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A MESO-SCALE MODEL OF PARTICLE REINFORCED TIMOSHENKO BEAM

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Keywords: multiscale modelling, homogenization, particle reinforced composites, transverse shear, Timoshenko beam

Мезомасштабная модель балки Тимошенко, усиленной частицами

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Ключевые слова: многомасштабное моделирование, гомогенизация, композиты усиленные частицами, поперечный сдвиг, балка Тимошенко

В этой статье, из соответствующих микро-скопических уравнений, выводятся мезо-скопические уравнения изгиба балки Тимошенко, усиленной частицами, когда масштабный параметр равный отношению радиуса частиц и толщины балки убывая стремится к нулю. Балка жестко заделана на одном конце, а к другому ее концу применяется осевая нормальная нагрузка постоянной интенсивности.

Применяя метод конечных элементов, численно определяются поле перемещений и напряженно-деформированное состояние балки для убывающих значений масштабного параметра и показывается сходимость решений моделированной задачи.

Մասնիկներով ուժեղացված Տիմոշենկոյի հեծանի մեզո-մասշտաբ մոդել

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Հիմնաբառեր. բազմամասշտաբ մոդելավորում, համասեռացում, մասնիկներով ուժեղացված բաղադրյալ նյութեր, լայնական սահք, Տիմոշենկոյի հեծան

Այս աշխատանքում, համապատասխան միկրո-մասշտաբ հավասարումներից ստացվում են մասնիկներով ուժեղացված Տիմոշենկոյի հեծանի ծռման մեզո-մասշտաբ հավասարումները, երբ մասնիկների շառավղի և հեծանի հաստության հարաբերությունը նկարագրող մասշտաբի պարամետրը ձգտում է զրոյի: Հեծանի մի ծայրը կոշտ ամրակցված է, իսկ դրա մյուս ծայրին կիրառված է հեծանի առանցքով ազդող հաստատուն ուժ:

Մոդելավորված խնդրի լուծումների զուգամիտությունը ցույց տալու համար՝ հեծանի տեղափոխության դաշտը և լարվածադեֆորմացիոն վիճակը հետազոտվում են վերջավոր տարրերի թվային եղանակով՝ մասշտաբի պարամետրի նվազող արժեքների համար:

In this paper, we derive the meso-scale equations of motion for particle reinforced Timoshenko beam from corresponding micro-scale equations by letting the scale parameter denoting the ratio of the particle radius and beam thickness decrease to zero. The beam is cantilevered at one end and is subject to a normal pressure of constant magnitude at the other end. The displacement field and the stress-strain state of the beam are determined for a decreasing sequence of values of the scale parameter to establish the convergence of solutions numerically using the finite element analysis.

Introduction

Materials with new target features and improved properties are of extremely high demand in all areas of engineering ranging from civil to aerospace and even medical. The reason is that most of the current needs of engineering are not fully satisfied by existing homogeneous or even some of the composite materials anymore stimulating new experimental and theoretical research in different areas of materials science.

Usually, composites are reinforced either by fibers or particles. Each of these two types has its own wide area of applications. Current material science has powerful tools of modelling reinforced composites with even very chaotic micro-structure. One of the main modelling tools for finer description of reinforced composites is the so-called multiscale approach [1]. In the most vast description, the multiscale approach develops the micro-scale description of the composite taking into consideration all possible micro-inhomogeneities and micro-defects [2, 3]. Then, the most appropriate homogenization tool [4, 5] is applied to derive the meso-scale of the macro-scale description of the composite. Nevertheless, application of the homogenization for specific cases is not that straightforward and each case may require an extensive research.

Particle-reinforced composites are widely used in many important areas of modern engineering. By appropriate choice of the reinforcing material properties of the volume fraction, it becomes possible to design in some sense optimal structures. This motivates us to study another important factor that may have influence on the desired properties of the composite- spatial distribution of particles. As of now, most of the research about modelling and characterization of PRC materials is carried out when the particle distribution follows a specific random distribution. However, evidences show that the uniform distribution of reinforcements may not always be optimal [20]. In other words, it may be possible to achieve better properties for the composite with a properly chosen particle distribution. For that purpose, the dependence of the target properties of PRCs (such as bending or flexural stiffness) from the particle distribution law must be analyzed. Apparently, this can be easily done in case when that dependence is explicit.

