ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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HYBRID OF RAYLEIGH AND GULYAEV-BLUESTEIN ELECTRO-ACOUSTIC WAVES NEAR THE INNER SURFACE OF A LAYERED PIEZOELECTRIC COMPOSITE

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Гибрид электроакустических волн Рэлея волн Гуляева-Блюстейна около внутренней поверхности слоистого пьезоэлектрического композита

Ключевые слова: гибрид волн, пьезоэлектрическая среда, неакустический контакт, электроакустическая поперечная волна, плоская деформация, локализация энергии волны, дисперсионное уравнение, фазовая скорость.

Исследуется гибридизация локализованных электроакустических волн Рэлея и Гуляева-Блюстейна у поверхности не акустического контакта двух пьезоэлектриков. Предложена простая схема двухслойного волновода, допускающая гибрид электроактивных локализованных волн Гуляева-Блюстейна и Рэлея. Показывается, что индуцированная, локализованная у свободной поверхности электроупругая сдвиговая волна Гуляева-Блюстейна в одном слое, может генерировать электроупругую волну Рэлея плоской деформации в другом слое и наоборот. Показано также, что соответствующий выбор материалов прилегающих полупространств может привести к усилению или ослаблению локализации энергии электроупругих волн вбизи поверхности немеханического контакта пьезоэлектриков.

Выявлено, что индуцированная в одном из слоев волна с нерезонансной частотой может вызвать внутренний резонанс или образование зон запрещенных частот в волноводе данной структуры.

Проведен сравнительный анализ результатов со случаями отсутствия пьезоэлектрического эффекта в одном из слоев композита.

Ավետիսյան Արա Ս., Ջիլավյան Հակոբ Ս.

Ռելեյի և Գուլյաև-Բլյուստեյնի էլեկտրաառաձգական ալիքների խաչասերումը շերտավոր բաղադրյալ պյեզոէլեկտրիկի ներքին մակերևույթի մոտ

Հիմնաբառեր. Ալիքների խաչասերում, պյեզոէլեկտրական միջավայր, անհպում կապ, էլեկտրաձայնային լայնական ալիք, հարթ դեֆորմացիա, ալիքային էներգիայի տեղայնացում, դիսպերսիայի հավասարում, ֆազային արագություն:

Հետազոտվում է Ռելեյի և Գուլյաև-Բլյուստեյնի էլեկտրաառաձգական տեղայնացված ալիքների խաչասեռումը երկու պյեզոէլեկտրիկների անհպում կապի հարթության մոտ: Ցույց է տրվում, որ մի պյեզոէլեկտրիկի ազատ եզրի մոտ հարուցելով Գուլյաև-Բլյուստեյնի տեղայնացված էլեկտրաառաձգական սահքի ալիք, կարելի է գրգռել Ռելեյի տեղայնացված հարթ դեֆորմացիայի էլեկտրաառաձգական ալիք մյուս պյեզոէլեկտրիկում, և հակառակը: Ցույց է տրված նաև, որ սահմանակցող պյեզոէլեկտրիկ նյութերի համապատասխան ընտրությանը կարող է բերել անհպում կապի հարթության մոտ էլեկտրաառաձգական ալիքների էներգիայի տեղայնացման մեծացմանը կամ թուլացմանը: Բացահայտված է, որ մի պյեզոէլեկտրիկում հարուցված ոչ ռեզոնանսային հաճախականությամբ ալիքը կարող է առաջացնել ներքին ռեզոնանս, կամ առաջացնել արգելված հաճախականությունների գոտի:

Կատարված են ստացված արդյունքների համեմատական վերլուծություն, շերտերից մեկն ու մեկում պյեզոէլեկտրական հատկության բացակայության դեպքերի հետ:

PPThe hybridization of localized electro-acoustic Rayleigh and Gulyaev-Bluestein waves at the surface of non-acoustic contact of two piezoelectrics is investigated. A simple scheme of a two-layer waveguide, which allows a hybrid of electro active localized waves of Gulyaev-Bluestein and Rayleigh is proposed. It is shown that the induced, localized near the free surface, Gulyaev-Bluestein shear wave in one layer can generate a Rayleigh electro-elastic plane strain wave in another layer and vice versa. It is also shown that the corresponding choice of materials of adjacent half-spaces can lead to an increase or decrease in the localization of the energy of electroelastic waves near the surface of the non-mechanical contact of piezoelectrics. It was revealed that a wave with a non-resonant frequency induced in one of the layers can cause internal resonance or the formation of bands of forbidden frequencies in the waveguide of this structure.
A comparative analysis of the results with cases of the absence of a piezoelectric effect in one of the layers of the composite is carried out.
Keywords: wave hybrid, piezoelectric medium, non-acoustic contact, electroacoustic shear wave, plane deformation, wave energy localization, dispersion equation, phase velocity.

Introduction.

In modern electronics, heterogeneous composite (in particular layered, piecewise homogeneous) waveguides made of piezoelectric crystals are widely used as converters, filters, or resonators of an electro-acoustic high-frequency wave signal. Qualitatively different interests are cases when the electro-acoustic signal overpasses through a transversely inhomogeneous layered structure, and when the electro-acoustic signal flows along the interface between the homogeneous layers of the structure.

Piezoelectric crystals are essentially anisotropic materials. Depending on the crystallographic symmetry of the piezoelectric, it is possible to excite a purely electroactive wave of pure shear or an electroactive wave of plane deformation in it. In the articles by Avetisyan A.S. **[1,2]** issues of separate excitation and propagation of electroelastic plane or electroelastic antiplane stress-strain states in homogeneous piezoelectric crystals are investigated. Necessary and sufficient conditions for the texture of piezoelectric crystals that allow separate excitation and propagation of an electroelastic wave signal of a specific type are determined. Material relations and quasistatic equations are derived for all piezoelectric textures in the corresponding sagittal planes.

If we also take into account the possibility of localizing wave energy under different boundary conditions near the surfaces of the composite elements of the composite waveguide, then these waves will be heterogeneous not only in the composition of the components, but also in the physicomechanical characteristics.

In 1968 **Bleustein J.L. [3]**, and in 1969 **Gulyaev Yu.V. [4]** showed that it is possible to localize the energy of an electroelastic shear wave near the mechanically free surface of a piezoelectric medium of certain symmetry, under different boundary conditions on accompanying electric field. The features of the localization of wave energy of purely shear electro-elastic wave of Gulyaev-Bluestein type are still being studied. Thousands of works are known, in parts of which the patterns of propagation of electroelastic shear waves in composite structures, or in media with complicated properties, are studied.

In particular, **Yang J.S. [5]** investigated the propagation of waves of the Gulyaev – Bluestein type in materials with complicated piezoelectric properties.

The propagation of waves of the Gulyaev-Bluestein type in a prestressed layered piezoelectric structure was considered by **Liu H., Kuang Z.B. & Cai Z.M. [6]**.

Vashishth A.K., Dahiya A., & Gupta V. [7] studied the propagation of a Bluestein-Gulyaev wave in a structure consisting of several layers and a half-space of porous piezoelectric materials. The specific form of waves that can propagate only in the layer above the half-space is investigated.

