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Механика

THE THEORY OF LINEAR AND NONLINEAR MAGNETOELECTRIC EFFECT IN LAYERED DISC-SHAPED MAGNETOSTRICTIVE-PIEZOELECTRIC COMPOSITES

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Ключевые слова: магнитоэлектрический эффект, магнитострикция, пьезоэлектричество, тонкие слои, электромеханический резонанс.

Բանալի բառեր. մագնիսաէլեկտրական էֆեկտ, մագնիսաստրիկցիա, պյեզոէլեկտրականություն, բարակ շերտեր, էլեկտրոմեխանիկական ռեզոնանս։

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Теория линейного и нелинейного магнитоэлектрического эффекта в слоистых магнитострикционнопьезоэлектрических композитах

В статье представлена теория линейного и нелинейного магнитоэлектрического эффекта в слоистых магнитострикционно-пьезоэлектрических мультиферроиках, построенная на основе решений материальных уравнений и уравнения движения среды для пьезоэлектрической и магнитострикционной подсистем по отдельности с учётом условий на границе раздела. Показано, что в слабых полях подмагничивания величина нелинейного эффекта сравнима с линейным, причём, наряду с основным резонансом, наблюдается дополнительный резонанс. Величина амплитуды этого резонанса не зависит от поля подмагничивания, а возбуждение происходит на частоте магнитного поля в два раза меньшей частоты основного резонанса.

Գալիչյան Տ.Ա., Ֆիլիպպով Դ.Ա., Ֆիռսովա Տ.Օ., Ռադչենկո Գ.Ս. Գծային և ոչ գծային մագնիսաէլեկտրական էֆեկտի տեսությունը շերտավոր մագնիսաստրիկցիոնպյեզոէլեկտրական բաղադրյալ համակարգերում

Անոտացիա. Հոդվածում նեկայացված է գծային և ոչ գծային մագնիսաէլեկտրական էֆեկտի տեսությունը մագնիսաստրիկցիոն-պյեզոէլեկտրական բազմաֆեռոիկներում։ Տեսությունը կառուցված է նյութական հավասարումների, պյեզոէլեկտրական և մագնիսաստրիկցիոն միջավայրերի շարժման համապատասխան հավասարումների լուծման հիման վրա, հաշվի առնելով եզրային պայմանները։ Ցույց է տրված, որ թույլ մագնիսականացնող դաշտերում ոչ գծային էֆեկտի մեծությունը համեմատական է գծային էֆեկտի մեծության հետ, ընդ որում հիմնական ռեզոնանսի հետ նկատվում է նաև լրացուցիչ ռեզոնանս։ Այդ ռեզոնանսի ամպլիտուդի մեծությունը կախված չէ մագնիսականացնող դաշտից, իսկ գռգռումը տեղի է ունենում հիմնական ռեզոնանսի համախությունից երկու անգամ փոքր մագնիսական դաշտի համախության դեպքում։

Abstract: The theory of linear and nonlinear magnetoelectric effect in layered magnetostrictive-piezoelectric multiferroics is presented based on the joint solution of the equations of motion and constitutive relations for the magnetostrictive and piezoelectric subsystems, considering the boundary conditions on the interface. It is shown that, in weak bias magnetic fields, the value of nonlinear effect is comparable with linear, and along with the main resonance, there is an additional resonance. Value of the resonance amplitude is not dependent on bias magnetic field and is excited at the frequency of magnetic field twice less the main resonance frequency. Trilayer nickel-

polymer–PZT–polymer-nickel disc-shaped structure is designed for the experiment. Frequency and field dependence of magnetoelectric effect is presented for this structure.

Introduction

Magnetoelectric (ME) effect, theoretically predicted in [1,2] and experimentally observed in works [3,4] over half a century ago, attracts an increasing number of researchers in recent years, evidenced by the growing number of publications on this subject [5].

In the past decade, the technology of producing composite ME multiferroics was improved. This allowed manufacturing structures with sufficient ME parameters to create a variety of electronic devices [6].