In this short note, we consider the homogenizaiton of a particle-reinforced, cantilever Timoshenko beam and show the convergence of the micro- and meso-scale displacements field numerically. The theory of the particle reinforced composites (PRCs) is well developed and currently includes results allowing to model anisotropic behavior, interface defects, material surface effects, two-phase materials, non-uniform and arbitrary distribution of particles (see [6–19]).

We start from micro-scopic description of the beam and apply the convergence definition given in [18, 19] to derive its meso-scopic description. In the micro-scopic description, the beam is represented as a continuum with spherical inclusions having specific geometry within the beam. The limiting description corresponds to a beam with point inhomogeneities. The finite element method is used to capture the displacement field of the beam for a decreasing sequence of the scale parameter denoting the ratio of the inclusion radius and the beam height. A clear convergence of the micro-scale displacements field to the meso-scale one is observed.

1 Main assumptions and beam model

In Cartesian system $Oxyz$, consider the beam $\mathrm{B} = \{\boldsymbol{x} \in \mathbb{R},\ 0 \leq x \leq l, 0 \leq y \leq h_1,$ $0 \leq z \leq h_2\}$ of constant, rectangular cross section. Let the beam be reinforced by a finite number of spherical particles $\mathrm{b}^{\varsigma} := \bigcup_{n=1}^{N} \mathrm{b}_n^{\varsigma} \subset \mathrm{B}$ with center $\boldsymbol{x}_{0n}$ and radius r_n of the n^{th} particle. Here, $\varsigma > 0$ is a scale parameter allowing to zoom in or zoom out the scale at which the composite is studied.

In order to be able to develop a consistent theory for the beam, we are going to accept the following assumptions.

Assumption 1. Suppose that $\mathrm{B}_b^{\varsigma} := \mathrm{B} \setminus \mathrm{b}$ is connected.

Assumption 2. We assume that both b^{ς} and B_b^{ς} are isotropic, linear elastic, homogeneous and free of all types of defects and voids.

Assumption 3. During the deformation of the beam, particles do not interact mechanically, meaning that for any $n_1 \neq n_2$,

$$\mathrm{b}_{n_1}^{\varsigma} \cap \mathrm{b}_{n_2}^{\varsigma} = \varnothing. \tag{1.1}$$

Assumption 4. For the sake of simplicity, the consideration is limited by infinitesimal strains such that for any $n_1 \neq n_2$,

$$\varepsilon_{ij} \ll \operatorname{dist}\left(\mathrm{b}_{n_1}^{\varsigma}, \mathrm{b}_{n_2}^{\varsigma}\right).$$

Here, ε_{ij} are the components of the strain tensor of the beam, $\operatorname{dist}(\cdot, \cdot)$ measures the distance in $\mathbb{R}^3$.

In addition to Assumptions above, with respect to the beam, we accept the Timoshenko assumptions [21].

It is important to emphasize that when changing the scale parameter, the volume fraction of the particles remains constant. In other words, by decreasing ς, we change only the visual representation of the composite corresponding to the current scale and not its geometric configuration.

1.1 Beam equations at micro-scale

Assume that the beam is subjected to an axial load of constant intensity F acting at the end of the beam, while its other end is cantilevered. The axial load is distributed uniformly over the whole end-section of the beam, so that, without losing the generality, we assume uniform displacement field over the width of the beam (see Figure 1).

