ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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TWO-DIMENSIONAL PROBLEMS OF ELECTROACOUSTICS IN HOMOGENEOUS PIEZOELECTRIC CRYSTALS

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Keywords: electroelastisity, stress-deformed electroactive state, anisotropy of piezocrystal, refined material relations, quasistatic equations.

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Двумерные задачи электроакустики в однородных пьезоэлектрических кристаллах

Ключевые слова: электроупругость, электроактивное напряжённо-деформированное состояние, анизотропия пьезокристалла, уточнённые материальные соотношения, квазистатические уравнения.

Исследуются вопросы возможности раздельного возбуждения и распространения электроупругого плоского или электроупругого антиплоского напряжённо-деформированных состояний в однородных пьезоэлектрических кристаллах.

Получены необходимые и достаточные условия раздельного возбуждения и распространения электроупругого плоского и электроупругого антиплоского напряжённо-деформированных состояний в пьезоэлектрических кристаллах.

Определены текстуры пьезодиэлектрических кристаллов, структуры обобщённого тензора электроупругости которых позволяют разделение плоского напряжённо-деформированного электроактивного состояния от неэлектроактивного антиплоского упругого состояния.

В рассматриваемых задачах получены материальные соотношения и квазистатические уравнения электроупругости для всех пьезоэлектрических текстур в каждой из всех трёх сагиттальных плоскостей соответствующей кристаллической решётки. Проведён сравнительный анализ вновь полученных соотношений напряжённо деформированных состояний с соответствующими соотношениями электроактивных состояний плоской и антиплоской деформаций.

Արա Ս. Ավետիսյան

Էլեկտրաակուստիկայի երկչափ խնդիրները համասեռ պիեզոէլեկտրական բյուրեղներում

Հիմնաբառեր. էլեկտրաառաձգականություն, էլեկտրականապես ակտիվ լարված-դեֆորմացիոն վիճակ, պիեզոբյուրեղի անիզոտրոպիա, ճշգրտված նյութական առնչություններ, քվազիստատիկ հավասարումներ:

Էլեկտրականապես ակտիվ, համասեռ պիեզոէլեկտրական բյուրեղներում հետազոտվում են հարթ կամ հակահարթ էլեկտրաառաձգական լարված-դեֆորմացիոն վիճակների առանձին-առանձին գրգռման և տարածման հնարավորության հարցերը:

Ստացված են առաձգական դեֆորմացվող միջավայրի անիզոտրոպիայով պայմանավորված էլեկտրաառաձգական հարթ լարվածադեֆորմացիոն վիճակի անջատ գրգռման և տարածման հնարավորության անհրաժեշտ և բավարար պայմանները:

Բացահայտված են պիեզոբյուրեղների այն դասերը, որոնց էլեկտրաառաձգականության թենզորի կառուցվածքը թույլ է տալիս էլեկտրականապես ակտիվ հարթ լարվածադեֆորմացիոն վիճակի անջատումը էլեկտրականապես ոչ ակտիվ հակահարթ դեֆորմացիայի վիճակից:

Ստացված են առաձգական դեֆորմացվող միջավայրի անիզոտրոպիայով պայմանավորված էլեկտրաառաձգական հակահարթ լարվածադեֆորմացիոն վիճակի անջատ գրգռման և տարածման հնարավորության անհրաժեշտ և բավարար պայմանները:

Բացահայտված են նաև պիեզոբյուրեղների այն դասերը, որոնց էլեկտրաառաձգականության թենզորի կառուցվածքը թույլ է տալիս էլեկտրականապես ակտիվ հակահարթ լարված-դեֆորմացիոն վիճակի անջատումը էլեկտրականապես ոչ ակտիվ հարթ դեֆորմացիայի վիճակից:

Բերված խնդիրներում ստացված են էլեկտրաառաձգականության նյութական առնչությունները և քվազի-ստատիկ հավասարումները պիեզոէլեկտրական միջավայրերի համար՝ պիեզոբյուրեղային կառուցվածքի բոլոր երեք ընտրված հարթություններում: Կատարված է նշված խնդիրներում ստացված առնչությունների ու էլեկտրականապես ակտիվ հարթ և հակահարթ դեֆորմացիոն վիճակների համապատասխան առնչությունների համեմատական վերլուծություն:

The problems of the possibility of separate excitation and propagation of electroelastic planar, or electroelastic anti-plane stress-strain states in homogeneous piezoelectric crystals are investigated.

The necessary and sufficient conditions for the separate excitation and propagation of electroelastic plane and electroelastic antiplane stress-strain states in piezoelectric crystals are obtained.

The textures of piezodielectric crystals are determined, the structures of the generalized electroelasticity tensor of which allow the separation of the plane stress-strain electroactive state from the non-electroactive anti-plane elastic state.

The textures of piezodielectric crystals, the structures of the generalized electroelasticity tensors of which allow the separation of the anti-flat stress-deformed electroactive state from the non-electroactive flat elastic state, are also determined.

In the considered problems, the material relations and quasistatic equations of electro elasticity for all the piezoelectric textures in each of all three sagittal planes of corresponding crystal lattice are obtained. A comparative analysis of the newly obtained ratios of stress-strain states with the corresponding ratios of electroactive states of plane and anti-plane deformations is carried out.

Introduction. In many structural schemes of modern electronic technology, various new crystalline elements, layered composite waveguides, formed from various natural or artificially grown piezoelectric materials with different physical and mechanical properties, are widely used. The operation of these elements is often based on the emission (or delay) of only electro-acoustic waves of a plane stress-strain state or only on the emission (or delay) of an electro-acoustic wave of anti-plane deformation (pure shear waves). As a rule, structural elements used in modern technology are thin-walled and in the formulation of two-dimensional problems, we must take into account possible approaches that allow separate formulations of the problems of electroactive plane deformation and electroactive anti-plane deformation.

In applied problems of studying stress-strain states in thin-walled elastic structural elements, two-dimensional problems of elasticity theory were modeled. In order to avoid taking into account the violation of the plane stress-strain state in the middle surface of a thin-walled element, scientists adopted various hypotheses Kirchhoff G. [1], Timoshenko S., Woinowsky-Krieger S. [2], Reissner E. [3], Ambartsumian S.A. [4]: the hypothesis of direct normals, the hypothesis of the absence of pressure of the plate layers on each other, the hypothesis of the inextensibility of the middle surface of the plate.

Hypotheses were accepted as additional restrictions, based on the nature of the distribution of the mechanical load on the element and the conditions of fastening of the end face of the elastic element.

The hypothetical approach has also been successfully implemented in the problems of electro-magneto-elasticity of thin plates and shells [5], where, along with hypothetical distributions of mechanical characteristics over the thickness of a thin-walled element, characteristic distributions of the electromagnetic field are also accepted.

In piezoelectric media, the electroelastic wave field is four-component. In addition to elastic displacements, the electric field potential is also a characteristic component. Separate excitation and propagation of electro-elastic waves of plane or anti-plane deformations in the composite continuum suggests the formation of different groups of interconnected wave components. It is obvious that in the case of a possible separation of elastic wave fields into plane or anti-plane stress-strain states, the electric field potential can be present only in one of the formed groups of elastic wave components. Then the divided second group of components will be non-electroactive.

The question of the possible separation of the electroactive wave of plane elastic deformation from the purely shear elastic (non-electroactive) wave in infinite homogeneous piezoelectric media, depending on the anisotropy of the physical properties of the material, was investigated in [6]. The question of the possible separation of the wave of electroactive antiplane elastic deformation from the wave of purely elastic plane deformation was also studied there. However, the material relationships and the equations of electroelasticity in all three sagittal planes of crystals are not given in the work.

The results of these studies are also given in the monograph [7].

In these studies, as in solving many specific problems of the separate excitation and propagation of the electroactive plane strain wave, the problem of the occurrence of the non-planar electroelastic stress state is not discussed. The occurrence of the third axial mechanical stress and the third component of the electric displacement vector will obviously violate the picture of planar changes in the parameters of the separated wave fields.

Naturally, the need to introduce additional restrictions (hypotheses) also arises when modeling two-dimensional problems on the propagation of electroelastic waves of plane or antiplane deformations in model semi-infinite waveguides. The formulation of the two-dimensional problem in the plane $x_\alpha 0 x_\beta$ also implies the invariance of deformations of any linear element perpendicular to this surface and each sagittal plane as a thin material layer should be in a plane stress state. These conditions make it possible to formulate the generalized plane stress-strain state equivalent to the state of plane deformation also in semi-infinite waveguides.

Similar to the hypothesis that the elastic surface is inextensible, when there are no acting forces in its plane, the assumption allows us to formulate the equivalent generalized antiplane stress-strain state.

The introduction of additional restrictions eliminates the possibility of simultaneous excitation and propagation of separated electroactive stress-strain states in homogeneous piezoelectric composites. The question of the possible separation of electroelastic wave fields into plane and anti-plane stress-strain states is naturally complicated in the case of composite, piecewise homogeneous bodies. This issue is relevant because in modern electronic technology there is a technical need: by exciting an electroactive wave of anti-plane deformation in a composite waveguide, obtain an electroactive wave of plane deformation from the receiver (or vice versa).

In the following works, it will be shown that this problem can be solved in layered, piecewise homogeneous composite waveguides, by choosing layers of different piezoelectric materials that are in the state of non-acoustic contact.

In this paper, we study the statements of two-dimensional problems of electroelasticity:

1. In what textures of homogeneous piezoelectrics is it possible to separately excite and propagate electroactive plane deformation, in which the electroelasticity problem will be plane-stress in the selected sagittal plane of the piezoelectric crystal?
2. In which textures of homogeneous piezoelectrics is it possible to separately excite and propagate electroactive antiplane deformation, in which there are no acting forces in the selected sagittal plane?

1. Basic concepts and relations of an electroelastic stress-strain state in homogeneous piezoelectric media

In the linear theory of electroelasticity of homogeneous piezoelectric media, we use the complete system of quasistatic equations

$$\partial\sigma_{ij}/\partial x_j = \rho\left(\partial^2 u_i/\partial t^2\right),\ \ \partial D_n/\partial x_n = 0\,, \tag{1.1}$$

taking into account the potentiality of the electric field $E_n = -\left(\partial\varphi/\partial x_n\right)$ and linear material relations of the medium

$$\sigma_{ij}=c_{(ij)(nk)}\left(\partial u_n/\partial x_k\right)+e_{m(ij)}\left(\partial\varphi/\partial x_m\right),\ D_m=e_{m(nk)}\left(\partial u_n/\partial x_k\right)-\varepsilon_{mk}\left(\partial\varphi/\partial x_k\right), \tag{1.2}$$

where the mechanical and electric fields are interconnected by the piezoelectric coefficient tensor $(\hat{e}_{j(mn)})$.

Physicomechanical constants of a homogeneous piezoelectric medium: elastic stiffness $c_{(ij)(nm)}$, piezoelectric coefficients $e_{j(mn)}$ and dielectric constant ε_{ik}, form a generalized electroelastic tensor of piezoelectric materials of the type $\left(\hat{\gamma}_{jn}\right)_{9\times9} = \left(\hat{c}_{ij}\right)_{6\times6} \cup \left(\hat{e}_{mn}\right)_{3\times6} \cup \left(\hat{\varepsilon}_{ik}\right)_{3\times3}$ [9,10]

$$\begin{pmatrix} \left(\hat{c}_{(ij)(nk)}\right)_{6\times6} & \left(\hat{e}_{(ij)m}\right)_{6\times3} \\ \left(\hat{e}_{m(ij)}\right)_{3\times6} & \left(\hat{\varepsilon}_{ik}\right)_{3\times3} \end{pmatrix} \tag{1.3}$$

In the equations of electroelasticity (1.1), in the material relations (1.2) and in the generalized tensor of linear electroelasticity of piezoelectric materials (1.3) the notations and known transitions from four-digit indices to two-digit indices $(\alpha\gamma) \rightleftarrows \alpha \ \ if \ \ \alpha = \gamma$ and $(\alpha\gamma) \rightleftarrows 9-\alpha-\gamma \ \ if \ \ \alpha \neq \gamma$ are used. It is also assumed that the indices $\{\alpha;\beta;\gamma\} \in \{1;2;3\}$, $\alpha \neq \beta$, $\beta \neq \gamma$ and $\gamma \neq \alpha$ indicated by the Greek letters, are dumb, and summation over them is not carried out.

The conditions permitting separate excitation and propagation of a plane or anti flat stress strain states in the uniform piezoelectric medium of this anisotropy, are imposed on the structure tensor of elastic material stiffness $\left(\hat{c}_{ij}\right)_{6\times6}$, as well as the corresponding structure tensor of piezoelectric coefficients $\left(\hat{e}_{nj}\right)$ and the coefficients of permittivity $\left(\hat{\varepsilon}_{ik}\right)_{3\times3}$.

The generalized linear electroelastic tensor of piezoelectric materials (1.3), as well as the material relations and basic equations are given following the rules for installing crystals according to syngonies (table 1) and according to the rules for choosing crystallographic axes in them (table 2) [9, 10]. These tables describe the order of the axes of symmetry (and / or inversion) and the anisotropy planes of the piezocrystals, the commensurability of the unit vectors and angles of the selected orthogonal system of base coordinates, as well as the order of alignment of the coordinate system with the base axes and planes of piezocrystals.

