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HYBRID CONTROL OF LINEAR MODEL OF AN UNMANNED AERIAL VEHICLE CARRYING A PENDULUM

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Keywords: dynamical Systems, Control, Optimal Stabilization, Quadcopter UAV

Гибридное управление линейной модели беспилотнлвго летательнвго аппарата, несущего маятника

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Ключевые слова: Динамические системы, управление, оптимальная стабилизация, квадрокоптер БЛА

Проблемы управления беспилотниками имеют важное теоретическое и прикладное назначение. В этой статье рассматривается задача управления беспилотного ЛА, когда под ним находится маятник. Представлена динамика как БПЛА, так и маятника. После линеаризации модели в системе применяется новый гибридный метод управления для решения задачы управления. Полученные результаты, то есть управляющие воздействия и фазовые траектории, показаны в виде графиков, которые были сгенерированы из виртуального моделирования.

Ճոճանակ կրող անօդաչու թռչող սարքի գծային մոդելի հիբրիդային ղեկավարում

Շահինյան Ա. Ս.

Հիմնաբառեր.դինամիկ համակարգեր, ղեկավարում, օպտիմալ ստաբիլացում, քառաթև ԱԹՍ

Անօդաչու թռչող սարքերի ղեկավարման խնդիրներն ունեն կարևոր տեսական և կիրառական նշանակություն: Աշխատանքում դիտարկվում է քառաթև անօդաչու թռչող սարքի ղեկավարման խնդիրը, երբ այն կրում է իրենից կախված ճոճանակ: Ներկայացված է և՛ ԱԹՍ-ի, և՛ ճոճանակի դինամիկան: Բերված հավասարումների գծային մոտավորության համար կիրառված է նոր՝ հիբրիդային ղեկավարման եղանակ և լուծված է համակարգի ղեկավարման խնդիրը: Ստացված են ղեկավարող ազդեցությունները և փազային հետագծերը: Կառուցված են դրանց գրաֆիկները:

Control problems of UAVs have important applications in both science and life. In this paper a control problem of UAV is considered when it has a pendulum hanging from it. The dynamics of both UAV and the pendulum is presented. After linearizing the model, a novel hybrid method of control is applied to the system to solve the control problem. The results we gained i.e. the control inputs and state trajectories are shown in form of graphs which were generated from virtual simulations.

1 Introduction

Control problems of UAVs have important applications in both science and life. The history of UAVs, the examination and the research about the UAVs is thoroughly discussed in [1]. In this paper dynamics of a UAV is considered alongside with a pendulum hanging below from the UAV. The dynamics of the pendulum is presented with respect to the UAV and then both models are combined into one. After linearizing the model, a novel hybrid method of control is applied to the system to solve the control problem.

The hybrid model we applied is as follows. We first stabilize optimally the pendulum using the motion of the UAV as control inputs and then we use the optimal stabilizing control inputs to drive the UAV-Pendulum system to a desired position.

The results we gained i.e., the control inputs and state trajectories are shown in form of graphs which were generated from virtual simulations. The results are compared with the case when an inverted pendulum is sticked to the top of the UAV. Used energy is calculated for same values for both cases (UAV with inverted pendulum and UAV with hanging pendulum) and it is shown that in the case when the pendulum is inverted the energy cost is almost two times as high as in the case when the pendulum is hanging down from the UAV.

2 Modelling of the System

To derive the pure theoretical dynamics of a UAV let us fix a coordinate system . Let be the origin. We will also need another coordinate system fixed in the center of mass of the UAV (Figure 1). The torques and forces generated by each of the propellers are shown in the Figure 1. The propellers are numbered 1 to 4 [2].

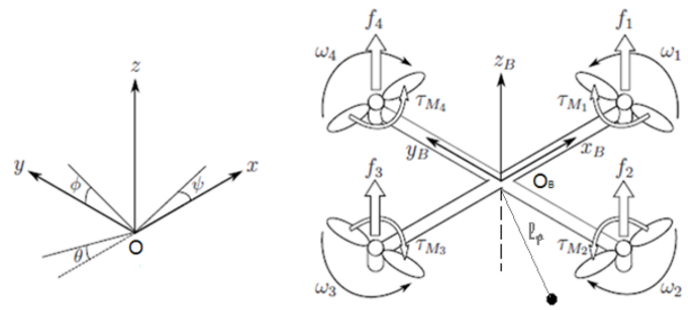


Figure 1

Let $\xi = \begin{pmatrix} x & y & z \end{pmatrix}^T$ be the coordinates of the center of mass of the UAV with respect to the system $Oxyz$. As mentioned above, the center of the mass of the UAV coincides with the origin of the coordinate system $O_B x_B y_B z_B$. Let us describe the inclined position of the UAV about the point O_B using yaw, pitch and roll angles. Let Φ be the pitch angle, Θ be the roll angle and, finally, let Ψ be the yaw angle.