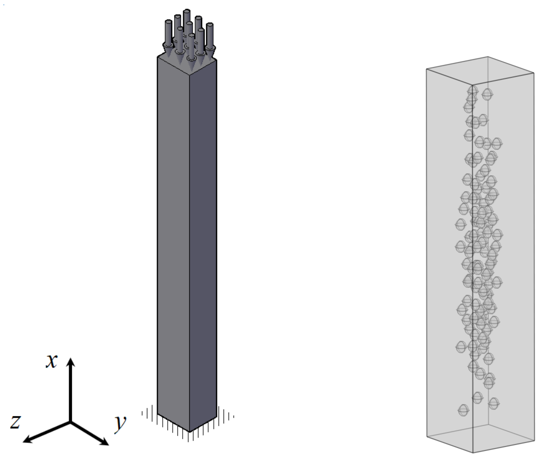

Figure 1: Schematic representation of the beam reinforced by spherical particles

Timoshenko assumptions lead to the following non-zero components of the strain tensor:

$$\varepsilon_{xx}^{\varsigma}(x,y,z) = -z\frac{\partial \varphi^{\varsigma}}{\partial x}, \quad \varepsilon_{xz}^{\varsigma}(x,y,z) = \frac{1}{2}\left(\frac{\partial w^{\varsigma}}{\partial x} - \varphi^{\varsigma}(x)\right).$$

Moreover, since the beam is isotropic, the non-zero stress components will be

$$\sigma_{xx}^{\varsigma}(x,y,z) = E^{\varsigma}(x,y,z)\,\varepsilon_{xx}^{\varsigma}(x,y,z), \quad \sigma_{xz}^{\varsigma}(x,y,z) = 2\mu^{\varsigma}(x,y,z)\,\varepsilon_{xz}^{\varsigma}(x,y,z).$$

Here, E^{ς} and μ^{ς} are the Young and shear moduli of the beam. At that,

$$\mu^{\varsigma} = \frac{E^{\varsigma}}{2(1+\nu)}.$$

Substituting the strain, the bending moment and the shear force are defined as [21]

$$M_{xx}^{\varsigma}(x) = \int\limits_A z\sigma_{xx}^{\varsigma}(x,y,z)\,\mathrm{d}A = -\frac{\partial \varphi^{\varsigma}}{\partial x}\int\limits_A z^2 E^{\varsigma}(x,y,z)\,\mathrm{d}A = -E_2^{\varsigma}(x)\frac{\partial \varphi^{\varsigma}}{\partial x}, \tag{1.2}$$

$$\begin{aligned} Q_x^{\varsigma}(x) &= \kappa\int\limits_A \sigma_{xz}^{\varsigma}(x,y,z)\,\mathrm{d}A = \frac{\kappa}{2(1+\nu)}\left(\frac{\partial w^{\varsigma}}{\partial x} - \varphi^{\varsigma}\right)\int\limits_A E^{\varsigma}(x,y,z)\,\mathrm{d}A = \\ &= \frac{\kappa}{2(1+\nu)}E_0^{\varsigma}(x)\left[\frac{\partial w^{\varsigma}}{\partial x} - \varphi^{\varsigma}\right], \end{aligned} \tag{1.3}$$

where

$$E_k^{\varsigma}(x) = \int\limits_A z^k E^{\varsigma}(x,y,z)\,\mathrm{d}A, \quad k = 0, 2. \tag{1.4}$$

Substituting (1.2) and (1.3) into the equilibrium equations of the Timoshenko beam,

$$\frac{\partial M_{xx}^{\varsigma}}{\partial x} - Q_x^{\varsigma} = 0, \qquad \frac{\partial Q_x^{\varsigma}}{\partial x} + F\frac{\partial w^{\varsigma}}{\partial x} = 0,$$

we get

$$\begin{aligned} &\frac{\kappa}{2(1+\nu)}\frac{\partial}{\partial x}\left[E_0^{\varsigma}(x)\left(\frac{\partial w^{\varsigma}}{\partial x} - \varphi^{\varsigma}\right)\right] = 0,\\ &F\frac{\partial w^{\varsigma}}{\partial x} + \frac{\partial}{\partial x}\left[E_2^{\varsigma}(x)\frac{\partial \varphi^{\varsigma}}{\partial x}\right] + \frac{\kappa}{2(1+\nu)}E_0^{\varsigma}(x)\left[\frac{\partial w^{\varsigma}}{\partial x} - \varphi^{\varsigma}\right] = 0. \end{aligned} \tag{1.5}$$

