ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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PROPAGATION OF ELASTIC WAVES IN A PLANE WAVEGUIDE LAYER ON THE BASIS OF A SIMPLIFIED MODEL OF THE COSSERAT CONTINUUM

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Keywords: micropolar material, Cosserat continuum, acoustic waves, elastic waveguide layer, wave energy localization, localized waves.
Բանալի բառեր. Միկրոպոլյար նյութ, Կոսերայի միջավայր, ակուստիկ ալիքներ, առաձգական ալիքատար շերտ, ալիքային էներգիայի տեղայնացում, տեղայնացված ալիքներ:
Ключевые слова: микрополярная среда, континуум Коссера, акустические волны, упругий слой-волновод, локализация волновой энергии, локализованные волны.

Համբարձումյան Ս.Ա.,Ավետիսյան Ա.Ս., Բելուբեկյան Մ.Վ.,
Առաձգական ալիքների տարածումը հարթ շերտ-ալիքատարում, Կոսերայի միջավայրի պարզեցված մոդելի հաշվառումով

Դիտարկվում է բարձր-հաճախականային և ցածր-հաճախականային ակուստիկ ալիքների տարածումը առաձգական հարթ շերտ-ալիքատարում, Կոսերայի միջավայրի պարզեցված մոդելի հաշվառմամբ: Հարթ և հակահարթ դեֆորմացիաների խնդիրներում, տարբեր եզրային պայմանների դեպքում, ձևակերպված են եզրային արժեքի խնդիրները հաշվի առնելով նյութի միկրոպոլյար հատկությունը: Երկար ալիքային և կարճ ալիքային մոտարկումների դեպքում ստացված արդյունքները համադրված են առաձգականության դասական տեսության արդյունքների հետ: Բացահայտված են ալիքատարի մակերևույթների մոտ ալիքային էներգիայի հնարավոր տեղայնացման պայմանները: Ցույց է տրված, որ հակահարթ դեֆորմացիայի խնդրում նյութի միկրոպոլյար հատկությունը սահքի ալիքների մակերևութային տեղայնացման չի բերում: Գտնվել են միկրոպոլյարության հաշվառման դեպքում հարմոնիկ ալիքների տարածման սեղմված հաճախականայի գոտիները: Ցույց է տրված, որ հարթ դեֆորմացիայի ալիքային ազդանշանի դեպքում միկրոպոլյար հատկությունը կարող է բերել նոր տեղայնացված ալիքի տարածման:

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Распространение упругих волн в плоском слое-волноводе с учётом упрощённой модели континуума Коссера

Рассматривается распространение высокочастотных и низкочастотных акустических волн в плоском упругом слое-волноводе на основе упрощённой модели среды Коссера. Учитывая наличие микро-полярности среды, для различных комбинаций граничных условий на поверхности волновода сформулированы граничные задачи плоской и антиплоской деформаций. В длинноволновом и коротковолновом приближениях полученные результаты сравниваются с результатами классической теории упругости. Найдены условия для возможной локализации волновой энергии вблизи поверхности волновода. Показано, что учёт микрополярности материала в задаче антиплоской деформации не приводит к существованию высокочастотных локализованных форм. В задаче плоской деформации учёт микро-полярности материала, при различных граничных условиях на поверхности волновода может вызвать как искажение частотного диапазона существования локализованных волн Рэлея, так и привести к появлению нового частотного диапазона возможных локализованных волн плоской деформации. Найдены частотные полосы локализованных и гармонических форм колебаний.

The problem of propagation of high-frequency and low-frequency acoustic waves in a plane elastic waveguide layer on the basis of a simplified model of the Cosserat continuum is considered. In view, the presence of micro polarity of the medium, boundary value problems for plane and antiplane deformations for different combinations of boundary conditions on the waveguide surface are formulated. In a long-wave and a short-wave approximations the obtained results are compared with the results of the classical theory of elasticity. The conditions for a possible localization of the wave energy near the surface of the waveguide are found. It is shown that in the antiplane deformation problem, the considering of micropolarity of the material is not leads to the possibility of existence of high localized forms. The frequency bands of localized and harmonic waveforms are found. In the plane deformation problem, considering of micropolarity of the material under different surface conditions may cause as a distortion of the frequency band of the localized Rayleigh wave existence so as the emergence of a new frequency band of possible localized waves.

