ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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LOCALIZED SHEAR WAVES IN PIEZOELECTRIC LAYER COVERED BY ELASTIC THIN COATING

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Key words: localized wave, piezoelectric, thin film, composite.

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Локализованные сдвиговые волны в пьезоэлектрическом слое с тонким упругим покрытием

Ключевые слова: локализованная волна, пьезоэлектрики, тонкое покрытие, композит.

В статье рассматривается распространение локализованной волны сдвига в двухфазной среде: упругое покрытие и пьезоэлектрический слой. В рамках прикладной модели тонкого слоя получено дисперсионное уравнение для частот связанных электроупругих волн. Обсуждается влияние упругого покрытия и пьезоэффекта на локализованные фазовые скорости волн.

Ղազարյան Կ.Բ., Պապյան Ա.Ա.

Տեղայնացված սահքի ալիքները բարակ առաձգական ծածկույթով պիեզոէլեկտրիկ շերտում

Հիմնաբառեր. տեղայնացված ալիքներ, պիեզոէլեկտրիկ, բարակ ծածկույթ, կոմպոզիտ.

Հոդվածում ուսումնասիրվում է սահքի տեղայնացված ալիքների տարածումը երկշերտ միջավայրում: Բաղկացած առաձգական ծածկույթից և պիեզոէլեկտրիկ շերտից,: Բարակ շերտի կիրառական մոդելի հիման վրա ստացվել է դիսպերսիոն հավասարում կապակցված էլեկտրաառաձգական ալիքների համար: Քննարկվել է առաձգական ծածկույթի և պիեզոէֆեկտի ազդեցությունը տեղայնացված ալիքի փուլային արագության վրա:

The paper focuses on shear localized wave propagation in two phase medium: elastic thin coating and piezoelectric layer. In the framework of an applied model of thin layer a dispersion equation is derived for coupled electro elastic wave phase speeds. The influence of elastic coating and piezo effect on localized wave phase speeds are discussed.

Introduction.

Piezoelectric composites that are made of by two or more of piezoceramic materials are widely studied and discussed in [1-11]. In the problems of wave propagation in the composites the perfect bonding at the interface between two materials is routinely assumed. The composite structures consisting of piezoelectric ceramics with several types of partial contacts at the interface between two materials i.e., the electrically shorted or electrically closed, mechanically compliant (sliding) interfaces are considered in [4-8]. Electro elastic shear surface (localized) waves in composite structures: inhomogeneous piezoelectric layer – piezoelectric substrate, dielectric layer- piezoelectric substrate, piezoelectric layer - inhomogeneous substrate are considered in [8-11]. The localized waves in piezoelectric layer with different electrical and mechanical boundary conditions at walls of layers are studied in [12]. A model of boundary contact for electro-magneto-elastic composites with interface roughness is proposed in [13].

In the paper an analytical solution is given for a problem of shear localized wave propagation in bi-material media constituted by thin elastic thin coating and piezoelectric layer. The interface between elastic coating and piezoelectric layer is considered to be elastically perfect and electrically shorted one. Analogous problem for pure elastic bi-material media is considered in [11]. For thin coating the applied model is used which brings to averaged boundary conditions at contact interface between piezoelectric and thin coating.

Statement of the problem

Let's consider shear wave propagation along a bi-material layer consisting from thin elastic coating and piezoelectric layer. The geometry of the bi-material layer in the Cartesian system (x, y, z), $-a_0 < x < a$, $-\infty < y < \infty$, $-\infty < z < \infty$ is depicted in Fig.1.

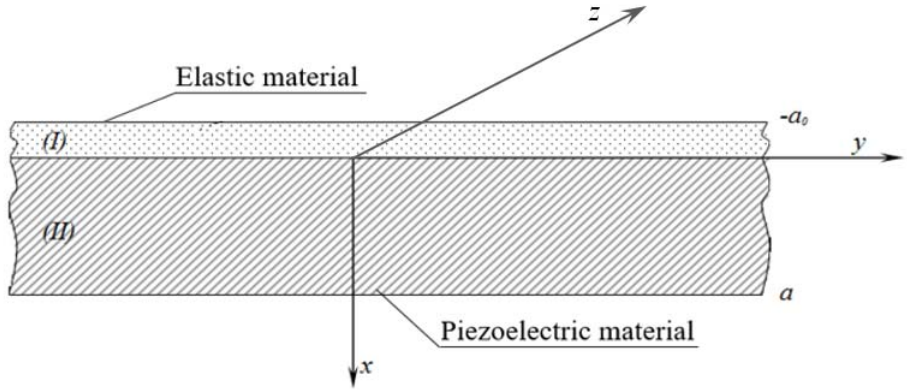

Fig1. Bi-material layer in Cartesian system (x, y, z)