The propagation of transverse surface waves in a functionally graduated substrate carrying a layer of piezoelectric material of hexagonal symmetry 6mm was studied by **Li P. & Jin F. [8].**
Avetisyan A.S., & Kamalyan A.A. [9] considered the propagation of an electro-elastic monochromatic wave signal in an inhomogeneous piezoelectric of hexagonal symmetry class 6mm.
The propagation of the Bluestein – Gulyaev waves in an unbounded piezoelectric half-space loaded with a layer of viscous fluid of finite thickness was considered in the framework of linear elasticity theories in **[10] Qian Z.-H., Jin F., Li P., Hirose S. ..**
By solving the equilibrium equations of piezoelectric materials and the diffusion equations of viscous fluid, exact solutions of the phase velocity equations are obtained both in the case of electrically open and in the case of electrically closed boundaries.
Although the localization of wave energy for plane deformation waves in the isotropic half-space **Rayleigh J.W. [11]** was discovered earlier than others, electroactive waves of Rayleigh type are relatively little studied.
In particular, **Singh B. & Singh R. [12]** examined the propagation of Rayleigh wave in a rotating initially strained piezoelectric half-space.
In the article by **Chaudhary S., Sahu S.A., Singhal A. [13]**, the authors proposed an analytical model for studying the propagation of Rayleigh waves in an orthotropic half-space with a piezoelectric layer.
The propagation of bound Rayleigh waves in a piezoelectric layer of the material of rhombic symmetry class 2mm over a porous piezo-thermoelastic half-space is investigated by **Vashishth, A.K., Sukhija, H. [13]**.
In the work by **Avetisyan A.S., Mkrtchyan S.H. [14]** the patterns of propagation of an electro-acoustic wave of plane deformation in a piezoelectric half-space are investigated. The problem of propagation of high-frequency electro-acoustic waves of plane deformation (electro-acoustic waves of Rayleigh type) under different electric boundary conditions on the mechanically free surface of a piezoelectric half-space is solved.
These electro-acoustic waves, which are heterogeneous in the composition of their components and physicomechanical characteristics, have different applications in technology. But the question of the possible hybrid of these dissimilar electro-acoustic waves is also obvious.
In **[15] Kuznetsova I.E., Zaitsev B.D., Borodina I.A., Teplykh A.A.,** conditions of hybridization of zero and high order acoustic radiation in a piezoelectric crystal plate are studied. It was found that hybridization occurs when the conductivity of the sheet exceeds a certain value, which can vary widely depending on the plate material and orientation.
The scheme of organization of a hybrid medium in a layered hydro-elastic waveguide is proposed in **[16] Choi H.K., Kim B.H. et al**. In this study, a hybrid of surface acoustic and electrohydrodynamic **(SAW-EHDA)** waves was introduced.
In **[17], Chow D.M., Beugnot J.-C., et al** the presence of surface and hybrid acoustic waves at various locations of conical fibers is experimentally confirmed and the first measurement of the distribution of surface acoustic waves is shown.
Obviously, with mechanical contact of piezoelectric and other elements of structure that allow heterogeneous electromechanical fields, lacking components of elastic displacements will appear in adjacent bodies. Foreign wave fields in adjacent bodies mix. In this case, the simultaneous localization of electro-acoustic waves of pure shear and waves of plane deformation, or the joint propagation of these foreign waves in one composite, becomes impossible.

In **[18] Avetisyan A.S., Khachatryan V.M.** the existence of a hybrid of one-dimensional electro-acoustic waves of pure shear and waves of pure dilatation in a composite, periodically transversely inhomogeneous piezoelectric medium from piezocrystals of hexagonal symmetry class $6mm$ (or tetragonal symmetry class $4mm$) and hexagonal symmetry class $\bar{6}m2$ is proved. It is shown that there are two groups of permissible discrete frequencies. Permissible discrete frequencies are resonant if the ratios of the widths of the bands and the velocities of the elastic waves in the bands are inverse.

A simple two-layer waveguide scheme is proposed here, allowing hybridization of localized waves of electroactive anti-plane deformation and electroactive plane deformation.

1. Problem modeling. Formulation of the mathematical boundary value problem.

We consider the propagation of high-frequency electroelastic waves in a two-layer piezoelectric body assigned to the Cartesian coordinate system $0xyz$. Composite piezoelectric waveguide $\Omega(x,y,z)=\Omega_1(x,y,z)\cup\Omega_2(x,y,z)$, where

$$\Omega_1(x,y,z)\triangleq\left\{|x|<\infty;\ 0\le y<h;\ |z|<\infty\right\},\ \Omega_2(x,y,z)\triangleq\left\{|x|<\infty;\ -h<y\le 0;\ |z|<\infty\right\},\ (1.1)$$

are designed so that the piezoelectric layers border the surface $y=0$ without acoustic contact (Fig. 1).

The crystallographic sections and orientations of the crystallographic axes of the strip materials are compared with the Cartesian coordinate system $0xyz$ so that in the coordinate plane $x0y$ of the adjacent layers $\Omega_1(x,y,z)$ and $\Omega_2(x,y,z)$ there are separate electroactive waves of antiplane and plane deformations, respectively.

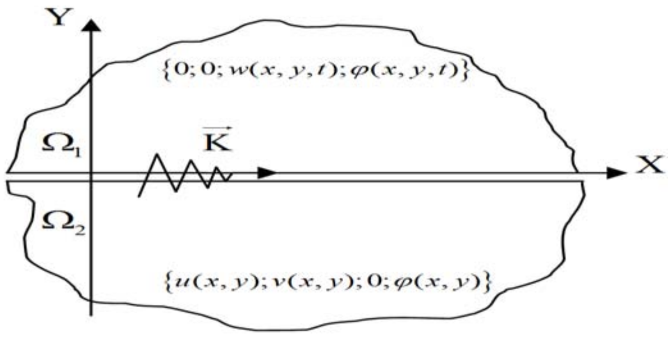


Fig. 1 Propagation pattern of a normal electroelastic wave signal in adjacent piezoelectric half-spaces without acoustic contact

Without disturbing the generality of reasoning, it is assumed that the material of the layer $\Omega_1(x,y,z)$ belongs to hexagonal symmetry class $6mm$, or tetragonal symmetry class $4mm$. The axis of symmetry of the sixth order $\vec{p}_6$ of the hexagonal piezocrystal, or, respectively, the axis of symmetry of the fourth order $\vec{p}_4$ of the tetragonal piezocrystal are aligned with the coordinate axis $0z$.

Then, the isotropic sagittal plane $x_1 0 x_2$ of the piezoelectric crystals is aligned with the coordinate plane $x0y$. In the case of antiplane deformation, for the above piezoelectric crystals, nonzero components of the tensor of mechanical stresses and the electric displacement vector on the coordinate plane $x0y$ are represented similarly [2]

$$\sigma_{zx}^{(1)}(x,y,t) = c_{44}^{(1)}\frac{\partial \mathrm{w}_1}{\partial x} + e_{15}^{(1)}\frac{\partial \varphi_1}{\partial x}, \quad \sigma_{xy}^{(1)}(x,y,t) = c_{44}^{(1)}\frac{\partial \mathrm{w}_1}{\partial y} + e_{15}^{(1)}\frac{\partial \varphi_1}{\partial y} \tag{1.2}$$

$$D_x^{(1)}(x,y,t) = e_{15}^{(1)}\frac{\partial \mathrm{w}_1}{\partial x} - \varepsilon_{11}^{(1)}\frac{\partial \varphi_1}{\partial x}, \quad D_y^{(1)}(x,y,t) = e_{15}^{(1)}\frac{\partial \mathrm{w}_1}{\partial y} - \varepsilon_{11}^{(1)}\frac{\partial \varphi_1}{\partial y} \tag{1.3}$$

The quasistatic equations of the electroactive antiplane stress-strain state describing the separate excitation and propagation of electro-elastic shear waves of type ***SH*** in this band are written as

$$\frac{\partial^2 \mathrm{w}_1(x,y,t)}{\partial x^2} + \frac{\partial^2 \mathrm{w}_1(x,y,t)}{\partial y^2} = \tilde{C}_{1t}^{-2}\cdot\frac{\partial^2 \mathrm{w}_1(x,y,t)}{\partial t^2},$$

$$\frac{\partial^2 \varphi_1(x,y,t)}{\partial x^2} + \frac{\partial^2 \varphi_1(x,y,t)}{\partial y^2} = \left(e_{15}^{(1)}/\varepsilon_{11}^{(1)}\right)\cdot\left[\frac{\partial^2 \mathrm{w}_1(x,y,t)}{\partial x^2} + \frac{\partial^2 \mathrm{w}_1(x,y,t)}{\partial y^2}\right] \tag{1.4}$$

where $\tilde{C}_{1t} = \sqrt{\tilde{c}_{44}^{(1)}/\rho_1}$ is the velocity of the electroelastic shear wave, $\tilde{c}_{44}^{(1)} = c_{44}^{(1)}(1+\chi_1^2)$ is reduced shear stiffness, $c_{44}^{(1)}$ is shear stiffness, $\chi_1^2 = (e_{15}^{(1)})^2/\left(c_{44}^{(1)}\varepsilon_{11}^{(1)}\right)$ is the coefficient of electro-mechanical coupling, $e_{15}^{(1)}$ is the piezoelectric module, $\varepsilon_{11}^{(1)}$ is the relative dielectric constant, ρ_1 is the density of the piezoelectric crystal.

For clarity, it is also assumed that the material of the layer $\Omega_2(x,y,z)$ belongs to hexagonal symmetry class $\overline{6}m2$. An electroactive plane stress-strain state is possible in the sagittal plane $x_3 0 x_1$ of the piezocrystal, combined with the coordinate plane $x0y$. The sixth-order inversion axis $\vec{\mathrm{p}}_{\overline{6}}$ of symmetry of the hexagonal piezocrystal is directed along the coordinate axis $0y$.