Introduction. In the classical theory of elasticity it is known that the reason of localization of high-frequency waves near the medium interface boundary is the disturbance of homogeneity of effective physical and mechanical characteristics of these fields. We first encounter the problem of wave energy localization in the primary sources [1÷4]. Within the framework of the classical theory of elasticity, more about localization of wave energy near the medium interface boundary can be found in [5÷7], and others. However, when changes in the microstructure of the body are essential (that is near the cracks and chipping, where the stress gradients are essential) there appears a discrepancy between the results of the classical theory of elasticity and the experiments' results. Such discrepancies also appear in the case of granular medium and multi molecular structures, such as polymers.

The influence of microstructure is particularly evident in the case of elastic oscillations of high frequency and short wavelength. W. Voigt [8] attempted to overcome the disadvantages of the classical theory of elasticity under the assumption that the interaction of two parts of the body through the area element is transmitted not only by the force vector, but also by the vector momentum. However, the complete theory of asymmetric elasticity was developed only in 1909 by Francois and Eugene Cosserat brothers [9]. Currently Cosserat theory is in rapid development.

There is an extensive literature on the study of mechanics problems based on the micropolar theory of elasticity (or based on the Cosserat continuum). General works of A. C. Eringen and others [10,11] and Vladimir Yerofeyev's work [12] should be noted.

In this article the problems of waves propagation in a flat elastic waveguide with due regard to the internal rotation of the medium particles are considered. The limiting cases of short and long waves (high and low frequency acoustic waves) on the basis of a simplified model of the Cosserat continuum are investigated.

1. Basic relations of a simplified model of the Cosserat continuum.

In general, the motion equations in the asymmetric elasticity theory are written as:

$$\begin{aligned} &\sigma_{ji,j} + X_i = \rho \ddot{u}_i \\ &\mu_{ji,j} + \in_{ijk} \sigma_{jk} + Y_i = J \ddot{\varphi}_i \end{aligned} \tag{1.1}$$

where σ_{ij} and μ_{ik} are force and moment stresses, respectively, X_i and Y_i are mass forces, $\in_{ijk}$ is the Levi-Civita tensor, ρ is the material density, u_i are the displacement vector components, φ_i are the rotation vector components at medium unit point, J is the rotary inertia.

The material relations of isotropic material for σ_{ji} force and μ_{ji} moment stresses are:

$$\begin{aligned} &\sigma_{ji} = (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ji} + \lambda\delta_{ji}\gamma_{kk} \\ &\mu_{ji} = (\gamma+\varepsilon)\omega_{ji} + (\gamma-\varepsilon)\omega_{ij} + \beta\delta_{ji}\omega_{kk} \end{aligned} \tag{1.2}$$

These relations (1.2) involve material constants $\lambda;\ \mu;\ \alpha;\ \beta$, which are independent, and Kronecker delta δ_{ik}. And besides, these material constants and their combinations are positive definite ones

$$\mu > 0\ ;\quad \gamma > 0\ ;\quad \alpha > 0\ ;\quad \varepsilon > 0\ ;\quad 3\lambda + 2\mu > 0\ ;\quad 3\beta + 2\gamma > 0\ .$$

Now, if we exclude σ_{ij} and μ_{ik} stresses from the motion equations (1.1), using the constitutive equations and defining relations for tensors

$$\gamma_{ji} = u_{i,j} - \in_{kji} \varphi_k\ ,\qquad \omega_{ji} = \varphi_{i,j}$$

we will obtain a system in vector form of six equations in terms of displacements $\vec{u} = \{u_i\}$ and rotations $\vec{\varphi} = \{\varphi_i\}$:

$$\begin{aligned} &\square_2 \vec{u} + (\lambda + \mu - \alpha)\,\mathrm{grad\,div}\,\vec{u} + 2\alpha\,\mathrm{rot}\,\vec{\varphi} + \vec{X} = 0 \\ &\square_4 \vec{\varphi} + (\beta + \gamma - \varepsilon)\,\mathrm{grad\,div}\,\vec{\varphi} + 2\alpha\,\mathrm{rot}\,\vec{u} + \vec{Y} = 0 \end{aligned} \qquad (1.3)$$

Where vector and operators $\square_2$ and $\square_4$ are given as

$$\square_2 = (\mu + \alpha)\Delta - \rho\partial_t^2\ ,\qquad \square_4 = (\gamma + \varepsilon)\Delta - 4\alpha J\partial_t^2\ .$$

Many authors have investigated the problem of distribution and localization of elastic waves by means of a system of general equations of the asymmetric elasticity theory. Ambartsumyan S.A. and Belubekyan M.V. [13] have also investigated the generalized Rayleigh waves in a micropolar continuous medium. V.R. Parfitt and A.C. Eringen [14], as well as J. Stefaniak [15] have investigated the reflection of a plane wave from a free boundary of the half space.

