ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 73, №3, 2020 Механика
УДК 539.376 Doi- http://doi.org/10.33018/73.3.2

On the main directions in creep mechanics of metallic materials

Holm Altenbach, Katharina Knape

Keywords: Creep, Relaxation, Classical Models, Advanced Creep Theories

Хольм Альтенбах, Катарина Кнапе
Об основных направлениях теории ползучести металлических материалов

Ключевые слова: Ползучесть, релаксация, классические модели, современные теории ползучести

Механика ползучести является частью инженерной механики. Развитие исследований в этой области было мотивировано некоторыми авариями, произошедшими в 19-ом веке. Первые теории были сформулированы как одномерные уравнения с несколькими параметрами. Позже эти уравнения были распространены на трёхмерные уравнения, при этом скалярные напряжения и деформации были заменены тензорными выражениями. Кроме того, для лучшего сравнения с одномерными результатами должны использоваться гипотезы эквивалентности для напряжений и деформаций.

До сих пор не существует теории ползучести, которая была бы настолько строгой, как механика сплошных сред. Однако существует много инженерных теорий, на основе которых получены очень и очень много решений для практических задач. Эта статья представляет собой обзор работ по механике ползучести металлических материалов и конструкций из этих материалов.

Հոլմ Ալտենբախ, Կատարինա Կնապե
Մետաղական նյութերի սողքի տեսության հիմնական ուղղությունների մասին
Հիմնաբառեր. Սողք, ռելաքսացիա, դասական մոդել, սողքի ժամանակակից տեսություններ

Սողքի մեխանիկան ճարտարագիտական մեխանիկայի մի մասն է: Այս ոլորտում հետազոտությունների զարգացումը պայմանավորված էր 19-րդ դարում տեղի ունեցած պատահարներով: Առաջին տեսությունները ձևակերպվել են որպես միաչափ հավասարումներ՝ մի քանի պարամետրերով: Հետագայում այս հավասարումները ընդհանրացվեցին եռաչափ հավասարումներ համար, փոխարինելով սկալյար լարումները և դեֆորմացիաները տենզորային արտահայտություններով: Բացի այդ, միաչափ արդյունքների հետ առավել լավ համեմատություն կատարելու նպատակով լարումների և դեֆորմացիաների համար պետք է օգտագործվեն համարժեքության վարկածներ:

Մինչ այժմ գոյություն չունի մի այնպիսի սողքի տեսություն, որը հոծ միջավայրի մեխանիկայի պես խիստ լինի: Այնուամենայնիվ, կան շատ ճարտարագիտական տեսություններ, որոնց հիման վրա ստացվել են շատ ու շատ կիրառական ծնդիրների լուծումներ: Այս հոդվածը իրենից ներկայացնում է ակնարկ մետաղական նյութերի և նրանցից պատրաստված կառուցվածքների սողքի մեխանիկայի վերաբերյալ:

Creep mechanics is a part of engineering mechanics. The developments in this research field were motivated by some failure cases in the 19th century. The first theories have been formulated as uniaxial equations with only a few parameters. Later, these equations were extended to three-dimensional equations, substituting the scalar stress and strain by tensorial expressions. In addition, for better comparison to one-dimensional results, equivalent statements for the stresses and the strains were introduced.

Up to now, there is no creep mechanics theory which is as strict as continuum mechanics. However, there are many engineering theories through which more and more solutions for practical cases can be obtained. The paper is a state of the art report of creep mechanics for metallic materials and structures composed from these materials.

1 Introduction

Creep mechanics is a part of engineering mechanics with a history of more than 100 years and numerous technical applications. In the first part of this paper, the motivation and a brief outline to the history are presented. After that, some approaches in creep mechanics are discussed. In the final part, references to several applications are given. For further reading, [1,2] can be recommended.

In the case of three-dimensional relationships, the direct (symbolic) tensor notations are used. The basics are presented, for example, in [2-4].

1.1 Motivation

Mechanics has been established as a science since antiquity. The first steps in this field were done by Archimedes of Syracuse (born c. 287 BC, Syracuse, Sicily, died c. 212 BC, Syracuse) which today are known as the Archimedes' principle, Archimedes' screw, hydrostatics, levers, infinitesimals, ... But the description of the mechanics' purpose that we are familiar with nowadays only began with the corresponding developments in mathematics during the 17th and 18th century (for example differential and integral calculus). Mechanics is often seen in connection with physics, as a branch of physics but at the same time, there is still a perception that mechanics can be viewed as an application for mathematical theories [5]. This is not correct any more, ever since the development of engineering mechanics for application purposes. Today, engineering mechanics can be seen as an independent scientific discipline, which in a special way, based on a theoretical foundation (increasingly formulated axiomatically), brings the problems of engineering practice to a solution.

If one investigates creep problems, three questions arise:

- A suitable material description must be found first of all. This task is not trivial, since the different concepts, based on considerations of material physics, materials science and continuum mechanics have advantages and disadvantages. It is important to ensure that the effort and benefit are in an appropriate relationship and that the identification of the parameters in the equations describing the material behavior can be solved in a satisfactory manner. It should be noted that not every conceivable experiment to determine material parameters can actually be carried out in the laboratory.
- Another problem is related to the fact that a suitable structural mechanical description must be made. Components are geometrically complex structures. Hence, their geometrical description and the structural mechanical implementation are usually associated with the introduction of models. These simplify reality and thus enable practical problems to be analyzed with less effort. However, it must be clarified if the use of certain models is allowed within the limits of the simplifications. For example, it is known that thin-walled components (they are typical for creep mechanics applications) can often be analyzed with two-dimensional equations. Which theory has to be used e.g. in the case of plates (Kirchhoff, Mindlin, Reissner, Ambartsumyan, von Kármán, ...) has been the subject of numerous studies.

- The third problem - the selection of a suitable numerical analysis method (finite element method, boundary element method, etc.) - is also important, but it is not supposed to be content of this paper. For further reading, [6] is recommended.

In this work, the focus is on creep of components, where typical thin- and thick-walled elements, which can be modeled as beams, plates, pipes, pipe bends, etc., are examined. These components can be found in, for example, power plants and in chemical apparatuses, where moderate mechanical loads but increased operating temperatures are typical. The material behavior of metals and corresponding alloys (these are the main used construction materials) is then characterized by irreversible time-dependent creep processes and material degradation. The long-term behavior is influenced by mechanisms of the time-dependent stress redistribution and the increasing damage, which occur especially in the areas of joints, connecting elements and welds. The continuum damage mechanics, that establishes the constitutive equations for the tensor of creep rates and the evolution equations for the phenomenological damage variables, leads to a nonlinear initial boundary value problem for structural mechanical analysis [7].

Creep analyses are particularly important at those points in a component where several parts have to be connected. Welded connections are often preferred, however this is a complex problem since the different creep behaviors in the base material, in the weld metal and in the heat affected zone have to be taken into account. In [8], it is reported about the lifespan increase in a petrochemical plant with respect to the creep behavior. The pipeline system considered was analyzed with the help of the methods of classic structural mechanics, however, only taking into account the change in thickness and ovality in the pipe bends lead to the additional effects and adjusted the values for the stresses and strains to the values observed in the experiment or in practice. A possible classification of the creep damage and the corresponding measures in operating systems is given in [9]. The starting point is the creep strain-time curve known from materials science. To determine the time-dependent changes of state of the structure, micrographs of the material are generated and evaluated. The following stages can be observed: (A) isolated cavities, (B) oriented cavities, (C) micro cracks, (D) macro cracks, (E) fracture. The following actions should be performed: (A) only observation, (B) observation with fixed inspection intervals, (C) limited operating time until repair, (D) repair. The last stage (E) should always be avoided.

1.2 History

A historical overview of creep mechanics is given for example in [10,11]. There were publications on creep mechanics already in the second half of the 19th century. However, systematic investigations were summarized for the first time in [12,13]. Until today, the Norton-Bailey law can be described as the most important creep law [14,15], which is a power law in the sense of mathematics. Even in this case, it can be shown that the material description in the creep range requires more effort in comparison to the elastic range. The uniaxial Hooke's law contains only one material parameter (elastic or Young's modulus) whereas two parameters are necessary for the Norton-Bailey law (magnitude and creep exponent). Various industrial applications from energy machine construction were initially in the focus of creep mechanics. Already in 1933, Stodola reported about applications in the area of gas turbine construction [16]. Because generally, mechanical loads are multidimensional, the stress and the strain states need to be, too. Therefore, Odqvist (1933-1936) [17] and Bailey [15] suggested a corresponding theory for isotropic material behavior using invariants of the stress and the strain tensors. A consistent tensorial description was made by Prager (1945) and Reiner (1945), which also includes anisotropy. Missing matches with experimental results led to the development of further modifications of the creep equations, e.g. the strain hardening theory presented by Nadai (1938) [18] and Soderberg

(1938) [19]. Due to applications related with stability problems, e.g. discussed by Hoff [20,21], elements of the geometrically nonlinear theory had to be developed. A new class of problems arose with the massive use of polymer materials, where analogies between viscoelasticity and creep can be seen. However, the viscoelastic behavior is often mathematically described with the help of integral equations, see Rabotnov (1948) [22] among others. Another application field is creep in concrete which was studied, for example by N.K. Arutyunyan [23].