Without loss of generality, we will formulate the problem in one of the sagittal planes $x_\alpha 0 x_\beta$, where all the components of the electroelastic field depend on the coordinates x_α and x_β, and there are no changes in the third base coordinate $\partial/\partial x_\gamma \equiv 0$.

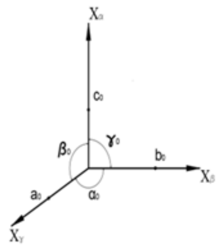
It should be noted that in the quasistatic problem of electroelasticity, the two-dimensional electric field is potential and plane

$$E_\alpha(x_\alpha,x_\beta,t) = -\partial\varphi(x_\alpha,x_\beta,t)/\partial x_\alpha \,,$$

$$E_\beta(x_\alpha,x_\beta,t) = -\partial\varphi(x_\alpha,x_\beta,t)/\partial x_\beta \,, \quad E_\gamma(x_\alpha,x_\beta,t) \equiv 0 \qquad (1.4)$$

Naturally, in the main relations (1.1) and (1.2), the electric field potential $\varphi(x_\alpha,x_\beta,t)$ can be present either only in the group of wave components characterizing the electroactive state of antiplane deformation $\{0;\ 0;\ u_\gamma(x_\alpha,x_\beta,t);\ \varphi(x_\alpha,x_\beta,t)\}$, or only in the group of wave components characterizing the state of electroactive plane deformation $\{u_\alpha(x_\alpha,x_\beta,t);\ u_\beta(x_\alpha,x_\beta,t);\ 0;\ \varphi(x_\alpha,x_\beta,t)\}$.

Table 1. Crystal installation rules by syngonies

	***Triclinic***	***Monoclinic***	***Rhombic***
	$X_\alpha, X_\beta, X_\gamma \rightleftarrows 1;\bar{1}$; $a_0 \neq b_0 \neq c_0$, $\alpha_0 \neq \beta_0 \neq \gamma_0 \neq 90^0$	$X_\beta \rightleftarrows 2$ *or m* $a_0 \neq b_0 \neq c_0$, $\alpha_0 = \gamma_0 = 90^0 \neq \beta_0$	$X_\alpha, X_\beta, X_\gamma \rightleftarrows 2$ *or m* , $a_0 \neq b_0 \neq c_0$, $\alpha_0 = \beta_0 = \gamma_0 = 90^0$
	Tetragonal	***Trigonal and Hexagonal***	***Cubic***
$\{X_\alpha, X_\beta, X_\gamma\}$ – *crystallographic axis of the texture,* $\{\alpha_0, \beta_0, \gamma_0\}$ - *angles in the sagittal planes,* $\{a_0, b_0, c_0\}$ - *measures of axial unit vectors*	$x, y \rightleftarrows 2; m$; $z \rightleftarrows 4; \bar{4}$ $a_0 = b_0 \neq c_0$, $\alpha_0 = \beta_0 = \gamma_0 = 90^0$	$x, y \rightleftarrows 2; m$; $z \rightleftarrows 3; \bar{3}; 6; \bar{6}$; $a_0 = b_0 \neq c_0$, $\alpha_0 = \beta_0 = 90^0$, $\gamma_0 = 120^0$	$x, y, z \rightleftarrows 4; \bar{4}; 2$; $a_0 = b_0 = c_0$, $\alpha_0 = \beta_0 = \gamma_0 = 90^0$

Therefore, in the case of the possible separate excitation and propagation of waves of elastic strains in an electroelastic medium, only one of them will be electroactive. In both cases, a group of wave components separated from a given electroelastic wave will already characterize the non-electroactive state of deformation.

The problems of the possible separate excitation and propagation of the wave of electroactive plane deformation from the non-electroactive elastic wave of anti-plane deformation, as well as the separate excitation and propagation of the wave of electroactive anti-plane deformation from the wave of non-electroactive elastic plane deformation in homogeneous piezoelectric materials, were studied in [1]. The article shows that if in elastic anisotropic homogeneous media the separation of the plane elastic deformation wave from the anti-plane elastic deformation wave in a selected sagittal plane $x_\alpha 0 x_\beta$ of the crystalline medium is ensured by the absence of the corresponding constant in the structure of the elastic stiffness tensor

$$c_{\alpha(\gamma\alpha)} = c_{\alpha(\beta\gamma)} = c_{\beta(\beta\gamma)} = c_{\beta(\gamma\alpha)} = c_{(\alpha\beta)(\gamma\alpha)} = c_{(\alpha\beta)(\beta\gamma)} \equiv 0 \qquad (1.5)$$

then it is possible to separate the wave of electroactive plane deformation from the non-electroactive elastic wave of anti-plane deformation when, along with conditions (1.5), the conditions for the absence of piezoelectric coefficients are satisfied

$$e_{\alpha(\gamma\alpha)} = e_{\alpha(\gamma\beta)} = e_{\beta(\gamma\alpha)} = e_{\beta(\gamma\beta)} \equiv 0 \quad (1.6)$$

Table 2. Rules for selecting crystallographic axes in

Syngony	x_α	x_β	x_γ
Triclinic	In a plane perpendicular to the direction [001]		[001]
Monoclinic	[100]	[010]	[001]
Rhombic	[100]	[010]	[001]
Tetragonal	[100]	[010]	[001]
Trigonal and Hexagonal	[100]	[010]	[001]
Cubic	[100]	[010]	[001]

Separation of the wave of electroactive anti-plane deformation from the non-electro-active elastic wave of plane deformation is possible when, along with conditions (1.5), the following conditions are satisfied in the piezoelectric coefficient tensor of the material

$$e_{\alpha\alpha} = e_{\alpha\beta} = e_{\alpha(\alpha\beta)} = e_{\beta\alpha} = e_{\beta\beta} = e_{\beta(\beta\alpha)} \equiv 0 \quad (1.7)$$

The problem of separate excitation and propagation of elastic waves of plane and antiplane deformation in a selected sagittal plane $x_\alpha 0 x_\beta$ of anisotropic homogeneous media (without taking into account any kind of coupled physical fields) was raised in [4]. The author showed that in elastic anisotropic homogeneous media, the separation of a plane elastic strain wave from an anti-plane elastic strain wave in a selected sagittal plane $x_\alpha 0 x_\beta$ of a crystalline medium is ensured by the absence of material constants in the tensor $(\hat{c}_{ij})$ of elastic stiffness (1.5).

The above conditions (1.5) and (1.6), or (1.5) and (1.7), as restrictions on the anisotropy of the medium for separate excitation and propagation of plane or antiplane stress-strain states, are necessary, but not sufficient.

Below we will discuss conditions additional to relations (1.5) and (1.6), in which case separate excitation and propagation of the plane electroactive stress-strain state is possible, when all components of the electroelastic field belong to the sagittal plane. We will also discuss conditions additional to relations (1.5) and (1.7), in which case separate excitation and propagation of antiplane electroactive deformation is possible, when all other components of the electroelastic field belong to the sagittal plane.

2. Electroactive plane stress-strain state in homogeneous piezoelectric textures

When only the conditions for the absence of the third component of elastic displacement $u_\gamma(x_\alpha, x_\beta, t) \equiv 0$ and derivatives of all other wave field characteristics $\partial/\partial x_\gamma \equiv 0$ are accepted, the basic relations of general planar electroactive deformation $\{u_\alpha(x_\alpha, x_\beta, t);\ u_\beta(x_\alpha, x_\beta, t);\ 0;\ \varphi(x_\alpha, x_\beta, t)\}$ are obtained from the quasistatic equations of electroelasticity (1.1), material relations (1.2), taking into account the structure of the generalized tensor of electroelasticity of piezoelectric materials (1.3). From the material relations obtained, taking into account the form of the electric field (1.4) and the corresponding conditions (1.5) and (1.6), it follows that, along with the non-zero stresses

$\sigma_{\alpha\alpha}(x_\alpha, x_\beta, t)$, $\sigma_{\beta\beta}(x_\alpha, x_\beta, t)$ и $\sigma_{\alpha\beta}(x_\alpha, x_\beta, t)$ characteristic to the plane stress state, axial mechanical stress also arises in the case of electroactive plane

$$\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t) = c_{\gamma\alpha}\left(\partial u_\alpha / \partial x_\alpha\right) + c_{\gamma\beta}\left(\partial u_\beta / \partial x_\beta\right) + \\ + c_{\gamma(\alpha\beta)}\left[\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)\right] + e_{\alpha(\gamma\gamma)}\left(\partial \varphi / \partial x_\alpha\right) + e_{\beta(\gamma\gamma)}\left(\partial \varphi / \partial x_\beta\right) \tag{2.1}$$

Along with the nonzero components of the electric displacement of the plane electric field $D_\alpha(x_\alpha, x_\beta, t)$ and $D_\beta(x_\alpha, x_\beta, t)$, the third component of the electric displacement vector can also arise

$$D_\gamma(x_\alpha, x_\beta, t) = e_{\gamma(\alpha\alpha)}\left(\partial u_\alpha / \partial x_\alpha\right) + e_{\gamma(\beta\beta)}\left(\partial u_\beta / \partial x_\beta\right) + e_{\gamma(\alpha\beta)}\left[\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)\right] \tag{2.2}$$

In all piezoelectric crystals for which conditions (1.5) and (1.6) are satisfied, the dielectric constant tensors $\left(\hat{\varepsilon}_{ik}\right)_{3\times 3}$ are diagonal. Therefore, in expression (2.2), the third component of the electric displacement is represented only by the elastic elongations $\left(\partial u_\alpha / \partial x_\alpha\right)$, $\left(\partial u_\beta / \partial x_\beta\right)$ and the shift $\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)$ in the sagittal plane.

It is known that in any basic plane $x_\alpha 0 x_\beta$ the elastic stiffnesses $c_{\gamma\alpha} \neq 0$ and $c_{\gamma\beta} \neq 0$, as well as the elastic compliance coefficients $s_{\gamma\alpha} = (-1)^{\alpha+\gamma} \cdot \Delta c_{\alpha\gamma} / \Delta^c$ and $s_{\gamma\beta} = (-1)^{\beta+\gamma} \cdot \Delta c_{\beta\gamma} / \Delta^c$ cannot be zeros. Therefore, the existence of a non-zero axial stress $\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t)$ can lead to the axial tensions (compressions) $r_{\gamma\gamma}(x_\alpha, x_\beta, t)$ in the direction of the axis $0x_\gamma$, violating the planar deformed state.

The existence of a non-zero axial component of the electric displacement vector $D_\gamma(x_\alpha, x_\beta, t)$ along the axis $0x_\gamma$ violates the plane electric field.

From relation (2.2) it is obvious that, taking into account the arbitrariness of elastic elongations $\left(\partial u_\alpha / \partial x_\alpha\right)$, $\left(\partial u_\beta / \partial x_\beta\right)$, and elastic shear $\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)$ in the sagittal plane, the axial component of the electric displacement vector $D_\gamma(x_\alpha, x_\beta, t)$ disappears in the piezoelectric crystals in which

$$e_{\gamma(\alpha\beta)} \equiv 0, \quad e_{\gamma(\alpha\alpha)} \equiv 0, \quad e_{\gamma(\beta\beta)} \equiv 0. \tag{2.3}$$

The fulfillment of identity (2.3) means the absence of a direct piezoelectric effect in the perpendicular direction to a given sagittal plane of the piezocrystal. Then the condition for compatibility of axial elastic elongations (compressions) and polarizations of the electric field in the basal plane $x_\alpha 0 x_\beta$ relative to axial elongations (compressions) can be written as

$$c_{\gamma\alpha}\left(\partial u_\alpha / \partial x_\alpha\right) + c_{\gamma\beta}\left(\partial u_\beta / \partial x_\beta\right) + c_{\gamma(\alpha\beta)}\left[\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)\right] = \\ = -e_{\alpha(\gamma\gamma)}\left(\partial \varphi / \partial x_\alpha\right) - e_{\beta(\gamma\gamma)}\left(\partial \varphi / \partial x_\beta\right) \tag{2.4}$$

If we take into account that in all piezocrystals for which conditions (1.5) and (1.6) are satisfied, there are no constants $c_{\gamma(\alpha\beta)}$ in the elastic stiffness tensor:

$$c_{\gamma(\alpha\beta)} \equiv 0, \tag{2.5}$$

then, the compatibility ratio of elastic non-zero axial elongations (compressions) is finally written in the form

$$\left(\partial u_\alpha / \partial x_\alpha\right) = -\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)\left(\partial u_\beta / \partial x_\beta\right) - \left[e_{\alpha(\gamma\gamma)}\left(\partial\varphi/\partial x_\alpha\right) + e_{\beta(\gamma\gamma)}\left(\partial\varphi/\partial x_\beta\right)\right]\Big/ c_{\gamma\alpha} \tag{2.6}$$

If there is also no inverse piezoelectric effect in the perpendicular direction to a given sagittal plane $x_\alpha 0 x_\beta$ of the piezocrystal, when $e_{\alpha(\gamma\gamma)} \equiv 0$ and $e_{\beta(\gamma\gamma)} \equiv 0$, the compatibility condition for elastic non-zero axial elongations (compressions) is written as in the case of non-electroactive elastic anisotropic medium

$$c_{\gamma\alpha}\left(\partial u_\alpha / \partial x_\alpha\right) = -c_{\gamma\beta}\left(\partial u_\beta / \partial x_\beta\right) \tag{2.7}$$