The same problem was discussed in the expanded paper of S. Kaliski, J. Kupelewski and C. Rymarz [16]. Propagation of waves in a plate and generalized Lamb waves have been considered in W.Nowacki and W.K. Nowacki articles [17-18].

In general, the equations and relations in the micropolar theory are quite complex, so far simple models [19 ÷ 21] are used often for solving some specific problems. On the other hand, the most significant effects, associated with moment stresses under consideration, occur in dynamic problems. For such problems, in particularly, where elastic wave propagation is studied, a simple model considering only the dynamics of the internal rotation of the particles, was proposed on the basis of the Cosserat model. Simplified Cosserat model for dynamic problems, apparently independently of one another, has been proposed in works [22 ÷ 24].

In the Cartesian coordinate system $\{x_i\}$ for a simplified Cosserat model the known linear motion equations of the classical theory of elasticity are applied

$$\frac{\partial \sigma_{ji}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2};\qquad i;\, j \in \{1; 2; 3\} \qquad (1.4)$$

However, in (1.4) the shear stresses are not symmetrical, and are defined as a generalization of the classical Hooke's law for isotropic material

$$\sigma_{ij} = 2\mu\gamma_{ij} + \delta_{ij} \nu\gamma_{kk} + J\left(\partial^2 \omega_{ij} / \partial t^2\right) \qquad (1.5)$$

where strain tensor γ_{ij} is defined by the usual way

$$\gamma_{ij}=\frac{1}{2}\left(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j}\right) \tag{1.6}$$

and the transposed tensor of additional rotations ω_{ij} defines the asymmetry of shear stresses $\left(\omega_{ij}=-\omega_{ji}\right)$

$$\omega_{ij}=\frac{1}{2}\left(\frac{\partial u_j}{\partial x_i}-\frac{\partial u_i}{\partial x_j}\right). \tag{1.7}$$

Consideration of two-dimensional dynamic problems, when all the physico-mechanical characteristics of the elastic field don't depend on the coordinate x_3, i.e. $\left(\partial/\partial x_3\equiv 0\right)$, is much easier. As in the general micropolar elasticity theory, in the simplified theory of the Cosserat continuum the constitutive equations (1.5) and the equations of motion (1.4) allow the separation of the problems to plane and antiplane deformations.

This model was used to solve a number of problems on propagation of acoustic waves, the reviews are given in [20, 25].

In the next, for convenience instead of $\{x_i\}$ coordinates we will use $\{x;y;z\}$ coordinates, and instead of the displacement vector components $\{u_i\}$ we will use $\{u;\mathrm{v};w\}$ notation.

Then, for plane strain problems from (1.4) in view of (1.5) ÷ (1.7) the following equations of motion are obtained:

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2}; \qquad \frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=\rho\frac{\partial^2 \mathrm{v}}{\partial t^2}. \tag{1.8}$$

The corresponding constitutive equations are

$$\begin{gathered}\sigma_{xx}=(\lambda+2\mu)\gamma_{xx}+\lambda\gamma_{yy}; \qquad \sigma_{yy}=(\lambda+2\mu)\gamma_{yy}+\lambda\gamma_{xx};\\ \sigma_{xy}=2\mu\gamma_{xy}+J\frac{\partial^2\omega_{xy}}{\partial t^2}; \qquad \sigma_{yx}=2\mu\gamma_{xy}+J\frac{\partial^2\omega_{yx}}{\partial t^2}.\end{gathered} \tag{1.9}$$

The defining relations are

$$\begin{gathered}\gamma_{xx}=\frac{\partial u}{\partial x}; \quad \gamma_{xx}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial \mathrm{v}}{\partial x}\right); \quad \gamma_{yy}=\frac{\partial \mathrm{v}}{\partial y};\\ \omega_{xy}=\left(\frac{\partial u}{\partial y}-\frac{\partial \mathrm{v}}{\partial x}\right)=-\omega_{yx}.\end{gathered} \tag{1.10}$$

For antiplane strain problems we obtain the equations of motion:

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}=\rho\frac{\partial^2 w}{\partial t^2} \tag{1.11}$$

the constitutive equations:

$$\sigma_{xz} = 2\mu\gamma_{xz} + J\frac{\partial^2\omega_{xz}}{\partial t^2}; \quad \sigma_{zx} = 2\mu\gamma_{zx} + J\frac{\partial^2\omega_{zx}}{\partial t^2}$$
$$\sigma_{yz} = 2\mu\gamma_{yz} + J\frac{\partial^2\omega_{yz}}{\partial t^2}; \quad \sigma_{zy} = 2\mu\gamma_{zy} + J\frac{\partial^2\omega_{zy}}{\partial t^2} \tag{112}$$

the defining relations:

$$\gamma_{xz} = \frac{1}{2}\frac{\partial w}{\partial x}; \quad \gamma_{yz} = \frac{1}{2}\frac{\partial w}{\partial y}; \quad \omega_{xz} = \frac{\partial w}{\partial x} = -\omega_{zx}; \quad \omega_{yz} = \frac{\partial w}{\partial y} = -\omega_{zy} \tag{1.13}$$

2. The antiplane deformation problem. Let us consider an elastic, homogeneous isotropic waveguide, which occupies the region $\Omega \triangleq \{-\infty < x < \infty; \ \ 0 \le y \le h; \ \ -\infty < z < \infty\}$
Equations of purely shear waves (1.11), with accounting the material equations (1.12) and defining relations (1.13) are reduced to the form:

$$\mu\Delta w + J\frac{\partial^2}{\partial t^2}\Delta w = \rho\frac{\partial^2 w}{\partial t^2}. \tag{2.1}$$

Presenting the solution of equation (2.1) in the form of normal harmonic waves

$$w(x,y,t) = w_0(y)\exp[i(\omega t - kx)] \tag{2.2}$$

where ω is the oscillation frequency, $k \triangleq 2\pi/\lambda$ is the wave number, λ is the length of the wave, $w_0(y)$ is amplitude function, which determines the distribution across the waveguide's thickness, we obtain the following ordinary differential equation:

$$w_0''(y) + k^2q^2w_0(y) = 0, \tag{2.3}$$

where

$$q^2 \triangleq \eta^2\big/\left(1-\beta_k\eta^2\right) - 1 \ ; \ \eta^2 \triangleq \omega^2/(k^2c_t^2) \ ; \ c_t^2 \triangleq \mu/\rho \ ; \ \beta_k \triangleq (Jk^2)/\rho \tag{2.4}$$

From these notations one can see that for all wave numbers k values $\eta > 0$ and $\beta_k > 0$ are positive.

The condition of the existence of harmonic oscillations $q^2 > 0$ is easily obtained from the first notation (2.4) in the form of

$$1/(1+\beta_k) < \eta^2 < 1/\beta_k \quad \text{or} \quad \sqrt{\frac{\mu k^2}{Jk^2+\rho}} < \omega < \sqrt{\mu/J} \tag{2.5}$$

If the condition (2.5) is valid, the general solution of equation (2.3) can be represented by trigonometric functions as

$$w_0(y) = A\sin(kqy) + B\cos(kqy) \tag{2.6}$$

It should be noted, that when the micro rotation does not take into account (if $J \equiv 0$ then $\beta_k \to 0$), the condition (2.5) takes the known form $\eta > 1$. From (2.5) it is also obvious that

for normal short waves, when $\lambda \ll 2\pi\sqrt{J/\rho}$, the frequency range is very narrow $\sqrt{\mu/J} - o(\lambda^2\rho/4\pi^2 J) < \omega < \sqrt{\mu/J}$.
Consequently, the condition for the existence of harmonic waves is transformed into

$$\omega_n \in \left(\sqrt{\mu/J} - o(\lambda^2\rho/4\pi^2 J);\ \sqrt{\mu/J}\right) \tag{2.7}$$

Here ω_n is the oscillation frequency of corresponding harmonic. The resulting waveforms and the corresponding frequencies are determined by the boundary conditions on the waveguide walls.
In particularly, the problems for a waveguide with boundary conditions of clamped or traction free wall, according to (2.6) lead to the definition of the phase velocity satisfying (2.5). Herewith consideration of the internal rotation reduces the given phase velocity $\sqrt{\eta}$. If waveguide walls $y=0$ and $y=h$ are clamped: $w_0(0)=0$ and $w_0(h)=0$, then from the dispersion equation the values of natural frequencies are obtained:

$$q_n \triangleq \frac{n\pi}{kh} \text{ ; where } n = 0;\ 1;\ 2;\ \ldots \tag{2.8}$$

$$\eta^2 = \left(1-q_n^2\right) \Big/ \left[1+\beta_k\left(1-q_n^2\right)\right] \ ;$$

The condition of existence (2.5) of the n[th] oscillations harmonic is transformed into

$$\sqrt{\frac{\mu k^2}{Jk^2+\rho}} < kc_t \cdot \sqrt{\frac{1+n^2\pi^2/k^2h^2}{1+(Jk^2/\rho)\cdot\left(1+n^2\pi^2/k^2h^2\right)}} < \sqrt{\mu/J} \tag{2.9}$$

and the value $n=0$ corresponds to the limiting wave, for which the frequency is defined as $\omega_{01} = \sqrt{k^2\mu/\left(\rho+Jk^2\right)}$. For higher harmonics when $n\to\infty$, the limiting frequency is $\omega_{02} = \sqrt{\mu/J}$ (Fig.1a).
From (2.9) it also follows that in this frequency range there always exist harmonics with numbers $n \geq \left[2h/\lambda\right]$.