Magnetoelectric effect in layered magnetostrictive-piezoelectric multiferroics occurs through mechanical interaction between the subsystems. Mechanical oscillations, resulting in a magnetic material in an alternating magnetic field are transmitted to the piezoelectric material, which leads to appearance of an electric field. There are peaks at the frequency dependence of the effect due to electromechanical resonance, because the mechanism of the ME effect is associated with the propagation of mechanical waves [7].

There are certain problems and inaccuracies in the theories of ME effect [8-18] currently available. The method of effective parameters proposed in [8] and developed further in [9-12] is applicable to structures where the characteristic sizes are much smaller than the length of the waves propagating in the composite. The disadvantage of the method of effective parameters is also the difficulty of determining the values of effective parameters themselves. A more accurate method is based on the solution of the equations of motion and the material equations separately for the magnetic and piezoelectric phases, then connecting these solutions using boundary conditions. Earlier, the theory of ME effect in magnetostrictive-piezoelectric structures was proposed using this approach in works [13-16]. However, there are some inaccuracies in the proposed theory. The interface between the phases was considered by introducing a coupling coefficient or by the shearlag model in [13-15], which were determined empirically. A perfect bonding between the layers was discussed in [16], and it was assumed that the displacement of the magnet and the piezoelectric media was the same. As will be shown below, this assumption takes place in case of thin layers, when the displacements change over the sample thickness can be neglected.

Theory of the ME effect was presented in [17,18] apparently considering the interface in bilayer magnetostrictive-piezoelectric structure with perfect bonded layers and glued magnetostrictive and piezoelectric layers in [19,20]. Magnetoelectric structures of Nickel and Metglas magnetostrictive layers attached to one free end of piezoelectric Pb(Zr,Ti)O₃ (PZT) cantilever was studied in [21]. In these papers, structures in the form of a thin plate were considered. On the other hand, disc-shaped structures are used in practice more often. The geometry of disc-shaped structures has a number of different characteristics compared to the plate, so the equations obtained in [17-21] for the frequency dependence of the ME effect are not directly applicable for such structures. Enhanced converse ME effect has been experimentally observed in cylindrical PZT-Terfenol-D piezoelectric-magnetostrictive bilayered composites [22]. The theory of linear ME effect is described sufficiently detailed, but only two papers [23-27] are devoted to the theory of nonlinear ME effect, where the nonlinear effect was studied in plate samples. In this paper, we present the theory of linear and nonlinear ME effect for disk-shaped structures considering the explicit account of the interface between the phase boundaries.

1. Model and basic equations

As a model, we consider a structure of disc-shaped layers of radius R, consisting of mechanically interacting magnet and piezoelectric layers of thickness t_m and t_p . Thin metal contacts are applied on the top and the bottom of the plate (Fig.1).



Fig. 1. Schematic view of disc-shaped layered sample

Longitudinal orientation of the electric and magnetic fields is investigated for this structure. In case of longitudinal fields the magnetic fields (constant H_{bias} and alternating H with the frequency ω) coincide in direction with the polarization vector P. Due to the symmetry of the problem, we choose a cylindrical coordinate system. The origin of the coordinates coincides with the boundary between the layers and the direction Z is perpendicular to that boundary. Preliminarily, the piezoelectric layer is polarized perpendicularly to the contact (Z axis). The interaction between the magnet and the piezoelectric is carried out via the interface by shear stresses. Due to the axial symmetry of the problem, the nonzero components of the stress tensor T_{ij} in cylindrical coordinates are

only T_{rr} , $T_{_{\theta\theta}}$, T_{rz} and $T_{_{\thetaz}}$. Consequently, the equations for the stress tensor and the electric induction are of the form

$$T_{rr}^{p} = \frac{Y_{p}}{\left(1 - v^{2}\right)} \left(S_{rr}^{p} + vS_{\theta\theta}^{p} - \left(1 + v\right)d_{31}E_{3}\right), \qquad (1)$$