For piezoelectric layer of piezo crystal of 6mm hexagonal class of symmetry with polling axis parallel to z coordinate direction, the anti-plane problem is described by the following equations and the constitutive material relations, based on the decoupled linear dynamic equations of theory of elasticity and quasi- static set of Maxwell equations [1,16]

$$\nabla\cdot\vec{\sigma}=\rho\frac{\partial^2 U}{\partial t^2},\qquad \nabla\cdot\vec{D}=0,\qquad \vec{E}=-\vec{\nabla}\cdot\varphi$$

$$\vec{\sigma}=\vec{\nabla}\left(GU+e_{15}\varphi\right)\ ,\quad \vec{D}=\vec{\nabla}\left(-\varepsilon\varphi+e_{15}U\right) \tag{1}$$

$$\vec{\sigma}=\left(\sigma_{xz}(x,y,t),\sigma_{yz}(x,y,t)\right);\vec{E}=\left(E_x(x,y,t),E_y(x,y,t),0\right);$$

$$\vec{D}=\left(D_x(x,y,t),D_y(x,y,t),0\right),$$

Here, σ_{xz} and σ_{yz} are shear stresses, $\vec{E}$ is the electric field intensity vector, $\vec{D}$ is the electrical displacement vector, $\varphi=\varphi(x,y,t)$ is the electric field potential, ρ is bulk density, $\vec{\nabla}$ is the nabla vector, G is the shear modulus, ε is electrical permittivity coefficient, e_{15} is the piezoelectric modulus.

Using (1) we get the following set of equations

$$c^2\Delta U-\frac{\partial^2 U}{\partial t^2}=0;\qquad \Delta\left(U-\frac{e_{15}}{\varepsilon}\varphi\right)=0\,,\quad \Delta\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{2}$$

For elastic layer the following equation and material relations are valid

$$\frac{\partial \sigma_{0xz}}{\partial x} + \frac{\partial \sigma_{0yz}}{\partial y} - \rho_0 \frac{\partial^2 V}{\partial t^2} = 0, \quad \sigma_{0xz} = G_0 \frac{\partial V}{\partial x}, \quad \sigma_{0yz} = G_0 \frac{\partial V}{\partial y}; \qquad (3)$$

In (2, 3) ρ_0 is the bulk density, G_0 are the shear modulus of piezoelectric material, $c = \sqrt{\left(G + \varepsilon^{-1} e_{15}^2\right)\rho^{-1}}$.

At the bi-material interface $y = 0$ we consider the partial contact conditions of electrically shorted contact for electric potential and continuous mechanical displacements and tractions

$$\varphi(0, y, t) = 0; \quad \sigma_{xz}(0, y, t) = \sigma_{0xz}(0, y, t); \qquad \mathrm{U}(0, y, t) = V(0, y, t) \qquad (4)$$

At the bi-material layer external surfaces we take the following conditions

$$\sigma_{0xz}(-a_0, y, t) = 0, \qquad \sigma_{xz}(a, y, t) = 0, \qquad \varphi(a, y, t) = 0. \qquad (5)$$

Assuming that $a_0 << a$ we average [10, 14, 15] the equation (3) by elastic layer thickness along coordinate x

$$\int_{-a_0}^{0} \left(\frac{\partial \sigma_{0xz}}{\partial x} + \frac{\partial \sigma_{0yz}}{\partial y} - \rho_0 \frac{\partial^2 V}{\partial t^2} \right) dx = 0. \qquad (6)$$

Taking in a view of the smallness of the thickness a_0, assuming that the displacement $V(x, y, t)$ do not vary along the thickness of the elastic layer and taking into account the boundary and contact conditions (4, 5) the averaged boundary condition at $x = 0$ interface can be cast as

$$\sigma_{xz}(0, y, t) + a_0 G_0 \frac{\partial^2 U}{\partial y^2} - a_0 \rho_0 \frac{\partial^2 U}{\partial t^2} = 0; \qquad x = 0. \qquad (7)$$

Presenting the solutions in the form of plane wave propagating along y direction

$$U(x, y, t) = U_0(x) \exp(iky - i\omega t), \varphi(x, y, t) = \varphi_0(x) \exp(iky - i\omega t) \qquad (8)$$

we get solutions for $U_0(x), \varphi_0(x)$ as

$$U_0(x) = C_1 \cos\left(x\sqrt{\eta^2 - 1}\right) + C_2 \sin\left(x\sqrt{\eta^2 - 1}\right) \qquad (9)$$

$$\varphi_0(x) = A_1 \exp(kx) + A_2 \exp(-kx) + \frac{e_{15}}{\varepsilon}\left(C_1 \cos\left(x\sqrt{\eta^2 - 1}\right) + C_2 \sin\left(x\sqrt{\eta^2 - 1}\right)\right)$$

Here C_1, C_2, A_1, A_2 are the arbitrary unknown constants, $\eta = \omega(ck)^{-1}$ is the dimensional phase speed of electro elastic vibrations.

