ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 73, №1, 2020 Механика
УДК 539.3 Doi- http://doi.org/10.33018/73.1.1

PROPAGATION OF HYBRID ELECTROELASTIC WAVES IN TRANSVERSALLY INHOMOGENEOUS PERIODIC PIEZOELECTRIC STRUCTURE

Avetisyan Ara S. , Khachatryan Vazgen M.

Keywords: electro-acoustic hybrid wave, periodic inhomogeneity, piezoelectric medium, wave signal, non-acoustic contact, resonant frequency.

Аветисян Ара С. , Хачатрян В.М.

Распространение гибридных электроупругих волн в периодически поперечно-неоднородной пьезоэлектрической структуре

Ключевые слова: электроакустическая гибридная волна, периодическая неоднородность, пьезоэлектрическая среда, волновой сигнал, не акустический контакт, резонансная частота.

В пьезоэлектрическом композиционном материале обнаружено существование новой гибридной волны, состоящей из сдвиговой электроакустической волны и электроакустической волны плоской деформации. Пьезоэлектрические слои изготовлены из разных материалов и находятся в неакустическом контакте. Доказано, что если один из материалов допускает раздельное возбуждение и распространение электроактивной волны чистого сдвига, а другой материал допускает раздельное возбуждение и распространение электроактивной волны дилатации, то в неоднородной структуре волновой сигнал приводит к распространению гибридной электроакустической волны. Исследованы распределения волновых мод компонент гибридной электроакустической волны, а также допустимые частоты его распространения.

Как частные случаи, также исследованы распространения электроакустического волнового сигнала в разных однородных пьезоэлектрических средах с системой бесконечных поперечных трещин.

Ավետիսյան Արա Ս., Խաչատրյան Վազգեն Մ.

Խաչասերված էլեկտրաառաձգական ալիքների տարածումը պարբերականորեն լայնական անհամասեռ պյեզոէլեկտրական կառուցվածքում

Հիմնաբառեր. էլեկտրաձայնային խաչասերված ալիք, պարբերական անհամասեռություն, պյեզոէլեկտրական միջավայր, ալիքային ազդանշան, անհպում կապ, ռեզոնանսային հաճախականություն:

Հետազոտված է էլեկտրաառաձգական ալիքային ազդանշանի տարածումը պարբերականորեն անհամասեռ, շերտավոր պյեզոէլեկտրական միջավայրում: Շերտերը տարբեր պյեզոէլեկտրական նյութերից են, և գտնվում են անհպում կապի մեջ: Ապացուցված է, որ եթե նյութերից մեկը թույլ է տալիս էլեկտրականապես ակտիվ, մաքուր սահքի ալիքի առանձնացված գրգռում և տարածում, իսկ մյուսը թույլ է տալիս էլեկտրականապես ակտիվ մաքուր դիլատացիայի ալիքի առանձնացված գրգռում և տարածում, ապա էլեկտրաառաձգական ալիքային ազդանշանը պարբերականորեն անհամասեռ, շերտավոր պյեզոէլեկտրական միջավայրում բերում է խաչասերված ալիքի տարածման: Հետազոտված են խաչասերված ալիքի բաղադրիչների բաշխումները շերտերի լայնությամբ, ինչպես նաև դրանց թույլատրելի հաճախականությունները:

Որպես մասնակի դեպքեր, ուսումնասիրված են նաև էլեկտրաձայնային ալիքային ազդանշանի տարածումը լայնական ճաքերի անվերջ համակարգով համասեռ պյեզոէլեկտրական տարբեր միջավայրերում:

In a piezoelectric composite material, existence of a new hybrid wave, which is consisting of shear electroacoustic wave and electroacoustic wave of plane deformation is revealed. In the composite, layers are made of different piezoelectric materials and they are in non-acoustic contact. It is proved that if one of the materials allows separate excitation and propagation of electroactive wave of pure shear, and the other material allows separate excitation and propagation of electroactive wave of dilatation, then the wave signal leads to the propagation of hybrid electro-acoustic wave in the heterogeneous structure. The distributions of the components of hybrid electro-acoustic wave modes, as well as the corresponding permissible frequencies are investigated.
As special cases, the propagation of electro-acoustic wave signal in different homogeneous piezoelectric media, with a system of infinite transverse cracks is studied.

Introduction

Inhomogeneous composite waveguides of piezoelectric crystals are widely used in nowadays high-accuracy technologies as transformators, filters and resonators of electroacoustic wave signal. For the comprehensive review of perspectives, current state and future areas of development in analysis of wave processes in periodic structures, see **Hussein et al. [1].** During the analysis of wave processes in periodic structures, the main attention is paid to the character of these structures as frequency filters or resonators of propagating waves.

Occurrence of frequency locking zones for one-directional wave in periodic elastic structure has been obtained in **Rayleigh [2].** The analysis of the impedance role on existence of locking zones has been carried out in **Avetisyan & Ghazaryan [3, 4]**. It has been shown also that when the impedance of periodically inhomogeneous 1D structure is constant, then locking zones do not exist.

The Floquet-Lyapunov theory has been applied for analyzing elastic wave propagation in periodic structures in **Lee [5]** and **Lee & Yang [6].** From mathematical point of view, the spectral theory of transversal vibrations of periodic elastic beams has been developed in **Papanicolaou [7, 8]**. The dispersion relations of SH-waves propagating in periodic piezoelectric layered composites has been derived and studied in **Qian et al [9]**. The spectrum of Floquet-Bloch type waves propagating in elastic periodic waveguides has been studied in **Adams et al. [10, 11]**. In mentioned papers, the wave-field is homogeneous and the character of normal wave propagating in the waveguide does not change.

The phenomenon of coupled (simultaneous) propagation of waves of heterogeneous elastic deformations can be applied in various areas of technical electronics and high-precision measuring equipment.

It is well-known that not in all anisotropic piezoelectric materials, electric vibrations that are accompanied by elastic deformations allow separate excitation and propagation of pure shear electroelastic wave or electroelastic wave of plane strain. The possibility of separate excitation and propagation of electroactive elastic fields in specific sagittal surfaces of anisotropy of piezoelectric crystals has been studied in **Avetisyan [12]** without taking into account the hypotheses of

i) undeformed normal of the sagittal surface,
ii) absence of pressure of material surfaces onto each other,
iii) non-extensible material sagittal surface.

Problems of separate excitation and propagation of electroelastic plane or anti-plane stress-strain states in homogeneous piezoelectric crystals has been studied also in **Avetisyan**

[13]. Formulations of 2D electroacoustic problems in homogeneous piezoelectric crystals are considered taking into account the above hypotheses. Necessary and sufficient conditions for separate excitation and propagation of electroacoustic states of mentioned types in anisotropic piezoelectric media are obtained. Constitutive laws and the quasi-static equations of electro elasticity, for all piezoelectric materials in all three sagittal surfaces of corresponding crystal texture are also received.

The electroactive waves of anti-plane deformation, after their discovery by **Bluestein J. L. [14],** have been extensively investigated. A thousand works are known.

In contrast, electro-elastic waves of plane deformation in piezoelectric crystals (electro-acoustic waves of the Rayleigh type) have been studied relatively little by **Singh B. & Singh R. [15], Chaudhary S., Sahu S.A. & Singhal A. [16], Vashishth A.K. & Sukhija, H., [17] et al**.

The problem of propagation of high frequency acoustic waves of plane strain (electro-acoustic Rayleigh waves) at different electric boundary conditions for mechanically free surface of a piezoelectric half-space are discussed in **Avetisyan & Mkrtchyan [18]**. The possibility of a new localization of wave's plane strain, under certain electrical conditions at a surface is shown. It is revealed that the presence of a concomitant electric field, the waves of plane strain result in both quantitative and qualitative changes of the characteristics of a localization of electro-acoustic Rayleigh waves.

In this paper, we suggest a simple scheme for analysis of propagation of 1D electroactive hybrid elastic waves in compound transversally inhomogeneous periodic piezoelectric medium.