In the case of the plane stress-strain state, for the above piezoelectric crystals, nonzero components of the tensor of mechanical stresses and the vector of electric displacement in the coordinate plane $x0y$ of the strips $\Omega_2(x,y,z)$ of a piezoelectric of hexagonal symmetry class $\overline{6}m2$ are presented in the form [2]

$$\sigma_{xx}^{(2)}(x,y) = c_{11}^{(2)}\frac{\partial \mathrm{u}_2(x,y)}{\partial x} + c_{13}^{(2)}\frac{\partial \mathrm{v}_2(x,y)}{\partial y} + e_{11}^{(2)}\frac{\partial \varphi_2(x,y)}{\partial x},$$

$$\sigma_{yy}^{(2)}(x,y) = c_{13}^{(2)}\frac{\partial \mathrm{u}_2(x,y)}{\partial x} + c_{33}^{(2)}\frac{\partial \mathrm{v}_2(x,y)}{\partial y},$$

$$\sigma_{xy}^{(2)}(x,y) = c_{44}^{(2)}\frac{\partial \mathrm{v}_2(x,y)}{\partial x} + c_{44}^{(2)}\frac{\partial \mathrm{u}_2(x,y)}{\partial y}, \tag{1.5}$$

$$D_x^{(2)}(x,y) = e_{11}^{(2)}\frac{\partial \mathrm{u}_2(x,y)}{\partial x} - \varepsilon_{11}^{(2)}\frac{\partial \varphi_2(x,y)}{\partial x},$$

$$D_y^{(2)}(x,y) = -\varepsilon_{33}^{(2)}\frac{\partial \varphi_2(x,y)}{\partial y}.$$

Taking into account the compatibility conditions for the axial stress $\sigma_{zz}^{(2)}(x,y,t)\equiv 0$ and the third component of the displacement of the electric field $D_z^{(2)}(x,y,t)\equiv 0$ in the formulation of two-dimensional problem of electro elasticity [2], in the plane $x0y$ leads to the relationship between the characteristics of the electro-acoustic field

$$c_{12}^{(2)}\frac{\partial \mathrm{u}_2(x,y)}{\partial x}+c_{13}^{(2)}\frac{\partial \mathrm{v}_2(x,y)}{\partial y}-e_{11}^{(2)}\frac{\partial \varphi_2(x,y)}{\partial x}=0 \tag{1.6}$$

The quasistatic equations of the electroactive plane stress-strain state, with respect to elastic displacements $\mathrm{u}_2(x,y,t)$, $\mathrm{v}_2(x,y,t)$ and the potential of the electric field $\varphi_2(x,y,t)$ are written in the simplified form

$$\left(c_{11}^{(2)}-\vartheta_2 c_{12}^{(2)}\right)\frac{\partial^2 \mathrm{u}_2}{\partial x^2}+c_{44}^{(2)}\frac{\partial^2 \mathrm{u}_2}{\partial y^2}-e_{11}^{(2)}\left(1+\vartheta_2\right)\frac{\partial^2 \varphi}{\partial x^2}=\rho_2\frac{\partial^2 \mathrm{u}_2}{\partial t^2} \tag{1.7}$$

$$c_{44}^{(2)}\frac{\partial^2 \mathrm{v}_2}{\partial x^2}+\left(c_{33}^{(2)}-\vartheta_1 c_{13}^{(2)}\right)\frac{\partial^2 \mathrm{v}_2}{\partial y^2}-e_{11}^{(2)}\vartheta_1\frac{\partial^2 \varphi_2}{\partial x\partial y}=\rho_2\frac{\partial^2 \mathrm{v}_2}{\partial t^2} \tag{1.8}$$

$$\varepsilon_{11}^{(2)}\frac{\partial^2 \varphi_2}{\partial x^2}+\varepsilon_{33}^{(2)}\frac{\partial^2 \varphi_2}{\partial y^2}-e_{11}^{(2)}\frac{\partial^2 \mathrm{u}_2}{\partial x^2}=0\,. \tag{1.9}$$

In relations (1.7)÷(1.9) $c_{11}^{(2)}$, $c_{12}^{(2)}$, $c_{44}^{(2)}$, $c_{13}^{(2)}$ and $c_{33}^{(2)}$ are the elastic stiffnesses, $e_{11}^{(2)}$ is the piezoelectric modulus, $\varepsilon_{11}^{(2)}$ and $\varepsilon_{33}^{(2)}$ are the relative permittivities, and ρ_2 is the density of the piezoelectric crystal. The introduced dimensionless coefficients $\vartheta_1=\left(c_{13}^{(2)}+c_{44}^{(2)}\right)/c_{12}^{(2)}$ and $\vartheta_2=\left(c_{13}^{(2)}+c_{44}^{(2)}\right)/c_{13}^{(2)}$ characterize the anisotropy of the piezoelectric of the class $\overline{6}m2$ in the plane $x0y$.

From equations (1.7)÷(1.9) it can be seen that according to the model of the generalized stress-strain state [2], the reduced elastic tensile stiffnesses decrease with respect to the natural axial stiffnesses c_{11} and c_{33}, accordingly,

$$c_{11}^{*}=c_{11}^{(2)}-\vartheta_2 c_{12}^{(2)},\qquad c_{33}^{*}=c_{33}^{(2)}-\vartheta_1 c_{13}^{(2)}\,. \tag{1.10}$$

The reduced coefficients of the direct piezoelectric effect increase accordingly

$$e_{11}^{*}=e_{11}^{(2)}\left(1+\vartheta_2\right),\qquad e_{12}^{*}=\vartheta_1 e_{11}^{(2)}\,. \tag{1.11}$$

When the piezoelectric layers in the composite are in non-acoustic contact, the conditions of the mechanically free surface for both piezoelectric half-spaces are satisfied on the surface $y=0$

$$\left.c_{44}^{(1)}\frac{\partial \mathrm{w}_1(x,y,t)}{\partial y}+e_{15}^{(1)}\frac{\partial \varphi_1(x,y,t)}{\partial y}\right|_{y=0}=0 \tag{1.12}$$

$$\left(\vartheta_5 c_{44}^{(2)}\frac{\partial \mathrm{u}_2(x,y,t)}{\partial x}+e_{11}^{(2)}\frac{\partial \varphi_2(x,y,t)}{\partial x}\right)_{y=0}=0 \tag{1.13}$$

$$\left(\frac{\partial \mathrm{u}_2(x,y,t)}{\partial y}+\frac{\partial \mathrm{v}_2(x,y,t)}{\partial x}\right)_{y=0}=0\,. \tag{1.14}$$

Condition (1.13), in which the notation $\vartheta_5=\left[(c_{13}^{(2)})^2-c_{12}^{(2)}c_{33}^{(2)}\right]/(c_{33}^{(2)}c_{44}^{(2)})$ is introduced, is obtained taking into account the second equation of material relations (1.5) and the compatibility condition for mechanical stresses (1.6).

The conditions for the conjugation of the electric field on the surfaces of the crack $y = 0$ between adjacent half-spaces, taking into account the zero gap width, can be written as

$$\left[\varphi_1(x,y,t) - \varphi_2(x,y,t)\right]\Big|_{y=0} = 0\,, \tag{1.15}$$

$$\left[e_{15}^{(1)}\frac{\partial \mathrm{w}_1(x,y,t)}{\partial y} - \varepsilon_{11}^{(1)}\frac{\partial \varphi_1(x,y,t)}{\partial y} + \varepsilon_{33}^{(2)}\frac{\partial \varphi_2(x,y,t)}{\partial y}\right]\Bigg|_{y=0} = 0 \tag{1.16}$$

Due to the conjugation of the electric field on the surfaces of the crack, the oscillations of the electric field, accompanying the wave signal of one type in the first medium, leak through the vacuum gap into another piezoelectric medium. In the second medium, another type of electroelastic wave is already generated (and vice versa)

$\{0;\ 0;\ \mathrm{w}_1(x,y,t);\ \varphi_1(x,y,t)\} \rightleftarrows \{\mathrm{u}_2(x,y,t);\ \mathrm{v}_2(x,y,t);\ 0;\ \varphi_2(x,y,t)\}$.