$$T_{\theta\theta}^{p} = \frac{Y_{p}}{(1-\nu^{2})} \left(\nu S_{rr}^{p} + S_{\theta\theta}^{p} - (1+\nu)d_{31}E_{3}\right),$$
(2)

$$D_{3} = \varepsilon_{33}E_{3} + d_{31}(T_{rr}^{p} + T_{\theta\theta}^{p}), \qquad (3)$$

$$T_{rr}^{m} = \frac{Y_{m}}{(1-\nu^{2})} \left(S_{rr}^{m} + \nu S_{\theta\theta}^{m} - (1+\nu)\lambda(H) \right), \tag{4}$$

$$T^{m}_{\theta\theta} = \frac{Y_{m}}{(1-\nu^{2})} \left(\nu S^{m}_{rr} + S^{m}_{\theta\theta} - (1+\nu)\lambda(H)\right),$$
(5)

where $Y_{\alpha}(\alpha = p, m)$ are the first order Young's moduli for the magnet and piezoelectric,

respectively, v is the Poisson's ratio, expected the same for both media. $S_{rr}^{\alpha} = \frac{\partial u_r^{\alpha}}{\partial r}$ and

 $S_{\theta\theta}^{\alpha} = \frac{1}{r} \frac{\partial u_{\theta}^{\alpha}}{\partial \theta} + \frac{u_{r}^{\alpha}}{r} \text{ are the components of deformation tensor, } u_{r}^{\alpha} \text{ and } u_{\theta}^{\alpha} \text{ are the components of medium displacement vector, } d_{31} \text{ is the piezo-electric tensor, } \lambda(H) \text{ is the magnetostriction of the magnet, } H \text{ and } E \text{ are the external magnetic and induced electric}$

Since the magnetostriction is a nonlinear function of the magnetic field, in general, the magnetic stress tensor will also be a nonlinear way dependent on the strength of the magnetic field. The dependence of magnetostriction for nickel and permendur materials will have the form shown in Fig.2, according to [28].



Fig. 2. Field dependence of magnetostriction $\lambda(H)$ for permendur (P) and nickel (Ni) [28]

In weak means, magnetostriction can be presented in the form of an expansion in degrees of magnetic field, and, as can be seen from Fig.2, it is sufficient to consider the first terms of that expansion, i.e.,

field.

$$\lambda(H) = \left(\frac{\partial \lambda}{\partial H}\Big|_{H=H_{bias}}\right) \times H + \frac{1}{2} \left(\frac{\partial^2 \lambda}{\partial H^2}\Big|_{H=H_{bias}}\right) \times H^2$$
(6)

Equation (6) for the magnetostriction can be written in the following form:

$$\lambda(H) = qH + gH^2, \tag{7}$$

where
$$q = \frac{\partial \lambda}{\partial H}\Big|_{H=H_{bias}}$$
 is the piezomagnetic coefficient and $g = \frac{1}{2} \frac{\partial^2 \lambda}{\partial H^2}\Big|_{H=H_{bias}}$ is the

magnetostriction coefficient.

It is clear that in the absence of the bias field, point $H_{bias} = 0$, the piezomagnetic coefficient is q = 0, whereas the magnetostriction coefficient is $g \neq 0$. Consequently, the linear ME effect is observed only at a bias magnetic field, while a nonlinear ME effect exists at $H_{bias} = 0$. As can be seen from Figure 2, piezomagnetic coefficient initially increases with the increasing bias field, and then decreases. This leads to an increase of the effect value with increasing bias field reaching its maximum and then to a decrease. The value of magnetostrictive coefficient g is not dependent on the bias magnetic field, unlike the piezomagnetic coefficient q, which follows from Eq. (6) in weak fields. This leads to a value of nonlinear ME effect not dependent on H_{bias} in weak fields, which is validated experimentally in works [16-19].