There are numerous textbooks and monographs on creep mechanics which mainly contain established research results. The authors prefer the engineering (inductive) approach meaning that based on experimental observations, creep equations in the simplest form are suggested and generalized step by step. It seems that there is no book that represents creep mechanics as strict as continuum mechanics (for the elasticity theory or plasticity theory there are several).

For studying creep mechanics, the following books can be recommended [2,6,10,24-36]. While reading, one notices that theories for static (or quasi-static) applications in the case of monotonous loads and under isothermal conditions are well established. Dynamic loads and their consequences for the creep behavior are under discussion and require further research.

Up to now, creep mechanics is in the focus of many research teams worldwide, selected conferences include presentations about the topics and activities devoted to creep problems. The International Union for Theoretical and Applied Mechanics (IUTAM) organises an IUTAM symposium *Creep in Structures* since 1960 every 10 years: 1960 - Stanford/U.S.A. [37], 1970 - Göteborg/Sweden [38], 1980 - Leicester/U.K. [39], 1990 - Kraków/Poland [40] and 2000 - Nagoya/Japan [41]. The rather long time interval between two symposia is relatively easy to explain: the research is related to time-dependent processes and their verification in the field of metals and alloys needs long-term tests over several years. Only recently, due to the arising questions concerning creep problems of plastics and composite materials, shorter time periods for experiments are possible, which was also accepted during the Nagoya meeting in 2000. However, this series of symposia is continued, but caused by some technical issues, the next after Nagoya was only held in 2012 in Paris/France [42].

There are more special conferences and courses devoted to selected creep problems, for example [43-45]. The International Association for Applied Mathematics and Mechanics (GAMM) offered plenary lectures and in 2001, one of the main topics was dedicated to creep mechanics [46]. In the next few years, further impulses for research are mainly expected from the following areas: power plant construction, aircraft construction and microsystem technology. The first two represent traditional fields of application. Since the performance and efficiency improvement leads to a further rising of temperature, the tendency to creep and damage also increases. In microsystem technology, the influence of temperature also cannot be neglected, like the different reactions of various materials to the loads and temperatures. In addition, in microsystem technology, all components are arranged in a confined space, so that their interactions become interesting as well.

2 Basic Model

The creep behavior is always analyzed from two different perpectives: of the material and of the component. Material creep always includes creep and creep recovery processes which are both accompanied by time-dependent microstructural modifications in the material as a result of moderate mechanical loads (below the yield limit) at elevated temperatures (> 0.3 of the melting temperature T_{m}). The creep in components is also a time-dependent process leading to changes in strain and stress states. One differentiates among other things between creep, recovery and stress redistribution. For this, multi-axial, inhomogeneous stress states are typical.

2.1 Description Possibilities

There are different ways to describe the creep behavior. Considerations of the materials science and physics, macroscopic observations or continuum mechanical methods are the starting point. Until today, special phenomenological approaches based on macroscopic experimental observation remain the beginning of the models. This concept is already described in [12] and still dominates the literature on structural mechanical analyses. The advantages and disadvantages of different description possibilities should be evaluated thoroughly. For example, material science approaches and creep equations based on them are most suitable to characterize processes at the microlevel. At the same time, three-dimensional generalization is often complicated because the necessary equivalent stress concepts only rely on engineering ideas, meaning there is no overriding principle in analogy to the balance equations, etc. The phenomenological description within engineering mechanics is not always strict enough to meet all aspects of the modeling requirements. On the other hand, the corresponding three-dimensional creep equations can be easily implemented into existing commercial finite element codes. A strict continuum mechanics formulation should be preferred but at the moment, we are far from solving this problem of including it into the existing calculation software.

2.2 Three Creep Stages

Creep curves show strains at constant mechanical load over time where the small elastic range is often neglected. Three stages of creep behavior can be observed. The first one (primary or delayed creep) is characterized by a decline in the slope of the creep curve, that means the creep rate decreases. This correlates with microstructural observations, since hardening occurs due to obstruction of dislocation movements. One can also observe relaxation, i.e. rearrangement of lattice defects. The subsequent secondary (or stationary) creep is indicated by an equilibrium of hardening and softening. The resulting creep rate takes a stationary value, which is also its minimum. The tertiary (or accelerated) creep is particularly denoted by damage (formation, growth and coalescence of cavities at the grain boundaries, microstructure aging, etc.). This rough classification, which can be found, for example, in [47-49] is a suitable basis for the formulation of phenomenological models of creep mechanics. Note that temperature effects are either neglected or, for the simplest models, are assumed to be constant because the temperature dependency is often very complex. In addition, the form of these three stages varies for different materials. Generally, in many cases, the secondary stage lasts significantly longer in comparison to the primary and tertiary creep ranges but there are also materials without any secondary creep range [50-53] (only the minimum value of the creep curve can be estimated). Regardless of the arguments given for classification and simplification, creep curves usually provide trustworthy statements for uniaxial creep tests, presuming material isotropy.

3 Extensions

In the case of extending the classical one-dimensional models, two aspects should be considered. Since the loading state is generally three-dimensional, a suitable three-dimensional description of the material behavior is needed. In addition, the anisotropies in material behavior are also important for creep processes. There are originally isotropic materials which may show anisotropic behavior (damage-induced anisotropy) in the tertiary creep stage but there are also à priori anisotropic materials (initial anisotropy).

The simplest creep test is an analogy to the tensile test for which only a normal stress is considered in the tensile direction resulting from a constant load (force). Another simple creep test is the torsion test, where a shear stress can be noticed as a result of a constant acting torsional moment. Both tests can be superposed, in this case, one gets a complex stress state described by the following stress tensor $\boldsymbol{\sigma}$ [2]

$$\boldsymbol{\sigma} = \sigma \mathbf{k} \otimes \mathbf{k} + \tau\left(\mathbf{e}_{\varphi} \otimes \mathbf{k} + \mathbf{k} \otimes \mathbf{e}_{\varphi}\right) \tag{3.1}$$

where $\boldsymbol{k}$ is a unit vector in the normal creep direction and $\boldsymbol{e}_{\varphi}$ denotes a unit vector in the circumferential direction. σ and τ are the normal stress in the $\boldsymbol{k}$-direction and the shear stress in the circumferential direction $\boldsymbol{e}_{\varphi}$, $\otimes$ is dyadic product. The inelastic material behavior is often assumed to be independent from the hydrostatic stress state. In addition to the stress tensor, the stress deviator $\boldsymbol{s}$ needs to be introduced

$$\boldsymbol{s} = \sigma\left(\boldsymbol{k} \otimes \boldsymbol{k} - \frac{1}{3}\boldsymbol{E}\right) + \tau\left(\boldsymbol{e}_{\varphi} \otimes \boldsymbol{k} + \boldsymbol{k} \otimes \boldsymbol{e}_{\varphi}\right) \tag{3.2}$$

with $\boldsymbol{E}$ as the second rank unit tensor. Furthermore, to ensure a better comparison of one-dimensional and three-dimensional states, an equivalence hypothesis should be found. In the simplest case, the von Mises equivalent stress can be suggested as

$$\sigma_{\mathrm{vM}} = \sqrt{\frac{3}{2}\boldsymbol{s} \cdot\cdot \boldsymbol{s}} = \sqrt{\sigma^2 + 3\tau^2} \tag{3.3}$$

with $\boldsymbol{s}$ as the stress deviator and the double scalar product $\cdot\cdot$. Considering the three creep stages, the uniaxial description is made based on the secondary stage with the following approach

$$\dot{\varepsilon}^{\mathrm{cr}}_{\min} = f(\sigma, T) \tag{3.4}$$

$\dot{\varepsilon}^{\mathrm{cr}}_{\min}$ is the minimal creep rate, σ is the existing stress responsible for the creep and T is the temperature which is assumed to be constant in order to simplify the model. The experimental verification is straightforward. From the literature (for example, [2]) several approximations for the function of the minimal creep strain rate are known. The power law is the one used the most. However, there are also reasons to use other approximations like the exponential function or a hyperbolic sine function.

