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RESONANCE AND LOCALIZED SHEAR VIBRATION OF BI-MATERIAL ELASTIC RESONATOR

Ghazaryan K.B., Papyan A.A.

Key words: Shear waves, localized waves, resonator, internal resonance.
Ключевые слова: сдвиговые волны, локализованные волны, внутренний резонанс.
Բանալի բառեր. սահքի ալիքներ, տեղայնացված ալիքներ, ներքին ռեզոնանս

Ղազարյան Կ.Բ., Պապյան Ա.Ա.

Ռեզոնանսային և տեղայնացված սահքի տատանումները բաղադրյալ առաձգական ռեզոնատորում

Աշխատանքը նվիրված է բաղադրյալ ռեզոնատորում սահքի տատանումների խնդրին, երբ ռեզոնատորը բաղկացած է երկու տարբեր առաձգական նյութերից և ունի ուղղանկյուն հատույթ: Դիտարկվել է այն դեպքը, երբ ռեզոնատորի մի կողմը ազատ է լարումներից, իսկ մնացած երեք կողմերը կոշտ ամրակցված են: Ցույց է տրված, որ գոյություն ունեն երկու տիպի տատանումներ`տեղայանացված և սեփական: Ցույց է տրված երկու տարբեր տիրույթներում տեղայանցված և սեփական տատանումների հաճախությունների համընկնման հնարավորությունը:

Казарян К.Б., Папян А.А.

Резонансные и локализованные сдвиговые колебания в упругом составном резонаторе

В работе рассмотрена задача сдвиговых колебаний составного резонатора прямоугольного сечения, состоящего из двух различных упругих материалов, когда одна из сторон резонатора свободна от напряжений, а остальные три жёстко закреплены. Установлено существование двух различных типов колебаний: локализованных и собственных. Показана возможность совпадения локализованных и собственных частот колебаний различных форм, приводяшее к эффекту внутреннего резонанса.

The paper is dedicated to the problem of shear vibration of compound resonator, made from two different elastic materials, with rectangular cross section, when one side of the resonator is traction free, three other sides are clamped. The existence of two different types of vibration, namely localized and natural types are established. Possibility of coinciding of localized and natural frequencies from two different spectrums are shown, resulting in the internal resonance occurrence that does not exist in one phase material resonator, with ordinary boundary conditions.

Introduction.

A number of studies and reviews devoted to specific cases of localized waves edge resonance in elastic systems are presented in [1].The correlation between effects of resonance and localisation of shear waves in elasic resonator were have been firstly reported in a modal problem [2], where was shown that due to vibration localisation frequencies the internal resonance can occur. In [3] classical compound systems are analyzed formed by the pairs of coupled resonators, including a system of elastically coupled masses, a system of rigid rods separated by a notch, and an optical system made by a pair of dielectric films separated by a thin metallic layer. Non linear effects in elastic resonators are considered in [4].

Statement of the problem.

In Cartesian system (x, y, z) a two phase bi-material elastic resonator is considered, occupying a region $-b \le x \le a;\ 0 \le y \le d;\ -\infty < z < \infty$. The resonator consists of the two different elastic materials: (1) of length b, bulk density $\rho^{(1)}$, shear modules $G^{(1)}$ and (2) of length a, bulk density $\rho^{(2)}$, shear modules $G^{(2)}$ (Fig.1).

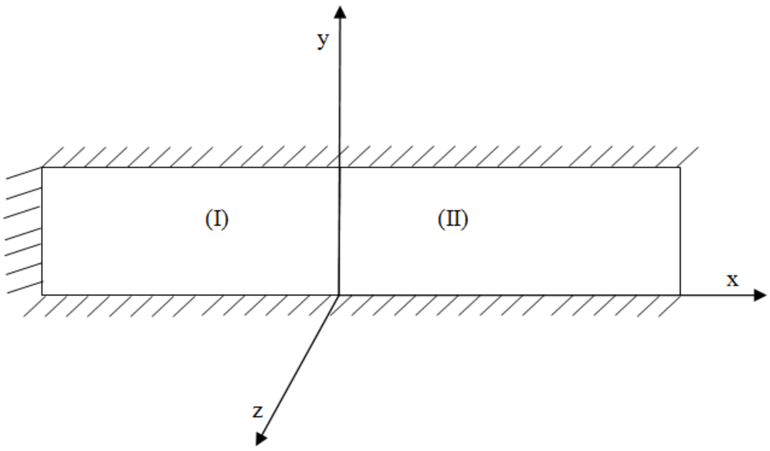

Fig.1. Resonator's cross-section

$s = 1,\ \ s = 2$ stand for first and second materials, correspondingly.