Here, κ is the Timoshenko shear factor. The cross section of the beam is assumed to be uniform, so that κ is considered to be constant. Hereinafter, it is assumed that [22]

$$\kappa = \frac{10(1+\nu)}{12+11\nu}.$$

Taking into account the microstructure of the beam, its Young's modulus at microscale, can be represented as

$$E^{\varsigma}(x,y,z) = \begin{cases} E_b, & (x,y,z) \in \mathrm{B}_b^{\varsigma},\\ E_p, & (x,y,z) \in \mathrm{b}^{\varsigma}, \end{cases}$$

where E_b and E_p are the Young's moduli of B_b^{ς} and b^{ς}. Moreover, using the definition of the characteristic function $\chi_{\mathrm{b}^{\varsigma}}$, we may write

$$E^{\varsigma}(x,y,z) = E_b\chi_{\mathrm{B}_b^{\varsigma}}(x,y,z) + E_p\chi_{\mathrm{b}^{\varsigma}}(x,y,z). \tag{1.6}$$

1.2 Beam equations at meso-scale

In the context of this paper, we will consider the case $\varsigma = \dfrac{r}{h}$ with $h = \min(h_1, h_2)$. Then, using the theory developed in recent papers [18, 19], we can prove that the meso-limit of system (1.5) with coefficients derived from (1.6) is the system

$$\begin{aligned} &\frac{\kappa}{2(1+\nu)}\frac{\partial}{\partial x}\left[E_0^{0}(x)\left(\frac{\partial w^{\varsigma}}{\partial x} - \varphi^{0}\right)\right] = 0,\\ &F\frac{\partial w^{0}}{\partial x} + \frac{\partial}{\partial x}\left[E_2^{0}(x)\frac{\partial \varphi^{0}}{\partial x}\right] + \frac{\kappa}{2(1+\nu)}E_0^{0}(x)\left[\frac{\partial w^{0}}{\partial x} - \varphi^{0}\right] = 0, \end{aligned} \tag{1.7}$$

where E_0^0 and E_2^0 are defined exactly as in (1.4) but using the following expression for E^0:

$$E^0(x,y,z) = (1-\phi_p)E_b + \phi_p \cdot \frac{E_p \mathrm{V_B}}{N}\sum_{n=1}^{N}\delta(x-x_{0n})\,\delta(y-y_{0n})\,\delta(z-z_{0n}).$$

Here, δ is the Dirac function, V_B is the volume of the beam, ϕ_p is the constant volume fraction of particles. See [18, 19] for details.

2 Numerical analysis of a cantilever beam subject to axial load

In order to make sure that solution to (1.5) converges to solution to (1.7), we involve the numerical method of finite elements. The beam is discretized into tetrahedral elements as shown in Figure 2. Note that the beam material is considered to be made from copper, whereas the material of the particles is made from steel.

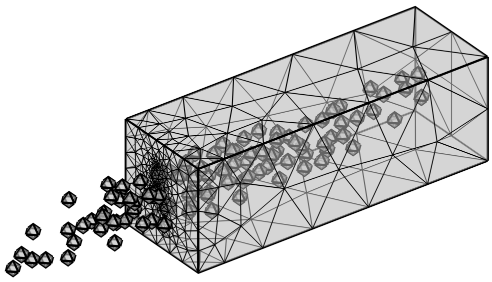

Figure 2: Finite element discretization of the beam and particles

Then, the displacement field of the beam described by (1.5) is captured for $\varsigma = 0.075$, 0.05 and when $\varsigma \to 0$. The latter case is modelled using the discretization of (1.7).

Figures 3-5 show the evolution of the normal displacement u_3^ς, normal stress σ_{xx}^ς and tangential stress σ_{xz}^ς when $\varsigma = 0.075$, 0.05 and when $\varsigma \to 0$.