The ansatz for the secondary creep can be extended by a hardening term (H denotes a hardening variable)

$$\dot{\varepsilon}^{\mathrm{cr}} = f(\sigma, H, T) \tag{3.5}$$

completed with an evolution equation

$$\dot{H} = \dot{H}(\sigma, H, T) \tag{3.6}$$

For tertiary creep, a damage variable ω can be introduced

$$\dot{\varepsilon}^{\mathrm{cr}} = f(\sigma, H, \omega, T) \tag{3.7}$$

and a damage evolution equation is postulated

$$\dot{\omega} = \dot{\omega}(\sigma, H, \omega, T) \tag{3.8}$$

The procedure presented here is not limited to uniaxial behavior. By introducing suitable tensor variables for the stress and the strain rate as well as corresponding equivalent variables, multiaxial constitutive and evolution laws can be established. It should still be noted that equivalence concepts for the stresses and the strains are always just engineering hypothesises, therefore, any concept and potential modifications must be examined again to determine whether the assumptions made are valid. More information on equivalence hypotheses is presented in [54,55]. The procedure is not limited to only one hardening variable and one damage variable, which could only be assigned to one mechanism.

The concept has to be changed, if the anisotropy must be included since anisotropy tensors should be justified and described. For the damage-induced anisotropy, evolution laws have to be added and presented mathematically in a proper manner. Models become very complex and the identification effort is increasing dramatically.

The derivation of the basic equations for isotropic creep behavior can also be performed as follows. Assuming constant temperature and constant or slightly variable loads, the infinitesimal creep rates are first introduced as tensor quantities

$$\dot{\boldsymbol{D}}^{cr} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{cr}} = \boldsymbol{f}(\boldsymbol{\sigma}) \tag{3.9}$$

$\boldsymbol{f}$ is an arbitrary second rank tensor function. The potential hypothesis with the creep condition [56]

$$\dot{\boldsymbol{D}}^{cr} = \frac{\partial W(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \tag{3.10}$$

and the creep potential W, taking into account the dissipation power

$$P = \dot{\boldsymbol{D}}^{cr} \cdot\cdot\, \boldsymbol{\sigma} \geq 0$$

then leads to a general isotropic creep equation. In addition, the isotropy conditions must be considered

$$\begin{aligned} \boldsymbol{f}(\boldsymbol{Q}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{Q}^{\mathrm{T}}) &= \boldsymbol{Q}\cdot\boldsymbol{f}(\boldsymbol{\sigma})\cdot\boldsymbol{Q}^{\mathrm{T}}, \\ W(\boldsymbol{Q}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{Q}^{\mathrm{T}}) &= W(\boldsymbol{\sigma}), \end{aligned} \qquad \forall \boldsymbol{Q}: \boldsymbol{Q}\cdot\boldsymbol{Q}^{\mathrm{T}} = \boldsymbol{E}, \quad det\boldsymbol{Q} = \pm 1 \tag{3.11}$$

$\boldsymbol{Q}$ is an orthogonal tensor. In the case of isotropy, $\boldsymbol{Q}$ is

$$\boldsymbol{Q} = \boldsymbol{E}\cos\omega + \boldsymbol{e}\otimes\boldsymbol{e}(1-\cos\omega) + \boldsymbol{e}\times\boldsymbol{E}\sin\omega$$

where $\boldsymbol{e}$ denotes the unit vector along an arbitrary axis of rotation with the arbitrary angle ω. Finally, the following tensorial non-linear creep equation can be established

$$\dot{\boldsymbol{D}}^{cr} = \boldsymbol{f}(\boldsymbol{\sigma}) = \dot{\alpha}_0\,\boldsymbol{E} + \dot{\alpha}_1\,\boldsymbol{\sigma} + \dot{\alpha}_2\,\boldsymbol{\sigma}^2, \qquad \dot{\alpha}_i = \dot{\alpha}_i(I_1, I_2, I_3), \tag{3.12}$$

with the invariants

$$I_1 = \mathrm{tr}\boldsymbol{\sigma}, \quad I_2 = \mathrm{tr}\boldsymbol{\sigma}^2, \quad I_3 = \mathrm{tr}\boldsymbol{\sigma}^3 \tag{3.13}$$

Assuming $W = W(I_1, I_2, I_3)$,

$$\dot{\boldsymbol{D}}^{cr} = \frac{\partial W}{\partial I_1}\boldsymbol{E} + 2\frac{\partial W}{\partial I_2}\boldsymbol{\sigma} + 3\frac{\partial W}{\partial I_3}\boldsymbol{\sigma}^2 \tag{3.14}$$

can be established. $\mathrm{tr}(...)$ denotes the trace of the tensor. The integrity condition for the $\dot{\alpha}_i$ is presented in [33].

The chosen set of invariants is not the only possible one. One can prove that for second rank tensors three linearly independent invariants exist. The different sets of invariants are discussed in the literature (e.g. [56,57]). It can be shown that from the mathematical point of view or material theory, a preference for a certain set of invariants cannot be justified. However, one can easily recognize that the form of presentation has consequences for the experimental verification of the parameters in the constitutive equations. So, the question arises if certain invariants have constructive interpretations for the experiment. It is obvious that the first invariant of the stress tensor relates to the hydrostatic stress state. Same thing applies to the equivalent stress according to von Mises, which is connected to the second invariant of the stress deviator. Unfortunately, the meaning of the third invariant is not straightforward. In this case, it is better to consider the Lode parameter [58] which has a practical meaning behind it. The considerations for the invariants are trivial in the case of isotropy. Introducing other constitutive laws regarding transverse isotropy or orthotropy, such considerations may help to structure the necessary experimental work since a lot of information can be drawn from the theory for planning the tests [34]. The general form of the isotropic creep law mentioned here is also given in [25,33].

Some special but yet elementary cases can be deduced. Using the separation of the stress tensor into hydrostatic and a deviatoric part

$$\boldsymbol{\sigma} = \sigma_{\mathrm{m}}\mathbf{E} + \mathbf{s}, \quad \mathrm{tr}\mathbf{s} = 0 \ \Rightarrow \quad \sigma_{\mathrm{m}} = \frac{1}{3}\mathrm{tr}\boldsymbol{\sigma} = \frac{1}{3}I_1, \tag{3.15}$$

the following representation of the potential is valid4

$$W = W(I_1, J_{2\mathrm{D}}, J_{3\mathrm{D}}), \quad J_{2\mathrm{D}} = -\frac{1}{2}\mathrm{tr}\boldsymbol{s}^2, \quad J_{3\mathrm{D}} = -\frac{1}{3}\mathrm{tr}\boldsymbol{s}^3 \tag{3.16}$$

Since the set of possible invariants is exchangeable, the invariants of the corresponding stress deviator can be used instead of the stress tensor's. That means the last representation of the potential is equivalent to $W(I_1, I_2, I_3)$. Then the tensor of creep strain rates can be expressed

$$\dot{\boldsymbol{D}}^{cr} = \frac{\partial W}{\partial I_1}\boldsymbol{E} - \frac{\partial W}{\partial J_{2\mathrm{D}}}\boldsymbol{s} + \frac{\partial W}{\partial J_{3\mathrm{D}}}\left(\boldsymbol{s}^2 - \frac{1}{3}\mathrm{tr}\boldsymbol{s}^2\boldsymbol{E}\right) \tag{3.17}$$

It is obvious that the assumption of classic material behavior (inelastic behavior is not influenced by the hydrostatic stress state [59,60]) can be considered directly: the dependence on the first invariant should be eliminated

$$\mathrm{tr}\dot{\boldsymbol{D}}^{cr} = 3\frac{\partial W}{\partial I_1} = 0 \tag{3.18}$$

The classic creep equations will result, if the following assumption is still valid

$$\frac{\partial W}{\partial J_{3\mathrm{D}}} = 0 \tag{3.19}$$

In this case, no more tensorial non-linearity is considered in the creep law and one gets

$$\dot{\boldsymbol{D}}^{cr} = \frac{3}{2}\frac{\dot{\varepsilon}_{\mathrm{vM}}}{\sigma_{\mathrm{vM}}}\boldsymbol{s}, \quad \sigma_{\mathrm{vM}}^2 = -3J_{2\mathrm{D}}, \quad \dot{\varepsilon}_{\mathrm{vM}} \equiv \frac{\partial W(\sigma_{\mathrm{vM}})}{\partial \sigma_{\mathrm{vM}}} \tag{3.20}$$

This expression does not differ from [11,25,31]. For application purposes, it is important that the latter law is not the only possible expression. For porous materials and materials with a similar microstructure, it is better to also consider the first invariant in order to adapt the results closer to the experiment. For materials that show tensorial nonlinear behavior as well as so-called second-order effects (which cannot be neglected), the inclusion of the third invariant is useful. It can be seen that the constitutive equations discussed here can also be used if large deformations occur. In this case, a suitable choice of the strain and the stress tensor and the time derivative is not trivial [61].