Statement -1: Electro-elastic state of plane deformation

$$\left\{u_\alpha(x_\alpha, x_\beta, t);\quad u_\beta(x_\alpha, x_\beta, t);\quad 0;\ \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\alpha;\ \partial\varphi(x_\alpha, x_\beta, t)/\partial x_\beta;\ 0\right\} \tag{2.8}$$

in the sagittal plane $x_\alpha 0 x_\beta$ of a homogeneous piezoelectric medium induces an electroactive generalized plane stress state

$$\begin{Bmatrix} \sigma_{\alpha\alpha}(x_\alpha, x_\beta, t);\ \sigma_{\beta\beta}(x_\alpha, x_\beta, t);\ 0;\ 0;\ \ 0;\quad \sigma_{\alpha\beta}(x_\alpha, x_\beta, t) \\ D_\alpha(x_\alpha, x_\beta, t);\ \ D_\beta(x_\alpha, x_\beta, t);\ \ 0; \end{Bmatrix} \tag{2.9}$$

in piezoelectric crystals, in the generalized electroelastic tensor of which the following conditions are satisfied

$$\begin{gathered} c_{\alpha(\gamma\alpha)}{\equiv}0,\ c_{\alpha(\beta\gamma)}{\equiv}0,\ c_{\beta(\beta\gamma)}{\equiv}0,\ c_{\beta(\gamma\alpha)}{\equiv}0,\ c_{(\alpha\beta)(\gamma\alpha)}{\equiv}0,\ c_{(\alpha\beta)(\beta\gamma)}{\equiv}0,\ c_{\gamma(\alpha\beta)}{\equiv}0, \\ e_{\alpha(\gamma\alpha)}\equiv 0,\ e_{\alpha(\gamma\beta)}{\equiv}0,\ e_{\beta(\gamma\alpha)}\equiv 0,\ e_{\beta(\gamma\beta)}{\equiv}0,\ \ e_{\gamma(\alpha\beta)}\equiv 0,\ \ e_{\gamma(\alpha\alpha)}{\equiv}0,\ \ e_{\gamma(\beta\beta)}\equiv 0. \end{gathered} \tag{2.10}$$

The compatibility condition for elastic non-zero axial elongations (compressions) in the sagittal plane, in this case, has the form

$$\left(\partial u_\alpha / \partial x_\alpha\right) = -\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)\left(\partial u_\beta / \partial x_\beta\right) - \left[e_{\alpha(\gamma\gamma)}\left(\partial\varphi/\partial x_\alpha\right) + e_{\beta(\gamma\gamma)}\left(\partial\varphi/\partial x_\beta\right)\right]\Big/ c_{\gamma\alpha} \tag{2.11}$$

The electroactive generalized planar stress-strain state of type (2.8) and (2.9), taking into account conditions (2.10) and compatibility conditions of elastic axial elongations (compressions) (2.11), is represented by the nonzero components of electromechanical stresses

$$\begin{aligned} \sigma_{\alpha\alpha}(x_\alpha, x_\beta) = {} & c^*_{\alpha\alpha}\left(\partial u_\alpha / \partial x_\alpha\right) + c_{\alpha(\alpha\beta)}\left[\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)\right] + \\ & + e^*_{\alpha\alpha}\left(\partial\varphi/\partial x_\alpha\right) + e^*_{\beta\alpha}\left(\partial\varphi/\partial x_\beta\right) \end{aligned} \tag{2.12}$$

$$\begin{aligned} \sigma_{\beta\beta}(x_\alpha, x_\beta) = {} & c^*_{\beta\beta}\left(\partial u_\beta / \partial x_\beta\right) + c_{\beta(\alpha\beta)}\left[\left(\partial u_\alpha / \partial x_\beta\right) + \left(\partial u_\beta / \partial x_\alpha\right)\right] + \\ & + e^*_{\alpha\beta}\left(\partial\varphi/\partial x_\alpha\right) + e^*_{\beta\beta}\left(\partial\varphi/\partial x_\beta\right) \end{aligned} \tag{2.13}$$

$$\sigma_{\alpha\beta}(x_\alpha,x_\beta)=c^*_{(\alpha\beta)\alpha}\left(\partial u_\alpha/\partial x_\alpha\right)+c_{(\alpha\beta)(\alpha\beta)}\left[\left(\partial u_\alpha/\partial x_\beta\right)+\left(\partial u_\beta/\partial x_\alpha\right)\right]+ \\ +e^*_{\alpha(\alpha\beta)}\left(\partial\varphi/\partial x_\alpha\right)+e^*_{\beta(\alpha\beta)}\left(\partial\varphi/\partial x_\beta\right) \quad (2.14)$$

and the non-zero components of electric displacement

$$D_\alpha(x_\alpha,x_\beta)=e^{**}_{\alpha\alpha}\left(\partial u_\alpha/\partial x_\alpha\right)+e_{\alpha(\alpha\beta)}\left[\left(\partial u_\alpha/\partial x_\beta\right)+\left(\partial u_\beta/\partial x_\alpha\right)\right]- \\ -\varepsilon^*_{\alpha\alpha}\left(\partial\varphi/\partial x_\alpha\right)-\varepsilon^*_{\beta\alpha}\left(\partial\varphi/\partial x_\beta\right) \quad (2.15)$$

$$D_\beta(x_\alpha,x_\beta)=e^{**}_{\beta\beta}\left(\partial u_\alpha/\partial x_\alpha\right)+e_{\beta(\alpha\beta)}\left[\left(\partial u_\alpha/\partial x_\beta\right)+\left(\partial u_\beta/\partial x_\alpha\right)\right]- \\ -\varepsilon^*_{\beta\alpha}\left(\partial\varphi/\partial x_\alpha\right)-\varepsilon^*_{\beta\beta}\left(\partial\varphi/\partial x_\beta\right) \quad (2.16)$$

In the newly obtained material relations (2.12)÷(2.16), the physicomechanical constants corresponding to the two-dimensional electroelastic problem are indicated with asterisks (table 3).

Table 3. Reduced physicomechanical constants of electroelastic generalized plane stress-strain state

$c^*_{\alpha\alpha}$	$c_{\alpha\alpha}-c_{\alpha\beta}\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)$	$e^*_{\alpha(\alpha\beta)}$	$e_{\alpha(\alpha\beta)}-\left(c_{(\alpha\beta)\beta}/c_{\gamma\beta}\right)e_{\alpha\gamma}$
$c^*_{\beta\beta}$	$c_{\beta\beta}-c_{\beta\alpha}\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)$	$e^*_{\beta(\alpha\beta)}$	$e_{\beta(\alpha\beta)}-\left(c_{(\alpha\beta)\beta}/c_{\gamma\beta}\right)e_{\beta\gamma}$
$c^*_{(\alpha\beta)\alpha}$	$c_{(\alpha\beta)\alpha}-c_{(\alpha\beta)\beta}\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)$	$e^{**}_{\alpha\alpha}$	$e_{\alpha\alpha}-\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)e_{\alpha\beta}$
$e^*_{\alpha\alpha}$	$e_{\alpha\alpha}-\left(c_{\alpha\beta}/c_{\gamma\beta}\right)e_{\alpha\gamma}$	$e^{**}_{\beta\beta}$	$e_{\beta\beta}-\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)e_{\beta\alpha}$
$e^*_{\beta\beta}$	$e_{\beta\beta}-\left(c_{\beta\alpha}/c_{\gamma\alpha}\right)e_{\beta\gamma}$	$\varepsilon^*_{\alpha\alpha}$	$\varepsilon_{\alpha\alpha}+\left(e_{\alpha\beta}e_{\alpha\gamma}/c_{\gamma\beta}\right)$
$e^*_{\beta\alpha}$	$e_{\beta\alpha}-\left(c_{\alpha\beta}/c_{\gamma\beta}\right)e_{\beta\gamma}$	$\varepsilon^*_{\beta\beta}$	$\varepsilon_{\beta\beta}+\left(e_{\beta\alpha}e_{\beta\gamma}/c_{\gamma\alpha}\right)$
$e^*_{\alpha\beta}$	$e_{\alpha\beta}-\left(c_{\beta\alpha}/c_{\gamma\alpha}\right)e_{\alpha\gamma}$	$\varepsilon^*_{\alpha\beta}$	$\left(e_{\alpha\beta}e_{\beta\gamma}/c_{\gamma\beta}\right)$
		$\varepsilon^*_{\beta\alpha}$	$\left(e_{\beta\alpha}e_{\alpha\gamma}/c_{\gamma\alpha}\right)$

From (2.12)÷(2.16) and from Table 3 it follows that the electroactive planar stress-strain state in the sagittal plane is formulated with reduced anisotropy. The reduced anisotropy in the selected sagittal plane differs from the natural anisotropy both qualitatively and quantitatively. New piezoelectric coefficients and dielectric coefficients may appear. The electromechanical properties of the medium in the modeled material plane can also change.

Taking into account the material relations (2.12)÷(2.16) and the expression of the new constants in Table 3, the quasistatic equations of the electroelastic plane stress-strain state can be written in a single form

$$\left[c_{\alpha\alpha}-\left(c_{\alpha\beta}+c_{(\alpha\beta)(\alpha\beta)}\right)\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)\right]\left(\partial^2u_\alpha/\partial x^2_\alpha\right)+c_{(\alpha\beta)(\alpha\beta)}\left(\partial^2u_\alpha/\partial x^2_\beta\right)+$$

$$+c_{\alpha(\alpha\beta)}\left(\partial^2u_\beta/\partial x^2_\alpha\right)+\left[c_{(\alpha\beta)\beta}-\left(c_{\alpha(\alpha\beta)}+c_{(\alpha\beta)\alpha}\right)\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)\right]\left(\partial^2u_\beta/\partial x^2_\beta\right)+$$

$$+\left[e_{\alpha\alpha}-e_{\alpha\gamma}\left(c_{\alpha\beta}+c_{(\alpha\beta)(\alpha\beta)}\right)/c_{\gamma\beta}\right]\left(\partial^2\varphi/\partial x^2_\alpha\right)+\left[e_{\beta\alpha}+e_{\alpha(\alpha\beta)}\right]\left(\partial^2\varphi/\partial x_\alpha\partial x_\beta\right)-$$

$$
\begin{aligned}
&-\left[e_{\alpha\gamma}\left(c_{\alpha(\alpha\beta)}+c_{(\alpha\beta)\beta}\right)/c_{\gamma\alpha}+e_{\beta\gamma}\left(c_{\alpha\beta}+c_{(\alpha\beta)(\alpha\beta)}\right)/c_{\gamma\beta}\right]\left(\partial^2\varphi/\partial x_\alpha\partial x_\beta\right)+\\
&+\left[e_{\beta(\alpha\beta)}-2e_{\beta\gamma}\left(c_{\alpha(\alpha\beta)}/c_{\gamma\alpha}\right)\right]\left(\partial^2\varphi/\partial x_\beta^2\right)=\rho\left(\partial^2 u_\alpha/\partial t^2\right),
\end{aligned}
\tag{2.17}
$$

$$
\begin{aligned}
&\left[c^*_{(\alpha\beta)\alpha}-c_{\beta(\alpha\beta)}\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)\right]\left(\partial^2u_\alpha/\partial x_\alpha^2\right)+c_{\beta(\alpha\beta)}\left(\partial^2u_\alpha/\partial x_\beta^2\right)+\\
&+c_{(\alpha\beta)(\alpha\beta)}\left(\partial^2u_\beta/\partial x_\alpha^2\right)+\left[c_{\beta\beta}-\left(c_{\beta\alpha}+c_{(\alpha\beta)(\alpha\beta)}\right)\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)\right]\left(\partial^2u_\beta/\partial x_\beta^2\right)+\\
&+\left[e_{\alpha(\alpha\beta)}-\left(e_{\alpha\gamma}/c_{\gamma\beta}\right)\left(c_{(\alpha\beta)\beta}+c_{\beta(\alpha\beta)}+c_{(\alpha\beta)(\alpha\beta)}\right)\right]\left(\partial^2\varphi/\partial x_\alpha^2\right)+\\
&+\left[e_{\beta\beta}-\left(e_{\beta\gamma}/c_{\gamma\alpha}\right)\left(c_{\beta\alpha}+c_{\alpha(\alpha\beta)}+c_{(\alpha\beta)(\alpha\beta)}\right)\right]\left(\partial^2\varphi/\partial x_\beta^2\right)+\\
&+\left(e^*_{\alpha\beta}+e^*_{\beta(\alpha\beta)}-e_{\alpha\gamma}\left(c_{(\alpha\beta)(\alpha\beta)}/c_{\gamma\alpha}\right)-e_{\beta\gamma}\left(c_{\beta(\alpha\beta)}/c_{\gamma\beta}\right)\right)\left(\partial^2\varphi/\partial x_\alpha\partial x_\beta\right)=\rho\left(\partial^2u_\beta/\partial t^2\right)
\end{aligned}
\tag{2.18}
$$

$$
\begin{aligned}
&\left[e^{**}_{\alpha\alpha}-e_{\beta(\alpha\beta)}\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)\right]\left(\partial^2u_\alpha/\partial x_\alpha^2\right)+e_{\beta(\alpha\beta)}\left(\partial^2u_\alpha/\partial x_\beta^2\right)+\\
&+\left[e^{**}_{\beta\beta}-e_{\alpha(\alpha\beta)}\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)\right]\left(\partial^2u_\beta/\partial x_\beta^2\right)+e_{\alpha(\alpha\beta)}\left(\partial^2u_\beta/\partial x_\alpha^2\right)-\\
&-\left[\varepsilon^*_{\alpha\alpha}+\left(e_{\beta(\alpha\beta)}e_{\alpha\gamma}/c_{\gamma\beta}\right)\right]\left(\partial^2\varphi/\partial x_\alpha^2\right)-\left[\varepsilon^*_{\beta\beta}+\left(e_{\alpha(\alpha\beta)}e_{\beta\gamma}/c_{\gamma\alpha}\right)\right]\left(\partial^2\varphi/\partial x_\beta^2\right)-\\
&-\left[\varepsilon^*_{\alpha\beta}+\varepsilon^*_{\beta\alpha}+\left(e_{\alpha(\alpha\beta)}e_{\alpha\gamma}/c_{\gamma\alpha}\right)\left(e_{\beta(\alpha\beta)}e_{\beta\gamma}/c_{\gamma\beta}\right)\right]\left(\partial^2\varphi/\partial x_\alpha\partial x_\beta\right)=0.
\end{aligned}
\tag{2.19}
$$

From the deduced quasistatic equations of the plane stress-strain state (2.17)÷(2.19) it can be seen that, taking into account the compatibility conditions of axial elastic elongations (compressions) (2.11):

– the equations do not contain mixed derivatives of the components of the elastic displacement,

– non-diagonal reduced coefficients $\varepsilon^*_{\alpha\beta}$ and $\varepsilon^*_{\beta\alpha}$ of permittivity may appear in the equations.