The equation of motion for the radial component of the displacement vector can be written in the following form:

$$\rho_{\alpha} \frac{\partial^2 u_r^{\alpha}}{\partial t^2} = \frac{\partial T_r^{\alpha}}{\partial r} + \frac{T_r^{\alpha} - T_{\theta\theta}^{\alpha}}{r} + \frac{\partial T_r^{\alpha}}{\partial z}, \qquad (8)$$

where T_{rz}^{α} is the tangential component of the stress tensor, arising from the interface between the phases The relation between the stress and its appropriate component of the strain tensor is described by the Hooke's law

$$T_{rz}^{\alpha} = G_{\alpha} S_{rz}^{\alpha} \,, \tag{9}$$

where $G_{\alpha} = \frac{Y_{\alpha}}{2(1+\nu)}$ are the shear moduli and $S_{rz}^{\alpha} = \frac{\partial u_r^{\alpha}}{\partial z}$ are the shear strains.

The solution of Eq. (8) can be written as follows:

$$u_r^{\alpha}(t,r,z) = u_r^{\alpha}(r,z)\exp(i\omega t), \qquad (10)$$

where $u_r^{\alpha}(r, z)$ are the coordinates part of the function, ω is the frequency of the medium oscillation. Medium oscillations with ω' frequency are caused after the sample is placed in an alternating magnetic field, whose frequency is $\omega_L = \omega'$ in case of linear ME effect and $\omega_{nl} = 2\omega'$ in case of nonlinear ME effect, due to the quadratic dependence.

It should be noticed, that here only the radial oscillations and ME effect caused by displacement of radial waves in structure (basic contribution of ME effect) is considered.

That is why the component $\frac{\partial u_z}{\partial z}$ is not taken into account in the Hooke's law in Eq. (9).

Coordinate part of the wave function can be written in the following form:

$$u_r^{\alpha}(r,z) = g_{\alpha}(z) \left(A J_1(kr) + B Y_1(kr) \right), \tag{11}$$

where $g_{\alpha}(z)$ is a function which describes the change of the amplitude of medium displacement along the axis of the disk, $J_1(kr)$, $Y_1(kr)$ are the Bessel functions of first order and second order, respectively, k is the wave number, A and B are the constants of integration.

This solution $g_{\alpha}(z)$ fully characterizes inhomogeneous displacement of the waves along the thickness of the sample, namely along Z axis.

We obtain the value for the constant integration B = 0, since the center of the disc is $u_r^{\alpha}(r, z) = 0$.

Substituting expression (11) into Eq. (8) we obtain the following expression for the function $g_{\alpha}(z)$:

$$g_{\alpha}(z)'' + \left(\frac{2}{1-\nu}\right) \left[\frac{\omega^2}{V_{\alpha}^2} - k^2\right] g_{\alpha}(z) = 0, \qquad (12)$$

where the prime of the $g_{\alpha}(z)$ function indicates differentiation with respect to the variable z. $V_{\alpha}^{2} = \frac{Y_{\alpha}}{\rho_{\alpha}(1-v^{2})}$ in Eq. (12) are the square of the velocities of propagation of elastic

waves in the magnet and piezoelectric phases. As can be seen in Eq. (12), the components of H external magnetic and E induced electric fields are not included after substituting $u_r^{\alpha}(r,z)$ into (8), by means of the second-order differential equation for the $g_{\alpha}(z)$ function. Value of the second bracket in Eq. (12) equal to zero gives the dispersion relation for the propagation of elastic waves in pure magnet ($\alpha = m$) or piezoelectric ($\alpha = p$). The wave velocities propagating in bilayer structure will be in the interval between the wave velocities in magnet and piezoelectric phases. Consequently, the value of the expression in square brackets in Eq. (12) for the first phase will be positive, while negative for the second one. Let us consider the most typical case, when the velocity of the waves in the piezoelectric is less than in the magnet. In this case, the solution of equation (12) has the form:

$$g_m(z) = C_1 \exp(\chi_m z) + C_2 \exp(-\chi_m z), \qquad (13)$$

$$g_{p}(z) = C_{3}\cos(\chi_{p}z) + C_{4}\sin(\chi_{p}z), \qquad (14)$$

where $C_1 \dots C_4$ are the constants of integration, $\chi_m^2 = -\frac{2}{1-\nu} \left(\frac{\omega^2}{V_m^2} - k^2 \right)$,