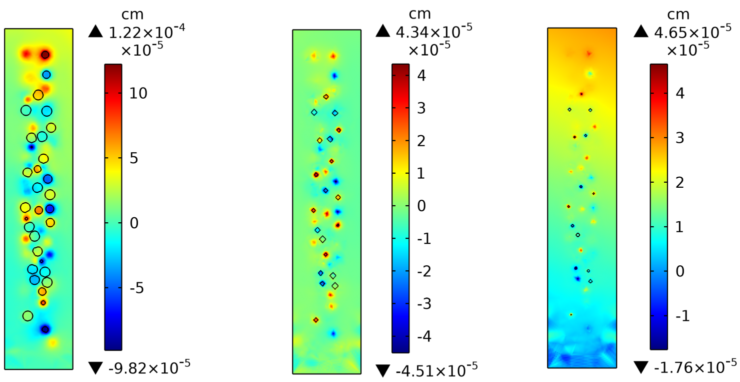

Figure 3: Distribution of w^ς

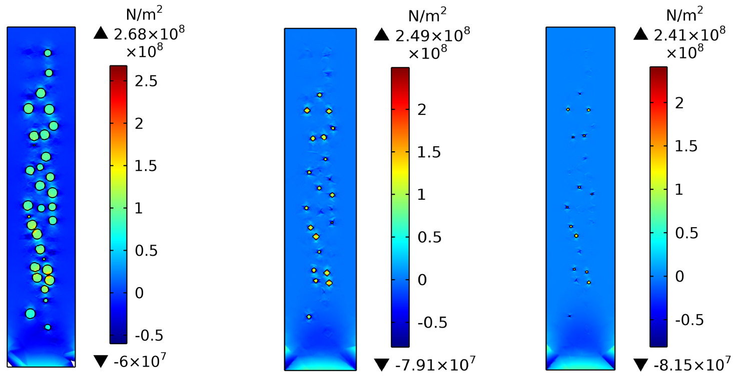

Figure 4: Distribution of σ^{ς}_{xx}

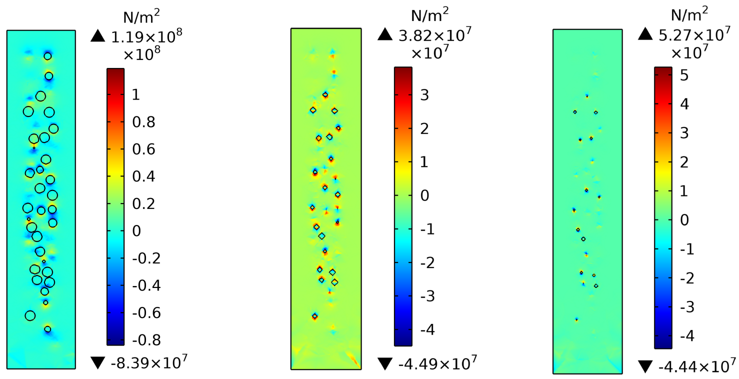

Figure 5: Distribution of σ^{ς}_{xz}

It is evident from the corresponding values of the quantities shown on plots above that as $\varsigma \to 0$

$$w^{\varsigma} \searrow w^{0}, \quad \sigma^{\varsigma}_{xx} \nearrow \sigma^{0}_{xx}, \quad \sigma^{\varsigma}_{xz} \searrow \sigma^{0}_{xz}.$$

3 Conclusions

In this study, we derive explicit dependence of PR Timoshenko beam stiffness on particles distribution and material properties giving enough theoretical base for optimal design of such beams by a proper choice of particles distribution. The method of multi-scale modelling allows to derive the meso-scale model of the beam from corresponding micro-scale model when the scale parameter describing the ratio of the particle radius to the beam thickness tends to zero. Using the numerical method of finite elements, the stress components and the displacement field of the beam are shown to converge to the corresponding quantities at meso-scale.

The model is important for many applications including, e.g., derivation of materials with improved target properties and material or structural optimization of PRC structures.

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