1. Problem statement. Electro elasticity equations and main relations for transversally inhomogeneous periodic piezoelectric medium

Let us consider the propagation of electroelastic 1D normal waves in transversally inhomogeneous periodic piezoelectric medium referred to Cartesian coordinate system $0xyz$ (see Figure 1). The medium consists of alternating infinite plates

$$\begin{aligned}\Omega_{1n}(x,y,z) &\triangleq \{x\in[n(a_1+a_2);a_1+n(a_1+a_2)],\ y\in(-\infty;+\infty), z\in(-\infty;+\infty)\}\\ \Omega_{2n}(x,y,z) &\triangleq \{x\in[a_1+n(a_1+a_2);(n+1)(a_1+a_2)],\ y\in(-\infty;+\infty), z\in(-\infty;+\infty)\}\end{aligned} \tag{1.1}$$

made of piezoelectric crystals. Linear sizes of these plates towards $0y$ and $0z$ are much larger than the wavelength of the propagating high-frequency wave.

Above, $n\in\mathbb{N}^+$ is the number of the repeating cell of two sub-layers with section $\Omega_0(x,y)=\Omega_{20}(x,y)\cup\Omega_{10}(x,y)$ in $x0y$ plane, where

$$\Omega_{10}(x,y)\triangleq\{x\in[0,a_1];\ y\in(-\infty,+\infty)\};\qquad \Omega_{20}(x,y)\triangleq\{x\in[-a_2,0];\ y\in(-\infty,+\infty)\}. \tag{1.2}$$

For the sake of definiteness, we assume that in $x0y$ plane, the material of layers with sections $\Omega_{1n}(x,y)$ allows separate excitation and propagation of electroactive elastic shear, while the material of layers with sections $\Omega_{2n}(x,y)$ allows separate excitation and propagation of electroactive plane stress-strain state.

It has been shown in [13] that there are 15 possible formulations of electroactive anti-plane states in distinct sagittal planes of piezoelectric crystals. It has also been shown therein that there are 10 possible formulations of electroactive generalized plane stress-strain state in distinct sagittal planes of piezoelectric crystals. Therefore, there are a large number of possible material pairs for preparation of the proposed inhomogeneous structure.

Without losing the generality of reasoning, we assume that the material of layers $\Omega_{1n}(x,y,z)$ allowing electroactive anti-plane stress-strain state is a piezoelectric of

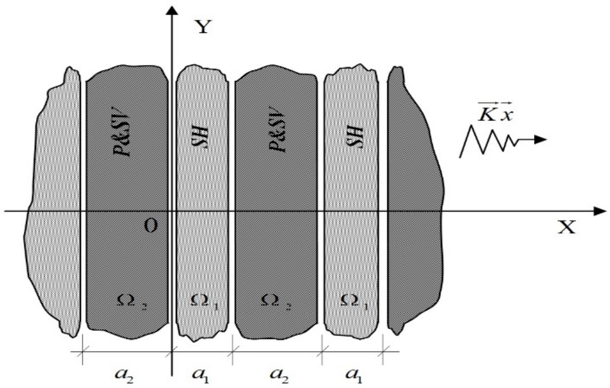


Figure 1. Transversally inhomogeneous periodic composite waveguide with non-acoustic contact between sub-layers

hexagonal symmetry class $6mm$ or is a piezoelectric of tetragonal symmetry class $4mm$. In these piezoelectrics, the anti-plane stress-strain state in $x0y$ plane is possible when the axis of 6[th] order symmetry of hexagonal piezocrystal, $\vec{p}_6$, or when the axis of 4[th] order symmetry of tetragonal piezocrystal, respectively, coincide with $0z$ axis. Then, the sagittal plane of piezoelectric crystals $x_1 0 x_2$ coincides with coordinate plane xOy.

The non-zero components of mechanical stress tensor and of electric displacement vector in layer with section $\Omega_{10}(x,y) \triangleq \{x \in [0,a_1];\ y \in (-\infty,+\infty)\}$ can be written as [13]

$$\sigma_{zx}^{(1)}(x,y,t)=c_{44}^{(1)}(\partial \mathrm{w}_1/\partial x)+e_{15}^{(1)}(\partial \varphi_1/\partial x);\ \sigma_{xy}^{(1)}(x,y,t)=c_{44}^{(1)}(\partial \mathrm{w}_1/\partial y)+e_{15}^{(1)}(\partial \varphi_1/\partial y) \quad (1.3)$$

$$D_x^{(1)}(x,y,t)=e_{15}^{(1)}(\partial \mathrm{w}_1/\partial x)-\varepsilon_{11}^{(1)}(\partial \varphi_1/\partial x);\ D_y^{(1)}(x,y,t)=e_{15}^{(1)}(\partial \mathrm{w}_1/\partial y)-\varepsilon_{11}^{(1)}(\partial \varphi_1/\partial y) \quad (1.4)$$

The quasi-static equations of electroactive anti-plane stress-strain state describing separate propagation of electroelastic waves of SH-type in current layer have the well-known form

$$\begin{aligned}&\partial^2 \mathrm{w}_1^{(1)}/\partial x^2+\partial^2 \mathrm{w}_1^{(1)}/\partial y^2=\tilde{C}_{1t}^{-2}\cdot\partial^2 \mathrm{w}_1^{(1)}/\partial t^2\\&\partial^2\varphi_1/\partial x^2+\partial^2\varphi_1/\partial y^2=\left(e_{15}^{(1)}/\varepsilon_{11}^{(1)}\right)\cdot\left[\partial^2 \mathrm{w}_1^{(1)}/\partial x^2+\partial^2 \mathrm{w}_1^{(1)}/\partial y^2\right]\end{aligned} \quad (1.5)$$

Coefficients $c_{44}^{(1)}$, $e_{15}^{(1)}$, $\varepsilon_{11}^{(1)}$, ρ_1 and speed $\tilde{C}_{1t}=\sqrt{(c_{44}^{(1)}/\rho_1)\left[1+(e_{15}^{(1)})^2/(c_{44}^{(1)}\varepsilon_{11}^{(1)})\right]}$ of SH-wave in (1.3)-(1.5) characterize the piezoelectric of the class $6mm$.

Without losing the generality of reasoning, we also assume that the material of $\Omega_{2n}(x,y,z)$ is a piezoelectric of hexagonal symmetry class $\bar{6}m2$. In piezocrystal of that class, the excitation of separate plane electroactive stress-strain state is possible both in sagittal plane $x_3 0 x_1$ and in $x_1 0 x_2$. For the sake of clarity, here we consider the case when the electroactive plane stress-strain state is possible in sagittal plane $x_3 0 x_1$ coinciding with $x0y$, and the inverse axis of 6[th] order symmetry of the hexagonal piezocrystal, $\vec{p}_{\bar{6}}$, is directed along $0y$. Then, non-zero components of the mechanical stress tensor and of electric displacement vector in $x0y$ plane of layers $\Omega_{2n}(x,y,z)$ made of piezoelectric of $\bar{6}m2$ class of hexagonal symmetry are as follows [13]:

$$\sigma_{xx}^{(2)}(x,y,t) = c_{11}^{(2)}\left(\partial \mathrm{u}_2/\partial x\right) + c_{13}^{(2)}\left(\partial \mathrm{v}_2/\partial y\right) + e_{11}^{(2)}\left(\partial \varphi_2/\partial x\right),$$

$$\sigma_{yy}^{(2)}(x,y,t) = c_{13}^{(2)}\left(\partial \mathrm{u}_2/\partial x\right) + c_{33}^{(2)}\left(\partial \mathrm{v}_2/\partial y\right), \tag{1.6}$$

$$\sigma_{xy}^{(2)}(x,y,t) = c_{44}^{(2)}\left(\partial \mathrm{v}_2/\partial x\right) + c_{44}^{(2)}\left(\partial \mathrm{u}_2/\partial y\right)$$

$$D_x^{(2)}(x,y,t) = e_{11}^{(2)}\left(\partial \mathrm{u}_2/\partial x\right) - \varepsilon_{11}^{(2)}\left(\partial \varphi_2/\partial x\right),$$

$$D_y^{(2)}(x,y,t) = -\varepsilon_{33}^{(2)}\left(\partial \varphi_2/\partial y\right). \tag{1.7}$$