$$\chi_p^2 = \frac{2}{1-\nu} \left(\frac{\omega^2}{V_p^2} - k^2 \right)$$

We use the boundary conditions to determine the constants of integration $C_1...C_4$. In the point z = 0 the components of the displacement vector $u_r^m(r,0) = u_r^p(r,0)$ and the tangential components of the stress tensor $T_{rz}^m(r,0) = T_{rz}^p(r,0)$. In points $z = -t_p$ and $z = t_m$ the tangential components of the stress tensor $T_{rz}^p(r,-t_p) = 0$ and $T_{rz}^m(r,t_m) = 0$. These conditions provide a system of four equations which solution gives an expression that defines the relationship between the frequency and the wave vector in the following form:

$$Y_m \chi_m \operatorname{th}(\kappa_m) = Y_p \chi_p \operatorname{tg}(\kappa_p), \qquad (15)$$

where $\kappa_m = \chi_m t_m$ and $\kappa_p = \chi_p t_p$ are non-dimensional parameters. It should be noted that a similar relation is derived for the plate in [16,17]. Dependence of the angular frequency ω on the wave vector *k*, as it follows from Eq. (15), is of a nonlinear character. For thin layers this relationship can be represented in an approximate expression by expanding functions in a series of small parameters in next form:

$$\omega = V(1+\delta)k \,. \tag{16}$$

where $\overline{V} = \sqrt{\frac{\overline{Y}}{\overline{\rho}(1-\nu^2)}}$ is the velocity of propagating elastic waves in a medium with averaged parameters, δ is a corrective which describes the deviation from the linear relationship between ω and k. Here $\overline{Y} = (Y_m t_m + Y_p t_p)/(t_m + t_p)$ is the average value the elasticity modulus, $\overline{\rho} = (\rho_m t_m + \rho_p t_p)/(t_m + t_p)$ is the average value of the medium density. In the first approximation, the δ corrective is given by the following form:

$$\delta = -\frac{(1+\nu)}{3} \frac{Y_m t_m \left[(\overline{V}/V_m)^2 - 1 \right]^2 (kt_m)^2 + Y_p t_p \left[(\overline{V}/V_p)^2 - 1 \right]^2 (kt_p)^2}{Y_m t_m + Y_p t_p}.$$
(17)

Finally, for the constants of integration C_1 ... C_4 we obtain the following expressions:

$$C_1 = 1, \ C_2 = \exp(2\kappa_m), \ C_3 = 1 + \exp(2\kappa_m), \ C_4 = -(1 + \exp(2\kappa_m)) \operatorname{tg}(\kappa_p).$$
(18)

Boundary conditions on the side surface of the disk are written in Eq. (19) using the condition of mechanical equilibrium in the following form:

$$\int_{-t_{p}}^{0} T_{rr}^{p}(R,z) dz + \int_{0}^{t_{m}} T_{rr}^{m}(R,z) dz = 0.$$
(19)

The constant of integration A_L can be obtained using (19) by carrying out integration for linear effect in next form:

$$A_{L} = \frac{1+\nu}{\Delta_{L}} \frac{R}{1+\exp(2\kappa_{m}^{L})} \frac{d_{31}Y_{p}t_{p}E_{3} + Y_{m}t_{m}q_{31}H_{3}}{Y_{p}t_{p}\frac{\mathrm{tg}\kappa_{p}^{L}}{\kappa_{p}^{L}} + Y_{m}t_{m}\frac{\mathrm{tg}\kappa_{m}^{L}}{\kappa_{m}^{L}}},$$
(20)

where $\kappa_L = k_L R$ and $\Delta_L = \kappa_L J_0(\kappa_L) - (1-\nu) J_1(\kappa_L)$.