From (2.9), taking into account (2.4), it follows that the phase velocity of the n^{th} harmonic is represented as

$$\mathrm{v}_f(k) \triangleq \frac{\omega}{k} = \sqrt{\mu\left(1+\left(n\pi/kh\right)^2\right)\Big/\left[\rho+Jk^2\left(1+\left(n\pi/kh\right)^2\right)\right]}$$

The behavior of harmonics phase velocity is shown on Fig. 1b.
In frequency intervals

$$0<\eta<1/\left(1+\beta_k\right) \quad \text{or} \quad \eta>1/\beta_k \tag{2.10}$$

according to (2.4) we get $q^2<0$. Then, non-harmonic solutions of the wave formation equation (2.3), with the notation $p \triangleq iq = \sqrt{1-\rho\left(\omega^2/k^2\right)/\left(\mu-J\omega^2\right)}$, are represented by hyperbolic functions

$$w_0(y) = A \cdot \mathrm{sh}(kpy) + B \cdot \mathrm{ch}(kpy) \tag{2.11}$$

For the frequency and phase velocity of the normal wave, we obtain the ranges

$$0 < \omega < \sqrt{\frac{\mu k^2}{Jk^2 + \rho}} \quad \text{or} \quad \omega > \sqrt{\mu/J} \tag{2.12}$$

$$0 < \mathrm{v}_f(k) < \sqrt{\frac{\mu}{Jk^2 + \rho}} \quad \text{or} \quad \mathrm{v}_f(k) > k^{-1} \cdot \sqrt{\mu/J}$$

Here $\sqrt{\mu k^2 / \left(Jk^2 + \rho\right)}$ and $\sqrt{\mu/J}$ are the limiting frequencies for harmonic formations of normal waves across the waveguide thickness.

For a waveguide with camped walls or traction free walls according to (2.12) the problem under consideration leads to determination of an oscillation frequency (or phase velocity) which does not satisfy condition (2.11).

3. The plane deformation problem. In the elastic, isotropic homogeneous waveguide $\Omega \triangleq \{-\infty < x < \infty;\ \ 0 \le y \le h;\ \ \infty < z < \infty\}$ the plane strain equations (1.8) in view of the material equations (1.9) and the defining relations (1.10) are reduced to the system of equations for the components $u(x, y, t)$ and $\mathrm{v}(x, y, t)$ of the displacement vector

$$\begin{aligned} &\mu\Delta u + (\lambda + \mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y}\right) - J\frac{\partial^3}{\partial y \partial t^2}\left(\frac{\partial \mathrm{v}}{\partial x} - \frac{\partial u}{\partial y}\right) = \rho\frac{\partial^2 u}{\partial t^2} \\ &\mu\Delta \mathrm{v} + (\lambda + \mu)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y}\right) - J\frac{\partial^3}{\partial x \partial t^2}\left(\frac{\partial \mathrm{v}}{\partial x} - \frac{\partial u}{\partial y}\right) = \rho\frac{\partial^2 \mathrm{v}}{\partial t^2} \end{aligned} \tag{3.1}$$

By means of Lame's transformation for plane strain problems

$$u \triangleq \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}\ ; \qquad \mathrm{v} \triangleq \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x} \tag{3.2}$$

the system of equations (3.1) gives a separate equations of longitudinal and transverse waves [28], for Lame's functions $\varphi(x, y, t)$ and $\psi(x, y, t)$

$$(\lambda + \mu)\Delta\varphi = \rho\frac{\partial^2 \varphi}{\partial t^2} \tag{3.3}$$

$$\mu\Delta\psi + J\frac{\partial^2}{\partial t^2}(\Delta\psi) = \rho\frac{\partial^2 \psi}{\partial t^2} \tag{3.4}$$