Taking into account the compatibility conditions of mechanical stress $\sigma_{zz}^{(2)}(x,y,t) \equiv 0$ and the third components of electric displacement vector $D_z^{(2)}(x,y,t) \equiv 0$ [13], the quasi-static equations of electroactive plane stress-strain state describing the separate excitation and propagation of P&SV-type waves have the following simplified form:

$$\left(c_{11}^{(2)} - c_{13}^{(2)}\vartheta_{12}\right)\left(\partial^2 \mathrm{u}_2/\partial x^2\right) + c_{44}^{(2)}\left(\partial^2 \mathrm{u}_2/\partial y^2\right) - e_{11}^{(2)}\left(1+\vartheta_{12}\right)\left(\partial^2 \varphi/\partial x^2\right) = \rho_2\left(\partial^2 \mathrm{u}_2/\partial t^2\right)$$

$$c_{44}^{(2)}\left(\partial^2 \mathrm{v}_2/\partial x^2\right) + \left(c_{33}^{(2)} - c_{12}^{(2)}\vartheta_{13}\right)\left(\partial^2 \mathrm{v}_2/\partial y^2\right) - e_{11}^{(2)}\vartheta_{13}\left(\partial^2 \varphi_2/\partial x\partial y\right) = \rho_2\left(\partial^2 \mathrm{v}_2/\partial t^2\right)$$

$$\varepsilon_{11}^{(2)}\left(\partial^2 \varphi_2/\partial x^2\right) + \varepsilon_{33}^{(2)}\left(\partial^2 \varphi_2/\partial y^2\right) - e_{11}^{(2)}\left(\partial^2 \mathrm{u}_2/\partial x^2\right) = 0 \tag{1.8}$$

Coefficients $c_{11}^{(2)}$, $c_{12}^{(2)}$, $c_{44}^{(2)}$, $c_{13}^{(2)}$, $c_{33}^{(2)}$, $e_{11}^{(2)}$, $\varepsilon_{11}^{(2)}$, $\varepsilon_{33}^{(2)}$, ρ_2, $\vartheta_{12} = \left(c_{13}^{(2)} + c_{44}^{(2)}\right)\Big/c_{12}^{(2)}$, $\vartheta_{13} = \left(c_{13}^{(2)} + c_{44}^{(2)}\right)\Big/c_{13}^{(2)}$ in (1.6), (1.7) and (1.8) characterize the piezoelectric of the symmetry class $\bar{6}m2$.

It is evident from equations (1.8) that, according to the model of generalized stress-strain state, the reduced elastic extension stiffnesses decrease and have the following form:

$$c_{11}^{*} = c_{11}^{(2)}\left[1 - \vartheta_{12}\left(c_{13}^{(2)}/c_{11}^{(2)}\right)\right], \quad c_{33}^{*} = c_{33}^{(2)}\left[1 - \vartheta_{13}\left(c_{12}^{(2)}/c_{33}^{(2)}\right)\right]. \tag{1.9}$$

The reduced coefficients of direct piezoelectric effect have the following form:

$$e_{11}^{*} = e_{11}^{(2)}\left(1+\vartheta_{12}\right); \quad e_{12}^{*} = \vartheta_{12}e_{11}^{(2)}; \tag{1.10}$$

Coordinate surfaces of non-acoustic contacts $x_{0n} = -a_2 + n(a_1+a_2)$, $x_{0n} = 0 + n(a_1+a_2)$ and $x_{2n} = a_1 + n(a_1+a_2)$ are free of mechanical stresses:

$$\sigma_{zx}^{(1)}(x,y,t) = 0; \quad \sigma_{xx}^{(2)}(x,y,t) = 0; \quad \sigma_{xy}^{(2)}(x,y,t) = 0; \tag{1.11}$$

and electric field conjugacy conditions are satisfied:

$$\varphi_1(x,y,t) = \varphi_2(x,y,t); \quad D_x^{(1)}(x,y,t) = D_x^{(2)}(x,y,t). \tag{1.12}$$

2. Solution of the mathematical boundary-value problem

In order to determine the characteristics of propagation of excited 1D forms of electroelastic hybrid waves in transversally inhomogeneous periodic piezoelectric structure, let us apply the Floquet-Lyapunov theory of periodic structures [9, 15]. Then, the mathematical boundary-value problem of normal wave signal propagation in inhomogeneous structure is formulated for the cell $\Omega_0(x,y)=\Omega_{20}(x,y)\cup\Omega_{10}(x,y)$.

The following system of one-dimensional quasi-static equations of anti-plane electroactive deformation obtained from (1.5) will be solved in layer with section $\Omega_{10}(x,y)$:

$$\partial^2 \mathrm{W}_1^{(1)}/\partial x^2=-\omega^2\tilde{C}_{1t}^{-2}\cdot \mathrm{W}_1^{(1)},\quad \partial^2\Phi_1^{(1)}/\partial x^2=\left(e_{15}^{(1)}/\varepsilon_{11}^{(1)}\right)\cdot\partial^2\mathrm{W}_1^{(1)}/\partial x^2 \qquad (2.1)$$

Here, $\tilde{C}_{1t}^2=\tilde{c}_{44}^{(1)}/\rho_1$ is the speed of transverse electroactive wave in the piezoelectric of the symmetry class $6mm$, $\tilde{c}_{44}^{(1)}=c_{44}^{(1)}(1+\chi_1^2)$ is the reduced shear stiffness and $\chi_1^2=(e_{15}^{(1)})^2/c_{44}^{(1)}\varepsilon_{11}^{(1)}$ is the electromechanical connection coefficient of the piezoelectric of the symmetry class $6mm$.

The following system of one-dimensional quasi-static equations of plane electroactive deformation obtained from (1.8) will be solved in layer with section $\Omega_{20}(x,y)$:

$$\begin{aligned}&\left(\partial^2\mathrm{U}_2/\partial x^2\right)=\left(-\omega^2/C_{2l*}^2\right)\mathrm{U}_2\\&\left(\partial^2\Phi_2/\partial x^2\right)=\left(e_{11}^{(2)}/\varepsilon_{11}^{(2)}\right)\left(\partial^2\mathrm{U}_2/\partial x^2\right)\end{aligned} \qquad (2.2)$$

$$\left(\partial^2\mathrm{V}_2/\partial x^2\right)=\left(-\omega^2/C_{2t}^2\right)\mathrm{V}_2 \qquad (2.3)$$

Here, $C_{2l*}^2=\left(1-\vartheta_{12}\,(c_{13}^{(2)}/c_{11}^{(2)})-\chi_2^2(1+\vartheta_{12})\right)\left(c_{11}^{(2)}/\rho_2\right)$ is the speed of longitudinal electroactive wave, $C_{2t}^2=c_{44}^{(2)}/\rho_2$ is the speed of transverse elastic wave in the piezoelectric of the symmetry class $\overline{6}m2$.

It is evident from the expression of the longitudinal electroactive wave speed C_{2l*}^2 that for values of the electromechanical connection coefficient satisfying $\chi_2^2>1-[(c_{13}^{(2)}+c_{44}^{(2)})(c_{11}^{(2)}+c_{13}^{(2)})]/[c_{11}^{(2)}(c_{12}^{(2)}+c_{13}^{(2)}+c_{44}^{(2)})]$, the tensile stiffness (or compression) becomes negative. This is possible in case of relation $c_{11}^{(2)}c_{12}^{(2)}>c_{13}^{(2)}\left(c_{13}^{(2)}+c_{44}^{(2)}\right)$ because $0<\chi_2^2<1$ always holds.