Then we will have two vectors describing the position of the UAV. Those are the following:

$$\xi = \begin{pmatrix} x & y & z \end{pmatrix}^T, \qquad \eta = \begin{pmatrix} \Phi & \Theta & \Psi \end{pmatrix}^T \tag{1}$$

In the coordinate system the linear velocities $\bar{V}_B$ and the angular velocities $\bar{v}$ are the following

$$\bar{V}_B = \begin{pmatrix} V_{Bx} & V_{By} & V_{Bz} \end{pmatrix}^T, \qquad \bar{v} = \begin{pmatrix} p & q & r \end{pmatrix}^T \tag{2}$$

In this setup we will have the dynamics of the system as given below [2; 3].

$$\begin{array}{l} \ddot{x} = \frac{T}{M} c_\Psi s_\Theta c_\Phi + \frac{T}{M} s_\Psi s_\Phi,\ \ddot{y} = \frac{T}{M} s_\Psi s_\Theta c_\Phi - \frac{T}{M} c_\Psi s_\Phi,\ \ddot{z} = -g + \frac{T}{M} c_\Theta c_\Phi, \\ \dot{\Phi} = p + \frac{s_\Phi s_\Theta}{c_\Theta} q + \frac{c_\Phi s_\Theta}{c_\Theta} r,\ \dot{\Theta} = c_\Phi q - s_\Phi r,\ \dot{\Psi} = \frac{s_\Phi}{c_\Theta} q + \frac{c_\Phi}{c_\Theta} r, \\ \dot{p} = \frac{(I_{yy} - I_{zz}) qr}{I_{xx}} - I_r \frac{q}{I_{xx}} \omega_\Gamma + \frac{\tau_\Phi}{I_{xx}},\ \dot{q} = \frac{(I_{zz} - I_{xx}) pr}{I_{yy}} - I_r \frac{p}{I_{yy}} \omega_\Gamma + \frac{\tau_\Theta}{I_{yy}} \\ \dot{r} = \frac{(I_{xx} - I_{yy}) pq}{I_{zz}} - I_r \frac{q}{I_{zz}} \omega_\Gamma + \frac{\tau_\Psi}{I_{zz}} \end{array} \tag{3}$$

where the following notations are used:

$$\begin{gathered} C_\alpha := \cos\alpha,\ S_\alpha := \sin\alpha, M = m_{UAV} + m_P \\ \tau_B = \begin{pmatrix} \tau_\Phi \\ \tau_\Theta \\ \tau_\Psi \end{pmatrix} = \begin{pmatrix} lk\left(-\omega_2^2 + \omega_4^2\right) \\ lk\left(-\omega_1^2 + \omega_3^2\right) \\ \sum_i \tau_i \end{pmatrix} \\ T = \sum_i F_i = \sum_i k\omega_i^2,\ \ \vec{T} = \begin{pmatrix} 0 & 0 & T \end{pmatrix}^T \end{gathered} \tag{4}$$

As for the mathematical model of the pendulum we will consider its dynamics in the coordinate system $O_B x_B y_B z_B$. So, the dynamics of the pendulum will be as shown below. [4]