Equation (3.4) coincides with equation (2.1) of the antiplane problem. Representing the solutions of equations (3.3) and (3.4) in a form of normal wave

$$\varphi(x, y, t) = \varphi_0(y)\exp\left[i(\omega t - kx)\right];\ \psi(x, y, t) = \psi_0(y)\exp\left[i(\omega t - kx)\right] \tag{3.5}$$

we obtain ordinary differential equations for amplitude functions $\varphi_0(y)$ and $\psi_0(y)$, which general solutions are

$$\varphi_0(y) = A\sin(kpy) + C\cos(kpy)\ ;\ \ \psi_0(y) = D\sin(kqy) + B\cos(kqy) \tag{3.6}$$

Where q has the same notation as in (2.4) and

$$p \triangleq \sqrt{\theta\eta - 1} \; ; \quad \theta \triangleq \mu/(\lambda + 2\mu) \tag{3.7}$$

Let us consider a wave process in a waveguide with different boundary conditions.

3.1. Navier Conditions on both walls of the waveguide. Suppose that Navier conditions are defined on the wall $y = 0$ of the waveguide

$$\varphi = 0; \quad \partial\psi/\partial y = 0 \tag{3.8}$$

Satisfying solution (3.6) to conditions (3.8), we obtain $C = D = 0$. Thus the solution forms for the required functions $\varphi_0(y)$ and $\psi_0(y)$ are simplified, by what problems with different conditions on the other surface of the waveguide can be investigated.
The simplest version of the boundary value problem is when Navier conditions (3.8) are also given at the wall $y = h$. In that case, as in the general classical theory of elasticity, the effect of micropolarity is included in the components of the elastic displacement, while the wave equations for Lame's function (3.3) and (3.4) are completely separated from each other.

$$u(x, y, t) = -k\left[A\mathrm{sh}(kp_1 y) + kqB\cos(kqy)\right]\exp\left[i(\omega t - kx)\right] \tag{3.9}$$

$$\mathrm{v}(x, y, t) = k\left[pA\sin(kpy) + B\mathrm{ch}(kq_1 y)\right]\exp\left[i(\omega t - kx)\right] \tag{3.10}$$

Here $p_1 = ip = \sqrt{1 - \theta\eta}$, and $q_1 = iq = \sqrt{1 - \eta^2/\left(1 - \beta_k\eta^2\right)}$ are the coefficients of formation in the plane strain problem.
Moreover, wave of (3.9) and (3.10) types will exist at the phase velocity $\mathrm{v}_{1f}(\lambda) \le (\lambda/2\pi)\sqrt{\mu/\left(J + \rho\left(\lambda^2/4\pi^2\right)\right)}$ for all permitted frequencies $\omega < \sqrt{\mu/J}$. For higher frequencies $\omega > \sqrt{\mu/J}$ the waves will exist with the phase velocity $(\lambda/2\pi)\sqrt{\mu/J} < \mathrm{v}_{2f}(\lambda) \le c_l$, which length is determined by the physical characteristics of the micropolar material. Phase zones of such waves' existence are shown in Fig. 2.
The second option of setting a boundary value problem assumes that with the boundary conditions (3.8) on $y = 0$, the clamped boundary conditions on the other wall of the waveguide must be satisfied

$$u(x, h, t) \equiv 0 \; ; \quad \mathrm{v}(x, h, t) \equiv 0 \tag{3.11}$$

Conditions (3.11) with the use of required functions $\varphi_0(y), \psi_0(y)$ and in the view of (3.5), are represented as:

$$-ik\varphi_0(h) + \psi_0'(h) = 0 \; ; \quad \varphi_0'(h) + ik\psi_0(h) = 0 \tag{3.12}$$

Satisfying solution (3.6) when $C = D = 0$ to satisfy conditions (3.12), we obtain a system of algebraic equations for arbitrary constants A and B. Condition for the existence of nontrivial solutions gives the dispersion equation

$$\mathrm{tg}(kph) = -pq \cdot \mathrm{tg}(kqh) \tag{3.13}$$

Equation (3.13) always has a solution satisfying to (2.5).

The question arise does the equation has any solution satisfying the condition $0<\eta<1/(1+\beta_k)$ of (2.10) Such decision would mean the existence of localized waves near the walls of the waveguide layer. To answer this question it's sufficient to consider equation (3.13) in a short-wave approximation ($kh \gg 1$). By substituting $p = ip_1$ and $q = iq_1$, equation (3.13) reduces to the equation with hyperbolic functions, where in a short-wave approximation we obtain

$$\mathrm{th}(khp_1) \approx khp_1 \quad \text{and} \quad \mathrm{th}(khq_1) \approx khq_1 \tag{3.14}$$

The condition of the existence of waves is obtained as

$$\eta = \frac{1+\theta}{(1+\beta_k)\theta} > \frac{1}{1+\beta_k}; \tag{3.15}$$

From (3.15) it follows that (3.13) cannot have roots, which satisfy to the first condition of (2.10).