3.1 Anisotropy

There are various application examples for anisotropic creep behavior. These include the fiber-reinforced materials (see e.g. [62,63]) and the single crystal alloys [64,65]. However, the description is associated with numerous problems. The presumed symmetries are difficult to verify, since the scatter of the measurement data may be up to 20%. The symmetries that appear during creep also depend on loads applied in the past and the damage already done [66]. There is also a clear temperature dependence (aluminum alloy D16AT - samples from rolled sheets: at 275°C anisotropic creep, at 300°C isotropic creep [67]).

Symmetry considerations in connection with the direct tensor notation allow an effective way to develop anisotropic creep equations in a simple manner. One has to distinguish between material and physical symmetries. Material symmetries are symmetries on the micro level (crystal symmetries in metals and alloys, symmetries as a result of the arrangement of fibers and particles, etc.) whereas physical symmetries are the symmetries of the constitutive equations and result from the experimental observations. Orthogonal tensors $\boldsymbol{Q}$ are used to describe them. The following symmetries occur most frequently: the transverse-isotropic symmetry and the orthotropy. In these cases, orthogonal tensors can be specified as follows. For a reflection, there is

$$\boldsymbol{Q}(\boldsymbol{n}) = \boldsymbol{E} - 2\boldsymbol{n} \otimes \boldsymbol{n} \tag{3.21}$$

with $\boldsymbol{n}$ as the normal to the mirror surface. For rotation follows

$$\boldsymbol{Q}(\varphi \boldsymbol{m}) = \boldsymbol{m} \otimes \boldsymbol{m} + \cos\varphi(\boldsymbol{E} - \boldsymbol{m} \otimes \boldsymbol{m}) + \sin\varphi \boldsymbol{m} \times \boldsymbol{E} \tag{3.22}$$

with the rotation axis $\boldsymbol{m}$. Both tensors are sufficient to characterize the corresponding symmetries.

Transverse-isotropic creep equations are discussed briefly below. The starting point is the condition

$$W\left(\boldsymbol{Q} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{Q}^{\mathrm{T}}\right) = W(\boldsymbol{\sigma}), \tag{3.23}$$

which should be proven for

$$\boldsymbol{Q}(\varphi \boldsymbol{m}) = \boldsymbol{m} \otimes \boldsymbol{m} + \cos\varphi(\boldsymbol{E} - \boldsymbol{m} \otimes \boldsymbol{m}) + \sin\varphi\, \boldsymbol{m} \times \boldsymbol{E} \tag{3.24}$$

with $-\pi < \varphi < \pi$, $\boldsymbol{m} = \mathrm{const}$, $\boldsymbol{m} \cdot \boldsymbol{m} = 1$. At first, one gets

$$(\mathbf{m} \times \boldsymbol{\sigma} - \boldsymbol{\sigma} \times \mathbf{m}) \cdot\cdot \frac{\partial W}{\partial \boldsymbol{\sigma}} = 0 \tag{3.25}$$

This differential equation yields the following characteristic system [68]

$$\frac{d\boldsymbol{\sigma}}{ds} = \mathbf{m} \times \boldsymbol{\sigma} - \boldsymbol{\sigma} \times \mathbf{m} \tag{3.26}$$

with the general solutions

$$\boldsymbol{\sigma}^k(s) = \mathbf{Q}(s\mathbf{m}) \cdot \boldsymbol{\sigma}_0^k \cdot \mathbf{Q}^T(s\mathbf{m}), \quad k = 1, 2, 3 \tag{3.27}$$

and the corresponding integrals lead to

$$\mathrm{tr}(\boldsymbol{\sigma}),\ \mathrm{tr}(\boldsymbol{\sigma}^2),\ \mathrm{tr}(\boldsymbol{\sigma}^3),\ \mathbf{m} \cdot \boldsymbol{\sigma} \cdot \mathbf{m},\ \mathbf{m} \cdot \boldsymbol{\sigma}^2 \cdot \mathbf{m},\ \mathbf{m} \cdot \boldsymbol{\sigma}^2 \cdot \left(\mathbf{m} \times \boldsymbol{\sigma} \cdot \mathbf{m}\right) \tag{3.28}$$

Such sets of transverse-isotropic invariants are also derived in [69]. From the mathematical viewpoint, the characteristic system only has 5 independent integrals [68]. In [70,71], it was shown that the six integrals introduced are not completely independent from one another.

The creep equations can be presented as follows. Firstly, the stress tensor should be splitted

$$\boldsymbol{\sigma} = \mathbf{m} \cdot \boldsymbol{\sigma} \cdot \mathbf{m}\mathbf{m} \otimes \mathbf{m} + \boldsymbol{\sigma}_{\mathbf{p}} + \boldsymbol{\tau}_{\mathbf{m}} \otimes \mathbf{m} + \mathbf{m} \otimes \boldsymbol{\tau}_{\mathbf{m}} \tag{3.29}$$

where $\boldsymbol{m}$ is the direction of the transverse isotropy (normal to the isotropy surface), $\boldsymbol{\sigma}_{\mathbf{p}}$ is the plane part of the tensor $\boldsymbol{\sigma}$ in the surface, which is orthogonal to the direction of the transversal isotropy, and $\boldsymbol{\tau}_{\mathbf{m}}$ is a shear stress vector. With the split of $\boldsymbol{\sigma}_{\mathbf{p}}$

$$\boldsymbol{\sigma}_{\mathbf{p}} = \mathbf{s}_{\mathbf{p}} + \frac{1}{2}\operatorname{tr}\boldsymbol{\sigma}_{\mathbf{p}}\left(\mathbf{E} - \mathbf{m}\otimes\mathbf{m}\right), \quad \operatorname{tr}\mathbf{s}_{\mathbf{p}} = 0 \tag{3.30}$$

the following invariants can be introduced

$$\begin{aligned} &I_{1\mathbf{m}} = \mathbf{m}\cdot\boldsymbol{\sigma}\cdot\mathbf{m}, \quad I_{2\mathbf{m}} = \frac{1}{2}\operatorname{tr}\boldsymbol{\sigma}_{\mathbf{p}}, \qquad I_{3\mathbf{m}} = \operatorname{tr}\mathbf{s}_{\mathbf{p}}^2, \\ &I_{4\mathbf{m}} = \boldsymbol{\tau}_{\mathbf{m}}\cdot\boldsymbol{\tau}_{\mathbf{m}}, \quad I_{5\mathbf{m}} = \boldsymbol{\tau}_{\mathbf{m}}\cdot\mathbf{s}_{\mathbf{p}}\cdot\boldsymbol{\tau}_{\mathbf{m}}, \quad I_{6\mathbf{m}} = \mathbf{m}\cdot\left(\boldsymbol{\tau}_{\mathbf{m}}\cdot\mathbf{s}_{\mathbf{p}}\times\boldsymbol{\tau}_{\mathbf{m}}\right) \end{aligned} \tag{3.31}$$

The constraint for the invariants yields

$$I_{6\boldsymbol{m}}^2 = I_{3\boldsymbol{m}}I_{4\boldsymbol{m}} - I_{5\boldsymbol{m}}^2 \tag{3.32}$$

The ansatz for the potential is

$$W = W\left(I_{1\boldsymbol{m}}, I_{2\boldsymbol{m}}, I_{3\boldsymbol{m}}, I_{4\boldsymbol{m}}, I_{5\boldsymbol{m}}\right) \tag{3.33}$$