$$\left\{ \begin{array}{c} \ddot{x}_p = \frac{1}{\left(L^2 - y_p^2\right)\zeta^2} \left(-x_p^4 \ddot{x} - \left(L^2 - y_p^2\right)\ddot{x} - 2x_p^2\left(y_p \dot{x}_p \dot{y}_p - \left(L^2 - y_p^2\right)\ddot{x}\right) + \right. \\ + x_p^3\left(\dot{y}_p^2 + y_p \ddot{y}_p + \zeta\left(g + \ddot{z}\right)\right) + x_p\left(-L^2 y_p \ddot{y}_p + y_p^3 \ddot{y}_p + y_p^2\left(\dot{x}_p^2 + \zeta\left(g + \ddot{z}\right)\right) + \right. \\ \left. \left. + L^2\left(-\dot{x}_p^2 - \dot{y}_p^2 - \zeta\left(g + \ddot{z}\right)\right)\right)\right) \\ \ddot{y}_p = \frac{1}{\left(L^2 - x_p^2\right)\zeta^2} \left(-y_p^4 \ddot{y} - \left(L^2 - x_p^2\right)\ddot{y} - 2y_p^2\left(x_p \dot{x}_p \dot{y}_p - \left(L^2 - x_p^2\right)\ddot{y}\right) + \right. \\ + y_p^3\left(\dot{x}_p^2 + x_p \ddot{x}_p + \zeta\left(g + \ddot{z}\right)\right) + \\ \left. + y_p\left(-L^2 x_p \ddot{x}_p + x_p^3 \ddot{x}_p + x_p^2\left(\dot{y}_p^2 + \zeta\left(g + \ddot{z}\right)\right) + L^2\left(-\dot{x}_p^2 - \dot{y}_p^2 - \zeta\left(g + \ddot{z}\right)\right)\right)\right) \end{array} \right. \tag{5}$$

Using the formula of center of mass of a system

$$\bar{X}_C = \frac{m_1 \bar{r}_1 + m_2 \bar{r}_2}{m_1 + m_2}$$

where $\bar{r}_1 = \bar{\xi} = \begin{pmatrix} x & y & z \end{pmatrix}^T$ and $\bar{r}_2 = \bar{r}_p = \begin{pmatrix} x + x_p & y + y_p & z - \xi \end{pmatrix}^T$, we can find the coordinates of center of mass of our UAV-Pendulum system in the coordinate system $Oxyz$. Let $m_1 = m_2 = 1$, then we will have

$$\begin{cases} x_c = x + \frac{1}{2}x_p \\ y_c = y + \frac{1}{2}y_p \\ z_c = z - \frac{1}{2}\sqrt{l_p^2 - x_p^2 - y_p^2} \end{cases}$$

To get the state space model of the UAV-Pendulum system we introduce the notations as shown below

$$\begin{array}{l} x_1 = x_c,\ x_2 = \dot{x}_c,\ x_3 = y_c,\ x_4 = \dot{y}_c,\ x_5 = z_c,\ x_6 = \dot{z}_c,\ x_7 = \Phi,\ x_8 = \Theta, \\ x_9 = \Psi,\ x_{10} = p,\ x_{11} = q,\ x_{12} = r,\ x_{13} = x_p,\ x_{14} = \dot{x}_p,\ x_{15} = y_p,\ x_{16} = \dot{y}_p \end{array} \quad (6)$$

We linearize the dynamics around the origin of the fixed coordinate system. So, we finally get.

$$\begin{array}{l} \dot{x}_1 = x_2,\ \dot{x}_2 = \frac{g}{2l_p}x_{13},\ \dot{x}_3 = x_4,\ \dot{x}_4 = \frac{g}{2l_p}x_{15},\ \dot{x}_5 = x_6,\ \dot{x}_6 = u_1,\ \dot{x}_7 = x_{10}, \\ \dot{x}_8 = x_{11},\ \dot{x}_9 = x_{12},\ \dot{x}_{10} = \frac{u_2}{I_{xx}} - \frac{g}{I_{xx}}x_{15},\ \dot{x}_{11} = \frac{u_3}{I_{yy}} - \frac{g}{I_{yy}}x_{13},\ \dot{x}_{12} = \frac{u_4}{I_{zz}}, \\ \dot{x}_{13} = x_{14},\ \dot{x}_{14} = -gx_8 - \frac{g}{l_p}x_{13},\ \dot{x}_{15} = x_{16},\ \dot{x}_{16} = gx_7 - \frac{g}{l_p}x_{15} \end{array} \quad (7)$$

where $u_1 = \frac{T}{M} - g$, $u_2 = \tau_\Phi$, $u_3 = \tau_\Theta$, $u_4 = \tau_\Psi$.
Using Kalman's rule one can check that the system (7) is fully controllable. So, now we are in a point where we can define the problem and we can go ahead to show the way we solved it.

3 Problem Definition

Given the system (7), the initial position of the system $x_1(0) = x_{1,0}, x_3(0) = x_{3,0}, x_5(0) = x_{5,0}$ and the final position $x_1(t_1) = x_{1,1}, x_3(t_1) = x_{3,1}, x_5(t_1) = x_{5,1}$, find control inputs u_1, u_2, u_3 such that it drives the system from the given initial position to the given final.