On the other hand, when

$$\beta_k > \theta \;; \quad \text{or} \quad k^2 > \rho\theta J^{-1} \tag{3.16}$$

the characteristic equation (3.13) has roots, which satisfy the second condition of (2.10)

$$\theta^{-1} > \eta > \beta_k^{-1} \quad \text{or} \quad (\lambda+2\mu)/\rho > \omega/k > \mu/(Jk^2) \tag{3.17}$$

The frequency of these waves will be limited for each length λ_0 in a way

$$2\pi(\lambda+2\mu)/(\lambda_0\rho) > \omega(\lambda) > \lambda_0\mu/(2\pi J) \tag{3.18}$$

3.2. Mixed boundary conditions on the surfaces of the waveguide. Suppose that in addition to Navier conditions (3.8) on the wall $y=0$, on the other wall $y=h$ of the waveguide the conditions of mechanically free boundary are defined

$$\sigma_{yy}(x,h,t) = 0 \quad \text{and} \quad \sigma_{yx}(x,h,t) = 0 \tag{3.19}$$

Conditions (3.19) by means of functions $\varphi(x,y,t)$ and $\psi(x,y,t)$, in the view of (3.5) reduce to

$$\begin{aligned} &(\lambda+2\mu)\varphi_0'' - k^2\lambda\varphi_0 + 2ik\mu\psi_0' = 0 \\ &-2ik\varphi_0' + (1-\beta_k\eta)\psi_0'' + k^2(1+\alpha\eta)\psi_0 = 0 \end{aligned} \tag{3.20}$$

Assuming that, in addition to conditions (3.19) on the other wall of the waveguide $y=0$ Navier conditions (3.8) are defined and using solutions (3.6) when $C = D = 0$, from (3.20) we obtain the equations for the arbitrary constants A and B. The dispersion equation

$$(2-\eta)[1+\beta_k\eta-(1-\beta_k\eta)q^2]\cdot \mathrm{tg}(khp) + 4pq\cdot \mathrm{tg}(khq) = 0 \tag{3.21}$$

is obtained from the condition of existence of nontrivial solutions.

To investigate the waves, localized at the free surface, which satisfy (2.5), it is more convenient to rewrite equation (3.21) as follows:

$$(2-\eta)[1+\beta_k\eta+(1-\beta_k\eta)q_1^2]\cdot \mathrm{th}(khp_1) - 4p_1q_1\cdot \mathrm{th}(khq_1) = 0 \tag{3.22}$$

where $p = ip_1$ and $q = iq_1$.

From equation (3.22) in a short-wave approximation, when $kh \gg 1$, we obtain the equation [26].

$$(2-\eta)[1+\beta_k\eta+(1-\beta_k\eta)q_1^2]-4p_1q_1=0 \tag{3.23}$$

If we ignore the internal rotation of the particles ($\beta_k \equiv 0$) from (3.23), we obtain the dispersion relation of Rayleigh waves [17].

In a long-wave approximation, when $k^2h^2 \ll 1$, assuming $\mathrm{th}z \approx z - z^3/3$, a well-known dispersion equation of one-dimensional bending oscillations of a plate can be obtained [27]

$$(1+4\beta_k)\eta = \frac{4(1-\theta)k^2h^2}{3} \tag{3.24}$$

From dispersion equation (3.24), the equation of oscillations of a plate can be restored as

$$D\frac{\partial^4 w}{\partial x^4} - 8hJ\frac{\partial^4 w}{\partial x^2 \partial t^2} + 2\rho h\frac{\partial^2 w}{\partial t^2} = 0, \tag{3.25}$$

where $D \triangleq \left(2Eh^3\right)/3(1-\nu^2)$.

The same analytical result was obtained in [27] on the basis of Kirchhoff's hypothesis.

4. Numerical analysis of the wave process behavior.

In the case of wave propagation for antiplane deformation in an elastic micropolar waveguide, the band of permitted frequencies is constrained by micropolarity of the material.