Assuming, incompressibility one gets

$$\frac{\partial W}{\partial\boldsymbol{\sigma}}\cdot\cdot\boldsymbol{E} = 0 \ \Rightarrow\ W = W\left(2I_{1\boldsymbol{m}} - I_{2\boldsymbol{m}}, I_{3\boldsymbol{m}}, I_{4\boldsymbol{m}}, I_{5\boldsymbol{m}}\right) \tag{3.34}$$

with $2I_{1\mathbf{m}} - I_{2\mathbf{m}} = 3\mathbf{m}\cdot\boldsymbol{\sigma}\cdot\mathbf{m} - \operatorname{tr}\boldsymbol{\sigma} = 3\mathbf{m}\cdot\mathbf{s}\cdot\mathbf{m}$. In addition, if there is an analogy to the classic von Mises-type material, the equivalent stress σ_{eq} will be introduced as quadratic with regard to the arguments: $2I_{1\boldsymbol{m}} - I_{2\boldsymbol{m}}, I_{3\boldsymbol{m}}, I_{4\boldsymbol{m}}, I_{5\boldsymbol{m}}$. Then, the definition is the following

$$\sigma_{\mathrm{eq}}^2 = J_0^2 + 3\alpha_1 J_1 + 3\alpha_2 J_2, \quad \alpha_1 > 0, \quad \alpha_2 > 0 \tag{3.35}$$

where the abbreviations below were introduced

$$\begin{aligned} &J_0 = \mathbf{m}\cdot\boldsymbol{\sigma}\cdot\mathbf{m} - \frac{1}{2}\operatorname{tr}\boldsymbol{\sigma}_{\mathbf{p}}, \quad \operatorname{tr}\boldsymbol{\sigma}_{\mathbf{p}} = \operatorname{tr}\boldsymbol{\sigma} - \mathbf{m}\cdot\boldsymbol{\sigma}\cdot\mathbf{m}, \\ &2J_1 = \operatorname{tr}\mathbf{s}_{\mathbf{p}}^2 = \operatorname{tr}\boldsymbol{\sigma}^2 - 2\mathbf{m}\cdot\boldsymbol{\sigma}^2\cdot\mathbf{m} + \left(\mathbf{m}\cdot\boldsymbol{\sigma}\cdot\mathbf{m}\right)^2 - \frac{1}{2}\left(\operatorname{tr}\boldsymbol{\sigma}_{\mathbf{p}}\right)^2, \\ &J_2 = \boldsymbol{\tau}_{\mathbf{m}}\cdot\boldsymbol{\tau}_{\mathbf{m}} = \mathbf{m}\cdot\boldsymbol{\sigma}^2\cdot\mathbf{m} - \left(\mathbf{m}\cdot\boldsymbol{\sigma}\cdot\mathbf{m}\right)^2 \end{aligned} \tag{3.36}$$

With the Norton-Bailey-Odqvist potential

$$W = \frac{a}{n+1}\sigma_{\mathrm{eq}}^{n+1} \tag{3.37}$$

one gets the creep equation

$$\dot{\mathbf{D}}^{cr} = \frac{3}{2}a\sigma_{\mathrm{eq}}^{n-1}\left[J_0\left(\mathbf{m}\otimes\mathbf{m} - \frac{1}{3}\mathbf{E}\right) + \alpha_1\mathbf{s}_{\mathbf{p}} + \alpha_2\left(\boldsymbol{\tau}_{\mathbf{m}}\otimes\mathbf{m} + \mathbf{m}\otimes\boldsymbol{\tau}_{\mathbf{m}}\right)\right] \tag{3.38}$$

The classical constitutive equation results from $\alpha_1 = \alpha_2 = 1$. The parameter identification is presented in [34,70].

The anisotropic creep law derived here for transverse isotropy (initial isotropy) can also be extended to the case of tertiary creep. The starting point is the creep law already discussed and a postulated damage that is described by its evolution. The effective stress concept is used as in the isotropic case

$$\boldsymbol{D}^{cr} = \frac{3}{2} a \tilde{\sigma}_{\text{eq}}^{n-1} \left[\tilde{J}_0 \left(\boldsymbol{m} \otimes \boldsymbol{m} - \frac{1}{3} \boldsymbol{E} \right) + \alpha_1 \tilde{\boldsymbol{s}}_{\boldsymbol{p}} + \alpha_2 \left(\tilde{\boldsymbol{\tau}}_{\boldsymbol{m}} \otimes \boldsymbol{m} + \boldsymbol{m} \otimes \tilde{\boldsymbol{\tau}}_{\boldsymbol{m}} \right) \right] \tag{3.39}$$

$\widetilde{(\ldots)}$ denotes effective values of the variables. However, it should be noted that there are no consistent suggestions on the formulation of these effective variables (see e.g. [72-74]). In addition, evolutionary equations for anisotropic damage have to be found but even here, the variety of options shows that there is still a need for further research. The current state of research is reported in [34,44].

4 Examples

The following examples on the one hand are supposed to demonstrate the application of creep mechanics to lower-dimension structural problems (shells, plates, beams, …). Sometimes the dimension reduction will yield unexpected difficulties, if initially three-dimensional creep damage laws are used. On the other hand, these examples can be seen as benchmarks for testing commercial finite element software.

4.1 Beams

The classic beam theory according to Euler-Bernoulli is based on simplified kinematics. It is assumed that cross-sections that were orthogonal and straight to the beam axis before the deformation still have these properties after the deformation. The validity of these assumptions can be proven experimentally for classic construction materials and linear-elastic material behavior for small deformations. In connection with creep damage problems, however, one must assume that the above-mentioned effects no longer exist. The reason for this is, among other things, that the creep processes are no longer expected to be distributed uniformly across the cross-section and that the deformations are non-linear in the direction of the cross-sectional coordinates. Therefore, the theory according to Euler-Bernoulli has to be improved. The simplest form is the Timoshenko theory.

If the elastic solution is neglected, the creep problem can be solved analytically in the simplest case. For a beam that is simply supported on both sides and loaded with a constant distributed transverse load, the Euler-Bernoulli theory yields a polynominal of the order $2n+2$, where n is the creep exponent. It is known from strength of materials theory that the elastic solution is a polynomial of 4th order. For classic creep materials, the value of the creep exponent ranges from $3 \leq n \leq 7$ (there are also examples where n goes up to 12 and higher). In conclusion, it can be said that for tasks about stationary creep treated with variation methods, the test functions need to at least be of $2n+2$ degree. It should also be emphasized that the degree is material-specific, since the polynomial order also depends on the creep exponent. If the elastic approximation is taken as a starting solution (as is often recommended), the results can be far from reality.

Even in the simplest case, no analytical solutions can be calculated for the creep damage case. The corresponding tasks have to be treated approximately with the use of semi-analytical methods. The qualitative and quantitative statements known from the stationary case are principally still valid.

For the first time, an improved theory considering the transverse shear was proposed in [75]. The basic idea is analogous to the elastic case but with an added rotatory degree of freedom (rotations of the cross section, so that it is plane, but no longer orthogonal after deformation with respect to the beam axis) to the translatory degree of freedom (deflections). A corresponding theory was derived based on the principle of virtual displacements, assuming specific distributions of the displacements and/or stresses over the thickness. From the classic elasticity theory, it is known that the normal stresses are linearly, the transverse shear stresses are parabolically and the normal stresses are cubically distributed in the transverse direction. Is an initially unspecified distribution law regarding the shear stress over the beam thickness chosen, one will get the usual equlibrium conditions after applying a mixed variational principle. In addition, there is an equation for the transverse shear force, which can be interpreted as a constitutive relationship. If this equation is written in the way known from elastic Timoshenko beams, it can be seen that this constitutive equation contains a correction factor (analogy to shear correction). It depends on the previously introduced and yet not specified distribution function. In the case of the postulated linear law, the value of the correction is $k = 1$. Using a cubic approach, it follows $k = 5/6$, i.e. the Reissner approximation is obtained. The best correction for station creep is

$$k = \frac{3n+2}{4n+2} \tag{4.1}$$

It is obvious that the correction of the material-specific creep exponent depends on n. The correction becomes smaller the more the creep exponent grows. The increase of the creep exponent is associated with an increase in the creep rate. Since the damage also is tied to this effect, it can be assumed that this results in a further decrease of the factor.

4.2 Pipe Bend

The second example should give a brief insight into the problems that can be expected when dealing with practical tasks. In [76], the creep damage analysis is extensively presented for a pipe bend. The focus was on the question whether this thin-walled component should be computed with two- or three-dimensional finite elements. The internal pressure load case is assumed, the temperature was kept constant. Just translatory (only displacements), but also the combination of translatory and independent rotatory degrees of freedom were chosen as boundary conditions. The pipe bend was made of 316 steel. The creep damage behavior was described using the Kachanov-Rabotnov-Leckie-Hayhurst model [77-79], whereby all parameters were known from the literature. For the case considered, the evolution of damage can be influenced by two stresses: the maximum tensile stress and the von Mises equivalent stress. It has been confirmed experimentally that the damage for the given steel is mainly influenced by the maximum tensile stress.