We take the following boundary conditions at the resonators walls

$$U^{(1)} = 0;\ \ U^{(2)} = 0 \ \ y = 0, \quad y = d, \tag{1}$$

$$U^{(1)} = 0 \ \ x = -b, \quad \frac{\partial U^{(2)}}{\partial x} = 0; \quad x = a. \tag{2}$$

We also take the ideal contact conditions of continuity for the displacements and the stresses of two different materials at the interface $x = 0$

$$U^{(1)} = U^{(2)}\ ; \quad G^{(1)} \frac{\partial U^{(1)}}{\partial x} = G^{(2)} \frac{\partial U^{(2)}}{\partial x}. \tag{3}$$

Solutions of the problem

The solutions of Eq.(1) satisfying boundary conditions at $y = 0, y = d$ we present in the form

$$U^{(s)}(x,y,t)=\sum_{n=1}^{\infty}U_{0n}^{(s)}(x)\sin(\lambda_n y)\exp(i\,\omega t)$$

$$\lambda_n=\frac{\pi n}{d};\; n=1,2,3..... \tag{4}$$

Functions $U_{0n}^{(s)}(x)$ satisfy the equations

$$\frac{d^2U_{0n}^{(1)}}{dx^2}+\lambda_1^2\left(\eta^2-n^2\right)U_0^{(1)}=0, \tag{5}$$

$$\frac{d^2U_{0n}^{(2)}}{dx^2}+\lambda_1^2\left(\alpha^2\eta^2-n^2\right)U_0^{(2)}=0,$$

Here $\eta=\frac{\omega}{\lambda_1 c_1};\quad c_1^2=\frac{G^{(1)}}{\rho^{(1)}};\; c_2^2=\frac{G^{(2)}}{\rho^{(2)}};\;\; \alpha=\frac{c_1}{c_2}$.

Solutions of Eq. (5) satisfying boundary condition $x=-b$ and contact conditions at $x=0$ can be written as

$$U_{0n}^{(1)}(x)=C\left(\sinh\left(x\lambda_1\sqrt{n^2-\eta^2}\right)+\tanh\left(b\lambda_1\sqrt{n^2-\eta^2}\right)\cosh\left(x\lambda_1\sqrt{n^2-\eta^2}\right)\right),$$

$$U_{0n}^{(2)}(x)=C\left(\frac{\gamma\sqrt{n^2-\eta^2}}{\sqrt{n^2-\alpha^2\eta^2}}\sinh\left(x\lambda_1\sqrt{n^2-\alpha^2\eta^2}\right)+\right.$$
$$\left.+\tanh\left(b\lambda_1\sqrt{n^2-\eta^2}\right)\cosh\left(x\lambda_1\sqrt{n^2-\alpha^2\eta^2}\right)\right) \tag{6}$$

Here C is an arbitrary constant, $\gamma=G^{(2)}/G^{(1)}$.

Satisfying solutions $U_{0n}^{(2)}(x)$ to the boundary condition at $x=a$ we get the dispersion equation defining dimensionless frequencies η

$$\frac{\gamma\sqrt{n^2-\eta^2}}{\sqrt{n^2-\alpha^2\eta^2}}+\tanh\left(b\lambda_1\sqrt{n^2-\eta^2}\right)\tanh\left(a\lambda_1\sqrt{n^2-\alpha^2\eta^2}\right)=0 \tag{7}$$

Analysis of dispersion equation

In the frequency regions $\eta<n$ if $\alpha\le 1$; $\eta<n\alpha^{-1}$ if $\alpha\ge 1$ the dispersion equation (7) has not solutions. In other regions of η the dispersion equation defines spectral correlations

for the resonator frequencies and may have two types of solution as it occurs in the problems of shear waves propogation in layered waveguides [5,6], and in the modal problem, considered in [2] .

The first type of solution gives a series of modes corresponding to natural vibration in the frequency regions $\eta > n\alpha^{-1}(\alpha < 1)$ or $\eta > n(\alpha > 1)$, $n = 1, 2.....$

The second type gives a series of modes corresponding to localized vibration in the frequency regions $m < \eta < m\alpha^{-1}$ if $\alpha < 1$ or $m\alpha^{-1} < \eta < m$ if $\alpha > 1$ $(m = 1, 2.....)$

In the frequency regions, $\eta > n\alpha^{-1}(\alpha < 1)$ or $\eta > n(\alpha > 1)$ we have the dispersion equation defining the spectrum of the resonator natural frequencies

$$\frac{\gamma\sqrt{\eta^2 - n^2}}{\sqrt{\alpha^2\eta^2 - n^2}} - \tan\left(b\lambda_1\sqrt{\eta^2 - n^2}\right)\tan\left(a\lambda_1\sqrt{\alpha^2\eta^2 - n^2}\right) = 0 \tag{8}$$