Hybridization of elastic heterogeneous waves, associated with the accompanying oscillations of the electric field, occurs.

In case of the propagation of high-frequency electromechanical waves (the propagation of ultrashort waves in thick layers, when the wavelength is much less than the thicknesses of adjacent layers $\lambda \ll \min\{h_1; h_2\}$), the above equations (1.4) and (1.7) ÷ (1.9) together with the boundary conditions (1.12) ÷ (1.16) and the damping conditions deep into half-spaces from the common surface of non-acoustic contact $y = 0$

$$\lim_{y\to\infty} \mathrm{w}_1(x,y,t) = 0, \qquad \lim_{y\to\infty} \varphi_1(x,y,t) = 0 \tag{1.17}$$

$$\lim_{y\to-\infty} \mathrm{u}_2(x,y,t) = 0; \qquad \lim_{y\to-\infty} \mathrm{v}_2(x,y,t) = 0; \qquad \lim_{y\to-\infty} \varphi_2(x,y,t) = 0 \tag{1.18}$$

constitute the complete boundary-value problem of the two-layer piezoelectric composite.

2. Solution of the boundary value problem of electro elasticity.

From the formulated mixed boundary-value problem, it is obvious that the induced normal electroelastic waves of elastic shear

$$\begin{Bmatrix} \mathrm{w}_1(x,y,t) \\ \varphi_1(x,y,t) \end{Bmatrix} = \begin{Bmatrix} \mathrm{w}_0(y) \\ \varphi_0(y) \end{Bmatrix} \cdot \exp[i(kx - \omega t)] \tag{2.1}$$

in the half-plane $\Omega_{10}(x,y) \triangleq \{|x| < \infty;\ 0 \le y < \infty\}$ as solutions of the system of equations (1.4), due to the conjugation of electric fields at the interface between half-spaces (1.15) and (1.16), can generate an electroelastic wave of plane deformation

$$\begin{Bmatrix} \mathrm{u}_2(x,y,t) \\ \mathrm{v}_2(x,y,t) \\ \varphi_2(x,y,t) \end{Bmatrix} = \begin{Bmatrix} \mathrm{u}_{20}(y) \\ \mathrm{v}_{20}(y) \\ \varphi_{20}(y) \end{Bmatrix} \cdot \exp[i(kx - \omega t)]\,. \tag{2.2}$$

in the half plane $\Omega_{20}(x,y) \triangleq \{|x| < \infty;\ -\infty < y \le 0\}$.

Taking into account the attenuation conditions (1.17), the solutions (2.1), damping deep into the half-space $\Omega_{10}(x,y)$, are written in the known form

$$\mathrm{w}_1(x,y,t) = A_1 \exp(-k\alpha_{1\mathrm{w}}(\omega,k)\cdot y)\cdot \exp[i(kx - \omega t)] \tag{2.3}$$

$$\varphi_1(x,y,t) = \left[C_1 \exp(-ky) + (e_{15}^{(1)}/\varepsilon_{11}^{(1)})A_1 \exp(-k\alpha_{1\mathrm{w}}(\omega,k)\cdot y)\right]\cdot \exp[i(kx - \omega t)] \tag{2.4}$$

In relations (2.3) and (2.4), the well-known notations are used: $\alpha_{1w} = \sqrt{1-\omega^2/k^2\tilde{C}_{1t}^2}$ - damping coefficient of elastic transverse vibrations in an antiplane deformation wave, $\tilde{C}_{1t}^2 = (\tilde{c}_{44}^{(1)}/\rho_1)$ - velocity of the electroelastic shear wave.

The permissible values of the phase velocity, as in the case of Gulyaev-Bluestein wave, have the form $\eta(\omega,k) = \omega/k < \tilde{C}_{1t}$.

Taking into account the decay conditions (1.18), solutions (2.2), which decay deep into the half-plane $\Omega_{20}(x,y)$, can be written as

$$\mathrm{u}_2(x,y,t) = \left[A_{\mathrm{u}}\exp[kq_{2\mathrm{u}}(\omega,k)\cdot y] + a_{\mathrm{u}\varphi}C_\varphi\exp[kq_{2\varphi}(\omega,k)\cdot y]\right]\cdot\exp[i(kx-\omega t)] \quad (2.5)$$

$$\mathrm{v}_2(x,y,t) = \begin{bmatrix} B_{\mathrm{v}}\exp[kq_{2\mathrm{v}}(\omega,k)\cdot y] + \\ +b_{\mathrm{vu}}A_{\mathrm{u}}\exp[kq_{2\mathrm{u}}(\omega,k)\cdot y] + b_{\mathrm{v}\varphi}C_\varphi\exp[kq_{2\varphi}(\omega,k)\cdot y] \end{bmatrix}\cdot\exp[i(kx-\omega t)] \quad (2.6)$$

$$\varphi_2(x,y,t) = \left[C_\varphi\exp[kq_{2\varphi}(\omega,k)\cdot y] + c_{\varphi u}A_{\mathrm{u}}\exp[kq_{2\mathrm{u}}(\omega,k)\cdot y]\right]\cdot\exp[i(kx-\omega t)] \quad (2.7)$$

The coefficients of the formation of an electroelastic wave of Rayleigh type $q_{2u}(\omega,k)$, $q_{2\mathrm{v}}(\omega,k)$ and $q_{2\varphi}(\omega,k)$ are obtained from the characteristic equation of the system (1.7)÷(1.9)

$$\left[q^2 - \left(\alpha_{2t}^2/\vartheta_3\right)\right]\cdot\left[\left(q^2-\alpha_{2\varphi}^2\right)\left[q^2-\left(\alpha_{2l*}^2/\vartheta_4\right)\right] - \tilde{\chi}_2^2\left(1+\vartheta_2\right)\right] = 0 \quad (2.8)$$

where $\vartheta_3 = \left(c_{33}^{(2)} - \vartheta_1 c_{13}^{(2)}\right)/c_{44}^{(2)}$ and $\vartheta_4 = c_{44}^{(2)}/\left(c_{11}^{(2)} - \vartheta_2 c_{12}^{(2)}\right)$ are the dimensionless anisotropy characteristics, and $\chi_2^2 = \left(e_{11}^{(2)}\right)^2/c_{44}^{(2)}\varepsilon_{11}^{(2)}$ and $\tilde{\chi}_2^2 = \left(\varepsilon_{11}^{(2)}/\varepsilon_{33}^{(2)}\right)\cdot\chi_2^2 = \left(e_{11}^{(2)}\right)^2/c_{44}^{(2)}\varepsilon_{33}^{(2)}$ are the reduced electromechanical coupling coefficients in the second medium

$$q_{2u}(\omega,k) = \sqrt{\frac{\left(\alpha_{2l*}^2/\vartheta_4\right)+\alpha_{2\varphi}^2}{2} + \sqrt{\frac{\left[\left(\alpha_{2l*}^2/\vartheta_4\right)-\alpha_{2\varphi}^2\right]^2}{4} + \tilde{\chi}_2^2\left(1+\vartheta_2\right)}}\ , \quad (2.9)$$

$$q_{2\mathrm{v}}(\omega,k) = \sqrt{\alpha_{2t}^2(\omega,k)/\vartheta_3}\ , \quad (2.10)$$

$$q_{2\varphi}(\omega,k) = \sqrt{\frac{\left(\alpha_{2l*}^2/\vartheta_4\right)+\alpha_{2\varphi}^2}{2} - \sqrt{\frac{\left(\left(\alpha_{2l*}^2/\vartheta_4\right)-\alpha_{2\varphi}^2\right)^2}{4} + \tilde{\chi}_2^2\left(1+\vartheta_2\right)}}\ . \quad (2.11)$$