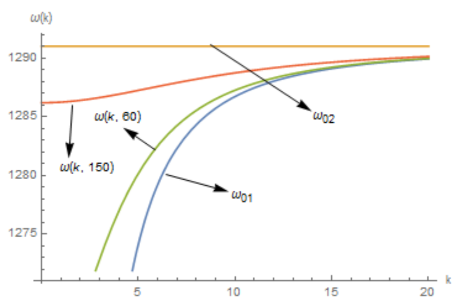

Fig. 1a. The region of frequencies of harmonic forms of shear wave's oscillations and the behavior of the natural frequencies

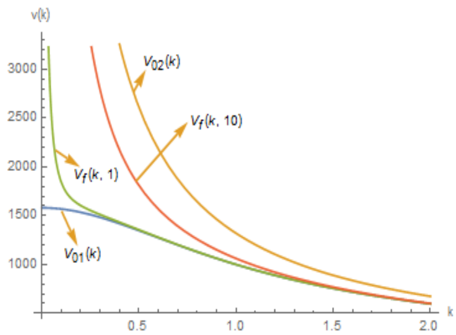

Fig. 1b. The zone and the behavior of the phase velocity of the shear wave's harmonic forms in a waveguide with mechanically free surfaces

In the case of ignoring microrotation (if $J \equiv 0$ then $\beta_k \to 0$) the condition of existence of harmonic waveforms takes the known form $\eta > 1$, while the microrotation account in the material narrows the band of the permitted frequencies and takes the form (2.9).

Figure 1a shows the frequency region of existence of waves with harmonic forms of oscillations for a waveguide of $h = 50 \cdot 10^{-3}$ m thickness from a material with physical-mechanical constants $\mu = 5 \times 10^9$ N/m², $J = 3 \times 10^3$ kg/m, , $\rho = 2 \times 10^3$ kg/m³, $\lambda = 12 \times 10^9$ N/ m².

Here we see that the natural frequencies of oscillations forms for all the harmonics are enclosed in the region $\omega_{01} < \omega_n(k) < \omega_{02}$.

For low harmonics the dispersion form with a phase velocity $v_{01}(k) = \sqrt{\mu / (Jk^2 + \rho)}$ is the limiting one, and for the higher harmonics the limiting dispersion form is the one with a phase velocity $v_{02}(k) = k^{-1}\sqrt{\mu/J}$ (Fig. 1b).

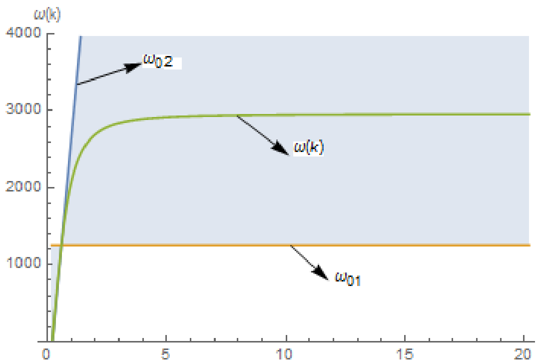

Fig. 2. The region of frequencies of plain deformation wave's localized form and the behavior of the frequency.

The calculations also show that the accounting of micropolarity of the material results in appearance of forbidden frequency zones for a shear wave harmonics in a waveguide with rigidly clamped or mechanically free surfaces $0 < \omega_n(k) < \sqrt{\mu k^2 / (Jk^2 + \rho)}$ or $\omega_n(k) > \sqrt{\mu / J}$.

In the case of plane strain wave propagation in an elastic micropolar waveguide the accounting of rotations leads to a possible localization of wave energy near the surface of the waveguide.

In frequency determination zone (3.17) the wave signal of plane deformation is localized near the surface of the waveguide and has a propagation frequency $\omega(k) = k\sqrt{(c_l^2 + c_t^2) / (1 + Jk^2/\rho)}$.

From Fig. 2 we can see that the localized wave signals of plain deformation have a wavelength $\lambda \le \pi (c_l / c_t) \sqrt{J/\rho}$.

Conclusion. On the basis of a simplified model of the Cosserat continuum, the conditions of possible localization of the wave energy with different boundary conditions on the surfaces of the elastic micropolar waveguide are obtained. The conditions for a possible localization of the wave energy near the surfaces of the waveguide are found. It is shown that in the antiplane deformation problem for a waveguide with clamped or mechanically free walls, the accoun of material micropolarity doesn't lead to the possibility of localized forms existence of high frequency. In the plain strain problem the micropolarity account of under different boundary conditions may cause a distortion of a frequency band of the existence of localized Rayleigh waves, and the emergence of a new frequency band of possible localized waves. Frequency bands of localized and harmonic waveforms are found.

In a long-wave and a short-wave approximation the obtained results are compared with the results of the classical theory of elasticity. Characteristic distribution of elastic displacement across the thickness of the waveguide with different combinations of boundary conditions is given.

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