In the frequency region $m < \eta < m\alpha^{-1}$, $m = 1, 2....$ when $\alpha < 1$ me have the following dispersion equations defining the spectrum of the resonator localized frequencies

$$\frac{\gamma\sqrt{\eta^2 - m^2}}{\sqrt{m^2 - \alpha^2\eta^2}} + \tan\left(b\lambda_1\sqrt{\eta^2 - m^2}\right)\tanh\left(a\lambda_1\sqrt{m^2 - \alpha^2\eta^2}\right) = 0 \tag{9}$$

When $\alpha > 1$ in region $m\alpha^{-1} < \eta < m$ $m = 1, 2.....$ the dispersion equation defining the spectrum of the resonator localized frequencies can be written as

$$\frac{\gamma\sqrt{m^2 - \eta^2}}{\sqrt{\eta^2 - \alpha^2 m^2}} - \tanh\left(b\lambda_1\sqrt{m^2 - \eta^2}\right)\tan\left(a\lambda_1\sqrt{\eta^2 - \alpha^2 m^2}\right) = 0 \tag{10}$$

When $\alpha = 1$ the dispersion equation of natural frequencies as the form

$$\gamma - \tan\left(b\lambda_1\sqrt{\eta^2 - n^2}\right)\tan\left(a\lambda_1\sqrt{\eta^2 - n^2}\right) = 0 \tag{11}$$

while the dispersion equation of localized frequencies

$$\gamma + \tanh\left(b\lambda_1\sqrt{m^2 - \eta^2}\right)\tanh\left(a\lambda_1\sqrt{m^2 - \eta^2}\right) = 0\sqrt{b^2 - 4ac} \tag{12}$$

has no solutions.

α		η_* $a\lambda_1 = 1$ $b\lambda_1 = 0.5$	η_* $a\lambda_1 = 0.5$ $b\lambda_1 = 1$
0.1		22.70	17.17
0.3		17.51	12.27
0.5		11.10	11.38
0.7		9.22	10.42
0.9		7.77	9.69
1.0		7.19	9.48
1.1		6.79	9.30
1.3		6.22	8.89
1.5		5.77	8.25
1.7		5.31	7.55
1.9		4.85	6.94
2.0		4.63	6.79

Table 1. Minimal natural frequencies of the resonator first mode

Based on the numerical analysis of the dispersion equations (8,11) defining the natural frequencies η_*, in the Table 1 the data for the minimal natural frequencies η_* related to dependence from geometrical parameter α are presented for first mode $n = 1$ of the resonator oscillation. The numerical calculations have been carried out for resonators with parameters γ =0.5 $a\lambda_1 = 1$, $b\lambda_1 = 0.5$. Data of the Table 1 shows that the minimal frequencies decreasing with increase of α. On the other hand, the localized vibration frequencies increasing with increase of mode number m and in some cases the frequency of m mode of localized vibration may coincide with minimal frequency of natural vibration of $n = 1$ mode. The coincidence of these frequencies results in the effect of an internal resonance.

$a\lambda_1$	$b\lambda_1$	α	$\eta_{nat} \approx \eta_*$	m
0.1	0.1	0.5	31.46	24
0.6	0.2	0.7	17.24	14
0.09	0.03	0.3	68.52	33
0.5	0.1	0.5	31.47	24
2.5	5.0	2.0	1.35	2
0.6	2.0	1.7	1.83	2
0.5	5.0	1.2	1.65	2
2.5	5.0	1.2	1.62	2

Table 2. Resonance frequencies data

On the Table 3 the resonance frequencies of localized and natural frequencies are presented for different cases where the internal resonance occur. The number m corresponds to localized vibration frequencies modes, the minimal frequencies of natural vibration correspond to $n = 1$. The numerical calculations have been carried out for resonators with γ=0.5, for different cases of $a\lambda_1, b\lambda_1$.

Conclusion

Shear vibration of bi-material elastic resonator with rectangular cross section is considered, when one side of the resonator is traction free, three other sides are clamped. The corresponding dispersion equations are obtained defining spectral correlations for resonator frequencies. It is shown that dispersion equation may have two different kinds of frequency spectrums, namely natural frequency spectrum and localized frequency spectrum. The equation of frequency spectrums are analyzed numerically in detail. Possibility of coinciding (internal resonance) of frequencies from two spectrums are shown. Based on numerical analysis the resonance frequencies of localized and natural frequencies are presented for different cases where the internal resonance occur.

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About authors

Ghazaryan Karen – Doctor of sciences, professor, principal researcher, Institute of mechanics NAN of Armenia. **Phones:** (374 10) 22 73 95, (374 955) 22 73 95. **E-mail:** ghkarren@gmail.com

Papyan Ararat – Phd, science researcher, Institute of mechanics NAN of Armenia. **Phone:** (374 93) 05 00 93. **E-mail:** aro088@mail.ru

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