In solutions (2.5)÷(2.7), notations of amplitude coefficients are introduced, which characterize the connection of the components of the electromechanical field in the second piezoelectric

$$b_{\mathrm{v}\varphi}(\omega,k) = i\frac{\left(e_{11}^{(2)}/c_{44}^{(2)}\right)\left(\vartheta_1/\vartheta_3\right)\cdot q_{2\varphi}(\omega,k)}{q_{2\varphi}^2(\omega,k) - q_{2\mathrm{v}}^2(\omega,k)}\ , \quad (2.12)$$

$$b_{\mathrm{vu}}(\omega,k) = i\frac{\chi_2^2\cdot\left(\vartheta_1/\vartheta_3\right)}{q_{2\mathrm{u}}^2(\omega,k) - q_{2\mathrm{v}}^2(\omega,k)}\cdot\frac{\alpha_{2\varphi}^2\cdot q_{2\mathrm{u}}(\omega,k)}{\alpha_{2\varphi}^2 - q_{2\mathrm{u}}^2(\omega,k)}\ , \quad (2.13)$$

$$a_{u\varphi}(\omega,k) = \frac{\left(1+\vartheta_2\right)\left(e_{11}^{(2)}/c_{44}^{(2)}\right)}{\left(\alpha_{2l*}^2(\omega,k)/\vartheta_4\right) - q_{2\varphi}^2(\omega,k)}\ , \qquad c_{\varphi u}(\omega,k) = \frac{\left(e_{11}^{(2)}/\varepsilon_{33}^{(2)}\right)}{\alpha_{2\varphi}^2 - q_{2u}^2(\omega,k)}\ . \quad (2.14)$$

In the expressions (2.9)÷(2.11) of the attenuation coefficients of the components of the electro-acoustic wave and the amplitude coefficients (2.12)÷(2.13), the attenuation coefficients of elastic dilatation $\alpha_{2l*} = \sqrt{1-\omega^2 C_{2l*}^{-2}/k^2}$, the attenuation coefficient of the transverse displacement $\alpha_{2t} = \sqrt{1-\omega^2 C_{2t}^{-2}/k^2}$, the attenuation coefficient of the electric field oscillations $\alpha_{2\varphi} = \sqrt{\varepsilon_{11}^{(2)}/\varepsilon_{33}^{(2)}}$, are written without taking into account the piezoelectric effect of the second medium. In these notations, $C_{2l*} = \sqrt{\left(c_{11}^{(2)} - \vartheta_2 c_{12}^{(2)}\right)/\rho_2}$ and $C_{2t} = \sqrt{c_{44}^{(2)}/\rho_2}$ are the velocities of purely elastic waves of dilatation and shear in the second medium.
From solutions (2.5)÷(2.7) it follows that in the second piezoelectric, the induced electroactive wave of dilatation $\mathrm{u}_2(x,y,t)$ and the oscillations of the accompanying electric field $\varphi_2(x,y,t)$ coincide in the propagation phase. The induced electroactive shear wave $\mathrm{v}_2(x,y,t)$ (second and third terms) is shifted from them by the propagation phase $\pi/2$.
From the solutions (2.5)÷(2.7) it is also obvious that the components of the induced electroelastic wave in the second piezoelectric are damped along the depth of the half-space in the zone of permissible phase velocities, when

$$\eta(\omega,k) < \min\left\{C_{2t}/\sqrt{\vartheta_3};\ C_{2l*}\cdot\sqrt{1-\chi_2^2\left(1+\vartheta_2\right)\vartheta_4}\right\} \quad \text{if} \quad \vartheta_3 > 0 \qquad (2.15)$$

or

$$C_{2t}\sqrt{1-\vartheta_3} < \eta(\omega,k) < C_{2l*}\cdot\sqrt{1-\chi_2^2\left(1+\vartheta_2\right)\vartheta_4} \qquad \text{if} \quad \vartheta_3 < 0 \qquad (2.16)$$

The existence of the second variant of conditions (2.16), when in an elastic medium $\vartheta_3 < 0$, is associated with the choice of the model of a plane stress-strain state in the statements of two-dimensional problems of electroacoustic in homogeneous piezoelectric crystals [2]. For the isotropic medium $\vartheta_3 \equiv 1$, and the longitudinal wave velocity will be $C_{2l} = \sqrt{c_{11}^{(2)}/\rho_2}$.
From the conditions for the existence of electroactive waves of the Rayleigh type (2.15) and (2.16) it follows that such waves in a medium can exist only in the case of a small value of the piezoelectric effect

$$\chi_2^2 < \left(c_{11}^{(2)} - \vartheta_2 c_{12}^{(2)}\right)/c_{44}^{(2)}\left(1+\vartheta_2\right) \qquad (2.17)$$

In elastic media (taking into account the piezoelectric effect or not) of the class $\bar{6}m2$ with anisotropy $c_{11}^{(2)} - \vartheta_2 c_{12}^{(2)} < 0$, Rayleigh waves do not exist.
In elasticity problems, when the piezoelectric effect in the medium is not taken into account $\tilde{\chi}_2^2 \equiv 0$, and the existing pressure of the selected coordinate surface on neighboring surfaces is not taken into account (condition (1.6)), the zone of permissible phase velocities (2.15) turns into the known relation $\eta(\omega,k) < \min\{\sqrt{c_{44}^{(2)}/\rho_2};\ \sqrt{c_{11}^{(2)}/\rho_2}\}$.
Substituting the decaying solutions (2.3)÷(2.7) into the boundary conditions (1.12)÷(1.16), we obtain the dispersion equation of electroelastic hybrid ***R&GB*** waves in the following form

$$\frac{\alpha_{1\mathrm{w}} - \tilde{\chi}_1^2}{\alpha_{1\mathrm{w}}} = -\left(\varepsilon_{11}^{(1)}/\varepsilon_{33}^{(2)}\right)\begin{pmatrix} c_{\varphi u} & 1 \\ \left(\vartheta_5 + e_{11}^{(2)}c_{\varphi u}\right) & \left(e_{11}^{(2)} + \vartheta_5 a_{u\varphi}\right)\end{pmatrix} \Big/ \begin{pmatrix} q_{2\mathrm{u}}c_{\varphi u} & q_{2\varphi} \\ \left(\vartheta_5 + e_{11}^{(2)}c_{\varphi u}\right) & \left(e_{11}^{(2)} + \vartheta_5 a_{u\varphi}\right)\end{pmatrix} \qquad (2.18)$$

If the piezoelectric effect is not taken into account in the second piezoelectric, i.e. when $e_{11}^{(2)} \equiv 0$, $\chi_2^2 \equiv 0$ and $\chi_{2*}^2 \equiv 0$, all amplitude coefficients (2.12)÷(2.14) are zero. The expressions of the shaping coefficients (2.9)÷(2.11) of the electro-elastic Rayleigh wave $q_{2u}(\omega,k) = \alpha_{2l*}(\omega,k)\big/\sqrt{\vartheta_4}$ and $q_{2\varphi}(\omega,k) = \alpha_{2\varphi} \equiv \sqrt{\varepsilon_{11}^{(2)}/\varepsilon_{33}^{(2)}}$ are also simplified.

To maintain solutions (2.3) and (2.4) in the first piezoelectric half-space, and the accompanying oscillations of the electric field in the second piezoelectric half-space

$$\varphi_2(x,y,t) = C_{2\varphi}\exp[k\alpha_{2\varphi}\cdot y]\cdot\exp[i(kx-\omega t)], \tag{2.19}$$

from the dispersion equation (2.17) of the hybrid wave, the dispersion equation of the Gulyaev – Bluestein problem remains [3],

$$\alpha_{1t} = \tilde{\chi}_1^2 \cdot \frac{\varepsilon_{33}^{(2)}}{\varepsilon_{33}^{(2)} + \varepsilon_{11}^{(1)}} \tag{2.20}$$

with the difference in the dielectric constant of the media $\varepsilon_{33}^{(2)} \rightleftarrows \varepsilon^{(e)} = 0.885\times10^{-11}\,F/m$.