As one can notice this control problem is not an optimal control problem.

Solution: Our approach to the problem solution was the following. First, we ensure that the pendulum remains at its lower equilibrium position. We do this by applying optimal control input stabilizers inside the coordinate system $O_B x_B y_B z_B$. And after we know that the pendulum will remain stable (will not oscillate with respect to the UAV) we proceed to the control problem. Let us now define a subproblem of optimal stabilization for the subsystem

$$\begin{cases} \dot{x}_{13} = x_{14} \\ \dot{x}_{14} = -gu_5 - \frac{g}{l_p}x_{13} \\ \dot{x}_{15} = x_{16} \\ \dot{x}_{16} = gu_6 - \frac{g}{l_p}x_{15} \end{cases} \quad (8)$$

Note that here we use the notation

$$\begin{cases} x_8 = u_5 \\ x_7 = u_6 \end{cases} \quad (9)$$

Now the subproblem will be the defined as follows.

4 Problem Definition

Given the system (8), the initial position of the system $x_i\,(0) = x_{i,0},\ i = \overline{13,16}$, find control inputs $u^0 = \begin{pmatrix} u_5^0 & u_6^0 \end{pmatrix}^T$ such that it drives the system from the given initial position to asymptotically stable position while minimizing the linear quadratic regulator

$$J\,[\bullet] = \int\limits_0^\infty \left(x_{14}^2 + x_{16}^2 + u_5^2 + u_6^2\right) d\tau$$

Solution: Notice that the system (8) can be divided into two subsystems which are

$$\begin{cases} \dot{x}_{13} = x_{14} \\ \dot{x}_{14} = -\frac{g}{l_p}x_{13} - gu_5 \end{cases} \tag{8.1}$$

$$\begin{cases} \dot{x}_{15} = x_{16} \\ \dot{x}_{16} = -\frac{g}{l_p}x_{15} + gu_6 \end{cases} \tag{8.2}$$

With optimality constraints

$$J\,[\bullet] = \int\limits_0^\infty \left(x_{14}^2 + u_5^2\right) d\tau \text{ and } J\,[\bullet] = \int\limits_0^\infty \left(x_{16}^2 + u_6^2\right) d\tau$$

respectively. We will show the solution steps for one of the systems (say (8.1)) as both of them are solved absolutely identically.

We choose to solve the optimal stabilization problem by using Lyapunov-Bellman method. In general, the method says that the optimal control input has to satisfy the optimization equation as given below

$$\min_u \left(\nabla V\,(x)\,(Ax + Bu) + \left(x^T Q x + u^T R u\right)\right) = 0 \tag{10}$$

Where

$$\mathfrak{B}[\bullet] = \nabla V\,(x)\,(Ax + Bu) + \left(x^T Q x + u^T R u\right) \tag{11}$$

(11) is Bellman's expression for the linear time-invariant control systems. So, in our case for the system (8.1) we will have

$$\mathfrak{B}\,[\bullet] = \frac{\partial V}{\partial x_{13}}x_{14} + \frac{\partial V}{\partial x_{14}}\left(-\frac{g}{l_p}x_{13} - gu_5\right) + x_{14}^2 + u_5^2 \tag{12}$$

It is obvious that the value of u_5^0 which optimizes (10) is the extremum of (12). Thus, we will have

$$u_5^0 = \frac{g}{2}\frac{\partial V}{\partial x_{14}} \tag{13}$$

By substituting (13) back into (12) we get the following.

$$\frac{\partial V}{\partial x_{13}}x_{14}-\frac{g}{l_p}x_{13}\frac{\partial V}{\partial x_{14}}-\frac{g^2}{4}\left(\frac{\partial V}{\partial x_{14}}\right)^2+x_{14}^2=0 \tag{14}$$

Here $V=V\left(x_{13},x_{14}\right)$ is the Lyapunov function for the system (8.1) and we search for it in the form

$$V=\frac{1}{2}\left(c_{11}x_{13}^2+2c_{12}x_{13}x_{14}+c_{22}x_{14}^2\right) \tag{15}$$

Putting (15) into (14) we get an equation which have the form

$$\begin{aligned}&\left(c_{11}x_{13}+c_{12}x_{14}\right)x_{14}-\frac{g}{l_p}\left(c_{12}x_{13}+c_{22}x_{14}\right)x_{13}-\\&-\frac{g^2}{4}\left(c_{12}x_{13}+c_{22}x_{14}\right)^2+x_{14}^2=0\end{aligned} \tag{16}$$