According to the theory of Floquet-Lyapunov, instead of electromechanical boundary conditions (1.11) and (1.12), considering the periodicity of the structure, on the finite interval $-a_2\le x\le a_1$, boundary conditions of quasi-periodicity of the considered cell are fulfilled. Then, based on one-dimensional problem, the full system of boundary conditions will have the following form:

$$c_{44}^{(1)}\frac{d\mathrm{W}_1(0)}{dx}+e_{15}^{(1)}\frac{d\Phi_1(0)}{dx}=0 \qquad (2.4)$$

$$c_{11}^{(2)}\frac{d\mathrm{U}_2(0)}{dx}+e_{11}^{(2)}\frac{d\Phi_2(0)}{dx}=0 \qquad (2.5)$$

$$c_{44}^{(2)}\frac{d\mathrm{V}_2(0)}{dx}=0 \qquad (2.6)$$

$$c_{44}^{(1)}\frac{d\mathrm{W}_1(a_1)}{dx}+e_{15}^{(1)}\frac{d\Phi_1(a_1)}{dx}=0, \qquad (2.7)$$

$$c_{11}^{(2)}\frac{d\mathrm{U}_2(-a_2)}{dx}+e_{11}^{(2)}\frac{d\Phi_2(-a_2)}{dx}=0 \tag{2.8}$$

$$c_{44}^{(2)}\frac{d\mathrm{V}_2(-a_2)}{dx}=0 \tag{2.9}$$

Conjugacy conditions for the electric field are as follows:

$$\Phi_1(0)=\Phi_2(0), \tag{2.10}$$

$$\Phi_1(a_1)=\mu\cdot\Phi_2(-a_2) \quad \text{or} \quad \Phi_2(a_1)=\mu\cdot\Phi_1(-a_2) \tag{2.11}$$

$$e_{15}^{(1)}\frac{d\mathrm{W}_1(0)}{dx}-\varepsilon_{11}^{(1)}\frac{d\Phi_1(0)}{dx}=e_{11}^{(2)}\frac{d\mathrm{U}_2(0)}{dx}-\varepsilon_{11}^{(2)}\frac{d\Phi_2(0)}{dx}, \tag{2.12}$$

$$e_{15}^{(1)}\frac{d\mathrm{W}_1(a_1,y,t)}{dx}-\varepsilon_{11}^{(1)}\frac{d\Phi_1(a_1,y,t)}{dx}=\mu\cdot\left(e_{11}^{(2)}\frac{d\mathrm{U}_2(-a_2,y,t)}{dx}-\varepsilon_{11}^{(2)}\frac{d\Phi_2(-a_2,y,t)}{dx}\right) \tag{2.13}$$

In (2.9) and (2.11), $\mu=\exp[ik(a_1+a_2)]$ is the Floquet parameter characterizing the periodicity of the structure, $k=2\pi/\lambda$ is the Floquet-Bloch wave number perpendicular to the interface of sub-layers, λ is the length of propagating wave and $a_1+a_2=L$ is the width of repeating cell.

2.1 Formation of electroacoustic hybrid waves in 1D transversally inhomogeneous periodic composite.

Considering the infiniteness of the structure in y direction, wave forms of 1D elastic displacements and electric field potential in homogeneous layers are represented as a plane wave: $f(x,y,t)=f_0\exp[i(kx-\omega t)]$.

Vibrations of electric shear and corresponding vibrations of electric potential in homogeneous layer $\Omega_{10}(x,y)$ made of piezoelectric of the symmetry class $6mm$ are characterized by system (2.1) along with boundary conditions (2.4) and (2.7) and are given by

$$W_1(x)=D_{1w}\cos\left(k_{1w}(\omega)\cdot x\right),$$

$$\Phi_1(x)=D_{1\varphi}\cos\left(k_{1\varphi}(\omega)\cdot x\right)+\gamma_1 D_{1w}\cos\left(k_{1w}(\omega)\cdot x\right) \tag{2.14}$$

Here, $k_w(\omega)$ and $k_{1\varphi}(\omega)$ are formation parameters or moduli of wave numbers of electroactive shear waves in $\Omega_{10}(x,y)$ perpendicular to the surface of non-acoustic contact, $\gamma_1=\left(e_{15}^{(1)}/\varepsilon_{11}^{(1)}\right)$ is the piezo-dielectric parameter of the piezoelectric of the symmetry class $6mm$ characterizing the connection of electromechanical fields.

Wave forms of 1D elastic extension and electric field potential in $\Omega_{20}(x,y)$ made of piezoelectric of the symmetry class $\overline{6}m2$ are described by system (2.2) along with the boundary conditions (2.5) and (2.8) and are given by:

$$\mathrm{U}_2(x)=D_{2u}\cos\left(k_u(\omega)\cdot x\right),$$

$$\Phi_2(x)=D_{2\varphi}\cos\left(k_{2\varphi}(\omega)\cdot x\right)+\gamma_2 D_{2u}\cos\left(k_u(\omega)\cdot x\right) \tag{2.15}$$

Taking into account (2.3) along with boundary conditions (2.6) and (2.9), we obtain

$$V_2(x) = D_{2v}\cos\left(k_v(\omega)\cdot x\right) \tag{2.16}$$

In (2.15) and (2.16), $k_u(\omega)$ and $k_{2\varphi}(\omega)$ are formation parameters or moduli of wave numbers of dilatation waves in $\Omega_{20}(x,y)$ perpendicular to the surface of non-acoustic contact, $\gamma_2 = \left(e_{11}^{(2)}/\varepsilon_{11}^{(2)}\right)$ is the piezo-dielectric parameter of the piezoelectric of the symmetry class $\bar{6}m2$ characterizing the connection of electromechanical fields, $k_v(\omega)$ is the formation parameters or modulus of wave numbers of shear waves in $\Omega_{20}(x,y)$ perpendicular to the surface of non-acoustic contact.

The eigenfrequencies of vibrations corresponding to wave forms in sublayers $\Omega_{10}(x,y)$ and $\Omega_{20}(x,y)$ are as follows:

$$\omega_{0w} = n\pi\tilde{C}_{1t}/a_1\,,\quad \omega_{0u} = n\pi C_{2l*}/a_2\,,\quad \omega_{0v} = n\pi C_{2t}/a_2\,,\quad n\in\mathbb{N}\,. \tag{2.17}$$

It is evident from (2.15) and (2.16) that 1D longitudinal elastic vibrations in $\Omega_{20}(x,y)$ are electroactive, while the transverse vibrations are not related to electric field vibrations. The corresponding electric field in that layer is related only to the displacement $U_2(x)\cdot\exp(-i\omega t)$. The character of connection of extensional (or compressing) displacements with electric potential $\Phi_2(x)\cdot\exp(-i\omega t)$ in $\Omega_{20}(x,y)$ is the same as the character of the connection between $W_1(x)\cdot\exp(-i\omega t)$ and $\Phi_1(x)\cdot\exp(-i\omega t)$ in $\Omega_{10}(x,y)$.