If the piezoelectric effect is not taken into account in the first piezoelectric half-space $e_{15}^{(1)} \equiv 0$ and $\tilde{\chi}_1^2 \equiv 0$, the dispersion equation (2.17) is simplified, taking the form of the dispersion equation of the electro-acoustic Rayleigh problem

$$\begin{aligned}&\left[q_*^2(\omega,k) + \vartheta_5\left(1+\vartheta_2\right)\right]\cdot q_{2u}(\omega,k) - \left[q_*^2(\omega,k) + \vartheta_5\left(1+\vartheta_2\right)\theta\right]\cdot q_{2\varphi}(\omega,k) - \\ &-\vartheta_5\left(1+\vartheta_2\right)\left(\theta-1\right)(\varepsilon_{11}^{(1)}/\varepsilon_{33}^{(2)}) = 0\end{aligned} \tag{2.21}$$

in order to maintain the solutions (2.5) and (2.7) in the second piezoelectric half-space, and associated electrical vibrations in the first piezoelectric half-space

$$\varphi_1(x,y,t) = C_{1\varphi}\exp[-k\alpha_{2\varphi}\cdot y]\cdot\exp[i(kx-\omega t)] \tag{2.22}$$

From the form of dispersion equation (2.17) it follows that it has a solution in the case of permissible wave signal frequencies at which the dispersion function in the second piezoelectric takes on values

$$\left(\frac{c_{\varphi u}}{\left(\vartheta_5 + e_{11}^{(2)}c_{\varphi u}\right)} \quad \frac{1}{\left(e_{11}^{(2)} + \vartheta_5 a_{u\varphi}\right)}\right)\Bigg/\left(\frac{q_{2u}c_{\varphi u}}{\left(\vartheta_5 + e_{11}^{(2)}c_{\varphi u}\right)} \quad \frac{q_{2\varphi}}{\left(e_{11}^{(2)} + \vartheta_5 a_{u\varphi}\right)}\right) \le \left(\varepsilon_{33}^{(2)}/\varepsilon_{11}^{(1)}\right)\cdot\left(1-\tilde{\chi}_1^2\right) \tag{2.23}$$

Relation (2.22) determines the zone of permissible phase velocities $\Omega_0\left(\omega/k(\omega)\right)$. The phase velocity of the formed electro-acoustic hybrid wave is determined by solving the dispersion equation of the hybrid of waves ***R&GB*** (2.17), taking into account the function (2.18).

From the form of solutions (2.3)÷(2.7) it follows that the wave electroacoustic signal of the type $\{0;\ 0;\ \mathrm{w}_1(x,y,t);\ \varphi_1(x,y,t)\}$ in the first piezoelectric of the class $6mm$ induces an electro-acoustic wave of dilatation of the type $\{\mathrm{u}_2(x,y,t);\ 0;\ 0;\ \varphi_2(x,y,t)\}$ in the second piezoelectric of the class $\overline{6}m2$, followed by a shear component $\mathrm{v}_2(x,y,t)$, with the phase shift $\pi/2$.

3. Numerical comparative analysis of the results.

The determination of the zones of permissible phase velocities and phase function of the resulting electro-acoustic hybrid wave is carried out by selecting neighboring materials with different physicomechanical constants (Table 1). Choosing the following material constants, we calculate the dimensionless parameters of the anisotropy and wave velocity in these materials for piezoelectrics of the class $\bar{6}m2$, P_{21}: $\vartheta_1 = 1.4297$, $\vartheta_2 = 1.2679$, $\vartheta_3 = 1.3819$, $\vartheta_4 = 0.6808$, $\theta = 0.4305$, $C_{2t} = 2.3358 \times 10^3\ m/s$ $C_{2l} = 6.0938 \times 10^3\ m/s$, $C_{2l*} = 3.9118 \times 10^3\ m/s$ и P_{22}: $\vartheta_1 = 3.4306$, $\vartheta_2 = 2.0756$, $\vartheta_3 = -0.6269$, $\vartheta_4 = 0.6131$, $\theta = 0.4465$, $C_{2t} = 2.6644 \times 10^3\ m/s$, $C_{2l} = 5.8498 \times 10^3\ m/s$, $C_{2l*} = 5.0924 \times 10^3\ m/s$ respectively.

Table 1. The different physicomechanical constants for piezoelectric materials

Physico mechanical constants	Piezoelectric materials			
	P_{11}, class $6mm$	P_{12}, class $6mm$	P_{21}, class $\bar{6}m2$	P_{22}, class $\bar{6}m2$
$c_{11}(N/m^2)$			8.682×10^{10}	6.17×10^{10}
$c_{12}(N/m^2)$			4.0258×10^{10}	0.72×10^{10}
$c_{13}(N/m^2)$			4.762×10^{10}	1.19×10^{10}
$c_{33}(N/m^2)$			8.571×10^{10}	3.28×10^{10}
$c_{44}(N/m^2)$	1.639×10^{10}	14.330×10^{10}	1.2756×10^{10}	1.28×10^{10}
$\rho(kg/m^3)$	5.302×10^3	4.820×10^3	2.338×10^3	1.803×10^3
$\varepsilon_{11}(F/m)$	$8{,}786 \times 10^{-11}$	9.312×10^{-11}	9.312×10^{-11}	49.8938×10^{-11}
$\varepsilon_{33}(F/m)$			19.293×10^{-11}	14.5081×10^{-11}
$e_{15}(C/m^2)$	0.534	2.723		
$e_{11}(C/m^2)$			0.576	1.225
$\tilde{\chi}_1^2$	0.1980	0.5556		
χ_2^2			0.6120	0.2349

For piezoelectric class: $6mm$ and P_{11}: $\tilde{C}_{1t} = 2.1063 \times 10^3\ m/s$ and P_{12}: $\tilde{C}_{1t} = 8.4822 \times 10^3\ m/s$.

From the expressions (2.3) and (2.4) it is obvious that when choosing different piezoelectrics of the class $6mm$, the coefficients of the Gulyaev-Bluestein wave components differ only quantitatively.

When choosing different piezoelectrics from a class $\bar{6}m2$ outside the hybrid, the coefficients of the components of a Rayleigh wave type differ both quantitatively and qualitatively.

The forming coefficients of the components of the constituents of the hybrid of Gulyaev-Bluestein type waves $\alpha_{1w}(\omega,k)$ from (2.3), (2.4) and Rayleigh type waves $q_{2u}(\omega,k)$, $q_{2v}(\omega,k)$ and $q_{2\varphi}(\omega,k)$ from (2.9), (2.10) and (2.11), are functions of the phase velocity $\omega/k(\omega)$. From their expressions, it is obvious that when choosing different piezoelectrics of the class $6mm$, the coefficients of the components of the constituents of the wave of Gulyaev-Bluestein type outside the hybrid, change only quantitatively. When choosing different piezoelectrics of the class $\bar{6}m2$, the coefficients of constituents the components of Rayleigh type wave outside the hybrid change both quantitatively and qualitatively.

From the numerical data it follows that conditions (2.16) and (2.17) are fulfilled for a piezoelectric P_{22} , and conditions (2.17) are not fulfilled for a piezoelectric P_{21} of hexagonal symmetry class $\bar{6}m2$. Consequently, the induced electro-acoustic wave in the second piezoelectric P_{21} will not decrease in depth of half-space.

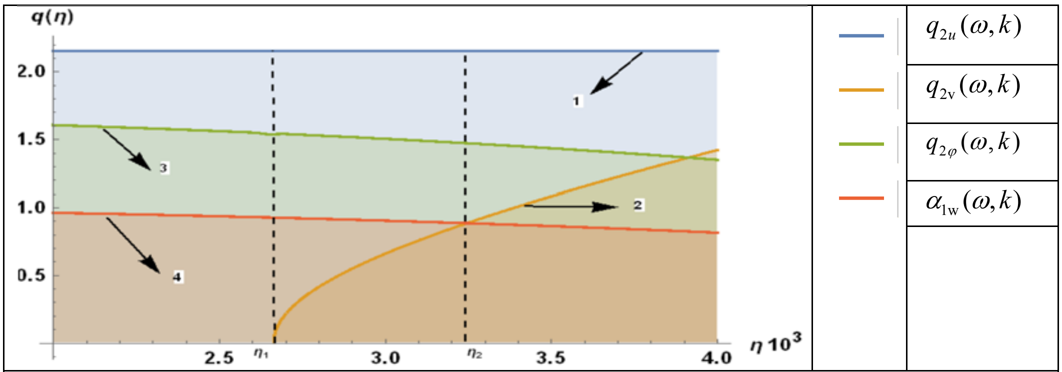


Fig. 2 ***Forming coefficients of the components of the hybrid wave and the zone of permissible phase velocities*** $[\eta_1(\omega);\ \eta_2(\omega)]$ ***in a composite of pairs of piezoelectrics -*** P_{12} ***of the class*** $6mm$ ***and*** P_{22} ***- of hexagonal symmetry class*** $\bar{6}m2$

Although in a composite of a pair of piezoelectrics P_{11} of the class $6mm$ and P_{22} of hexagonal symmetry class $\bar{6}m2$, wave forms decaying along the depth of half-spaces are formed, but the zones of permissible phase velocities for them are different. In a composite of a pair of piezoelectrics P_{12} of the class $6mm$ and P_{22} of hexagonal symmetry class $\bar{6}m2$, electroactive wave forms that decay along the depth of half-spaces have a common zone of permissible phase velocities (Fig. 2).