From (16) the following system of algebraic equations will follow

$$\begin{cases}-\frac{g}{l_p}c_{12}-\frac{g^2}{4}c_{12}^2=0\\ c_{12}-\frac{g^2}{4}c_{22}^2+1=0\\ c_{11}-\frac{g}{l_p}c_{22}-\frac{g^2}{2}c_{12}c_{22}=0\end{cases}\Rightarrow\begin{cases}c_{11}=\frac{2}{l_p}\\ c_{12}=0\\ c_{22}=\frac{2}{g}\end{cases} \tag{17}$$

Where the shown solutions are the ones which make $V=V\left(x_{13},x_{14}\right)$ positive definite. Finally, to get $u_5^0=u_5^0\left(x_{13},x_{14}\right)$ we put (17) into (15) and put what we get into (13). That gives us

$$u_5^0=x_{14} \tag{18}$$

To obtain $u_5^0=u_5^0\left(t\right)$ we simply need to substitute (18) into (8.1) and integrate the system. Under the initial conditions

$$x_{13}\left(0\right)=0.5,\ \ x_{14}\left(0\right)=0$$

we will get

$$u_5^0=\frac{0.5\left(\mathrm{e}^{\frac{\left(-gl_p-\sqrt{gl_p}\sqrt{-4+gl_p}\right)t}{2l_p}}-\mathrm{e}^{\frac{\left(-gl_p+\sqrt{gl_p}\sqrt{-4+gl_p}\right)t}{2l_p}}\right)\sqrt{g}}{\sqrt{l_p}\sqrt{-4+gl_p}} \tag{19}$$

Taking the exact same steps for the system (8.2) we will get $u_6^0=-x_{16}$, and finally $u_6^0=u_6^0\left(t\right)$ which will be.

$$u_6^0=-\frac{0.5\left(\mathrm{e}^{\frac{\left(-gl_p-\sqrt{gl_p}\sqrt{-4+gl_p}\right)t}{2l_p}}-\mathrm{e}^{\frac{\left(-gl_p+\sqrt{gl_p}\sqrt{-4+gl_p}\right)t}{2l_p}}\right)\sqrt{g}}{\sqrt{l_p}\sqrt{-4+gl_p}} \tag{20}$$

Under the initial conditions $x_{15}\left(0\right)=0.5,\ \ x_{16}\left(0\right)=0$.

5 Back to Core Problem

Now, that we have the solution for the subproblem, we can proceed to our main problem. Recall that the control inputs in the sub problem which are $u_5^0 = u_5^0(t)$ and $u_6^0 = u_6^0(t)$ are actually x_7 andx_8 in the system (7). In that case one can notice that two subsystems of (7) can be simply integrated. Those subsystems are the following.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{g}{2l_p} x_{13} \\ \dot{x}_8 = x_{11} \\ \dot{x}_{11} = \frac{u_3}{I_{yy}} - \frac{g}{I_{yy}} x_{13} \end{cases} \tag{7.1}$$

$$\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{g}{2l_p} x_{15} \\ \dot{x}_7 = x_{10} \\ \dot{x}_{10} = \frac{u_2}{I_{xx}} - \frac{g}{I_{xx}} x_{15} \end{cases} \tag{7.2}$$

As we already have $x_7 = x_7(t)$ and $x_8 = x_8(t)$ we can simply derive $x_{11} = x_{11}(t)$ from system (7.1) and $x_{10} = x_{10}(t)$ from system (7.2). As for $x_i = x_i(t),\ i = \overline{1,4}$ we will obtain by integrating $\dot{x}_2 = \frac{g}{2l_p} x_{13}$ and $\dot{x}_4 = \frac{g}{2l_p} x_{15}$ under the consideration of desired edge conditions. As a result, we will have the desired state trajectories of the UAV and the control inputs $u_2 = u_2(t)$ and $u_3 = u_3(t)$ which will drive the system through the desired trajectories. Of course, those control inputs are not optimal because of the absence of constraint.