2.2. Propagation of 1D electroacoustic forms in transversally inhomogeneous periodic composite.

Substituting expressions (2.14) ÷ (2.16) into boundary conditions of quasi-periodicity (2.10) ÷ (2.13), we obtain the following dispersion equation of propagation (frequency filtration) of electroacoustic waves (as in [4] and [19]):

$$\cos[(a_1+a_2)\cdot k(\omega)] = \frac{\left[1-\left(\cos k_w(\omega)a_1\right)\right]^2 + \left[1-\cos\left(k_u(\omega)a_2\right)\right]^2}{2\left[1-\cos\left(k_w(\omega)a_1\right)\right]\cdot\left[1-\cos\left(k_u(\omega)a_2\right)\right]} \tag{2.18}$$

Substituting the expressions of formation coefficients $k_w(\omega)$ and $k_u(\omega)$ into (2.18), the solution of the dispersion equation can be represented as

$$k(\omega) = \frac{1}{(a_1+a_2)}\cdot\arccos\left[\frac{\left[1-\cos\left(\omega a_2/C_{2l*}\right)\right]^2 + \left[1-\cos\left(\omega a_1/C_{1t}\right)\right]^2}{2\left[1-\cos\left(\omega a_1/C_{1t}\right)\right]\cdot\left[1-\cos\left(\omega a_2/C_{2l*}\right)\right]}\right] \tag{2.19}$$

The region of allowed frequencies in the layered structure is determined from the obvious constraint $\left|\cos[k(a_1+a_2)]\right|\le 1$. Since for the right-hand side of (2.18), the inequality

$$\frac{\left[1-\left(\cos k_w(\omega)a_1\right)\right]^2 + \left[1-\cos\left(k_u(\omega)a_2\right)\right]^2}{2\left[1-\cos\left(k_w(\omega)a_1\right)\right]\left[1-\cos\left(k_u(\omega)a_2\right)\right]} \ge 1\,, \tag{2.20}$$

holds only when $\cos\left(k_{w}(\omega)a_1\right)=\cos\left(k_u(\omega)a_2\right)$, then the set of allowed frequencies consists of the following two groups:

$$\omega^*_{0u}\in\left\{\frac{2m_2\pi C_{2l*}}{a_2\left[1-\left(a_1C_{2l*}/a_2C_{1t}\right)\right]}\right\},\ \omega^*_{0w}\in\left\{\frac{2m_1\pi\tilde{C}_{1t}}{a_1\left[1-\left(a_2C_{1t}/a_1C_{2l*}\right)\right]}\right\},\ m_1,m_2\in\mathbb{N} \quad (2.21)$$

It is evident from the obtained dispersion equation of frequency filtration that the electroacoustic wave signal propagates in the layered composite under study as an electroactive hybrid wave of pure shear and as a wave of pure dilatation with two groups of discrete frequencies (2.21).

Taking into account the values of eigenfrequencies (2.17), from (2.21) it follows that the main allowed frequency ω^*_{0u} with number $m_2=1$ is always smaller than main discrete eigenfrequencies ω_{0u} and ω_{0w} with number $n=1$. Therefore, the main frequency $\omega^*_{0u}(m_2\geq 2)$ may be of resonant type if and only if

$$\omega^*_{0u}(m_2\geq 2)\geq\min\left\{\omega_{0u}(n<m_2)=n\pi C_{2l*}/a_2;\ \omega_{0w}(n<m_2)=n\pi\tilde{C}_{1t}/a_1\right\} \quad (2.22)$$

In electronics and related fields, piezoelectric plates of magnitude of 1.0 mm or even thinner are used for which $\{[\![a_1]\!];[\![a_2]\!]\}\leq 10^{-3}$ m, and elastic wave speed has the amplitude of order $\{[\![C_{1t}]\!];[\![C_{2l*}]\!]\}\sim 10^3$ m/s. Then, from (2.17) and (2.22) it follows that the first main frequency $\omega^*_{0u}(1)$ of electroacoustic hybrid vibrations has the order $\omega_{0l}\sim 10^6$ Hz. Choosing the parameters of the composite according to $a_1C_{2l*}\sim a_2C_{1t}$, it is possible to achieve greater values for the second main frequency of electroacoustic hybrid vibrations: $\omega_{0h}\sim 10^8\div 10^{10}$ Hz. In these cases, the second main frequency $\omega^*_{0w}(1)$ of the electroacoustic hybrid vibrations can always be of resonant type, i.e.,

$$\omega^*_{0w}(m_1\geq 1)\geq\min\left\{\omega_{0u}(n)=n\pi C_{2l*}/a_2;\ \omega_{0w}(n)=n\pi\tilde{C}_{1t}/a_1\right\} \quad (2.23)$$

Comparably larger values of allowed frequencies $\omega^*_{0w}(1)$ will be of resonant type in sub-layer $\Omega_{10}(x,y)$ when the geometric and material characteristics of the composite satisfy $a_2/a_1=C_{2l*}/2C_{1t}$. On the other hand, comparably larger values of allowed frequencies $\omega^*_{0w}(1)$ will be of resonant type in sub-layer $\Omega_{20}(x,y)$ when $a_1/a_2=3C_{1t}/C_{2l*}$.

From (2.19), we obtain the length of propagating mode of electroacoustic hybrid waves as $\lambda_m(\omega)=4(a_1+a_2)/(1+2m)$. The groups of allowed frequencies are determined as intersection points of phase curves and line $\lambda(0)=4(a_1+a_2)$ (see Figure 2).

In numerical computations below, we consider the following values of characteristics:
- for widths of sublayers - $a_1=10^{-3}m$ and $a_2=2\times 10^{-3}m$,
- for the piezoelectric material of hexagonal symmetry class *6mm* –

$c^{(1)}_{44}=1,639\times 10^{10}\ N/m^2$, $\rho_1=5,302\times 10^3\ kg/m^3$, $\varepsilon^{(1)}_{11}=8,786\times 10^{-11}\ F/m$, $e^{(1)}_{15}=0,23\ C/m^2$, $C_{1t}=1,7585\times 10^3\ m/s$.

- for the piezoelectric of hexagonal symmetry class $\overline{6}m2$ - $c_{11}^{(2)} = 8,612\times10^{10}\ N/m^2$, $c_{12}^{(2)} = 4,852\times10^{10}\ N/m^2$, $c_{13}^{(2)} = 1,045\times10^{10}\ N/m^2$, $c_{33}^{(2)} = 10,71\times10^{10}\ N/m^2$, $c_{44}^{(2)} = 5,86\times10^{10}\ N/m^2$, $\rho_2 = 2,648\times10^3\ kg/m^3$, $\varepsilon_{11}^{(2)} = 4,25\times10^{-11}\ F/m$, $\varepsilon_{11}^{(2)} = 4,63\times10^{-11}\ F/m$, $e_{11}^{(1)} = 3,21\ C/m^2$, $\vartheta_{12} = 1,4231$, $C_{2l*} = 2,189\times10^3\ m/s$.

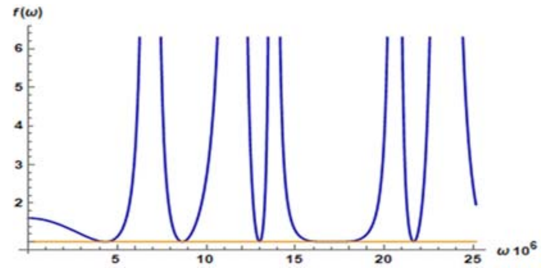
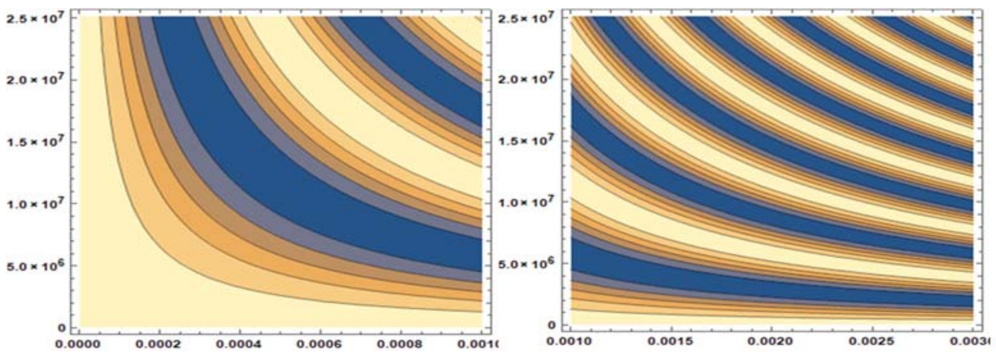


Figure 2. Intersection points of phase curves and the line determines the groups of allowed frequencies



a) Wave surface $W_1(x,\omega)$ of electroelastic shear in sub-layer $\Omega_{10}(x,y)$

