybrid of the electroactive and the magneto active SH elastic waves

Based on attenuation conditions (1.9), the solutions both of the equations (1.5) and (1.6) can be represented as

$$\begin{aligned} u_3^{(1)}(x_1, x_2, t) &= A_1 \cdot \exp(-k\alpha_1 x_2) \cdot \exp[ik(x_1 - vt)], \\ \varphi^{(1)}(x_1, x_2, t) &= [B_1 \cdot \exp(-kx_2) - (e_{15}/\varepsilon_{11}) \cdot A_1 \cdot \exp(-k\alpha_1 x_2)] \cdot \exp[ik(x_1 - vt)] \end{aligned} \quad (2.1)$$

$$u_3^{(2)}(x_1, x_2, t) = A_2 \cdot \exp(k\alpha_2 x_2) \cdot \exp[ik(x_1 - vt)] \quad (2.2)$$

Here  $\alpha_1^2 = 1 - v^2/c_1^2 > 0$  and  $\alpha_2^2 = 1 + a_2^2/c_2^2 - v^2/c_2^2 > 0$  are attenuation coefficients of waves in half-spaces,  $v(\omega) = \omega/k$  is the unknown phase velocity of a surface hybrid wave localized at the contact interface of the adjacent half-spaces of the composite media.

Substituting solutions (2.1) into boundary conditions (1.7) and (1.8), we obtain a system of homogeneous algebraic equations for arbitrary constant amplitudes. From the condition for the existence of nontrivial solutions to a system of homogeneous algebraic equations, the dispersion equation is obtained

$$(1 - v^2(\omega)/c_1^2)^{1/2} + (G_1/G_2) \cdot (1 + a_2^2/c_2^2 - v^2(\omega)/c_2^2)^{1/2} = \chi^2 \quad (2.3)$$

The condition for the existence of real roots for the dispersion equation (2.3) is  $v^2(\omega) < \min\{c_1^2, c_2^2 + a_2^2\}$ .

The left side of equation (2.3) is a monotonically decreasing function of the phase velocity  $v(\omega)$ . This makes it possible to obtain the conditions for the existence of a hybrid of surface waves depending on the ratio of the shear wave velocities in the media and on the magnetic field intensity. Let's consider different cases:

i) Let the speed of an elastic transverse wave in the piezoelectric medium be less than the speed of an elastic transverse wave in the perfect conducting medium  $c_1 \leq c_2$ . Since the parameter on the right-hand side of Eq. (2.3) must be between the extreme values of the function on the left-hand side of the Eq. (2.3). The condition for the existence of surface waves in this case can be written in the following form

$$(G_1/G_2) \cdot (1 + (a_2^2 - c_1^2)/c_2^2)^{1/2} \leq \chi^2 \leq 1 + (G_1/G_2) \cdot (1 + a_2^2/c_2^2)^{1/2} \quad (2.4)$$

Since the coefficient of electromechanical coupling of the piezoelectric medium is less than unity  $0 < \chi^2 < 1$ , the condition (2.4) can be reduced to the form

$$H_0 \leq 2\sqrt{\pi\rho^{(2)}} \cdot \left[ (G_2/G_1)^2 \cdot \chi^4 - 1 \right] c_2^2 + c_1^2 \quad (2.5)$$

It follows from this condition that in the absence of an external magnetic field, ( $H_{01} = 0$ ), the surface elastic shear waves of phase velocity  $v(\omega) < c_1$ , can exist in the composite if  $\chi^2 \geq (G_1/G_2) \cdot \sqrt{c_2^2 - c_1^2}/c_2 < 1$ .

The weak external magnetic field, satisfying to condition (2.5) does not eliminate the surface waves. The strong external magnetic field can lead to elimination of the existing hybrid of surface shear waves, at the following values of the magnetic field intensities

$$H_0 > 2\sqrt{\pi\rho^{(2)}} \cdot [c_1^2 - (1 - \chi^4 G_1^2 G_2^{-2}) \cdot c_2^2]^{1/2} \quad (2.6)$$

ii) In velocity ratios  $c_1^2 \geq c_2^2 + a_2^2$ , the condition for the existence of a hybrid of surface shear waves can be defined as

$$(1 - (c_2^2 + a_2^2)/c_1^2)^{1/2} \leq \chi^2 \leq 1 + (G_2/G_1) \cdot (1 + a_2^2/c_2^2)^{1/2} \quad (2.7)$$