In non-piezoelectric anisotropic media, for which $e_{j(mn)}\equiv 0$ the necessary and sufficient conditions for the possible separation of the plane stress-strain state are the conditions of the first row (2.10) and the compatibility ratio of axial elastic elongations (compressions) (2.7).

In this case, in Table 3 there are only expressions of reduced elastic stiffnesses. The material relation (2.12)÷(2.14) and equations of electroelasticity (2.17) ÷ (2.19) are simplified.

$$
\begin{aligned}
&\sigma_{\alpha\alpha}(x_\alpha,x_\beta)=c^*_{\alpha\alpha}\left(\partial u_\alpha/\partial x_\alpha\right)+c_{\alpha(\alpha\beta)}\left[\left(\partial u_\alpha/\partial x_\beta\right)+\left(\partial u_\beta/\partial x_\alpha\right)\right],\\
&\sigma_{\beta\beta}(x_\alpha,x_\beta)=c^*_{\beta\beta}\left(\partial u_\beta/\partial x_\beta\right)+c_{\beta(\alpha\beta)}\left[\left(\partial u_\alpha/\partial x_\beta\right)+\left(\partial u_\beta/\partial x_\alpha\right)\right],\\
&\sigma_{\alpha\beta}(x_\alpha,x_\beta)=c^*_{(\alpha\beta)\alpha}\left(\partial u_\alpha/\partial x_\alpha\right)+c_{(\alpha\beta)(\alpha\beta)}\left[\left(\partial u_\alpha/\partial x_\beta\right)+\left(\partial u_\beta/\partial x_\alpha\right)\right].
\end{aligned}
\tag{2.20}
$$

The equations of electro elasticity (2.17) and (2.18) take the form

$$\left[c^{*}_{\alpha\alpha} - c_{(\alpha\beta)(\alpha\beta)}\left(c_{\gamma\alpha}/c_{\gamma\beta}\right)\right]\left(\partial^2 u_\alpha/\partial x_\alpha^2\right) + c_{(\alpha\beta)(\alpha\beta)}\left(\partial^2 u_\alpha/\partial x_\beta^2\right) + \\ + c_{\alpha(\alpha\beta)}\left(\partial^2 u_\beta/\partial x_\alpha^2\right) + c_{(\alpha\beta)\beta}\left(\partial^2 u_\beta/\partial x_\beta^2\right) = \rho\left(\partial^2 u_\alpha/\partial t^2\right) \tag{2.21}$$

$$c_{(\alpha\beta)\alpha}\left(\partial^2 u_\alpha/\partial x_\alpha^2\right) + c_{\beta(\alpha\beta)}\left(\partial^2 u_\alpha/\partial x_\beta^2\right) + c_{(\alpha\beta)(\alpha\beta)}\left(\partial^2 u_\beta/\partial x_\alpha^2\right) + \\ + \left[c^{*}_{\beta\beta} - c_{(\alpha\beta)(\alpha\beta)}\left(c_{\gamma\beta}/c_{\gamma\alpha}\right)\right]\left(\partial^2 u_\beta/\partial x_\beta^2\right) = \rho\left(\partial^2 u_\beta/\partial t^2\right) \tag{2.22}$$

In the two other sagittal planes of piezoelectric crystals $x_\gamma 0 x_\alpha$ and $x_\beta 0 x_\gamma$, the conditions for the existence of an electroelastic generalized plane stress-strain state, the material relations of non-zero electromechanical components, the reduced electromechanical coefficients, as well as the quasistatic equations of electroelasticity for the corresponding plane stress-strain states of the type (2.8) and (2.9)

$$\begin{Bmatrix} u_\alpha(x_\gamma,x_\alpha,t);\ 0;\ u_\gamma(x_\gamma,x_\alpha,t);\ \partial\varphi(x_\gamma,x_\alpha,t)/\partial x_\alpha;\ 0;\ \partial\varphi(x_\gamma,x_\alpha,t)/\partial x_\gamma; \\ \sigma_{\alpha\alpha}(x_\gamma,x_\alpha,t);0;\ \sigma_{\gamma\gamma}(x_\gamma,x_\alpha,t);\ 0;\ \sigma_{\alpha\gamma}(x_\gamma,x_\alpha,t);\ 0; D_\alpha(x_\gamma,x_\alpha,t);\ 0;\ D_\gamma(x_\gamma,x_\alpha,t) \end{Bmatrix} \tag{2.23}$$

and

$$\begin{Bmatrix} 0;\ u_\beta(x_\beta,x_\gamma,t);\ \ u_\gamma(x_\beta,x_\gamma,t);\ \ 0;\ \ 0;\ \ \partial\varphi(x_\beta,x_\gamma,t)/\partial x_\beta;\ \ \partial\varphi(x_\beta,x_\gamma,t)/\partial x_\gamma \\ 0;\ \sigma_{\beta\beta}(x_\beta,x_\gamma,t);\ \sigma_{\gamma\gamma}(x_\beta,x_\gamma,t);\ \sigma_{\beta\gamma}(x_\beta,x_\gamma,t);\ 0;\ 0;\ 0;\ D_\beta(x_\beta,x_\gamma,t);\ D_\gamma(x_\beta,x_\gamma,t) \end{Bmatrix} \tag{2.24}$$

are obtained by rotating silent indexes $\{\gamma,\alpha,\beta\} \rightleftarrows \{\alpha,\beta,\gamma\} \rightleftarrows \{\beta,\gamma,\alpha\}$ in relations (2.1)÷(2.19), respectively.

2.1. Material relations and quasistatic equations of an electroelastic generalized plane stress-strain state in the sagittal plane $x_1 0 x_2$ **.**

In this sagittal plane of piezocrystalline textures, the conditions for the existence of an electroelastic plane stress-strain state (2.10) take the form

$$c_{14} \equiv 0,\ c_{15} \equiv 0,\ c_{24} \equiv 0,\ c_{25} \equiv 0,\ c_{64} \equiv 0,\ c_{65} \equiv 0,\ c_{36} \equiv 0, \\ e_{14} \equiv 0,\ e_{15} \equiv 0,\ e_{24} \equiv 0,\ e_{25} \equiv 0,\ e_{36} \equiv 0,\ e_{31} \equiv 0,\ e_{32} \equiv 0. \tag{2.25}$$

Only the generalized tensors of electroelasticity of piezoelectric media from classes $\bar{6}$, $\bar{6}m2$ of hexagonal symmetry correspond to such restrictions.

Taking into account the compatibility conditions of elastic non-zero axial elongations (compressions)

$$\left(\partial u_1/\partial x_1\right) = -\left(c_{23}/c_{31}\right)\left(\partial u_2/\partial x_2\right) - \left(e_{13}/c_{31}\right)\left(\partial\varphi/\partial x_1\right) - \left(e_{23}/c_{31}\right)\left(\partial\varphi/\partial x_2\right), \tag{2.26}$$

the material relations of non-zero electromechanical components in these media can be written in the general form

$$\sigma_{11} = \left(c_{11} - c_{12}\right)\left(\partial u_1/\partial x_1\right) + e^{*}_{11}\left(\partial\varphi/\partial x_1\right) + e^{*}_{21}\left(\partial\varphi/\partial x_2\right), \\ \sigma_{22} = \left(c_{11} - c_{12}\right)\left(\partial u_2/\partial x_2\right) + e^{*}_{12}\left(\partial\varphi/\partial x_1\right) + e^{*}_{22}\left(\partial\varphi/\partial x_2\right), \\ \sigma_{12} = c^{*}_{44}\left[\left(\partial u_1/\partial x_2\right) + \left(\partial u_2/\partial x_1\right)\right] + e^{*}_{16}\left(\partial\varphi/\partial x_1\right) + e^{*}_{26}\left(\partial\varphi/\partial x_2\right). \tag{2.27}$$

$$D_1=e_{11}^*(\partial u_1/\partial x_1)+e_{12}^*(\partial u_2/\partial x_2)+e_{16}^*\left[(\partial u_1/\partial x_2)+(\partial u_2/\partial x_1)\right]-\varepsilon_{11}(\partial\varphi/\partial x_1),$$
$$D_2=e_{21}^*(\partial u_1/\partial x_1)+e_{22}^*(\partial u_2/\partial x_2)+e_{26}^*\left[(\partial u_1/\partial x_2)+(\partial u_2/\partial x_1)\right]-\varepsilon_{11}(\partial\varphi/\partial x_2). \quad (2.28)$$

In the selected sagittal plane of these classes of media, it is also convenient to write down the quasistatic equations of electro elasticity (2.17)÷(2.19) in general form

$$c_{66}^*\left(\partial^2 u_1/\partial x_1^2\right)+c_{66}^*\left(\partial^2 u_1/\partial x_2^2\right)-\rho\left(\partial^2 u_1/\partial t^2\right)=$$
$$=-e_{11}^*\left(\partial^2\varphi/\partial x_1^2\right)-\left(e_{21}^*+e_{16}^*\right)\left(\partial^2\varphi/\partial x_1\partial x_2\right)-e_{26}^*\left(\partial^2\varphi/\partial x_2^2\right),$$
$$c_{66}^*\left(\partial^2 u_2/\partial x_1^2\right)+c_{66}^*\left(\partial^2 u_2/\partial x_2^2\right)-\rho\left(\partial^2 u_2/\partial t^2\right)=$$
$$=-e_{16}^*\left(\partial^2\varphi/\partial x_1^2\right)-\left(e_{26}^*+e_{12}^*\right)\left(\partial^2\varphi/\partial x_1\partial x_2\right)-e_{22}^*\left(\partial^2\varphi/\partial x_2^2\right), \quad (2.29)$$
$$\left(e_{11}^*-e_{26}^*-e_{12}^*\right)\left(\partial^2 u_1/\partial x_1^2\right)+e_{26}^*\left(\partial^2 u_1/\partial x_2^2\right)+e_{16}^*\left(\partial^2 u_2/\partial x_1^2\right)+$$
$$+\left(e_{22}^*-e_{16}^*-e_{21}^*\right)\left(\partial^2 u_2/\partial x_2^2\right)=\varepsilon_{11}\left(\partial^2\varphi/\partial x_1^2\right)+\varepsilon_{11}\left(\partial^2\varphi/\partial x_2^2\right).$$

The coefficients with asterisks in the relations (2.23)÷(2.25) are given in Table 4.

Table 4. Values of reduced coefficients with asterisks of the simulated two-dimensional problem in the sagittal plane ***x₁0x₂***

		c_{66}^*	e_{11}^*	e_{12}^*	e_{21}^*	e_{22}^*	e_{16}^*	e_{26}^*
Classes	$\bar{6}$	$(c_{11}-c_{12})/2$	e_{11}	$-e_{11}$	$-e_{22}$	e_{22}	$-e_{22}$	$-e_{11}$
	$\bar{6}m2$	$(c_{11}-c_{12})/2$	e_{11}	$-e_{11}$	0	0	0	e_{11}

In view of Table 4, it is obvious that the material relations of the non-zero electromechanical components (2.23) and (2.24), as well as the equations of electroelasticity (2.25) for simulated two-dimensional problems in the sagittal plane $x_1 0 x_2$ of the media of class $\bar{6}m2$ have more simplified forms than for the media of class $\bar{6}$. Despite this, from the derived material relations and the equations of electroelasticity it follows that in the media of class $\bar{6}m2$, axial elongations (compression) $\partial u_1/\partial x_1=-\partial u_2/\partial x_2$ lead to electric polarization with induction $D_1(x_1,x_2,t)=e_{11}\left(\partial u_1/\partial x_1\right)$, and the elastic shear $\left(\partial u_1/\partial x_2\right)+\left(\partial u_2/\partial x_1\right)$ leads to electric polarization with induction $D_2(x_1,x_2,t)=e_{11}\left[\left(\partial u_1/\partial x_2\right)+\left(\partial u_2/\partial x_1\right)\right]$.