Only the first of the remaining two subsystems of (7) which are

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = u_1 \end{cases} \tag{7.3}$$

$$\begin{cases} \dot{x}_9 = x_{12} \\ \dot{x}_{12} = \frac{u_4}{I_{zz}} \end{cases} \tag{7.4}$$

are discussed in this paper. The reason is that the second subsystem will have trivial solution for in the scope of this problem and, hence will not affect the energy spent for the control process. What refers to the subsystem (7.3) is that it describes the movement of the system along Z-axis. We will assume the system goes up the Z-axis with a constant speed for simplicity.

6 Simulating the Results

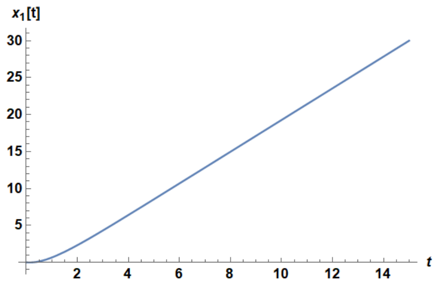
We have chosen to check the theoretical result of this paper by simulating the motion of the UAV and recording state trajectories in form of graphs with time being the independent variable. For the simulation purposes the following values have been chosen for the parameters.

$$g = 9.81 \ m \ s^{-2}, \quad l_p = 1 \ m, \quad I_{xx} = I_{yy} = 0.4856 \ Kgm^2 \tag{21}$$

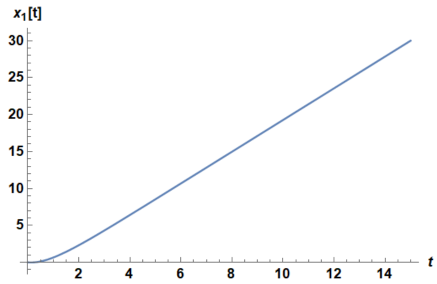
As for the initial and final positions of the system we have chosen the following values.

$$x_1\left(0\right) = 0, x_3\left(0\right) = 0, x_5\left(0\right) = 0, x_6\left(0\right) = 1 \ x_1\left(15\right) = 30, x_3\left(15\right) = 30, x_5\left(15\right) = 15$$

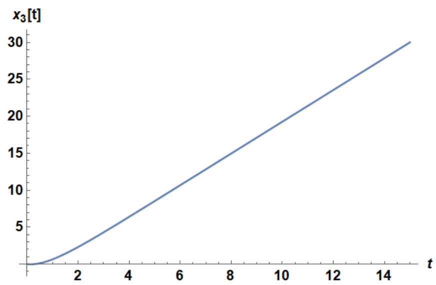
Finally, we are ready to present the graphs describing the motion of the quadcopter (shown below).



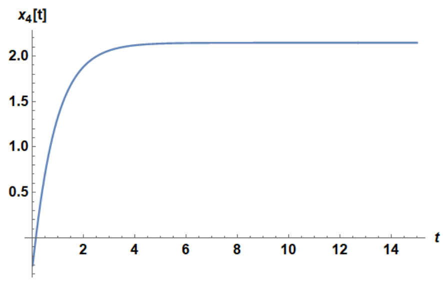
(a) The trajectory of $x_1\left(t\right)$



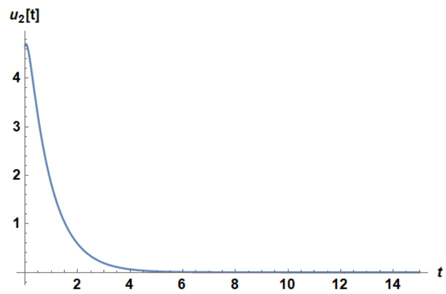
(b) The trajectory of $x_2\left(t\right)$



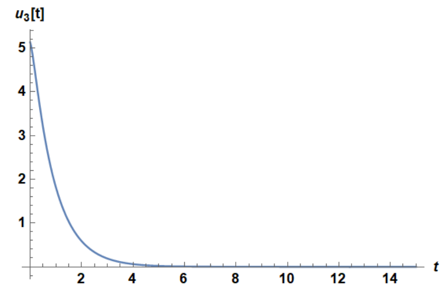
(a) The trajectory of $x_3\left(t\right)$



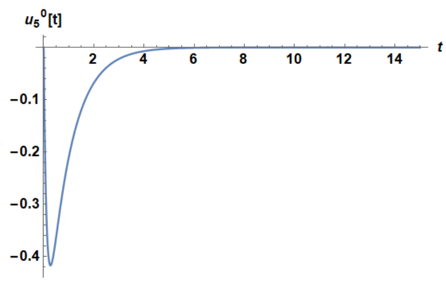
(b) The trajectory of $x_4\left(t\right)$