The constant of the integration A_{NL} for the nonlinear effect have the next form:

$$A_{NL} = \frac{1+\nu}{\Delta_{NL}} \frac{R}{1+\exp(2\kappa_m^{NL})} \frac{d_{31}Y_p t_p E_3 + Y_m t_m g_{31}H_3^2}{Y_p t_p \frac{\mathrm{tg}\kappa_p^{NL}}{\kappa_p^{NL}} + Y_m t_m \frac{\mathrm{tg}\kappa_m^{NL}}{\kappa_m^{NL}}},$$
(21)

where $\kappa_{_{NL}} = k_{_{NL}}R$ and $\Delta_{_{NL}} = \kappa_{_{NL}}J_0(\kappa_{_{NL}}) - (1-\nu)J_1(\kappa_{_{NL}})$.

The principal difference between the variables with indexes *L* and *NL*, such as κ_m^L , κ_m^{NL} and κ_p^L , κ_p^{NL} is that parameter k_L is included in the variables with the index *L* in the expression for linear effect. Parameter k_L is determined from the relation (16), in which the frequency of the oscillations of the medium is $\omega_L = \omega'$, where ω' is the frequency of the external magnetic field. Variables with index, on the other hand, include parameter k_{NL} , which is determined from the relation (16), where the frequency of the oscillations of the medium is $\omega_L = \omega'$.

2. Magnetoelectric effect

Potential difference between electrodes of the sample can be obtained from the expression

$$U = \int_{-t_p}^{0} E_3 dz \,. \tag{22}$$

The potential difference generated between electrodes in case of the linear effect can be obtained expressing the electric field through the stress tensor in Eq. (3) and using Eqs. (1) and (2) taking into account expression (11). The final expression can be derived for the potential difference using the open circuit condition $\int_{0}^{r} r dr \int_{0}^{2\pi} D_{3} d\theta = 0$ which can be given by the following form:

$$U_{L} = \frac{2d_{31}q_{31}(1+\nu)Y_{p}t_{p}}{\varepsilon_{33}(1-\nu)} \left[\frac{Y_{m}t_{m}}{Y_{p}t_{p}} \frac{\mathrm{tg}\kappa_{p}^{L}}{\kappa_{p}^{L}} + Y_{m}t_{m}\frac{\mathrm{th}\kappa_{m}^{L}}{\kappa_{m}^{L}} \frac{\mathrm{tg}\kappa_{p}^{L}}{\omega_{p}^{L}} \right] \frac{J_{1}(\kappa_{L})}{\Delta_{a}^{L}}H_{3}, \qquad (23)$$

with $K_p^2 = \frac{d_{31}^2 Y_p}{\varepsilon_{33} (1-\nu)}$ the squared coefficient of electromechanical coupling for radial

oscillations. Designation given in the equation (23) has the next form:

$$\Delta_a^L = \Delta_L (1 - K_p^2) + 2(1 + \nu) K_p^2 \frac{Y_p t_p}{Y_p t_p \frac{\operatorname{tg} \kappa_p^L}{\kappa_p^L} + Y_m t_m} \frac{\operatorname{tg} \kappa_p^L}{\kappa_p^L} J_1(\kappa_L).$$
(24)

The condition $\Delta_a^L = 0$ determines the value of the wave vector, and, consequently, the values of the frequencies at which the resonant increase of the linear ME effect takes place. The values of these frequencies in turn depend on the dispersion relation between ω_L and

 k_L . From Eq. (24) it follows that in the low-frequency region the value of induced voltage does not depend on the frequency and is given by the form:

$$U_{L}^{Low} = \frac{2d_{31}q_{31}Y_{p}t_{p}}{\varepsilon_{33}(1-\nu)} \frac{Y_{m}t_{m}}{Y_{p}t_{p} + Y_{m}t_{m}} \frac{1}{1-K_{p}^{2}(1-\frac{2Y_{m}t_{m}}{Y_{p}t_{p} + Y_{m}t_{m}})}H_{3}.$$
(25)