2.2. Material relations and quasistatic equations of the electroelastic generalized plane stress-strain state in the sagittal plane $x_2 0 x_3$.

In this sagittal plane of piezocrystalline textures, the conditions for the existence of an electroelastic plane stress-strain state (2.10) take the form

$$c_{25}\equiv 0,\ c_{26}\equiv 0,\ c_{35}\equiv 0,\ c_{36}\equiv 0,\ c_{45}\equiv 0,\ c_{46}\equiv 0,\ c_{14}\equiv 0,$$
$$e_{25}\equiv 0,\ e_{26}\equiv 0,\ e_{35}\equiv 0,\ e_{36}\equiv 0,\ e_{14}\equiv 0,\ e_{12}\equiv 0,\ e_{13}\equiv 0. \quad (2.30)$$

Such restrictions on the anisotropy of the medium correspond to the structure of the generalized tensors of electroelasticity of piezoelectric media, only from the classes $6mm$ of hexagonal, $4mm$ of tetragonal and $mm2$ of rhombic symmetries.

Taking into account the compatibility condition of elastic non-zero axial elongations (compressions)

$$(\partial u_2/\partial x_2) = -(c_{13}/c_{12})(\partial u_3/\partial x_3) - (e_{21}/c_{12})(\partial \varphi/\partial x_2) - (e_{31}/c_{12})(\partial \varphi/\partial x_3) \quad (2.31)$$

the material relations of non-zero electromechanical components in these media can be written in the general form

$$\begin{aligned}
\sigma_{22} &= c_{11}(\partial u_2/\partial x_2) + c_{13}(\partial u_3/\partial x_3) + e_{32}^*(\partial \varphi/\partial x_3),\\
\sigma_{33} &= c_{13}(\partial u_2/\partial x_2) + c_{33}(\partial u_3/\partial x_3) + e_{33}^*(\partial \varphi/\partial x_3),\\
\sigma_{23} &= c_{44}((\partial u_3/\partial x_2) + (\partial u_2/\partial x_3)) + e_{24}^*(\partial \varphi/\partial x_2).\\
D_2 &= e_{24}^*((\partial u_3/\partial x_2) + (\partial u_2/\partial x_3)) - \varepsilon_{22}^*(\partial \varphi/\partial x_2),\\
D_3 &= e_{32}^*(\partial u_2/\partial x_2) + e_{33}^*(\partial u_3/\partial x_3) - \varepsilon_{33}^*(\partial \varphi/\partial x_3).
\end{aligned} \quad (2.32)$$

In the selected sagittal plane of these classes of media, it is also convenient to write down the quasistatic equations of electro elasticity (2.17)÷(2.19) in general form

$$\begin{aligned}
&(c_{11} - c_{12}^*)(\partial^2 u_2/\partial x_2^2) + c_{44}^*(\partial^2 u_2/\partial x_3^2) - \rho(\partial^2 u_2/\partial t^2) = -(e_{32}^* + e_{24}^* - e_{31}^*)(\partial^2 \varphi/\partial x_2 \partial x_3),\\
&(c_{33} - c_{13}^*)(\partial^2 u_3/\partial x_3^2) + c_{44}^*(\partial^2 u_3/\partial x_2^2) - \rho(\partial^2 u_3/\partial t^2) =\\
&= -e_{24}^*(\partial^2 \varphi/\partial x_2^2) - (e_{33}^* - e_{31}^*)(\partial^2 \varphi/\partial x_3^2),\\
&\varepsilon_{22}^*(\partial^2 \varphi/\partial x_2^2) + \left[\varepsilon_{33} - e_{31}(e_{24}^* + e_{32}^*)/c_{12}\right](\partial^2 \varphi/\partial x_3^2) =\\
&= e_{24}^*(\partial^2 u_3/\partial x_2^2) + (e_{33}^* - (e_{24}^* + e_{32}^*)\vartheta_{23})(\partial^2 u_3/\partial x_3^2).
\end{aligned} \quad (2.33)$$

The coefficients with asterisks in the relations (2.32)÷(2.33) are given in Table 5.

Table 5. Values of reduced coefficients with asterisks of a simulated two-dimensional problem in the sagittal plane $x_2 0 x_3$

		c_{13}^*	c_{12}^*	e_{31}^*	e_{32}^*	e_{24}^*	ε_{22}^*
Classes	$6mm$	$(c_{13}+c_{44})$	$(c_{12}c_{13}^*)/c_{13}$	$e_{31}(c_{13}^*/c_{12})$	e_{31}	e_{15}	ε_{11}
	$4mm$	$(c_{13}+c_{44})$	$(c_{12}c_{13}^*)/c_{13}$	$e_{31}(c_{13}^*/c_{12})$	e_{31}	e_{15}	ε_{11}
	$mm2$	$(c_{13}+c_{44})$	$(c_{12}c_{13}^*)/c_{13}$	$e_{31}(c_{13}^*/c_{12})$	e_{32}	e_{24}	ε_{22}

In view of Table 5, it is obvious that the material relations of the non-zero electromechanical components (2.32) and the electroelasticity equations (2.33) of the simulated two-dimensional problems for the classes $6mm$ of hexagonal and $4mm$ of tetragonal symmetries in the sagittal plane $x_2 0 x_3$ coincide.

From the derived material relations and equations of electroelasticity it also follows that axial elongations (compression) in these media lead to the electric polarization with induction $D_3(x_2,x_3,t)$, and the elastic shear $\left(\partial u_3/\partial x_2\right)+\left(\partial u_2/\partial x_3\right)$ leads to the electric polarization with induction $D_2(x_2,x_3,t)$. The equations also include the compatibility coefficient of axial elongations (compressions) $\vartheta_{23}=\left(c_{13}/c_{12}\right)$ (Poisson's ratio).

2.3. Material relations and quasistatic equations of the electroelastic generalized plane stress-strain state in the sagittal plane $x_3 0 x_1$.

In this sagittal plane of piezocrystalline textures, the conditions for the existence of an electroelastic plane stress-strain state (2.10) take the form

$$\begin{gathered}c_{14}\equiv 0,\ c_{16}\equiv 0,\ c_{34}\equiv 0,\ c_{36}\equiv 0,\ c_{54}\equiv 0,\ c_{56}\equiv 0,\ c_{25}\equiv 0,\\ e_{14}\equiv 0,\ e_{16}\equiv 0,\ e_{34}\equiv 0,\ e_{36}\equiv 0,\ e_{25}\equiv 0,\ e_{23}\equiv 0,\ e_{21}\equiv 0.\end{gathered} \tag{2.34}$$

Such restrictions on the anisotropy of the medium correspond to the structures of the generalized tensors of electroelasticity of piezoelectric media, only from the classes *6mm* and $\bar{6}m2$ of hexagonal, *4mm* and *422* of tetragonal and *mm*2 of rhombic symmetries.

Taking into account the compatibility condition of elastic non-zero axial elongations (compressions)

$$\left(\partial u_1/\partial x_1\right)=-\left(c_{23}/c_{21}\right)\left(\partial u_3/\partial x_3\right)-\left(e_{12}/c_{21}\right)\left(\partial\varphi/\partial x_1\right)-\left(e_{32}/c_{21}\right)\left(\partial\varphi/\partial x_3\right) \tag{2.35}$$

the material relations of non-zero electromechanical components in these media can be written in the general form:

$$\begin{aligned}&\sigma_{11}=c_{11}\left(\partial u_1/\partial x_1\right)+c_{13}\left(\partial u_3/\partial x_3\right)+e_{11}^{*}\left(\partial\varphi/\partial x_1\right)+e_{31}^{*}\left(\partial\varphi/\partial x_3\right),\\ &\sigma_{33}=c_{13}\left(\partial u_1/\partial x_1\right)+c_{33}\left(\partial u_3/\partial x_3\right)+e_{33}^{*}\left(\partial\varphi/\partial x_3\right),\\ &\sigma_{31}=c_{55}^{*}\left[\left(\partial u_3/\partial x_1\right)+\left(\partial u_1/\partial x_3\right)\right]+e_{15}^{*}\left(\partial\varphi/\partial x_1\right).\\ &D_1=e_{11}^{*}\left(\partial u_1/\partial x_1\right)+e_{15}^{*}\left[\left(\partial u_3/\partial x_1\right)+\left(\partial u_1/\partial x_3\right)\right]-\varepsilon_{11}\left(\partial\varphi/\partial x_1\right),\\ &D_3=e_{31}^{*}\left(\partial u_1/\partial x_1\right)+e_{33}^{*}\left(\partial u_3/\partial x_3\right)-\varepsilon_{33}\left(\partial\varphi/\partial x_3\right).\end{aligned} \tag{2.36}$$

In the selected sagittal plane of these classes of media, it is also convenient to write down the quasistatic equations of electro elasticity (2.17)÷(2.19) in general form

$$\begin{aligned}&\left[c_{11}-c_{12}\left(c_{13}+c_{55}^{*}\right)/c_{13}\right]\left(\partial^2 u_1/\partial x_1^2\right)+c_{55}^{*}\left(\partial^2 u_1/\partial x_3^2\right)-\rho\left(\partial^2 u_1/\partial t^2\right)=\\ &=\left[e_{11}^{*}-e_{12}^{*}\left(c_{13}+c_{55}^{*}\right)/c_{23}^{*}\right]\left(\partial^2\varphi/\partial x_1^2\right)-\left[e_{15}^{*}-e_{31}^{*}\left(c_{55}^{*}/c_{13}\right)\right]\left(\partial^2\varphi/\partial x_1\partial x_3\right),\\ &c_{55}^{*}\left(\partial^2 u_3/\partial x_1^2\right)+\left[c_{33}-c_{23}^{*}\left(c_{13}+c_{55}^{*}\right)/c_{12}\right]\left(\partial^2 u_3/\partial x_3^2\right)+\\ &+e_{15}^{*}\left(\partial^2\varphi/\partial x_1^2\right)+\left[e_{33}^{*}-e_{32}^{*}\left(c_{13}+c_{55}^{*}\right)/c_{12}\right]\left(\partial^2\varphi/\partial x_3^2\right)+\\ &+\left[e_{12}^{*}\left(c_{13}+c_{55}^{*}\right)/c_{12}\right]\left(\partial^2\varphi/\partial x_1\partial x_3\right)=\rho\left(\partial^2 u_3/\partial t^2\right),\end{aligned} \tag{2.37}$$

$$\varepsilon_{11}\left(\partial^2\varphi/\partial x_1^2\right)+\left(\varepsilon_{33}-e_{31}^*\left(e_{15}^*+e_{31}^*\right)/c_{12}\right)\left(\partial^2\varphi/\partial x_3^2\right)-e_{11}^*\left(e_{15}^*+e_{31}^*\right)/c_{12}\left(\partial^2\varphi/\partial x_1\partial x_3\right)$$
$$=e_{11}^*\left(\partial^2 u_1/\partial x_1^2\right)+e_{15}^*\left(\partial^2 u_3/\partial x_1^2\right)+\left(e_{33}^*-\left(e_{15}^*+e_{31}^*\right)\left(c_{13}/c_{12}\right)\right)\left(\partial^2 u_3/\partial x_3^2\right).$$

The coefficients with asterisks in the ratios (2.36) and (2.37) are given in Table 6.

From the material relations of non-zero electromechanical components (2.36) and electroelastic equations (2.37), taking into account Table 5 it follows that in the sagittal plane $x_3 0 x_1$ of piezoelectric classes $\bar{6}m2$ of hexagonal and 422 of tetragonal symmetry classes, direct and inverse piezoelectric effects are formed by axial polarization $\partial\varphi/\partial x_1$.

Table 6. Values of the reduced coefficients with asterisks of the simulated two-dimensional problem in the sagittal plane $x_2 0 x_3$

		c_{23}^*	c_{55}^*	e_{11}^*	e_{12}^*	e_{15}^*	e_{31}^*	e_{32}^*	e_{33}^*
Chryslographic classes	$6mm$	c_{13}	c_{44}	0	0	e_{15}	e_{31}	e_{31}	e_{33}
	$\bar{6}m2$	c_{13}	c_{44}	e_{11}	$-e_{11}$	0	0	0	0
	$4mm$	c_{13}	c_{44}	0	0	e_{15}	e_{31}	e_{31}	e_{33}
	422	c_{13}	c_{44}	0	0	e_{15}	0	0	0
	$mm2$	c_{13}	c_{55}	0	0	e_{15}	e_{31}	e_{32}	e_{33}

In media with the $\bar{6}m2$ hexagonal symmetry class texture, the axial polarization $\partial\varphi/\partial x_1$ results in axial elongations only. In media with the 422 tetragonal symmetry class texture, the axial polarization $\partial\varphi/\partial x_1$ leads only to elastic shear $\left(\partial u_3/\partial x_1\right)+\left(\partial u_1/\partial x_3\right)$ in the plane $x_3 0 x_1$.