All calculations were carried out with the help of the commercial finite element system ANSYS, using the elements SHELL43 and SOLID45 recommended for plastic and creep calculations. First, the elements were tested for the elastic case. Here, the calculations based on the two-dimensional and the three-dimensional elements showed a very good agreement.

The transition to creep damage calculations did not result in such a correspondence any more. The first calculation was made for the correct material model (damage caused by the maximum stress) and the obvious boundary conditions (purely translatory). The results for the stresses and for the damage showed no similarity especially with regard to the critical areas (maximum stress, maximum damage). Further calculations, for which the boundary conditions or the material model were changed, brought a qualitatively and quantitatively satisfactory agreement, i.e. an "incorrect material model" and "non-classical boundary conditions" led to this.

Those effects are difficult to explain. In this case, a brief analysis of the material model shows that the von Mises equivalent stress is not sensitive to tension and pressure or generally to the type of stress state. However, creep damage processes are highly sensitive to the type of stress state, if the tension stress is increasing, so does the damage while the pressure does not "heal". The effects associated with the change in boundary conditions indicate that the cross sections are simulated too stiff using only translatory boundary conditions. With the introduction of a computational model in the sense of Timoshenko (shear-soft model), the two-dimensional and three-dimensional calculations could be adapted much better. In addition, it was demonstrated that the integration over the thickness using the SHELL43 element is not provided with enough Gauss points (here 5). Test calculations with other commercial software showed that 17 integration points yield a very good match [34]. Other interesting effects in thick-walled pipes are described in [80].

5 Further Developments

As it was shown in the previous sections, the application possibilities of the presented theories are suggested. Below, some brief information is added.

5.1 Rheological Models

The method of rheological modeling is one possibility to establish constitutive equations for complex material behavior. The basics are presented, for example, in [60]. The approach was later developed for general continuum mechanics purposes in [81,82] (note that [82]} is the translation of the Russian original book from 1976). In [83], a phase mixture model is suggested for simulating the mechanical behavior of tempered martensitic steels at high temperatures. Assuming only two phases (hard phase and soft phase) with an unified description of the rate-dependent deformation including hardening and softening, the model starts from an iso-strain approach (similar to composite mechanics) along with a hard and a soft constituent. For both phases, a two-element model (elastic and inelastic branch connected in series) was suggested and the elastic part in both branches was assumed to be identical. Finally, they were connected in parallel, the parameters were calibrated for the uniaxial model, which can easily be extended to the three-dimensional case.

After the implementation of the model in a finite element code and testing the correctness of the numerical solution by simple benchmarks, the behavior of an idealized steam turbine rotor during a cold start and a subsequent hot start was simulated. The heat transfer analysis was conducted, while prescribing the nonstationary steam temperature and the heat transfer coefficients. The resulting temperature fields served as input for the structural analysis of the rotor.

The original constitutive and evolution equations were proposed in [50,51], computational tasks were discussed in [84] and the calibration procedure was described in detail in [85]. Note that a similar approach was used in [86] for POM.

5.2 Double Power Law

The Norton-Bailey law is used in many technical applications - the law is simple, so is its calibration and it can easily be extended to the primary and tertiary creep ranges. The disadvantage of this law is the validity in a limited stress range. With respect to the need to simulate creep behavior even for small and moderate stresses, in [87], a double power law was suggested. Published experimental data for advanced heat resistant steels indicate that the high creep exponent (in the range 7–12) may decrease to the low value of approximately unity within the stress range which is relevant for engineering structures like the transition from the power law to the viscous law and vice versa. The double power law matches the behavior in both ranges in an acceptable manner and the transition region itself is described by a smooth function.

5.3 Hyperbolic Sine Stress Response Function

A classic conventional material behavior model can be extended, if varying thermo-mechanical loading should be taken into account in wide stress ranges. In [88], a creep constitutive law in the form of a hyperbolic sine stress response function is used. The original proposal was suggested by Nadai [18]. For the analysis of a failure case in a power plant, the original model was extended assuming a damage process described by a scalar damage parameter and appropriate evolution equation in the sense of Kachanov–Rabotnov [77-78]. In addition, several parameters were added to reflect the hardening and recovery effects under cyclic loading. The uniaxial simulations were compared to cyclic stress–strain diagrams and other experimental data (creep curves, tensile stress–strain diagrams, relaxation curves, etc.) for the austenitic steel AISI type 316 at 600°C in a wide stress range.

6 Outlook

The outlook formulated below is purely subjective and does not claim to be complete. Only perspectives on the questions discussed in the article are given. Likewise important problems, such as the behavior during dynamic load changes, etc., are not taken into account, although there is still a great need for research. Those load changes are the usual case in practice, however, the experimental validation of certain facts and the theoretical basis for their description have not yet been sufficiently developed. Adapting static solutions by modifying the equations to dynamic processes is not the most elegant way out.

6.1 Own Challenges

Creep mechanics is still not entirely discovered, since numerous questions have not been adequately or finally clarified. Taking the three tasks described in the beginning as a starting point, the following main problems can be formulated. In connection with the description of the constitutive behavior, an open question is the formulation of uniform laws for the low, moderate and higher stress ranges. Different creep mechanisms occur in the range of lower stresses in comparison to higher ones. As part of the phenomenological concept, an analytical equation must be formulated which applies to the entire range of stresses. It should be as simple as possible (linear laws are used at low stresses, the power law is usually applied at higher stresses). As simple as possible also means that not too many material parameters have to be determined and that the expansion to three-dimensional stress states and non-isothermal processes can be carried out elementarily. First approaches to include the temperature dependencies are shown in [89-91]. Since creep processes are sensitive to the type of stress state, new ideas concerning this must be employed. Finally, it should be clarified whether the concept treating anisotropy introduced in [70] can also be transferred to all cases of anisotropy. In the contribution named above, only the case of transverse isotropic creep is discussed even though at least the orthotropy would be of special interest for practical applications.

6.2 Open Questions

There are numerous problems with the open questions, from the theoretical point of view which also have not been solved yet. This includes for example a consistent continuum mechanical representation of the theoretical foundation considering thermodynamics and the theoretical justification of creep equations. A lot of creep mechanics problems are related to thin-walled structures, which in the sense of structural mechanics are treated as one-dimensional or two-dimensional models. The starting point for such models is often the three-dimensional theory. The derivation of the governing low-dimensional equations can be realized with the help of hypotheses or mathematical techniques. Also, the direct formulation of one- or two-dimensional structural mechanics equations can be applied. In the case of creep problems, there are still no fully satisfying approaches. This is due to the fact that creep-damage yields strong inhomogeneities over the thickness. In addition to the known problems with the derivation or the establishment of two- or one-dimensional equations, the direct approach results in the difficulty of finding suitable constitutive laws. Finally, it also is not clear whether micro-macro approaches in analogy to the plasticity theory can be applied. These concept are focused on solving the problem within a reduced representative volume first and then using appropriate homogenizations known from composite mechanics to get the solution on the macro level. However, the limits and possibilities are not yet well enough described so that it remains open whether this approach can be used in creep mechanics.