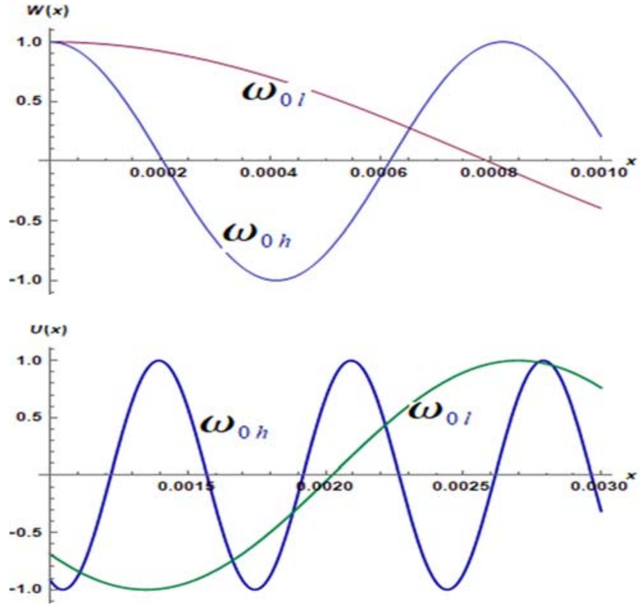
b) Wave surface $U_2(x,\omega)$ of electroelastic dilatation in sub-layer $\Omega_{20}(x,y)$

Figure 3. Wave surfaces of elastic displacements of hybrid electroactive wave in composite cells

Distributions of elastic displacements $W_1(x)\cdot\exp(-i\omega t)$ and $U_2(x)\cdot\exp(-i\omega t)$, as well as the potentials of electric field $\Phi_1(x)\cdot\exp(-i\omega t)$, $\Phi_2(x)\cdot\exp(-i\omega t)$ in sub-layers $\Omega_{10}(x,y)$ and $\Omega_{20}(x,y)$ are determined by (2.14) and (2.15), respectively.

In the case of comparably smaller and comparably greater allowed frequencies of wave signal, these distributions are plotted in Figures 3.a, 3.b, 4.a and 4.b, respectively.

From boundary-value problem (2.5), (2.7) and (2.14) it follows that in the case of layered piezoelectric medium, the shear displacement $v_2(x,y,t)$ does not occur in the piezoelectric of the symmetry class $\bar{6}m2$. The corresponding electric field in that sub-layer is connected with $u_2(x,y,t)$ only. The character of the connection between extensional displacement and electric potential in that sub-layer is the same as that between shear displacement $w_1(x,y,t)$ and electric potential in the first sub-layer.



a) Forms of electroelastic shear $W_1(x,\omega_{0h})$ and $W_1(x,\omega_{0l})$ in cases of allowed frequencies of composite

b) Forms of electroelastic shear $U_2(x,\omega_{0h})$ and $U_2(x,\omega_{0l})$ in cases of allowed frequencies of composite

Figure 4. Formation of elastic displacements of hybrid electroactive wave

3.1 Electroactive *SH*-waves in piezoelectric medium with system of infinite cracks

When the layers of the composite are made of piezoelectric material of the symmetry class $6mm$ (or $4mm$) only, and separated from each other by a system of infinite cracks, planes of which are parallel to the polarization axis of the piezocrystal $\vec{p}_6$ (resp. $\vec{p}_4$), then the quasi-periodicity boundary conditions are simplified to

$$e_{15}^{(1)}\frac{dW_1(0_-)}{dx}-\varepsilon_{11}^{(1)}\frac{d\Phi_1(0_-)}{dx}=e_{15}^{(1)}\frac{dW_1(0_+)}{dx}-\varepsilon_{11}^{(1)}\frac{d\Phi_1(0_+)}{dx}, \tag{3.1}$$

$$c_{44}^{(1)}\frac{dW_1(0_\pm)}{dx}+e_{15}^{(1)}\frac{d\Phi_1(0_\pm)}{dx}=0, \tag{3.2}$$

$$\Phi_1(a_{1\pm})=\mu\cdot\Phi_1(0_\pm), \tag{3.3}$$

$$e_{15}^{(1)}\frac{dW_1(a_{1\pm})}{dx}-\varepsilon_{11}^{(1)}\frac{d\Phi_1(a_{1\pm})}{dx}=\mu\cdot\left(e_{15}^{(1)}\frac{dW_1(0_\pm)}{dx}-\varepsilon_{11}^{(1)}\frac{d\Phi_1(0_\pm)}{dx}\right). \tag{3.4}$$

Nonetheless, the solution of (2.1) will have the same form (2.14). Note that the continuity conditions of the electric potential, as well as the periodicity conditions of mechanical stresses on all cut-offs $x = na_1$ are fulfilled.

The dispersion equation for determination of phase velocity is obtained by substituting (2.14) into (3.1) ÷ (3.4) and has a simpler form

$$\cos(a_1 k) = \left(1 + \cos^2(\omega a_1 / \tilde{C}_{1t})\right) \Big/ 2\cos(\omega a_1 / \tilde{C}_{1t}) \tag{3.5}$$

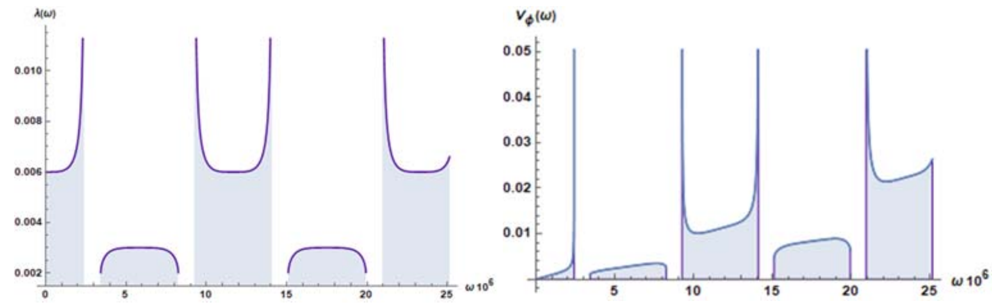
From (3.5) it follows that electroactive SH-wave can propagate in this case as well. Solution of dispersion equation (3.5) for wavelengths is obtained as

$$\lambda(\omega) = 2\pi a_1 \cdot \left[\arccos\left[\left(1 + \cos^2(\omega a_1 / \tilde{C}_{1t})\right) \Big/ \left(2\cos(\omega a_1 / \tilde{C}_{1t})\right)\right]\right]^{-1} \tag{3.6}$$

Therefore, the phase velocity will be of dispersion type

$$V_\phi(\omega) = \left[a_1 \omega \Big/ \arccos\left[\left(1 + \cos^2(\omega a_1 / \tilde{C}_{1t})\right) \Big/ \left(2\cos(\omega a_1 / \tilde{C}_{1t})\right)\right]\right] \tag{3.7}$$

Despite the case of homogeneous space (space without periodic cracks), in this case, zones of forbidden frequencies occur (see Figure 5.a). Actually for short waves, when they are of the order of width layer, there are two groups of forbidden frequency zones. It is clear that in one group of zones of permissible frequencies, the phase velocity first decreases, and then increases. In the second group of zones of permissible frequencies, the phase velocity first increases and then decreases (see Figure 5.b).



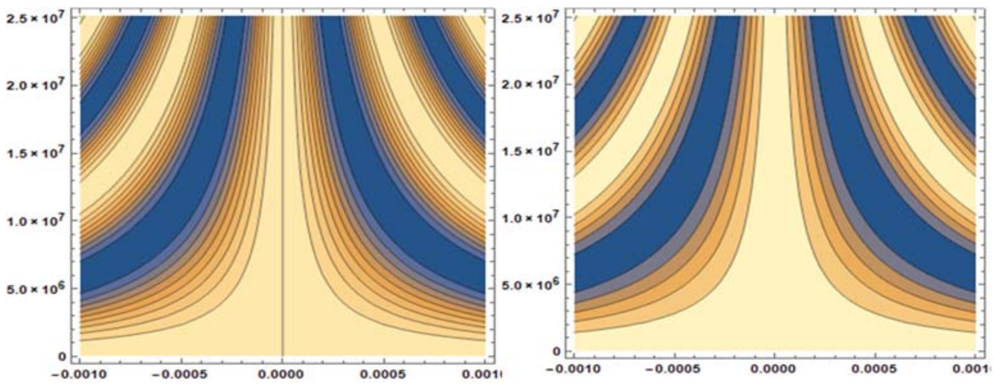
a) Zones of allowed (and forbidden) lengths $[\![\lambda(\omega)]\!]$ of electroelastic shear wave

b) Phase velocity of electroelastic shear wave $V_{1\phi}(\omega)$

Figure 5. The case of propagation of electroactive shear wave in piezoelectric with a system of infinite parallel cracks

In Figures 6.a and 6.b, distributions of elastic displacements and electric potential along width of two neighboring piezoelectric sub-layers respectively are shown.