3. Anti-flat electroactive stress-strain state in homogeneous piezoelectric textures

When the conditions for the absence of two components of elastic displacement $u_\alpha(x_\alpha,x_\beta,t)\equiv 0$ and $u_\beta(x_\alpha,x_\beta,t)\equiv 0$ and derivatives of all other characteristics of the electroactive stress-strain field $\partial[\cdot]/\partial x_\gamma \equiv 0$ are accepted only, the main relations of electroactive antiplane deformation

$$\left\{0;\;\; 0;\;\; u_\gamma(x_\alpha,x_\beta,t);\; E_\alpha(x_\alpha,x_\beta,t);\; E_\beta(x_\alpha,x_\beta,t);\; 0\right\} \tag{3.1}$$

in the sagittal plane $x_\alpha 0 x_\beta$ of an anisotropic piezoelectric medium are obtained from the equations of electroelasticity (1.1) and material relations (1.2), taking into account the structure of the tensor (1.3). Taking into account only the corresponding conditions (1.5) and (1.7), it follows that in this case, along with the nonzero pure shear stresses $\sigma_{\gamma\alpha}(x_\alpha,x_\beta,t)$, $\sigma_{\beta\gamma}(x_\alpha,x_\beta,t)$ characteristic to antiplanar stress state and nonzero components of electric displacement $D_\alpha(x_\alpha,x_\beta,t)$ and $D_\beta(x_\alpha,x_\beta,t)$

$$\sigma_{\beta\gamma}(x_\alpha, x_\beta, t) = \\ = c_{(\beta\gamma)(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + c_{(\gamma\beta)(\beta\gamma)}\left(\partial u_\gamma / \partial x_\beta\right) + e_{\alpha(\beta\gamma)}\left(\partial\varphi / \partial x_\alpha\right) + e_{\beta(\beta\gamma)}\left(\partial\varphi / \partial x_\beta\right) \quad (3.2)$$

$$\sigma_{\gamma\alpha}(x_\alpha, x_\beta, t) = \\ = c_{(\alpha\gamma)(\beta\gamma)}\left(\partial u_\gamma / \partial x_\beta\right) + c_{(\alpha\beta)(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + e_{\alpha(\alpha\gamma)}\left(\partial\varphi / \partial x_\alpha\right) + e_{\beta(\alpha\gamma)}\left(\partial\varphi / \partial x_\beta\right) \quad (3.3)$$

$$D_\alpha(x_\alpha, x_\beta, t) = e_{\alpha(\gamma\alpha)}\left(\partial u_\gamma / \partial x_\alpha\right) + e_{\alpha(\beta\gamma)}\left(\partial u_\gamma / \partial x_\beta\right) - \varepsilon_{\alpha\alpha}\left(\partial\varphi / \partial x_\alpha\right) \quad (3.4)$$

$$D_\beta(x_\alpha, x_\beta, t) = e_{\beta(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + e_{\beta(\gamma\beta)}\left(\partial u_\gamma / \partial x_\beta\right) - \varepsilon_{\beta\beta}\left(\partial\varphi / \partial x_\beta\right) \quad (3.5)$$

axial mechanical stress $\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t)$ and the third component of the electric displacement vector $D_\gamma(x_\alpha, x_\beta, t)$ also occur

$$\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t) = \\ = c_{\gamma(\gamma\beta)}\left(\partial u_\gamma / \partial x_\beta\right) + c_{\gamma(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + e_{\alpha(\gamma\gamma)}\left(\partial\varphi / \partial x_\alpha\right) + e_{\beta(\gamma\gamma)}\left(\partial\varphi / \partial x_\beta\right) \quad (3.6)$$

$$D_\gamma(x_\alpha, x_\beta, t) = e_{\gamma(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + e_{\gamma(\beta\gamma)}\left(\partial u_\gamma / \partial x_\beta\right) \quad (3.7)$$

As in the case of an electroactive plane stress-strain state, the existence of axial stress $\sigma_{\gamma\gamma}(x_\alpha, x_\beta, t)$ can lead to axial tensions (compressions) disrupting the anti-plane strain state. The existence of the axial component of the electric displacement vector $D_\gamma(x_\alpha, x_\beta, t)$ can lead to axial polarization, violating the plane electric field.

To exclude the violation of the antiplanar stress–strain state of the sagittal plane, it is necessary, similarly to the hypothesis that there are no pressure of the thin layers (sagittal planes $x_\alpha 0 x_\beta = \text{const}$) of the continuum on each other, to accept the compatibility of elastic shears and the third component of the axial polarization of the electric field.

The compatibility condition is obtained from the condition for the existence of nontrivial solutions of the system of algebraic equations

$$\begin{cases} c_{\gamma(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + c_{\gamma(\gamma\beta)}\left(\partial u_\gamma / \partial x_\beta\right) + e_{\alpha(\gamma\gamma)}\left(\partial\varphi / \partial x_\alpha\right) + e_{\beta(\gamma\gamma)}\left(\partial\varphi / \partial x_\beta\right) = 0 \\ e_{\gamma(\alpha\gamma)}\left(\partial u_\gamma / \partial x_\alpha\right) + e_{\gamma(\beta\gamma)}\left(\partial u_\gamma / \partial x_\beta\right) = 0 \end{cases} \quad (3.8)$$

From (3.8) it follows that nonzero elastic shifts $\left(\partial u_\gamma / \partial x_\alpha\right)$ and $\left(\partial u_\gamma / \partial x_\beta\right)$ are possible in piezoelectrics, in the electroelastic tensor (1.3) of which there are no coefficients of piezoelectric constants $e_{\gamma(\alpha\gamma)}$ and $e_{\gamma(\beta\gamma)}$

$$e_{\gamma(\alpha\gamma)} \equiv 0, \qquad e_{\gamma(\beta\gamma)} \equiv 0. \quad (3.9)$$

In contrast to the case of plane deformation, included in the first equation of system (3.8), the elastic stiffnesses $c_{\gamma(\alpha\gamma)}$ and $c_{\gamma(\gamma\beta)}$ are not imperatively nonzero. Therefore, the linear relation with respect to arbitrary non-zero elastic shifts $\left(\partial u_{\gamma}/\partial x_{\alpha}\right)$, $\left(\partial u_{\gamma}/\partial x_{\beta}\right)$, and electric field strengths $\left(\partial\varphi/\partial x_{\alpha}\right)$ and $\left(\partial\varphi/\partial x_{\beta}\right)$, is equal to zero at zero coefficients

$$c_{\gamma(\alpha\gamma)} \equiv 0, \quad c_{\gamma(\gamma\beta)} \equiv 0, \quad e_{\alpha(\gamma\gamma)} \equiv 0, \quad e_{\beta(\gamma\gamma)} \equiv 0. \tag{3.10}$$

In the cases of restrictions (3.9) and (3.10) on the anisotropy of the medium, the compatibility condition for nonzero elastic shifts and the electric field under which there are no pressures of the thin layers (sagittal planes $x_{\alpha}0x_{\beta} = \mathrm{const}$) of the continuum on each other is automatically satisfied.

If in the electroelastic tensor (1.3) of the piezoelectrics at least one of the elastic stiffnesses $c_{\gamma(\alpha\gamma)}$ and $c_{\gamma(\gamma\beta)}$, or the piezoelectric constants $e_{\alpha(\gamma\gamma)}$, $e_{\beta(\gamma\gamma)}$, $e_{\gamma(\alpha\gamma)}$ and $e_{\gamma(\beta\gamma)}$ is nonzero, then the elastic displacements $\left(\partial u_{\gamma}/\partial x_{\alpha}\right)$, $\left(\partial u_{\gamma}/\partial x_{\beta}\right)$ and the electric field strength $\left(\partial\varphi/\partial x_{\alpha}\right)$ и $\left(\partial\varphi/\partial x_{\beta}\right)$ will not correspond to the non-zero antiplane stress-strain state.

In the absence of the piezoelectric effect in the medium, the compatibility condition of elastic shifts becomes the hypothesis that there is no pressure of the thin layers (sagittal planes $x_{\alpha}0x_{\beta} = \mathrm{const}$) of the continuum on each other, when

$$c_{\gamma(\alpha\gamma)} \equiv 0, \quad c_{\gamma(\gamma\beta)} \equiv 0. \tag{3.11}$$

Statement-2: State of electro-elastic anti-flat deformation

$$\left\{0;\ 0;\ u_{\gamma}(x_{\alpha},x_{\beta},t);\ \partial\varphi(x_{\alpha},x_{\beta},t)/\partial x_{\alpha};\ \partial\varphi(x_{\alpha},x_{\beta},t)/\partial x_{\beta};\ 0\right\} \tag{3.12}$$

in the sagittal plane $x_{\alpha}0x_{\beta}$ of the homogeneous piezoelectric medium induces an electroactive generalized antiplane stress state

$$\left\{0;\ 0;\ 0;\ \sigma_{\gamma\beta}(x_{\alpha},x_{\beta},t);\ \sigma_{\gamma\alpha}(x_{\alpha},x_{\beta},t);\ 0;\ D_{\alpha}(x_{\alpha},x_{\beta},t);\ D_{\beta}(x_{\alpha},x_{\beta},t);\ 0\right\} \tag{3.13}$$

in piezoelectric crystals, in the generalized electroelastic tensor of which the following conditions are satisfied

$$\begin{gathered} c_{\alpha(\gamma\alpha)} \equiv 0,\ c_{\alpha(\beta\gamma)} \equiv 0,\ c_{\beta(\beta\gamma)} \equiv 0,\ c_{\beta(\gamma\alpha)} \equiv 0,\ c_{(\alpha\beta)(\gamma\alpha)} \equiv 0,\ c_{(\alpha\beta)(\beta\gamma)} \equiv 0, \\ e_{\alpha\alpha} \equiv 0,\ \ e_{\alpha\beta} \equiv 0,\ \ e_{\alpha(\alpha\beta)} \equiv 0,\ \ e_{\beta\alpha} \equiv 0,\ \ e_{\beta\beta} \equiv 0,\ \ e_{\beta(\beta\alpha)} \equiv 0, \\ c_{\gamma(\alpha\gamma)} \equiv 0,\ c_{\gamma(\gamma\beta)} \equiv 0,\ e_{\gamma(\alpha\gamma)} \equiv 0,\ e_{\gamma(\beta\gamma)} \equiv 0,\ e_{\alpha(\gamma\gamma)} \equiv 0,\ e_{\beta(\gamma\gamma)} \equiv 0. \end{gathered} \tag{3.14}$$

Compatibility conditions for elastic non-zero shifts and electric field strength in the sagittal plane, in this case, are performed automatically.

The electroactive generalized antiplane stress-strain state of the type (3.12) and (3.13), taking into account conditions (3.14), is represented by the nonzero components of electromechanical stresses

$$\sigma_{\beta\gamma}(x_{\alpha},x_{\beta},t) = c_{(\gamma\beta)(\beta\gamma)}\left(\partial u_{\gamma}/\partial x_{\beta}\right) + e_{\alpha(\gamma\beta)}\left(\partial\varphi/\partial x_{\alpha}\right) + e_{\beta(\gamma\beta)}\left(\partial\varphi/\partial x_{\beta}\right) \tag{3.15}$$

$$\sigma_{\gamma\alpha}(x_{\alpha},x_{\beta},t) = c_{(\alpha\gamma)(\alpha\gamma)}\left(\partial u_{\gamma}/\partial x_{\alpha}\right) + e_{\alpha(\gamma\alpha)}\left(\partial\varphi/\partial x_{\alpha}\right) + e_{\beta(\gamma\alpha)}\left(\partial\varphi/\partial x_{\beta}\right) \tag{3.16}$$

and nonzero components of the displacement of the electric field

$$D_\alpha(x_\alpha, x_\beta, t) = e_{\alpha(\gamma\beta)}\left(\partial u_\gamma / \partial x_\beta\right) + e_{\alpha(\gamma\alpha)}\left(\partial u_\gamma / \partial x_\alpha\right) - \varepsilon_{\alpha\alpha}\left(\partial\varphi / \partial x_\alpha\right) \quad (3.17)$$

$$D_\beta(x_\alpha, x_\beta, t) = e_{\beta(\gamma\beta)}\left(\partial u_\gamma / \partial x_\beta\right) + e_{\beta(\gamma\alpha)}\left(\partial u_\gamma / \partial x_\alpha\right) - \varepsilon_{\beta\beta}\left(\partial\varphi / \partial x_\beta\right) \quad (3.18)$$

The form of material relations (3.15)÷(3.18) allows us to formally preserve the general form of the system of quasistatic equations of the electroactive antiplane stress-strain state

$$c_{(\alpha\gamma)(\alpha\gamma)}\left(\partial^2 u_\gamma / \partial x_\alpha^2\right) + c_{(\gamma\beta)(\beta\gamma)}\left(\partial^2 u_\gamma / \partial x_\beta^2\right) + e_{\alpha(\gamma\alpha)}\left(\partial^2\varphi / \partial x_\alpha^2\right) +$$
$$+\left(e_{\alpha(\gamma\beta)} + e_{\beta(\gamma\alpha)}\right)\left(\partial^2\varphi / \partial x \partial x_{\alpha\beta}\right) + e_{\beta(\gamma\beta)}\left(\partial^2\varphi / \partial x_\beta^2\right) = \rho\left(\partial^2 u_\gamma / \partial t^2\right) \quad (3.19)$$

$$e_{\alpha(\gamma\alpha)}\left(\partial^2 u_\gamma / \partial x_\alpha^2\right) + \left(e_{\alpha(\gamma\beta)} + e_{\beta(\gamma\alpha)}\right)\left(\partial^2 u_\gamma / \partial x_\alpha \partial x_\beta\right) + e_{\beta(\gamma\beta)}\left(\partial^2 u_\gamma / \partial x_\beta^2\right) -$$
$$-\varepsilon_{\alpha\alpha}\left(\partial^2\varphi / \partial x_\alpha^2\right) - \varepsilon_{\beta\beta}\left(\partial^2\varphi / \partial x_\beta^2\right) = 0 \quad (3.20)$$

Obviously, the material relations (3.17)÷(3.20) and the quasistatic equations (3.21) and (3.22) of the generalized antiplanar stress-strain state coincide, respectively, with the material relations and the equations of electroelastic antiplane deformation [1].