References

[1] Altenbach H., Eisenträger J., Introduction to creep mechanics. Encyclopedia of Continuum Mechanics (Eds H. Altenbach & A. Öchsner), 2020, Berlin, Heidelberg, Springer, pp. 1337-1344

[2] Naumenko K., Altenbach H., *Modeling High Temperature Materials Behavior for Structural Analysis. Part I: Continuum Mechanics Foundations and Constitutive Models*. Cham, Springer, 2016 (Advanced Structured Materials series, vol. 28)

[3] Lurie A.I., *Theory of Elasticity*. Berlin, Springer, 2005

[4] Lebedev L., Cloud M., Eremeyev V., *Tensor Analysis with Applications to Mechanics*. World Scientific Publishing Company, 2010

[5] Altenbach H., Zu einigen Aspekten der klassischen Kontinuumsmechanik und ihrer Erweiterungen// Technische Mechanik 1990, vol. 11, issue 2, pp. 95-105

[6] Naumenko K., Altenbach H., *Modeling High Temperature Materials Behavior for Structural Analysis. Part II: Solution Procedures and Structural Analysis Examples*. Cham, Springer, 2019 (Advanced Structured Materials series, vol. 112)

[7] Hayhurst D.R., Wong M.T., Vakili-Tahami F., The use of CDM analysis techniques in high temperature creep failure of welded structures// JSME Int. J. Series A 2002, vol. 45, pp. 90-97

[8] Le May I., da Silveria T.L., Cheung-Mak S.K.P., Uncertainties in the evaluations of high temperature damage in power stations and petrochemical plant// Int. J. of Pressure Vessels & Piping 1994, vol. 59, pp. 335-343

[9] Neubauer B., Wedel U., Restlife estimation of creeping components by means of replicas. Advances in Life Prediction Methods (Eds D.A. Woodford & J.R. Whitehead), 1983, ASME, pp. 307-314

[10] Odqvist F.K.G.; Hult J., *Kriechfestigkeit metallischer Werkstoffe*. Berlin u.a., Springer, 1962

[11] Odqvist F.K.G.: Historical survey of the development of creep mechanics from its beginnings in the last century to 1970. Creep in Structures (Eds A.R.S. Ponter \& D.R. Hayhurst), 1981, Berlin, Springer, pp. 1-12

[12] da C. Andrade E.N., On the viscous flow of metals, and allied phenomena// Proc. R. Soc. Lond. A 1910, vol. LXXXIV, issue 567, pp. 1-12

[13] da C. Andrade E.N., The flow in metals underlarge constant stresses// Proc. R. Soc. Lond. A 1910, vol. XC, issue 619, pp. 329-342

[14] Norton F.H., *Creep of Steel at high Temperatures*. New York, McGraw-Hill, 1929

[15] Bailey R.W., The utilization of creep test data in engineering design// Proc. Inst. Mech. Eng. 1935, vol. 131, issue 1, pp. 131–349

[16] Stodola A., Die Kriecherscheinungen, ein neuer technisch wichtiger Aufgabenkreis der Elastizittstheorie// Zeitschrift für angewandte Mathematik und Mechanik 1933, vol. 13, issue 2, pp. 143-146

[17] Odqvist F.K.G., *Mathematical Theory of Creep and Creep Rupture*. Oxford, Oxford University Press, 1974

[18] Nadai A., The influence of time upon creep. The hyperbolic sine creep law. Contributions to the Mechanics of Solids dedicated to Stephen Timoshenko by his friends on the occasion of his sixties birthday anniversary. Macmillian, 1938, pp. 155-170

[19] Soderberg C.R., Plasticity and Creep in Machine Design. Contributions to the Mechanics of Solids dedicated to Stephen Timoshenko by his friends on the occasion of his sixties birthday anniversary. Macmillian, 1938, pp. 197-210

[20] Hoff N.J., The necking and the rupture of rods subjected to constant tensile loads// Trans. ASME Journal of Applied Mechanics 1953, vol. 20, issue 1, pp. 105-108

[21] Hoff N.J., Buckling at high temperature// Journal of the Royal Aeoronautic Society 1957, vol. 61, issue 563, pp. 756-774

[22] Rabotnov Yu.N., *Elements of Hereditary Solid Mechanics*. Moscow, Mir, 1980

[23] Arutyunyan N.K., *Some Problems in the Theory of Creep*. Oxford, Pergamon Press, 1966

[24] Hult J.A., *Creep in Engineering Structures*. Waltham, Blaisdell Publishing Company, 1966

[25] Rabotnov Yu.N., *Creep Problems in Structural Members*. Amsterdam, North-Holland, 1969

[26] Penny R.K., Mariott D.L., *Design for Creep*. London, Chapman & Hall, 1995

[27] Kraus H., *Creep Analysis*. New York, Wiley & Sons, 1980

[28] Malinin N.N., *Raschet na polzuchest' konstrukcionnykh elementov (Creep calculations of structural elements, in Russ.)*. Moskva, Mashinostroenie, 1981

[29] Boyle J.T., Spence J., *Stress Analysis for Creep*. London, Butterworth, 1983

[30] Lemaitre J., Chaboche J.-L., *Mechanics of Solid Materials*. Cambridge, Cambridge University Press, 1990

[31] Skrzypek J.J., *Plasticity and Creep*. Boca Raton, CRC Press, 1993

[32] Skrzypek J.J., Gancarski A., *Modelling of Material Damage and Failure of Structures*. Berlin, Springer 1998 (Foundation of Engineering Mechanics)

[33] Betten J., *Creep Mechanics*. Berlin, Springer, 2005

[34] Naumenko K., Altenbach, H., *Modeling of Creep for Structural Analysis*. Berlin, Springer, 2007 (Foundations of Engineering Mechanics)

[35] Hyde T.H., Sun W., Hyde J.C., *Applied Creep Mechanics*. New York, McGraw Hill Education, 2013

[36] Lokoshchenko A.M., *Polzuchest' i dlitel'naya prochnost' metallov (in Russ., Creep and Long-term Strength of Metals)*. Moscow, FIZMATLIT, 2016

[37] Hoff N.J. (Ed.), *Creep in Structures*. Berlin, Springer, 1962

[38] Hult J. (Ed.), *Creep in Structures*. Berlin, Springer, 1972

[39] Ponter A.R.S., Hayhurst D.R. (Eds), *Creep in Structures*. Berlin, Springer, 1981

[40] Życzkowski M. (Ed.), *Creep in Structures*. Berlin, Springer, 1991

[41] Murakami S., Ohno N. (Eds), *IUTAM Symposium on Creep in Structures*. Dordrecht, Kluwer, 2001

[42] Altenbach H., Kruch S. (Eds), *Advanced Materials Modelling for Structures*. Berlin-Heidelberg, Springer, 2013 (Advanced Structured Materials series, Vol. 19)

[43] Altenbach H., Skrzypek J.J. (Eds), *Creep and Damage in Materials and Structures*. - Wien, New York, Springer, 1999 (CISM Courses and Lectures No. 399)

[44] Skrzypek J.J., Ganczarski, A. (Eds), *Anisotropic Behaviour of Damaged Materials*. Berlin, Springer, 2003

[45] Ohno N., Uehara T. (Eds), *Engineering Plasticity and Its Applications from Nanoscale to Macroscale*. Stafa-Zürich, Trans. Tech. Publications Ltd., 2007

[46] Altenbach H., Creep analysis of thin-walled structures// Zeitschrift für angewandte Mathematik und Mechanik 2002, vol. 82, issue 8, pp. 507-533

[47] Ashby M.F., Gandhi C., Taplin D.M.R., Fracture-mechanism maps and their construction for f.c.c. metals and alloys// Acta Metall. 1979, vol. 27, pp. 699-729

[48] Nabarro F.R.N., de Villiers H.L., *The Physics of Creep. Creep and Creep-resistant Alloys*. London, Taylor & Francis, 1995

[49] Roesler J., Harders H., Baeker M. *Mechanical Behaviour of Engineering Materials - Metals, Ceramics, Polymers, and Composites*. Berlin, Heidelberg, Springer, 2007

[50] Naumenko K., Kutschke A., Kostenko Y., Rudolf T. Multi-axial thermo-mechanical analysis of power plant components from 9-12%Cr steels at high temperature// Engineering Fracture Mechanics 2011, vol. 78, pp. 1657-1668

[51] Naumenko K., Altenbach H., Kutschke A. A Combined Model for Hardening, Softening, and Damage Processes in Advanced Heat Resistant Steels at Elevated Temperature// International Journal of Damage Mechanics 2011, vol. 20. issue 4, pp. 578–597

[52] Naumenko K., Gariboldi E. A phase mixture model for anisotropic creep of forged Al–Cu–Mg–Si alloy// Materials Science and Engineering A 2014, vol. 618, pp. 368–376

[53] Eisenträger J., Naumenko K., Altenbach H., Gariboldi E. Analysis of temperature and strain rate dependencies of softening regime for tempered martensitic steel// The Journal of Strain Analysis for Engineering Design 2017, vol. 52, issue 4, pp. 226-238

[54] Altenbach H., Bolchoun A., Kolupaev V.A., Phenomenological Yield and Failure Criteria. Plasticity of Pressure-Sensitive Materials (Eds H. Altenbach & A. Öchsner). Berlin, Heidelberg, Springer, 2014, pp. 49-152, (Engineering Materials series)

[55] Kolupaev V.A. *Equivalent Stress Concept for Limit State Analysis*. Springer, 2018 (Advanced Structured Materials series, Vol. 86)

[56] Życzkowski, M.: *Combined Loadings in the Theory of Plasticity*. - PWN, Warszawa, 1981