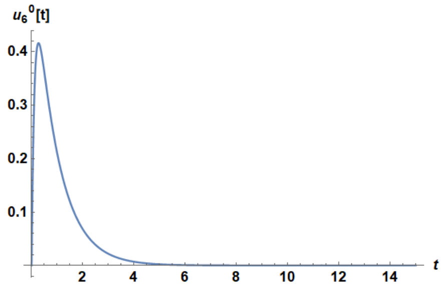
(a) The graph of $u_2\left(t\right)$



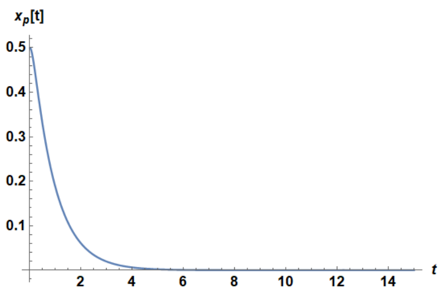
(b) The graph of $u_3\left(t\right)$



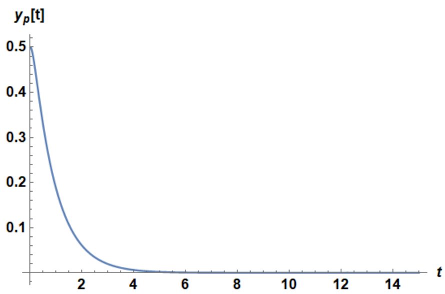
(a) The graph of $u_5^0\,(t)$



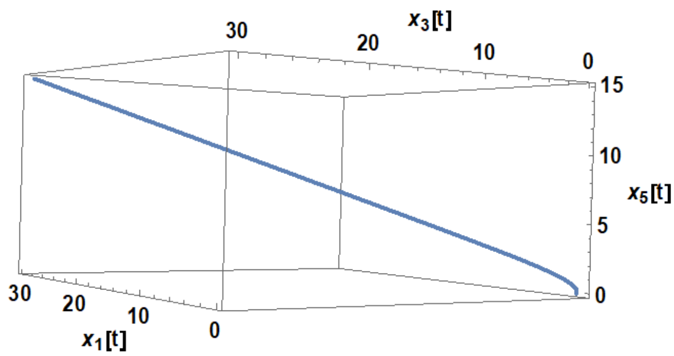
(b) The graph of $u_6^0\,(t)$



(a) Trajectory of pendulum along x-axis on the coordinate system $Oxyz$



(b) Trajectory of pendulum along y-axis on the coordinate system $Oxyz$



The real Trajectory of the UAV in 3D Space.

Now, that we have seen a numerical example, we can proceed to compare the results with another case scenario that is when the UAV carries an inverted pendulum. Namely we are interested in comparing the energy usage in both cases. For the case of current paper, we can calculate energy usage using the energy integral as shown below

$$\int_{t_0}^{t_1} \left(\sum_{i=1}^{16} x_i^2 + \sum_{j=1}^{4} u_j^2 \right) dt = 9699.85 \text{ units.} \tag{22}$$

As for the other case scenario we can use the result of [1] to calculate the amount of energy used. Using again the energy integral we will have

$$\int_{t_0}^{t_1} \left(\sum_{i=1}^{16} x_i^2 + \sum_{j=1}^{4} u_j^2 \right) dt = 18427.4 \text{ units.} \tag{23}$$

So, we see that the energy consumed for controlling the UAV with a pendulum hanging underneath is almost twice as easy as in the case when the UAV carries the pendulum inverted on its top. Of course, this result was expected and it is quite natural, that we have this huge difference.

Conclusion

The dynamics of the pendulum is presented with respect to the UAV and then both models are combined into one. The model is then linearized and the control problem is solved using proposed hybrid method, which means, we first stabilized optimally the pendulum using the motion of the UAV as control inputs and then we used the optimal stabilizing control inputs to drive the UAV-Pendulum system to a desired position. The results we gained are shown in form of graphs which were generated from virtual simulations. Then, we calculated the energy spent during the control process for the same values of parameters for both cases: when the UAV carries an inverted pendulum on top of it and when the UAV carries a pendulum hanging down from it. It is shown that the amount of energy used to control the UAV with an inverted pendulum is almost twice the energy used to control the UAV with a hanging pendulum.

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