Substituting these solutions into the boundary conditions we obtain a homogeneous set of algebraic equations, with respect to the arbitrary constants C_1, C_2, A_1, A_2. For nontrivial solutions, the determinant of this set has to vanish. Equating the determinant to zero we obtain the following dispersion equation determining phase speed η.

$$
\begin{aligned}
&\left(-\frac{2\left(\chi^2+\chi\right)}{\cosh\left(K\sqrt{1-\eta^2}\right)\cosh\left(K\right)}+2\left(\chi^2+\chi\right)\right)+\\
&+\left(\left(\eta^2-1\right)\left(\chi+1\right)^2-\chi^2\right)\frac{\tanh\left(K\sqrt{1-\eta^2}\right)\tanh\left(K\right)}{\sqrt{1-\eta^2}}+\\
&+\frac{\rho_0}{\rho}K_0\left(\frac{c_0^2}{c^2}-\eta^2\right)\left(\chi\frac{\tanh\left(K\sqrt{1-\eta^2}\right)}{\sqrt{1-\eta^2}}-\left(\chi+1\right)\tanh\left(K\right)\right)=0
\end{aligned}
\tag{10}
$$

Here the following notations are used

$$\chi=\frac{e_{15}^2}{G\varepsilon}\,,\;K=ka,\;K_0=ka_0;\;c_0^2=G_0/\rho_0\,.$$

The dispersion equation (10) defines a localised wave phase speed $\eta(K)$ as a function of the dimensionless width K and the elastic and electromechanical coefficients of the layered structure.

Now we shall restrict ourselves to consideration the case of localized vibrations of the piezo elastic layer when $\eta<1$ only.

Elastic layer dispersion equation

When the thin elastic coating is absent ($a_0\to 0$), the equation (10) results in

$$
\begin{aligned}
&f_1(\eta)\,f_2(\eta)=0\\
&f_1(\eta)=-(\chi+1)\sqrt{1-\eta^2}\,\tanh\left(\frac{K\sqrt{1-\eta^2}}{2}\right)+\chi\tanh\left(\frac{K}{2}\right);\\
&f_2(\eta)=\chi\tanh\left(\frac{K\sqrt{1-\eta^2}}{2}\right)-(\chi+1)\sqrt{1-\eta^2}\,\tanh\left(\frac{K}{2}\right)
\end{aligned}
\tag{11}
$$

The equations (11) have been obtained first in [8].

Here we will only mention that the first equation of (11) corresponds to a dispersion equation of symmetric vibration of layer with boundary condition

$$\sigma_{xz}\left(0,y,t\right)=0,\;\varphi\left(0,y,t\right)=0;\;\sigma_{xz}\left(\frac{a}{2},y,t\right)=0,\;D_x\left(\frac{a}{2},y,t\right)=0 \tag{12}$$

and the second equation of (11) corresponds to a dispersion equation of anti-symmetric vibration of layer with boundary condition

$$\sigma_{xz}\left(0,y,t\right)=0,\ \varphi\left(0,y,t\right)=0;\ U\left(\frac{a}{2},y,t\right)=0,\ \varphi\left(\frac{a}{2},y,t\right)=0; \tag{13}$$

The phase speed –wave number (dimension less thickness) dispersion curves $\eta\left(K\right)$ are depicted on the Fig.2 for piezoceramic PZT-5H, where δ_0 is the well-known Bluestein-Gulyaev solution for semi- space with electrically shorted and mechanically free interface [12]., δ_{01} is the limiting value of localized wave speed at $K \to 0$

$$\delta_0=\sqrt{1+2\chi}\Big/\left(1+\chi\right)=\sqrt{1-\chi_0^4}\ ,\quad \delta_{01}=\left(1+\chi\right)^{-\frac{1}{2}}<\delta_0 \tag{14}$$

The notation $\chi_0=\sqrt{e_{15}^2\left(G\varepsilon+e_{15}^2\right)^{-1}}$ was used by Bluestein in [12].