[57] Altenbach H., Altenbach J., Zolochevsky A., *Erweiterte Deformationsmodelle und Versagenskriterien der Werkstoffmechanik*. Stuttgart, Deutscher Verlag für Grundstoffindustrie, 1995

[58] Lode W., Versuche über den Einfluß der mittleren Hauptspannung auf das Fließen der Metalle Eisen, Kupfer und Nickel// Zeitung Phys. 1926, vol. 36, pp. 913-939

[59] Odqvist F.K.G., *Mathematical Theory of Creep and Creep Rupture*. Oxford, Oxford University Press, 1974

[60] Reiner M., *Deformation and Flow. An Elementary Introduction to Rheology*. London: H.K. Lewis & Co., 1969

[61] Billington E.W., *Introduction to the Mechanics and Physics of Solids*. - Bristol, Hilger, 1986

[62] Robinson D.N., Binienda W.K., Ruggles M.B., Creep of polymer matrix composites. I: Norton/Bailey Creep Law for transverse isotropy// Trans. ASCE. J. Engng Mech. 2003, vol. 129, issue 3, pp. 310-317

[63] Robinson D.N., Binienda W.K., Ruggles M.B., Creep of polymer matrix composites. II: Monkman-Grant failure relationship for transverse isotropy// Trans. ASCE. J. Engng Mech. 2003, vol. 129, issue 3, pp. 318-323

[64] Bertram A., Olschewski J., Anisotropic modelling of the single crystal superalloy SRR99// Comp. Mat. Sci. 1996, vol. 5, pp. 12-16

[65] Mahnken R., Anisotropic creep modeling based on elastic projection operators with applications to CMSX-4 superalloy// Int. J. Mech. Sci. 1998, vol. 14, pp. 235-280

[66] El-Magd E., Betten J., Palmen P., Auswirkung der Schädigungsanisotropie auf die Lebensdauer von Stählen bei Zeitstandbeanspruchung// Mat.-wiss. u. Werkstofftechn. 1996, vol. 27, pp. 239-245

[67] Konkin V.N., Morachkovskij O.K., Polzuchest' i dlitel'naya prochnost' legkikh splavov, proyavlyayushchikh anizotropnye svoistva (Creep and long-term strength of light alloys with anisotropic properties, in Russ.)// Problemy prochnosti 1987, issue 5, pp. 38-42

[68] Courant R., Hilbert D., *Methods of Mathematical Physics, Partial Differential Equations, Vol. 2*. New York, Wiley Interscience, 1989

[69] Bruhns O., Xiao H., Meyers A., On representation of yield functions for crystals, quasicrystals and transversely isotropic solids// Eur. J. Mech. A/Solids 1999, vol. 18, pp. 47-67

[70] Naumenko K., Altenbach H. A phenomenological model for anisotropic creep in a multi-pass weld metal// Arch. Appl. Mech. 2005, vol. 74, pp. 808-819

[71] Altenbach H., Naumenko K., Zhilin P.A., A note on transversely-isotropic invariants. Zeitschrift für angewandte Mathematik und Mechanik 2006, vol. 86, issue 2, pp. 162-168

[72] Cordebois J., Sidoroff F., Damage induced elastic anisotropy. Mechanical Behaviors of Anisotropic Solids (Ed. J.P. Boehler). Boston, Martinus Nijhoff, 1983. - pp. 761-774

[73] Betten J., Materialgleichungen zur Beschreibung des sekundären und tertiären Kriechverhaltens anisotroper Stoffe// Zeitschrift für angewandte Mathematik und Mechanik 1984, vol. 64, pp. 211-220

[74] Murakami S., Sanomura Y., Hattori M., Modelling of the coupled effect of plastic damage and creep damage in Nimonic 80A// Int. J. Solids Struct. 1986, vol. 22, issue 4, pp. 373-386

[75] Naumenko K., On the use of the first order shear deformation models of beams, plates and shells in creep lifetime estimations// Technische Mechanik 2000, vol. 20, issue 3, pp. 215-226

[76] Altenbach H., Kushnevsky V., Naumenko K., On the use of solid- and shell-type finite elements in creep-damage predictions of thinwalled structures// Arch. Appl. Mech. 2001, vol. 71, pp. 164-181

[77] Kachanov L.M., O vremeni razrusheniya v usloviyakh polzuchesti (in Russ., On the time to rupture under creep conditions)// Izv. AN. SSSR. Otd. Tekh. Nauk 1958, issue 8, pp. 26-31

[78] Rabotnov Yu.N., O mechanizme dlitel'nogo razrusheniya (in Russ., A mechanism of the long term fracture). Voprosy Prochnosti Materialov i Konstruktsii, AN SSSR, 1959, pp. 5-7

[79] Hahurst D., Leckie F., The effect of creep constitutive and damage relationships upon rupture time of solid circular torsion bar// Journal of the Mechanics and Physics of Solids 1973, vol. 21, pp. 431-446

[80] Altenbach H., Gorash Y., Naumenko K., Steady-state creep of a pressurized thick cylinder in both the linear and the power law ranges// Acta Mechanica 2008, vol. 195, issue 1-4, pp. 263-274

[81] Krawietz A., *Materialtheorie*. Berlin, Heidelberg, Springer, 1986

[82] Palmov V., *Vibrations of Elasto-Plastic Bodies*. Berlin, Heidelberg, Springer, 1998 (Foundations of Engineering Mechanics)

[83] Eisenträger, Naumenko K., Kostenko Y., Altenbach H. Analysis of a Power Plant Rotor Made of Tempered Martensitic Steel Based on a Composite Model of Inelastic Deformation. Advances in Mechanics of High-Temperature Materials (Eds K. Naumenko & M. Krüger). Springer, 2020 (Advanced Structured Materials series, Vol. 117)

[84] Eisenträger J., Naumenko K., Altenbach H., Numerical implementation of a phase mixture model for rate-dependent inelasticity of tempered martensitic steels// Acta Mechanica 2018, vol. 229, pp. 3051–3068

[85] Eisenträger J., Naumenko K., Altenbach H., Calibration of a phase mixture model for hardening and softening regimes in tempered martensitic steel over wide stress and temperature ranges// J. Strain Anal. Eng. Des. 2018, vol. 53, pp. 156–177

[86] Altenbach H., Girchenko A., Kutschke A., Naumenko K., Creep Behavior Modeling of Polyoxymethylene (POM) Applying Rheological Models. Inelastic Behavior of Materials and Structures Under Monotonic and Cyclic Loading (Eds H. Altenbach & M. Brünig), Cham, Springer, 2015, pp. 1–15 (Advanced Structured Materials series, vol. 57)

[87] Naumenko K., Altenbach H., Gorash, Y., Creep analysis with a stress range dependent constitutive model// Archive of Applied Mechanics 2009, vol. 79, issue 6-7, pp. 619–630

[88] Gorash Y., Altenbach H., Lvov G., Modelling of high-temperature inelastic behaviour of the austenitic steel AISI type 316 using a continuum damage mechanics approach// The Journal of Strain Analysis for Engineering Design 2012, vol. 47, issue 4, pp. 229–243

[89] Kostenko Y., Lvov G., Gorash E., Altenbach H., Naumenko K., Power plant component design using creep-damage analysis. Proc. of 2006 ASME Int. Mech. Engng Congress and Exposition, ASME, 2006, IMECE2006-13710, pp. 1-10

[90] Altenbach H., Gorash Y., Naumenko K., Creep analysis for a wide stress range based on relaxation data// Modern Physics Letters B 2008, vol. 22, issue 31 & 32, pp. 5413-5418

[91] Naumenko K., Altenbach H., Gorash Y., Creep analysis with a stress range dependent constitutive model// Arch. Appl. Mech. 2009, vol. 79, issue 6/7, pp. 619-630

Information about the authors

Holm Altenbach, Dr. of Technical Sciences, Professor, Dr.h.c.mult., Foreign member of the Russian Academy of Sciences
Affiliation: Chair of Engineering Mechanics, Institute of Mechanics, Faculty of Mechanical Engineering, Otto-von-Guericke-Universität Magdeburg
Address: Universitätsplatz 2, 39106 Magdeburg, Germany
Tell: +49 391 67–58814
E-mail: holm.altenbach@ovgu.de

Katharina Knape, M.Sc.
Affiliation: Chair of Engineering Mechanics, Institute of Mechanics, Faculty of Mechanical Engineering, Otto-von-Guericke-Universität Magdeburg
Address: Universitätsplatz 2, 39106 Magdeburg, Germany
Tell: +49 391 67–51985
E-mail: katharina.knape@ovgu.de

Received 10.08.2020