Obviously, in the absence of an external magnetic field, when  $H_{01} = 0$ , for composite materials where  $c_1 \geq c_2$  the surface shear waves always exist. Such waves

can be eliminated outside of the narrow interval of external magnetic field intensities

$$2\sqrt{\pi\rho^{(2)}} \cdot [(1 - \chi^4) \cdot c_1^2 - c_2^2]^{1/2} \leq H_0 < 2\sqrt{\pi\rho^{(2)}} \cdot (c_1^2 - c_2^2)^{1/2} \quad (2.8)$$

iii) When  $c_2^2 < c_1^2 < c_2^2 + a_2^2$ , instead of relation (2.7), the existence condition for the surface shear waves can be written as

$$(G_2/G_1) \cdot (1 - (c_1^2 - a_2^2)/c_2^2)^{1/2} < \chi^2 < 1 + (G_2/G_1) \cdot (1 + a_2^2/c_2^2)^{1/2} \quad (2.9)$$

In this case, in the absence of a magnetic field, the condition for of the surface shear wave existence is

$$(1 - c_2^2/c_1^2)^{1/2} \leq \chi^2 \leq 1 + (G_2/G_1) \quad (2.10)$$

The external magnetic field can lead to the elimination of existing surface waves if

$$H_0 < 2\sqrt{\pi\rho^{(2)}} \cdot [c_1^2 - [1 - (G_2/G_1)^2 \cdot \chi^4]c_2^2]^{1/2} \quad (2.11)$$

From condition (2.10) it follows, that in case of absence of an external magnetic field, shear surface waves always exist.

In the cases both of a weak magnetic field, when  $H_0 < 2\sqrt{\pi\rho^{(2)}} \cdot (c_1^2 - c_2^2)^{1/2}$  and a strong magnetic field, when  $H_0 > 2\sqrt{\pi\rho^{(2)}} \cdot [c_1^2 - (1 - \chi^4 G_1^2 G_2^{-2}) \cdot c_2^2]^{1/2}$ , the elastic surface shear waves disappears.

An external magnetic field can lead to appearance of waves, if

$$2\sqrt{\pi\rho^{(2)}} \cdot (c_1^2 - c_2^2)^{1/2} < H_0 < 2\sqrt{\pi\rho^{(2)}} \cdot [c_1^2 - (1 - \chi^4 G_1^2 G_2^{-2}) \cdot c_2^2]^{1/2} \quad (2.12)$$

## Numerical Analysis

For piezoelectric medium, we take the material Cadmium Sulfide CdS, with the following material parameters:  $\rho_1 = 4.82 \cdot 10^3 \text{ kg/m}^3$  - bulk density,  $c_{44} = 1.43 \cdot 10^{10} \text{ N/m}^2$  - shear modulus,  $\chi^2 = 0,047$  - coefficient of electro-mechanical coupling,  $G_1 = 1.4972 \cdot 10^{10} \text{ N/m}^2$  - piezoelectrically stiffened elastic shear modulus and  $c_1 = 1.7625 \cdot 10^3 \text{ m/sec}$  is the speed of bulk shear waves.

For perfect conducting medium we take the material Tin Sn, with material parameters:  $\rho_2 = 7,29 \cdot 10^3 \text{ kg/m}^3$  is the bulk density and  $c_2 = 1.7563 \cdot 10^3 \text{ m/sec}$  are the shear stiffness of, and the velocity of bulk shear waves is  $G_2 = 2.2487 \cdot 10^{10} \text{ N/m}^2$ .

For this selected materials  $c_1 > c_2$  and the condition for the existence of waves (2.10) is also satisfied. In the absence of an external magnetic field, the surface elastic shear waves propagate in both of half-spaces. In the piezoelectric half-space, the surface shear wave is accompanied by oscillations of the electric field, and in the electro conducting half-space we have a purely elastic shear wave. For the weak magnetic field, when  $H_0 < 3.558 \cdot 10^6 \text{ a/m}$  or strong magnetic field, when  $H_0 > 3.797 \cdot 10^6 \text{ a/m}$ , the surface waves in the composite do not exist.

For the intensities of the external magnetic field satisfying to condition (2.12):  $3.558 \cdot 10^6 \text{ a/m} < H_0 < 3.797 \cdot 10^6 \text{ a/m}$ , there exist the hybrid both of the magnetically

active surface elastic shear wave in electro conducting medium and the electrically active surface elastic shear wave in a piezoelectric localized at the contact interface of these media.

## Conclusion

In a composite consisting of 6mm class of hexagonal symmetry piezoelectric medium contacting with perfect conducting medium, in the absence of external constant magnetic field the elastic shear surface waves are always exist localizing at contact interface. In the piezoelectric medium, this wave is accompanied by oscillations of the electric field. The presence of an external magnetic field parallel to the interface can eliminate these surface waves.

At other values of the external magnetic field intensities, the hybrid both of a magnetically active elastic surface shear wave in perfect conducting medium and an electrically active elastic shear wave in a piezoelectric medium is exist localized at the contact interface of these media.

This kind of compound composite makes it is possible, by means of changing intensities of external magnetic field, to control propagation modes of electro-magneto-elastic surface waves in composite devises.

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*Dedicated to the memory of our Teacher: To Professor Mels Vagharshak Belubekyan*

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