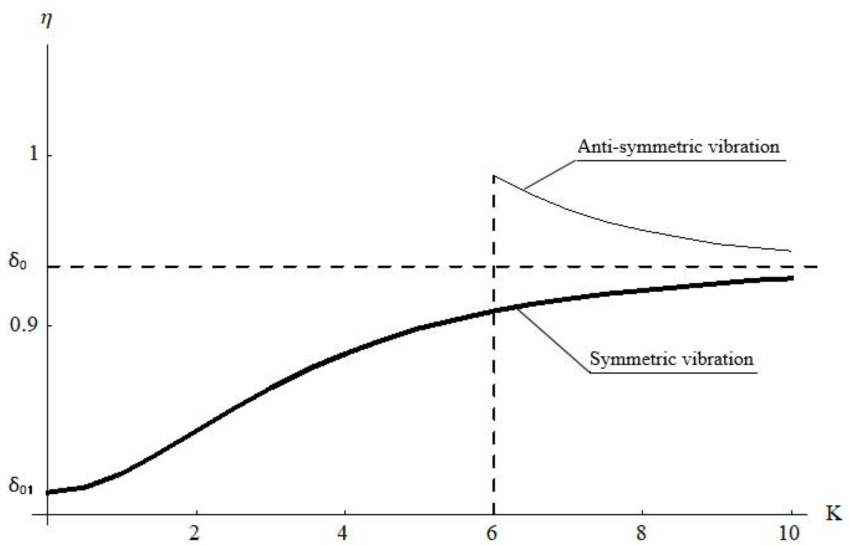

Fig.2. Dispersion curves $\eta\left(K\right)$ for elastic layer, $\delta_0=0.935,\ \delta_{01}=0.803$

Modal structure of the localised wave in the bi-material layer

First it will be noted that the bi-material layer dispersion equation does not decoupled into symmetric and anti-symmetric types of vibration which means that two phase structure may not in some cases support two types of localized waves.
In the case of $K>>1$ the dispersion (10) can be rewritten as

$$\frac{K_0G_0}{G}\left(1-\gamma^{-1}\eta^2\right)+\left(\left(1+\chi\right)\sqrt{1-\eta^2}-\chi\right)=0 \tag{15}$$

For thin layer $K<<1$ instead of dispersion equation (9) we get

$$\eta = \sqrt{\frac{1+\gamma\frac{a_0\rho_0}{a\rho}}{(1+\chi)+\frac{a_0\rho_0}{a\rho}}} \qquad (16)$$

Here $\gamma = c_0^2/c^2$ is the ratio coefficient of phase speeds of bulk shear waves in the elastic layer and piezoelectric layer. The analysis of the dispersion equations (11, 15, 16) reveals that its solutions are very sensitive to coefficient γ. For the «soft» elastic coating $\gamma \le 1$ we have that $\eta < 1$ for any values of K, but in the case of the «hard» elastic coating $\gamma > 1$ the equation (9) may have no localised solutions for small values of K. Moreover, in this case, for some classes of materials, only one localized wave can exist in a layered structure, in contrast to the case of a «soft» elastic layer.

These circumstances and more are illustrated in Table 1, for certain class of materials (η_1, η_2 are the speeds of localized waves).

Table 1

	$\chi = 0$		$\chi = 0.3$		$\chi = 0.5$		$\chi = 0$		$\chi = 0.3$		$\chi = 0.5$	
K	η_1	η_2	η_1	η_2	η_1	η_2	η_1	η_2	η_1	η_2	η_1	η_2
0.5	-	-	-	-	-	-	0.885	-	0.805	-	0.762	-
1.0	-	-	-	-	-	-	0.933	-	0.841	-	0.793	-
3.0	-	-	0.980	-	0.936	-	0.973	-	0.897	-	0.856	-
5.0	-	-	0.973	-	0.938	-	0.981	-	0.924	-	0.890	-
7.0	-	-	0.973	-	0.941	-	0.985	-	0.936	-	0.905	0.962
10	-	-	0.973	-	0.942	-	0.987	-	0.943	0.980	0.911	0.945
$\rho_0/\rho = 2,\ c_0/c = 4 \quad K_0 = 0.1$							$\rho_0/\rho = 2,\ c_0/c = 0.2 \quad K_0 = 0.1$					

Conclusion

The localized vibration of layered structure is considered consisting from elastic coating and piezoelectric layer of piezo crystals of 6mm hexagonal symmetry class. Based on an applied model of thin elastic coating the dispersion equation is obtained for phase speed as a function of the layer dimensionless width and the elastic and electromechanical coefficients of the layered structure. It is defined that the solutions of the dispersion equation are very sensitive to the coefficient of ratio of phase speeds of bulk shear waves in the elastic layer and piezoelectric layer. For some classes of materials it is shown that the layered structure with the “hard” elastic coating can support only one localized wave, contrary to the case of the “soft” elastic coating where two localized waves may exist.

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