The automatic fulfillment of the compatibility conditions for nonzero elastic shifts and electric field strength in the sagittal plane of the chosen piezoelectric textures does not change the material relations and quasistatic equations, but can narrow the quantity of textures where an anti-plane stress-strain state is possible.

In the other two sagittal planes of piezoelectric crystals $x_\gamma 0 x_\alpha$ and $x_\beta 0 x_\gamma$, the conditions for the existence of an antiplane stress-strain state, the material relations of nonzero electromechanical components, the reduced elastic stiffnesses and piezoelectric coefficients, as well as the equations of electroelasticity for the corresponding antiplane stress-strain states of the type (3.13) and (3.14)

$$\left\{\begin{matrix} 0;\ \ u_\beta(x_\alpha, x_\gamma, t);\ \ 0;\ \ \partial\varphi(x_\alpha, x_\gamma, t)/\partial x_\alpha;\ \ 0;\ \ \partial\varphi(x_\alpha, x_\gamma, t)/\partial x_\gamma \\ 0;\ 0;\ 0;\ \sigma_{\gamma\beta}(x_\alpha, x_\gamma, t);\ 0;\ \sigma_{\alpha\beta}(x_\alpha, x_\gamma, t);\ D_\alpha(x_\alpha, x_\gamma, t);\ 0;\ D_\gamma(x_\alpha, x_\gamma, t) \end{matrix}\right\} \quad (3.21)$$

or

$$\left\{\begin{matrix} u_\alpha(x_\beta, x_\gamma, t);\ \ 0;\ \ 0;\ \ 0;\ \ \partial\varphi(x_\beta, x_\gamma, t)/\partial x_\beta;\ \ \partial\varphi(x_\beta, x_\gamma, t)/\partial x_\gamma; \\ 0;\ 0;\ 0;\ 0;\ \sigma_{\gamma\alpha}(x_\beta, x_\gamma, t);\ \sigma_{\alpha\beta}(x_\beta, x_\gamma, t);\ 0;\ D_\beta(x_\beta, x_\gamma, t);\ D_\gamma(x_\beta, x_\gamma, t) \end{matrix}\right\} \quad (3.22)$$

are obtained by rotating silent indexes $\{\gamma, \alpha, \beta\} \rightleftarrows \{\alpha, \beta, \gamma\} \rightleftarrows \{\beta, \gamma, \alpha\}$ in all relations (3.1)÷(3.22).

3.1. Material relationships and quasistatic equations of the electroelastic generalized antiplanar stress-strain state in the sagittal plane $x_1 0 x_2$.

In this sagittal plane of piezocrystalline textures, the conditions for the existence of an electroelastic antiplane stress-strain state (3.14) take the form

$$c_{14} \equiv 0,\ c_{15} \equiv 0,\ c_{24} \equiv 0,\ c_{25} \equiv 0,\ c_{64} \equiv 0,\ c_{65} \equiv 0,\ c_{35} \equiv 0,\ c_{34} \equiv 0,\ e_{34} \equiv 0,$$
$$e_{11} \equiv 0,\ e_{12} \equiv 0,\ e_{13} \equiv 0,\ e_{16} \equiv 0,\ e_{21} \equiv 0,\ e_{22} \equiv 0,\ e_{23} \equiv 0,\ e_{26} \equiv 0,\ e_{35} \equiv 0. \quad (3.23)$$

Only the generalized tensors of electroelasticity of piezoelectric media from the classes $\bar{4}3m$ and 23 of cubic, 622, 6, $6mm$ of hexagonal, 4, $\bar{4}$, $4mm$, $\bar{4}2m$ of tetragonal and 222, $mm2$ of rhombic symmetries correspond to these restrictions.

Material relationships of non-zero electromechanical components in these media can be written in the general form

$$\sigma_{23}(x_1,x_2,t)=c_{44}\left(\partial u_3/\partial x_2\right)+e^*_{14}\left(\partial\varphi/\partial x_1\right)+e^*_{24}\left(\partial\varphi/\partial x_2\right)$$
$$\sigma_{31}(x_\alpha,x_\beta,t)=c^*_{55}\left(\partial u_3/\partial x_1\right)+e^*_{15}\left(\partial\varphi/\partial x_1\right)+e^*_{25}\left(\partial\varphi/\partial x_2\right) \tag{3.24}$$

$$D_1(x_1,x_2,t)=e^*_{14}\left(\partial u_3/\partial x_2\right)+e^*_{15}\left(\partial u_3/\partial x_1\right)-\varepsilon_{11}\left(\partial\varphi/\partial x_1\right)$$
$$D_2(x_1,x_2,t)=e^*_{24}\left(\partial u_3/\partial x_2\right)+e^*_{25}\left(\partial u_3/\partial x_1\right)-\varepsilon^*_{22}\left(\partial\varphi/\partial x_2\right) \tag{3.25}$$

In the selected sagittal plane of these classes of media, it is convenient to write down also the quasistatic equations of electroelasticity in general form

$$c^*_{55}\left(\partial^2 u_3/\partial x_1^2\right)+c_{44}\left(\partial^2 u_3/\partial x_2^2\right)+\left(e^*_{14}+e^*_{25}\right)\left(\partial^2\varphi/\partial x_1\partial x_2\right)+$$
$$+e^*_{15}\left(\partial^2\varphi/\partial x_1^2\right)+e^*_{24}\left(\partial^2\varphi/\partial x_2^2\right)=\rho\left(\partial^2 u_3/\partial t^2\right),$$
$$e^*_{15}\left(\partial^2 u_3/\partial x_1^2\right)+\left(e^*_{14}+e^*_{25}\right)\left(\partial^2 u_3/\partial x_1\partial x_2\right)+e^*_{24}\left(\partial^2 u_3/\partial x_2^2\right)-$$
$$-\varepsilon_{11}\left(\partial^2\varphi/\partial x_1^2\right)-\varepsilon^*_{22}\left(\partial^2\varphi/\partial x_2^2\right)=0, \tag{3.26}$$

The coefficients with asterisks in the relations (3.24)÷(3.26) are given in Table 7.

Table 7. Values of physicomechanical coefficients with asterisks for the problem of antiplane stress-strain state in the sagittal plane ***x*₁0*x*₂**

		c^*_{55}	e^*_{14}	e^*_{15}	e^*_{24}	e^*_{25}	ε^*_{22}
Piezoelectric Textures	$\bar{4}3m/23$	c_{44}	e_{14}	0	0	e_{14}	ε_{11}
	$6mm$	c_{44}	0	e_{15}	e_{15}	0	ε_{11}
	6	c_{44}	e_{14}	e_{15}	e_{15}	$-e_{14}$	ε_{11}
	622	c_{44}	e_{14}	0	0	$-e_{14}$	ε_{11}
	4	c_{44}	e_{14}	e_{15}	e_{15}	$-e_{14}$	ε_{11}
	$\bar{4}$	c_{44}	e_{14}	e_{15}	$-e_{15}$	e_{14}	ε_{11}
	$4mm$	c_{44}	0	e_{15}	e_{15}	0	ε_{11}
	$\bar{4}2m$	c_{44}	e_{14}	0	0	e_{14}	ε_{11}
	222 ,	c_{55}	e_{14}	0	0	e_{25}	ε_{22}
	$mm2$	c_{55}	0	e_{15}	e_{24}	0	ε_{22}

From Table 7 it is seen that the material relations of the electromechanical components (3.24) and (3.25), as well as the equations of electroelasticity (3.26) of the problems of the antiplanar stress-strain state in the sagittal plane $x_1 0 x_2$ for media from the classes $\bar{4}3m$ and 23 of cubic, 622 of hexagonal, $\bar{4}2m$ of tetragonal and 222 of rhombic symmetries fundamentally coincide.

In the sagittal plane $x_1 0 x_2$, the material relationships and the equations of electroelasticity of the problems of antiplanar stress-strain state for media from the classes $6mm$ of hexagonal, $4mm$ of tetragonal and $mm2$ of rhombic symmetries also fundamentally coincide.

3.2. Material relationships and quasistatic equations of the electroelastic generalized antiplanar stress-strain state in the sagittal plane $x_2 0 x_3$.

In this sagittal plane of piezocrystalline textures, the conditions for the existence of an electroelastic antiplane stress-strain state

$$\left\{\begin{matrix} u_1(x_2,x_3,t); \quad 0; \quad 0; \quad 0; \quad \partial\varphi(x_2,x_3,t)/\partial x_2; \quad \partial\varphi(x_2,x_3,t)/\partial x_3; \\ 0;\ 0;\ 0;\ 0;\ \sigma_{13}(x_2,x_3,t);\ \sigma_{12}(x_2,x_3,t);\ 0;\ D_2(x_2,x_3,t);\ D_3(x_2,x_3,t) \end{matrix}\right\} \tag{3.27}$$

are written as follows

$$c_{16} \equiv 0,\ c_{15} \equiv 0,\ c_{26} \equiv 0,\ c_{25} \equiv 0,\ c_{35} \equiv 0,\ c_{36} \equiv 0,\ c_{46} \equiv 0,\ c_{45} \equiv 0, \tag{3.28}$$
$$e_{14} \equiv 0,\ e_{16} \equiv 0,\ e_{21} \equiv 0,\ e_{22} \equiv 0,\ e_{23} \equiv 0,\ e_{24} \equiv 0,\ e_{31} \equiv 0,\ e_{32} \equiv 0,\ e_{33} \equiv 0,\ e_{34} \equiv 0.$$

Only the generalized tensors of electroelasticity of piezoelectric media from the classes $\bar{6}m2$ of hexagonal and 422 of tetragonal symmetries correspond to these restrictions.

Material relationships of non-zero electromechanical components in these media can be written in the general form

$$\begin{aligned} \sigma_{13}(x_1,x_2,t) &= c_{55}^{*}\left(\partial u_1/\partial x_3\right) + e_{15}^{*}\left(\partial\varphi/\partial x_1\right) + e_{35}^{*}\left(\partial\varphi/\partial x_3\right) \\ \sigma_{12}(x_\alpha,x_\beta,t) &= c_{66}^{*}\left(\partial u_1/\partial x_2\right) + e_{16}^{*}\left(\partial\varphi/\partial x_1\right) + e_{36}^{*}\left(\partial\varphi/\partial x_2\right) \end{aligned} \tag{3.29}$$

$$\begin{aligned} D_1(x_1,x_3,t) &= e_{15}^{*}\left(\partial u_1/\partial x_3\right) + e_{16}^{*}\left(\partial u_1/\partial x_2\right) - \varepsilon_{11}\left(\partial\varphi/\partial x_2\right) \\ D_3(x_1,x_2,t) &= e_{35}^{*}\left(\partial u_1/\partial x_3\right) + e_{36}^{*}\left(\partial u_1/\partial x_2\right) - \varepsilon_{33}^{*}\left(\partial\varphi/\partial x_1\right) \end{aligned} \tag{3.30}$$

In the selected sagittal plane of these classes of media, it is convenient to write down also the quasistatic equations of electro elasticity in general form

$$\begin{aligned} & c_{66}^{*}\left(\partial^2 u_1/\partial x_2^2\right) + c_{55}^{*}\left(\partial^2 u_1/\partial x_3^2\right) + \left(e_{25}^{*} + e_{36}^{*}\right)\left(\partial^2\varphi/\partial x_2\partial x_3\right) + \\ & + e_{26}^{*}\left(\partial^2\varphi/\partial x_2^2\right) + e_{35}^{*}\left(\partial^2\varphi/\partial x_3^2\right) = \rho\left(\partial^2 u_1/\partial t^2\right), \\ & e_{36}^{*}\left(\partial^2 u_1/\partial x_2^2\right) + \left(e_{36}^{*} + e_{25}^{*}\right)\left(\partial^2 u_1/\partial x_2\partial x_3\right) + e_{35}^{*}\left(\partial^2 u_1/\partial x_3^2\right) - \\ & - \varepsilon_{22}^{*}\left(\partial^2\varphi/\partial x_2^2\right) - \varepsilon_{33}^{*}\left(\partial^2\varphi/\partial x_3^2\right) = 0, \end{aligned} \tag{3.31}$$

Table 8. Values of physicomechanical coefficients with asterisks of the problem of antiplanar stress-strain state in the sagittal plane $x_2 0 x_3$

		c_{55}^{*}	c_{66}^{*}	e_{15}^{*}	e_{16}^{*}	e_{35}^{*}	e_{36}^{*}	ε_{22}^{*}	ε_{33}^{*}
Classes	$\bar{6}m2$	c_{44}	c_{66}	0	0	0	0	ε_{11}	ε_{33}
	422	c_{44}	c_{66}	e_{15}	0	0	0	ε_{11}	ε_{33}

The coefficients with asterisks in the relations (3.24)÷(3.26) are given in Table 8.