From Figure 2 it follows that in a piezoelectric P_{22} of the class $\bar{6}m2$, the second decaying form of elastic shear with the coefficient $q_{2v}(\omega,k)$ is formed, starting from a certain value of the wave propagation velocity $\eta_1(\omega_0) = C_{2t} = 2.6644\times10^3\ m/s$. Highlighted by vertical lines in Figure 2, the zone of permissible phase velocity values $\eta(\omega)\in[\eta_1(\omega);\ \eta_2(\omega)]$ is determined by condition (2.16).

From Figure 2 it also follows that the corresponding frequency ω_{01} , when $q_{2v}(\omega_{01},k_r) = \alpha_{1w}(\omega_{01},k_r)$ for the limiting phase velocity $\eta_2(\omega_{01})$, will be resonant in the second piezoelectric P_{22} .

The formation of the electro-acoustic hybrid ***R&GB*** (a combination of electro-acoustic waves of the Rayleigh type and the Gulyaev-Bluestein type) is determined by the solution of dispersion equation (2.18) in the zone of permissible values of the phase velocity of the decaying components of the hybrid wave (2.16) and relation (2.23). Consequently, for different duets of adjacent piezoelectric materials, the zones of permissible frequencies of the electro-acoustic wave hybrid are determined differently.

The numerical solution of dispersion equation (2.18) in the case of a pair of piezoelectrics P_{12} of the class $6mm$ and P_{22} of hexagonal symmetry class $\overline{6}m2$ is shown in Figure 3. The figure shows that in the case of the selected pair of media, we have the solution of the dispersion equation in the zone of permissible phase velocities $\eta_0(\omega) \in [\eta_1(\omega);\ \eta_2(\omega)]$.

The phase velocity of the hybrid $\eta_0(\omega,k)$, which satisfies the conditions for localization of electro-acoustic components (2.16), leads to the joint propagation of electroactive waves of Gulyaev-Bluestein type in piezoelectrics P_{12} of the class $6mm$ and electroactive waves of Rayleigh type in piezoelectrics P_{22} of the class $\overline{6}m2$.

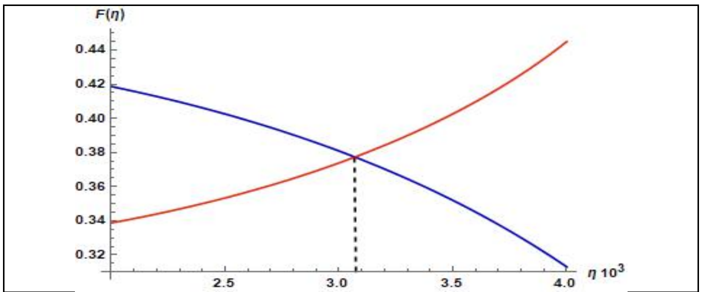
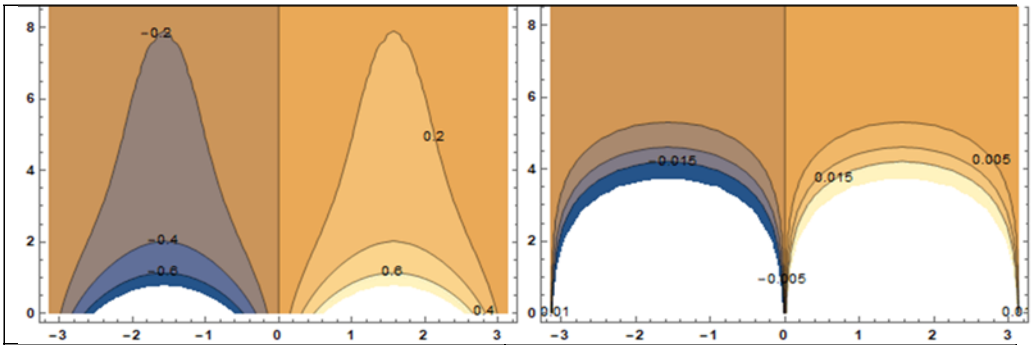


Fig. 3 ***Numerical solution of the dispersion equation of an electro-acoustic hybrid wave*** $\eta_0(\omega,k) = 3.074\times10^3\ m/s$

In the piezoelectric P_{12} of the class $6mm$, electroactive shear waves of length $\lambda < 8.4822(2\pi/\omega)$ will propagate. In the piezoelectric P_{22} of the class $\overline{6}m2$ electroactive waves of plane deformation of length $3.3984\cdot(2\pi/\omega) < \lambda < 4.5103\cdot(2\pi/\omega)$ will propagate.



a) in the case of the free surface of the piezoelectric

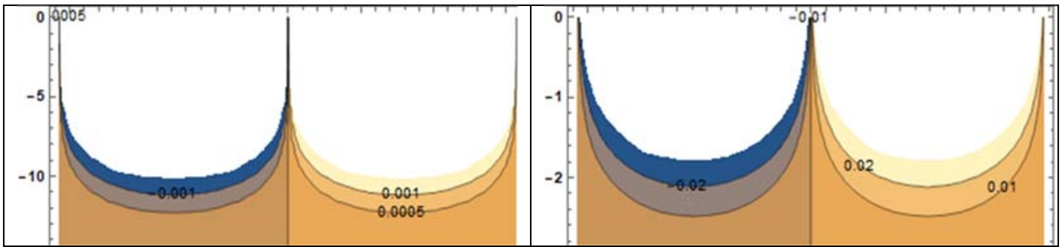
b) in the composition of the hybrid wave in the composite

Fig. 4 ***Type of localization of accompanying oscillations of the electric field in piezoelectric*** P_{12} ***of the class*** $6mm$

Taking into account the obtained numerical values of the forming coefficients of the components of the hybrid electro-acoustic wave, the figures below show the comparative analysis of the changes in the distribution of wave components.

When propagating, the electro-acoustic wave signal of Gulyaev-Bluestein type strongly changes the form of localization near the non-acoustic contact between piezoelectrics (Fig. 4.a), 4.b) and Fig. 6.a), 6.b)).

Near the surface of the piezoelectric P_{12} of the class $6mm$, the oscillations of the electric field, accompanying the elastic shear signal in the hybrid, are localized at shallow depth and with a smaller amplitude (Fig. 4.b) than in the case of the free surface of the piezoelectric (Fig. 4.a).



a) in the case of the free surface of the piezoelectric

b) in the composition of the hybrid wave in the composite

Fig. 5 ***Localization of accompanying oscillations of the electric field in a piezoelectric*** P_{22} ***of the class*** $\bar{6}m2$

Along with this, in the piezoelectric P_{22} of the class $\bar{6}m2$, the localization of the accompanying electric field oscillations in the case of the free surface of the piezoelectric occurs at a depth of approximately $3.5 \cdot \lambda$ (Fig. 5.a)), and in the composition of the hybrid, the localization of the accompanying electric field oscillation occurs quickly, already at a depth of approximately $0.75 \cdot \lambda$ (Fig. 5 b).

In the composition of the hybrid, the localization of both components of displacements occurs near the non-acoustic contact of piezoelectrics (Fig. 7.a) and Fig. 7.b)).

Figures 6.a) and 6.b) show the types of localization of the electro-acoustic displacement of the shear in the piezoelectric P_{12} of the class $6mm$ and the electro-acoustic dilatation in the piezoelectric P_{22} of the class $\bar{6}m2$ in the case of the free surface of the piezoelectrics.

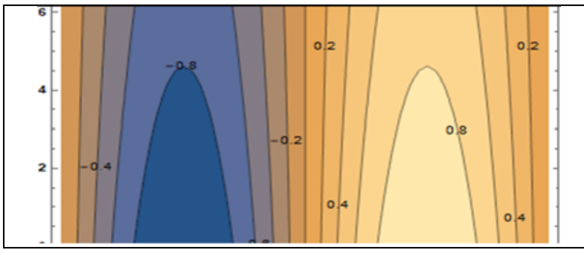


Fig. 6.a ***Localization of the electro-acoustic displacement of the shear in the piezoelectric*** P_{12} ***of the class*** $6mm$ ***in the case of the free surface of the piezoelectric***

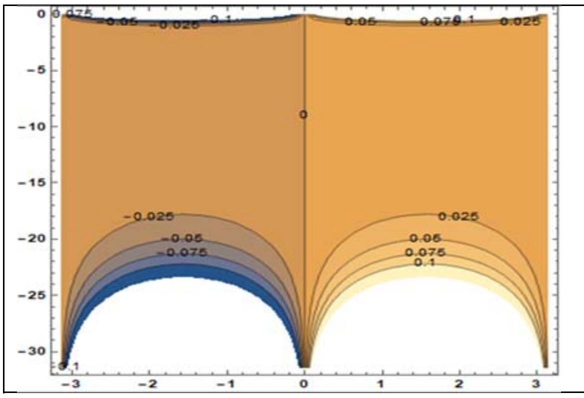


Fig. 6.b ***Localization of electro-acoustic dilatation in the piezoelectric*** P_{22} ***of the class*** $\bar{6}m2$ ***in the case of the free surface of the piezoelectric***

If the electro-acoustic displacement of the shear in the piezoelectric P_{12} has a canonical localization form, then the electro-acoustic displacement of the dilatation in the piezoelectric P_{22} has localization both near the free surface of the piezoelectric and in the depth of about 5λ of the half-spaces.