For the voltage which is induced on the plates of the sample due to the nonlinear effect, we have the following expression:

$$U_{NL} = \frac{2d_{31}g_{31}(1+\nu)Y_{p}t_{p}}{\varepsilon_{33}(1-\nu)} \left[\frac{Y_{m}t_{m}}{Y_{p}t_{p}\frac{tg\kappa_{p}^{NL}}{\kappa_{p}^{NL}} + Y_{m}t_{m}\frac{\mathrm{th}\kappa_{m}^{NL}}{\kappa_{m}^{NL}}} \frac{\mathrm{tg}\kappa_{p}^{NL}}{\varepsilon_{p}^{NL}} \right] \frac{J_{1}(\kappa_{NL})}{\Delta_{a}^{NL}}H_{3}^{2},$$
(26)

where Δ_a^{NL} is defined by the expression analogous to expression (24), wherein index *L* is replaced by the index *NL*. Total voltage induced in the sample is $U = U_L + U_{NL}$. Fig.3 shows the frequency dependence of the effect for nickel-lead zirconate titanate structure in an alternating magnetic field H = 2 Oe at a value of magnetizing field equal to the field of the Earth ($H_{bias} = 0.2$ Oe), and in the bias value $H_{bias} = 10$ Oe.

The following parameters of the structure were used during the calculations: disk radius is R = 4.5 mm, thickness of the piezoelectric $t_p = 0.4$ mm, thickness of the magnet $t_m = 0.32$ mm.

Parameters of the material: for piezoelectric (PZT) – $\rho_p = 7800 \text{ kg}/\text{m}^3$, $Y_p = 62 \text{ GPa}$, v = 0.3, $\varepsilon_{33} = 1750$, $d_{31} = 175 \text{ pC}/\text{N}$; the magnet nickel (Ni): $\rho_m = 8900 \text{ kg}/\text{m}^3$, $Y_m = 205 \text{ GPa}$, v = 0.3.



Fig. 3. The frequency dependence of the voltage induced on the plates by the linear ME effect -1 and nonlinear ME effect -2 in bias field: a) $H_{bias} = 0.2$ Oe (Earth field), b) $H_{bias} = 10$ Oe

The magnetostrictive curve shown in Figure 3, in the initial part was approximated by the expression $\lambda(H) = gH^2$, where the magnetostrictive coefficient had the value of $g = 1.2 \cdot 10^{-9} \text{ Oe}^{-2}$.

3. Experimental results

Disc-shaped nickel-polymer-PZT-polymer-nickel structures were manufactured for the experiment by the bonding method. Samples had the following parameters: diameter D = 8.7mm, thickness of the piezoelectric PZT layer $t_p = 0.32$ mm, thickness of the nickel layer $t_m = 0.25$ mm, thickness of the bonding polymer layer was of some micrometers.

The frequency dependence of the ME effect in low-frequency region and in the electromechanical resonance region is shown in Fig.4.



Fig. 4. Frequency dependence of the ME effect in disc-shaped trilayer structure of nickel-polymer-PZT-polymer-nickel. Bias magnetic field $H_{bias} = 50$ Oe



Fig. 5. Field dependence of the ME effect in disc-shaped trilayer structure of nickelpolymer-PZT-polymer-nickel

As can be seen from Fig.4, the main resonance is observed at frequency f = 338kHz. There is an additional resonance at frequency f = 169kHz, due to the nonlinear ME effect, which is in full accordance with the theory.

Field dependence of the ME effect is shown in Fig.5.

4. Conclusions

Magnetoelectric effect in composite multiferroics is a result of the mechanical interaction of the structure layers, which is carried out by tangential stresses, accompanied by shear strain. This leads to inhomogeneous change of the oscillation amplitude in the direction perpendicular to the interface. The compatibility condition for solving the equations of motion for magnetostrictive and piezoelectric phases resulting from the boundary conditions, leads to a nonlinear relationship between the frequency and the wave number. Linear ME effect and nonlinear ME effect of external magnetic field occurs due to the nonlinear dependence of the magnetostriction on the magnetic field. In contrast to the linear ME effect, it is nonzero in absence of the magnetizing field and its value is comparable with the linear ME effect in weak magnetization fields.

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