From the figures it follows that a periodic cell is formed of two identical interlayers. At the edges of cracks $x_{0n} = 0 \pm 2na_1$ between the piezoelectric layers, the elastic displacement has an underlined maximum, and the electric field potential has an increased background.



a) Electroelastic shear displacement $W_1(x,y)$ in two neighboring piezoelectric sub-layers

б) Electric potential $\Phi_1(x,y)$ in two neighboring piezoelectric sub-layers

Figure 6. Distributions of elastic displacements and electric potential in case of propagation of electroactive shear wave

At the edges of the cracks $x_{1n} = a_1 \pm 2na_1$ between the piezoelectric layers, the elastic displacement and the electric field potential smoothly pass into the next cell.

A similar picture is obtained in problem of the propagation of electroactive dilatation waves in a piezoelectric medium of the hexagonal symmetry class $\overline{6}m2$, with a system of transverse infinite cracks.

3.2 Electroactive elastic waves of dilatation in piezoelectric medium with system of infinite cracks

When the layers of the composite are made of piezoelectric material of the symmetry class $\overline{6}m2$ only, taking into account one-dimensional solutions (2.13), (2.14) and quasi-periodicity boundary conditions (2.4)÷(2.11), the plane stress-strain state of the layered structure is described by

$$c_{11}^{(2)} \frac{dU_2(0_\pm)}{dx} + e_{11}^{(2)} \frac{d\Phi_2(0_\pm)}{dx} = 0 \tag{3.8}$$

$$e_{11}^{(2)} \frac{dU_2(0_-)}{dx} - \varepsilon_{11}^{(2)} \frac{d\Phi_2(0_-)}{dx} = e_{11}^{(2)} \frac{dU_2(0_+)}{dx} - \varepsilon_{11}^{(2)} \frac{d\Phi_2(0_+)}{dx} \tag{3.9}$$

$$\Phi_2(a_2) = \mu \cdot \Phi_2(0) \tag{3.10}$$

$$e_{11}^{(2)} \frac{dU_2(0)}{dx} - \varepsilon_{11}^{(2)} \frac{d\Phi_2(0)}{dx} = \mu^{-1} \left(e_{11}^{(2)} \frac{dU_2(a_2)}{dx} - \varepsilon_{11}^{(2)} \frac{d\Phi_2(a_2)}{dx} \right) \tag{3.11}$$

In this case also, the continuity conditions for electric potential and the periodicity conditions for mechanical stresses are fulfilled on cut-offs $x = 0 \pm na_2$.

The obvious similarity of solutions (2.14) and (2.15) and of boundary conditions (3.8) ÷ (3.11) and (3.1) ÷ (3.4), implies a dispersion equation similar to (3.5). The reduced velocity $\eta_2(\omega)$ takes the form

$$\eta_2(\omega)=\frac{V_{2\phi}(\omega)}{\tilde{C}_{2l*}}=\frac{\left(a_2\omega/\tilde{C}_{2l*}\right)}{\arccos\left[1+\cos^2\left(\omega a_2/\tilde{C}_{2l*}\right)/4\cos\left(\omega a_2/\tilde{C}_{2l*}\right)\right]} \tag{3.12}$$

The wave of dilatation is characterized by the same properties, as the share wave in the case described in section 3.1.

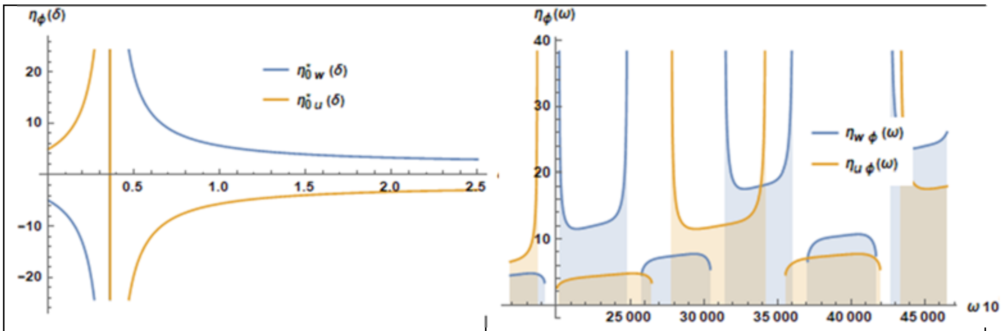
In both cases, phase velocities are dispersive. They differ from phase velocities of hybrid waves as qualitatively, as well as quantitatively. Figure 7.b shows the dependence of the phase velocity of electro-acoustic shear waves in a piezoelectric medium of the symmetry class $6mm$ with an infinite system of transverse periodic cracks

$$\eta_{w\phi}(\omega)=\left[\left(a_1\omega/\tilde{C}_{1t}\right)\Big/\arccos\left[\left(1+\cos^2(\omega a_1/\tilde{C}_{1t})\right)\Big/\left(2\cos(\omega a_1/\tilde{C}_{1t})\right)\right]\right] \tag{3.13}$$

and the dependence of the phase velocity of electro-acoustic shear waves in a piezoelectric medium of the symmetry class $\bar{6}m2$, with an infinite system of transverse periodic cracks

$$\eta_{u\phi}(\omega)=\left[\left(a_2\omega/\tilde{C}_{2l*}\right)\Big/\arccos\left[1+\cos^2\left(\omega a_2/\tilde{C}_{2l*}\right)/2\cos\left(\omega a_2/\tilde{C}_{2l*}\right)\right]\right] \tag{3.14}$$

respectively.



a) Reduced phase velocities $\eta^*_{0w}(\delta)$ and $\eta^*_{0u}(\delta)$ of electroelastic hybrid waves respectively

b) Reduced phase velocities $\eta_{w\phi}(\omega)$ and $\eta_{u\phi}(\omega)$ of electroelastic share dilatation waves respectively

Figure 7. Phase velocities of hybrid waves as well as of separately propagating electroactive shear waves and of dilatation wave in piezoelectric with a periodic system of infinite parallel cracks

The zones of permissible frequencies for phase velocities $\eta_{w\phi}(\omega)$ and $\eta_{u\phi}(\omega)$ from (3.13) and (3.14), respectively, will be $\pi\tilde{C}_{1t}/a_1\le\omega_{0w}\le 2\pi\tilde{C}_{1t}/a_1$ and $\pi\tilde{C}_{2l*}/a_2\le\omega_{0u}\le 2\pi\tilde{C}_{2l*}/a_2$ From Figure 7.b it is obvious, that the zones of permissible frequencies of the electro-elastic shear wave can intersect with the zones of permissible frequencies of electro-elastic wave of dilatation.

From (2.21) it follows that the allowed frequencies of hybrid waves are not dispersive. However, they depend on the ratio of widths of piezoelectric sublayers $\delta=\left(a_2/a_1\right)$.