Table 8 shows that the problem of an antiplane stress-strain state in the sagittal plane $x_2 0 x_3$ is not electroactive for media of the hexagonal symmetry class $\bar{6}m2$.

3.3. Material relationships and quasistatic equations of the electroelastic generalized antiplane stress-strain state in the sagittal plane $x_3 0 x_1$.

In this sagittal plane of piezocrystalline textures, the conditions for the existence of an electroelastic antiplane stress-strain state

$$\left\{\begin{matrix} 0; \quad u_2(x_1,x_3,t); \quad 0; \quad \partial\varphi(x_1,x_3,t)/\partial x_1; \quad 0; \quad \partial\varphi(x_1,x_3,t)/\partial x_3 \\ 0;\ 0;\ 0;\ \sigma_{23}(x_1,x_3,t);\ 0;\ \sigma_{12}(x_1,x_3,t);\ D_1(x_1,x_3,t);\ 0;\ D_3(x_1,x_3,t) \end{matrix}\right\} \tag{3.32}$$

take the following form

$$\begin{matrix} c_{14}\equiv 0,\ c_{16}\equiv 0,\ c_{24}\equiv 0,\ c_{26}\equiv 0,\ c_{34}\equiv 0,\ c_{36}\equiv 0,\ c_{54}\equiv 0,\ c_{56}\equiv 0,\ e_{24}\equiv 0, \\ e_{11}\equiv 0,\ e_{12}\equiv 0,\ e_{13}\equiv 0,\ e_{15}\equiv 0\ e_{31}\equiv 0,\ e_{32}\equiv 0,\ e_{33}\equiv 0,\ e_{35}\equiv 0,\ e_{26}\equiv 0, \end{matrix} \tag{3.33}$$

Only the generalized tensors of electroelasticity of piezoelectric media from the classes $\bar{4}3m$ and 23 of cubic, $\bar{4}2m$ of tetragonal, and 222 of rhombic symmetries correspond to these restrictions.

Material relationships of non-zero electromechanical components in these media can be written in general form

$$\begin{matrix} \sigma_{23}(x_1,x_3,t) = c_{44}\left(\partial u_2/\partial x_3\right) + e^*_{14}\left(\partial\varphi/\partial x_1\right) + e^*_{34}\left(\partial\varphi/\partial x_3\right) \\ \sigma_{12}(x_1,x_3,t) = c^*_{66}\left(\partial u_2/\partial x_1\right) + e^*_{16}\left(\partial\varphi/\partial x_1\right) + e^*_{36}\left(\partial\varphi/\partial x_3\right) \end{matrix} \tag{3.34}$$

$$\begin{matrix} D_1(x_1,x_3,t) = e^*_{14}\left(\partial u_2/\partial x_3\right) + e^*_{16}\left(\partial u_2/\partial x_1\right) - \varepsilon_{11}\left(\partial\varphi/\partial x_1\right) \\ D_3(x_1,x_3,t) = e^*_{34}\left(\partial u_2/\partial x_3\right) + e^*_{36}\left(\partial u_2/\partial x_1\right) - \varepsilon^*_{33}\left(\partial\varphi/\partial x_3\right) \end{matrix} \tag{3.35}$$

In the selected sagittal plane of these classes of media, it is convenient to write down also the quasistatic equations of electro elasticity in general form

$$\begin{matrix} c^*_{66}\left(\partial^2 u_2/\partial x_1^2\right) + c_{44}\left(\partial^2 u_2/\partial x_3^2\right) + \left(e^*_{14}+e^*_{36}\right)\left(\partial^2\varphi/\partial x_1\partial x_3\right) + \\ + e^*_{16}\left(\partial^2\varphi/\partial x_1^2\right) + e^*_{34}\left(\partial^2\varphi/\partial x_3^2\right) = \rho\left(\partial^2 u_2/\partial t^2\right) \end{matrix},$$

$$\begin{matrix} e^*_{16}\left(\partial^2 u_2/\partial x_1^2\right) + \left(e^*_{14}+e^*_{36}\right)\left(\partial^2 u_2/\partial x_1\partial x_3\right) + e^*_{34}\left(\partial^2 u_2/\partial x_3^2\right) - \\ -\varepsilon_{11}\left(\partial^2\varphi/\partial x_1^2\right) - \varepsilon^*_{33}\left(\partial^2\varphi/\partial x_3^2\right) = 0 \end{matrix}, \tag{3.36}$$

The coefficients with asterisks in the relations (3.34)÷(3.36) are given in Table 9.

Table 9. Values of the physicomechanical coefficients with asterisks of the problem of anti-planar stress-strain state in the sagittal plane ***x₁0x₃***

		c^*_{66}	e^*_{14}	e^*_{16}	e^*_{34}	e^*_{36}	ε^*_{33}
Classes	$\bar{4}3m/23$	c_{44}	e_{14}	0	0	e_{14}	ε_{11}
	$\bar{4}2m$	c_{66}	e_{14}	0	0	e_{36}	ε_{33}
	222	c_{66}	e_{14}	0	0	e_{36}	ε_{33}

From Table 9 it follows that in all three cases in the equations of electroelasticity (3.36) of the anti-planar stress-strain state for media from the classes $\bar{4}3m$ and 23 of cubic, $\bar{4}2m$ of tetragonal and 222 of rhombic symmetries, the coupling of the elastic and electric fields occurs through mixed derivatives of the electric field potential and elastic displacement, respectively. From the equations of electroelasticity (3.36) it also follows that in problems of the antiplanar stress-strain state for media from the classes $\bar{4}3m$ and 23 of cubic symmetry, in contrast to problems for media from the classes $\bar{4}2m$ of tetragonal and 222 of rhombic symmetries, the piezoelectric effect is equivalent in coordinates x_1 and x_3.

4. Comparative analysis.

Comparative analysis shows that, taking into account the conditions of compatibility of nonzero elastic axial elongations (compressions) and electric field strengths (2.11) and (3.8), the sets of piezocrystalline classes of anisotropy, which allow separate excitation and propagation of electroelastic planar or electroelastic antiplane stress deformed states in piezoelectric media do not change accordingly.

Taking into account the conditions of compatibility of nonzero elastic axial elongations (compressions) and electric field strength, leads to refinement of the notation of material relations, electroelastic equations, and necessary and sufficient conditions that allow separate excitation and propagation of a plane (or anti-planar) electro-elastic stress-strain state in the corresponding problems.

Using the simple interaction of mechanical and electric fields as an example, we see that the two-dimensional problem of plane deformation in the sagittal plane $x_3 0 x_1$ for the class $\bar{6}m2$ is formulated by material relations of non-zero electro-mechanical characteristics

$$\begin{aligned}
&\sigma_{11} = c_{11}\left(\partial u_1/\partial x_1\right) + c_{13}\left(\partial u_3/\partial x_3\right) + e_{11}\left(\partial\varphi/\partial x_1\right),\\
&\sigma_{22} = c_{12}\left(\partial u_1/\partial x_1\right) + c_{13}\left(\partial u_3/\partial x_3\right) - e_{11}\left(\partial\varphi/\partial x_1\right),\\
&\sigma_{33} = c_{13}\left(\partial u_1/\partial x_1\right) + c_{33}\left(\partial u_3/\partial x_3\right), \quad \sigma_{31} = c_{44}\left[\left(\partial u_3/\partial x_1\right) + \left(\partial u_1/\partial x_3\right)\right],\\
&D_1 = e_{11}\left(\partial u_1/\partial x_1\right) - \varepsilon_{11}\left(\partial\varphi/\partial x_1\right), \qquad D_3 = -\varepsilon_{33}\left(\partial\varphi/\partial x_3\right).
\end{aligned} \tag{3.37}$$

Quasistatic equations in this case are written as follows

$$\begin{aligned}
&c_{11}\left(\partial^2 u_1/\partial x_1^2\right) + c_{44}\left(\partial^2 u_1/\partial x_3^2\right) + \left(c_{13}+c_{44}\right)\left(\partial^2 u_3/\partial x_1\partial x_3\right) +\\
&+e_{11}\left(\partial^2\varphi/\partial x_1^2\right) = \rho\left(\partial^2 u_1/\partial t^2\right)
\end{aligned} \tag{3.38}$$

$$c_{44}\left(\partial^2 u_3/\partial x_1^2\right) + c_{33}\left(\partial^2 u_3/\partial x_3^2\right) + \left(c_{13}+c_{44}\right)\left(\partial^2 u_1/\partial x_1\partial x_3\right) = \rho\left(\partial^2 u_3/\partial t^2\right) \tag{3.39}$$

$$e_{11}\left(\partial^2 u_1/\partial x_1^2\right) - \varepsilon_{11}\left(\partial^2\varphi/\partial x_1^2\right) - \varepsilon_{33}\left(\partial\varphi^2/\partial x_3^2\right) = 0 \tag{3.40}$$

The problem of the plane stress-strain state in the same plane for the class $\bar{6}m2$ is formulated by the material relations of non-zero electromechanical characteristics (4.1) with the compatibility condition of elastic non-zero axial elongations (compressions) $\sigma_{22} \equiv 0$:

$$c_{12}\left(\partial u_1/\partial x_1\right) + c_{13}\left(\partial u_3/\partial x_3\right) - e_{11}\left(\partial\varphi/\partial x_1\right) = 0 \tag{3.41}$$

In view of (4.5), equations (4.2)÷(4.4) can be written as follows

$$\left(c_{11}-c_{12}^{*}\right)\frac{\partial^2 u_1}{\partial x_1^2}+c_{44}\frac{\partial^2 u_1}{\partial x_3^2}-e_{11}\left[1+\left(c_{12}^{*}/c_{12}\right)\right]\frac{\partial^2 \varphi}{\partial x_1^2}=\rho\frac{\partial^2 u_1}{\partial t^2},$$

$$c_{44}\frac{\partial^2 u_3}{\partial x_1^2}+\left(c_{33}-c_{13}^{*}\right)\frac{\partial^2 u_3}{\partial x_3^2}-e_{11}\left(c_{13}^{*}/c_{13}\right)\frac{\partial^2 \varphi}{\partial x_1 \partial x_3}=\rho\frac{\partial^2 u_3}{\partial t^2}, \quad (3.42)$$

$$e_{11}\frac{\partial^2 u_1}{\partial x_1^2}-\varepsilon_{11}\frac{\partial^2 \varphi}{\partial x_1^2}-\varepsilon_{33}\frac{\partial^2 \varphi}{\partial x_3^2}=0,$$

where it is clearly seen that both the new stiffness coefficients $c_{13}^{*}=\left(c_{13}+c_{44}\right)c_{13}/c_{12}$ and $c_{12}^{*}=\left(c_{13}+c_{44}\right)c_{12}/c_{13}$ "soften" the longitudinal stiffnesses and "tighten" the direct piezoelectric effect in the medium.

The equivalence of two-dimensional problems of electroelasticity of plane deformation and plane stress-strain state is ensured by taking into account the electro-mechanical conditions in the infinitely distant ends $x_\gamma = \pm\infty$ of a long waveguide.

5. Conclusion.

The necessary and sufficient conditions are derived that allow separate excitation and propagation of the electroelastic plane and also anti-plane stress-strain states in piezoelectric media:

- Conditions of type (2.10) and the compatibility condition of non-zero elastic axial elongations (compressions) and electric field strength (2.11) allow separate excitation and propagation of a plane electroelastic stress-strain state of type {(2.8), (2.9)} in the sagittal plane $x_\alpha 0 x_\beta$.
- Conditions of the type (3.14) allow separate excitation and propagation of the antiplane electroelastic stress-strain state {(3.12), (3.13)} in the sagittal plane $x_\alpha 0 x_\beta$.

The introduced compatibility conditions for elastic non-zero axial elongations (compressions) and electric field strengths (2.11) and (3.8) correspond to the requirements of hypotheses about the nature of the distribution of the mechanical field in thin-walled structural elements:

- In the case of a plane electroelastic stress-strain state of the type {(2.8), (2.9)}, all the characteristics of the wave field lie in the sagittal plane $x_\alpha 0 x_\beta$ and there is no pressure of parallel material planes $x_\gamma = \mathrm{const}$ on each other.
- In the case of an antiplane electroelastic stress-strain state of the type {(3.12), (3.13)}, the mechanical characteristics of the wave field do not distort the sagittal plane $x_\alpha 0 x_\beta$, the characteristics of the electric field lie in the sagittal plane and there is no pressure of parallel material planes $x_\gamma = \mathrm{const}$ on each other.

In all sagittal planes of all textures of piezoelectric crystals, the material relations of nonzero electromechanical components and quasistatic equations of electroelasticity are derived. This allows to choose different combinations of piezoelectric materials in layered waveguides in studies of the joint propagation of electroactive waves of plane and antiplane stress-strain state.

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