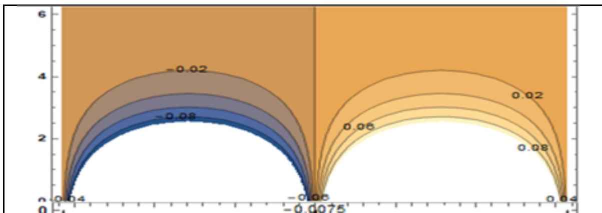


Fig. 7.a ***Localization of the electro-acoustic displacement of the shear as part of the hybrid wave in the piezoelectric*** P_{12} ***of the class*** $6mm$ ***in the composition of the hybrid wave***

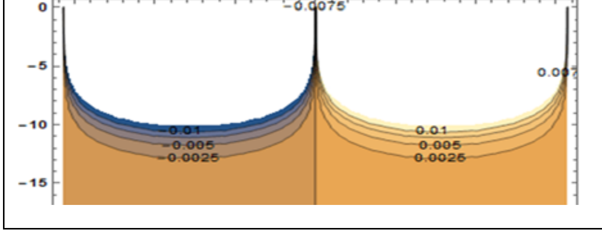


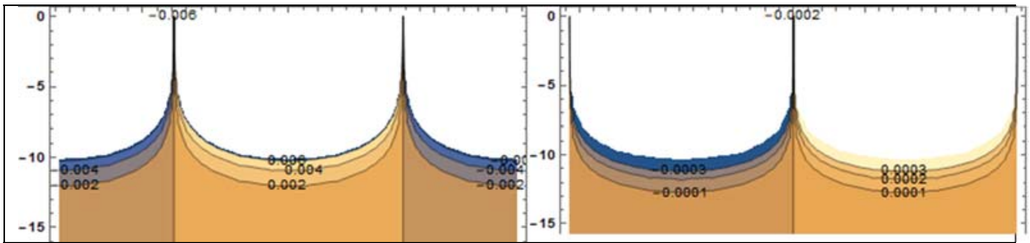
Fig. 7.b ***Localization of electro-acoustic dilatation as part of the hybrid wave in the piezoelectric*** P_{22} ***of the class*** $\bar{6}m2$ ***in the composition of the hybrid wave***

Moreover, the localization of the electro-acoustic shift in the composition of the hybrid is located approximately at the depth $1.25 \cdot \lambda$ of the first half-space, and the localization of the electro-acoustic dilation in the composition of the hybrid is located approximately at the depth $4.0 \cdot \lambda$ of the second half-space.

The phase velocity of the hybrid wave is about 2.5 times less than the speed of the Gulyaev-Bluestein wave in the first piezoelectric with the free surface.

From Figure 7.b) it can be seen that in the piezoelectric P_{22} of the class $\bar{6}m2$ the form of electro-acoustic dilatation in the composition of the hybrid qualitatively changes compared to the form of electro-acoustic dilatation in the composition of the Rayleigh electro-acoustic wave (Fig. 6.b)).

The phase velocity of the hybrid wave is approximately 1.5 times lower than the dilatation velocity in the Rayleigh electro-acoustic wave in the piezoelectric P_{22} with the free surface. In the piezoelectric P_{22} of the class $\overline{6}m2$, two electroactive forms of shear propagate in the composition of the hybrid. One term propagates simultaneously with dilatation, and the second term propagates with a phase shift $\pi/2$ (Fig. 7). Moreover, the amplitude of the late form is greater than the amplitude of the synchronous form about 20 times.
The phase velocity of the hybrid wave is approximately 13.3% higher than the velocity of the shear component in the Rayleigh electro-acoustic wave in the piezoelectric P_{22} with the free surface. In the absence of the piezoelectric effect in the medium of the second half-space, when $e_{11}^{(2)} \equiv 0$, the elastic plane deformation and the electric field in this medium are separated.



a) the form propagates with a phase shift $\pi/2$

b) the form is propagates simultaneously with the dilatation

Fig. 8 ***Localization of the electro-acoustic displacement of shear in the piezoelectric*** P_{22} ***of the class*** $\overline{6}m2$***, in the composition of the electro-acoustic hybrid wave***

With this in mind, the formation coefficients of the electro-elastic Rayleigh wave $q_{2u}(\omega,k)$ and $q_{2\varphi}(\omega,k)$ are simplified.

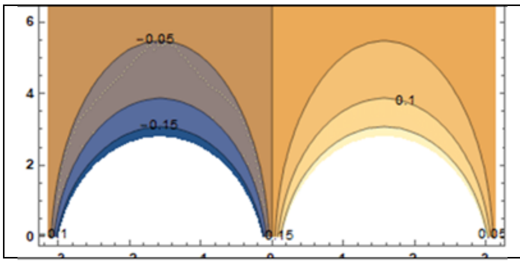


Fig. 9.a ***Localization of accompanying oscillations of the electric field in the piezoelectric*** P_{12} ***of the class*** $6mm$***, in the case of the presence of a dielectric near the free surface of the piezoelectric***

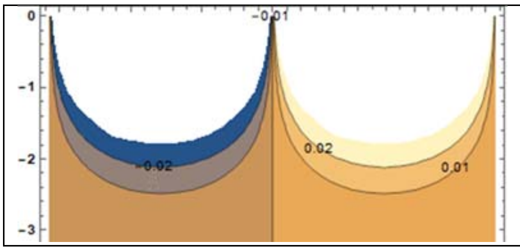


Fig. 9.b ***Localization of the accompanying oscillations of the electric field in the piezoelectric P_{22} of the class $\bar{6}m2$, in the case of the presence of a dielectric near the free surface of the piezoelectric***

In this case, in the first half-space of the piezoelectric P_{12} of the class $6mm$, a two-component electro-elastic Gulyaev-Bluestein wave (2.3) and (2.4) propagates and the accompanying oscillations of the electric field in the second layer

$$\varphi_2(x,y,t) = C_{2\varphi}\exp[k\,(\varepsilon_{11}^{(2)}/\varepsilon_{33}^{(2)})\cdot y]\cdot\exp[i(kx-\omega t)] \tag{2.24}$$

The phase velocity of this wave is determined from the dispersion equation

$$\sqrt{1-\left(V_{GB}^2/\tilde{C}_{1t}^2\right)} = \tilde{\chi}_1^2\cdot\varepsilon_{33}^{(2)}\Big/\left(\varepsilon_{11}^{(1)}+\varepsilon_{33}^{(2)}\right) \tag{2.25}$$

The distributions of the accompanying electroactive shear $w_{01}(x,y)$ of electrical vibrations $\varphi_{01}(x,y)$ and $\varphi_2(x,y)$ are shown in Figures 9.a) and 9.b), respectively.

Conclusion.

In a layered piezoelectric composite, by choosing pairs of different piezoelectrics with a non-acoustic contact, a hybrid of localized electro-acoustic waves of the Rayleigh and Gulyaev-Bluestein types can be obtained at the surface of the non-acoustic contact.

It is shown that the localized electroelastic shear wave signal of the Gulyaev-Bluestein wave type near the surface of the non-acoustic contact in one medium can generate an electroelastic plane deformation wave of the Rayleigh wave type in another medium and vice versa.

It is also shown that the corresponding choice of materials of adjacent half-spaces can lead to an increase or decrease of the localization of the energy of electroelastic waves in the vicinity of the no mechanical contact surface of two piezoelectrics.

It was revealed that an electro-acoustic wave signal induced with the non-resonant frequency in one of the media can cause internal resonance, or the formation of forbidden frequency bands in the waveguide of given structure.

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