The phase velocities of hybrid electro-acoustic waves in an inhomogeneous layered structure constructed of the same piezoelectrics are determined as

$$\eta_{0w}^{*}(\delta)=\tilde{C}_{1t}(1+\delta)\Big/\left[1-\left(C_{1t}/C_{2l*}\right)\delta\right] \tag{3.15}$$

$$\eta_{0u}^{*}(\delta)=\tilde{C}_{2l*}(1+\delta)\Big/\left[\delta+\left(\tilde{C}_{2l*}/\tilde{C}_{1t}\right)\right] \tag{3.16}$$

which are plotted in Figure 7.a.
From the graphs of the phase velocities of the hybrid wave, it is obvious that, depending on the parameter $\delta=\left(a_2/a_1\right)$, there is a critical value of frequency $\omega_0^{*}(\delta)$. In the frequency interval $0<\omega<\omega_0^{*}(\delta)$, the hybrid wave propagates with the phase velocity $\eta_{0u}^{*}(\delta)$, and in the frequency interval $\omega_0^{*}(\delta)<\omega<\infty$, the hybrid wave propagates with the phase velocity $\eta_{0w}^{*}(\delta)$.

The shear component of the wave $v_2(x,y,t)\equiv 0$, as in the case of piecewise homogeneous composite.

Conclusions

Non-acoustic contact within the piezoelectric layers results in hybridization of dissimilar electroacoustic waves.

The propagation of 1D electroactive hybrid elastic waves of pure share and pure dilatation in compound transversally inhomogeneous periodic piezoelectric space made of piezocrystalls of hexagonal symmetry class $6mm$ (or of tetragonal symmetry class $4mm$) and $\bar{6}m2$ is possible. There exist two groups of allowed discrete frequencies. When the ratio of widths and elastic wave speeds in layers are inverse to each other, then the allowed discrete frequencies are of resonant type.

The propagation of 1D electroactive shear waves in piezoelectric medium of the symmetry class $6mm$ (or $4mm$) with infinite transverse cut-offs is possible with zones of allowed and forbidden frequencies.

The propagation of 1D electroactive dilatation waves in piezoelectric medium of the symmetry class $\bar{6}m2$ with infinite transverse cut-offs is possible with zones of allowed and forbidden frequencies.

The study was carried out with the financial support of the Committee on Science of the Republic of Armenia within the research project 18T-2C195.

REFERENCES

1. M.I.Hussein, M.J. Leamy, M. Ruzzene, Dynamics of phononic materials and structures: historical origins, recent progress, and future outlook, (2014), Applied Mechanics Reviews, v. 66 (4): p.040802/1-38, http://doi.org.10.1115/1.4026911.
2. Lord Rayleigh, On the maintenance of vibrations of forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure, Phil. Mag., 24 (1887), pp. 145–159, http://doi.org.10.1080/14786448708628074.

3. Аветисян А.С., Казарян К.Б. Волны в «фонон-фотон» кристаллах и импеданс, Сб. «Проблемы механики деформируемого твердого тела» К 90-летию академика НАН Армении С.А. Амбарцумяна. Ереван. 2012, с.15-22. Avetisyan A.S., Ghazaryan K.B., Waves in «phonon-photon» crystals and impedance, Sat. «Problems of the mechanics of a deformable solid», Dedicated 90th anniversary of the academician of NAS of Armenia S.A. Ambartsumian. Yerevan. 2012, pp.15-22, (in Russian).
4. Piliposyan D.G., Ghazaryan K.B., Piliposyan G.T. and Avetisyan A. S., Wave Propagation in Periodic Piezoelectric Elastic Waveguides, (2012), ASME Conference on Smart Materials, Adaptive Structures and Intelligent Systems, Georgia, USA, pp.1-9., http://doi.org./10.1115/SMASIS2012-7911.
5. Lee E. H., A survey of variation methods for elastic wave propagation analysis in composites with periodic structures, in Dynamics of Composite Materials, E.H. Lee, ed., ASME, New York, 1972, pp.122–138.
6. Lee E.H. and Yang W. H., On waves in composite materials with periodic structure, (1973), SIAM Journal on Applied Mathematics, vol.25(3), pp.492–499, http://doi.org/10.1137/0125049.
7. Papanicolaou V.G., The periodic Euler–Bernoulli equation, (2003), Trans. AMS 355(09), pp. 3727–3759, , http://doi.org.10.1090/S0002-9947-03-03315-4.
8. V.G. Papanicolaou, An Inverse Spectral Result for the Periodic Euler-Bernoulli Equation, Indiana University Mathematics Journal (2003), Volume: 53(1), pp. 223-242, http://doi.org.10.1512/iumj.2004.53.2493.
9. Qian Z.H., Jin F., Wang Z.K., Kishimoto K., Dispersion relations for SH-wave propagation in periodic piezoelectric composite layered structures, (2004), Int. J. Eng. Sci., vol. 42(7), pp.673-689, , http://doi.org.10.1016/j.ijengsci.2003.09.010.
10. Adams S., Craster R., Guenneau S., Bloch waves in periodic multi-layered acoustic waveguides, (2008), Proc. Royal Society A Math. Phys. And Engin. Sciences 464 (2098) p.2669-2692, http://doi.org.10.1098/rspa.2008.0065.
11. Craster R.V., Guenneau S., Adams S., Mechanism for slow waves near cutoff frequencies in periodic waveguides, (2009), Physical Review B, Condensed matter 79(4), http://doi.org.10.1103/PhysRevB.79.045129.
12. Аветисян А.С. К задаче распространения сдвиговых волн в пьезоэлектрической среде. //Изв. НАН Армении. Механика. 1985, т.38, №1, с.12-19. Avetisyan A.S., About the Problem of the Propagation of Transversal Waves in Piezoelectric Bodies, Procc. of NAS of Armenia, Mechanics, (1985), vol.38, №1, pp.12-19, (in Russian).
13. Avetisyan Ara S., Two Dimensional Problems of Electro Acoustics in Homogeneous Piezoelectric Crystals, Procc. of NAS of Armenia, Mechanics, (2019), vol.72, №3, pp. 56-79, http://doi.org/10.33018/72.3.4.
14. Bluestein, J. L. A New Surface Wave in Piezoelectric Medium. //Appl. Phys. Lett., vol.13 (1968), p. 412-413, https://doi.org/10.1063/1.1652495.

15. Singh B., Singh R., Rayleigh wave in a rotating initially stressed piezoelectric half-space (2013), Jour. of Theor. And Appl. Mechanics, vol.43, Iss. 2, https://doi.org/10.2478/jtam-2013-0014.
16. Chaudhary S., Sahu S.A., Singhal A., Analytic model for Rayleigh wave propagation in piezoelectric layer overlaid orthotropic substratum, (2017), Acta Mechanica, vol. 228, Iss. 2, pp. 495–529, https://doi.org/10.1007/s00707-016-1708-0.
17. Vashishth, A.K., Sukhija, H., Coupled Rayleigh waves in a 2-mm piezoelectric layer over a porous piezo-thermoelastic half-space, (2017), Acta Mechanica, vol. 228, pp.773–803, https://doi.org/10.1007/s00707-016-1733-z.
18. Аветисян А.С., Мкртчян С.А., Электроупругие волны Рэлея в волноводе с элрктрически закрытыми или открытыми поверхностями. //Изв. НАН Армении. Механика. 2018, т.71, №1, с.12-30. http://doi.org/10.33018/71.1.2 Avetisyan A.S., Mkrtchyan S.H., The electro elastic Rayleigh waves in the waveguide, with an electrically closed or open surfaces. Procc. of NAS of Armenia, Mechanics, (2018), vol. 71, №1, pp. 12-30, (in Russian), http://doi.org/10.33018/71.1.2.
19. Piliposyan G.T., Avetisyan A.S., Ghazaryan K.B., Shear wave propagation in periodic phononic/photonic piezoelectric medium, Wave Motion (2012), vol.49(1), pp.125-134 http://doi.org/10.1016/j.wavemoti.2011.08.001.

Information about the authors:

Avetisyan Ara S. – Department of Dynamics of Deformable Systems and Coupled Fields, Institute of Mechanics of NAS of Armenia, **E-mail: ara.serg.avetisyan@gmail.com,**

Khachatryan Vazgen M. Department of Dynamics of Deformable Systems and Coupled Fields, Institute of Mechanics of NAS of Armenia, **E-mail: khachvaz@gmail.com**

Received 17.01.2020