LOAD TRANSFER FROM TWO PARALLEL ELASTIC INFINITE AND FINITE STRINGERS TO ELASTIC HOMOGENEOUS INFINITE PLATE

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Բեռի փոխանցումը անվերջ և վերջավոր երկու զուգահեռ առաձգական վերադիրներից համասեռ առաձգական անվերջ սալին

Աշխատանքում դիտարկված է բեռի փոխանցման կոնտակտային խնդիր՝ անվերջ և վերջավոր երկու զուգահեռ առաձգական վերադիրներից իզոտրոպ համասեռ առաձգական անվերջ սալին: Ենթադրվում է, որ հորիզոնական առանցքի նկատմամբ անհամաչափ դասավորված վերադիրները կոշտ ամրացված (սոսնձված) են առաձգական սալին և ունեն տարբեր առաձգական հատկություններ: Սալ-վերադիր կոնտակտային զույգը դեֆորմացվում է առաձգական վերադիրների վրա կիրառված կենտրոնացված ուժերի ու անվերջում առաձգական սալի վրա գործող հաստատուն ինտենսիվությամբ հավասարապաշտ բաշխված հորիզոնական ձգող լարումների ազդեցությամբ շնորհիվ: Խնդրի լուծումը մաթեմատիկորեն ձևակերպված է առաջին սեռի սինգուլյար ինտեգրալ հավասարումների համակարգի տեսքով՝ որոշակի պայմանների առկայության դեպքում: Ֆուրիեի ինտեգրալ ձևափոխության և Չեբիշկի բազմանդամների հայտնի մաթեմատիկական ապարատի օգնությամբ այդ համակարգի լուծումը բերված է քվազիլիովին ռեգուլյար անվերջ հանրահաշվական հավասարումների համակարգի լուծմանը: Որոշված են կոնտակտային շոշափող ուժերի բաշխման ինտենսիվությունները վերադիրների տակ: Անվերջ վերադիրի համար ստացված են ասիմպտոտիկ բանաձևեր, որոնք բնութագրում են լարումների վարքն ինչպես ուժի կիրառման կետի շրջակայքում, այնպես էլ՝ դրանից հեռու կետերում:

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Передача нагрузки от двух параллельных упругих бесконечного и конечного стрингеров к упругой однородной бесконечной пластине

В работе рассматривается контактная задача о передаче нагрузки от двух параллельных упругих бесконечного и конечного стрингеров к изотропной однородной упругой бесконечной пластине. Предполагается, что несимметрично относительно горизонтальной оси расположенные упругие стрингеры, жёстко сцеплены (приклеены) к упругой пластине и имеют разные упругие свойства. Контактирующая пара (пластина-стрингер) деформируется под воздействием сонаправленных и осевых сосредоточенных сил, приложенных к стрингерам, а также равномерно распределённых горизонтальных растягивающих напряжений постоянной интенсивности, действующих на пластину в бесконечности. Решение контактной задачи математически сформулирована в виде системы сингулярных интегральных уравнений первого рода при определённых условиях, с ядрами, состоящими из сингулярной и регулярной частей. При помощи

известного математического аппарата преобразования Фурье и многочленов Чебышева, решение этой системы сводится к решению квазивполне регулярной бесконечной системы линейных алгебраических уравнений. Определены интенсивности распределения тангенциальных контактных усилий под стрингерами. Для бесконечного стрингера получены асимптотические формулы, описывающие поведение напряжений как вблизи, так и вдали от точки приложения силы.

In the present paper a contact problem on load transfer from infinite and finite two parallel elastic stringers to isotropic homogeneous elastic infinite plate is considered. It is assumed, that the elastic stringers are placed non-symmetrically with respect to the horizontal axis, are attached to the plate rigidly and have different elastic characteristics and cross-sectional areas. The bodies in contact (plate-stringers) are deformed under influence of axial concentrated forces, applied on the stringers, as well as uniformly distributed horizontal tensions of constant intensity, acting at the infinity of the plate. Solution of the problem is mathematically formulated as a system of constrained singular integral equations of the first kind with moving singularity with kernels consisted of singular and regular parts. The solution of that system is reduced to solution of quasi completely regular infinite system of linear algebraic equations using the known mathematical techniques of Fourier transform and Chebyshev polynomials. Intensities of tangential contact stresses distributions are determined. Asymptotic formulas for infinite stringer describing the behavior of stresses near and far from force application point are obtained.

Introduction

Investigation of problems of interaction between massive deformable bodies containing stress concentrators such as cracks, thin-walled inclusions and stringers with homogeneous or composite (piecewise - homogeneous) massive deformable bodies is one of the priority directions of the contact and mixed problems of elasticity theory.

Since such problems often arise in mechanics of composites, rock mechanics, measurement technology, problems of load transfer from thin-walled elements to massive deformable bodies, and in other fields of applied mechanics, therefore their study is one of the modern problems in both theoretical and applied aspects. Taking into account the interaction between different types of stress concentrators often leads to new statements of contact and mixed problems, qualitatively changes the character of the stress concentrations, significantly affects stress intensity factors.

The problems of interaction between thin-walled elements in the form of stringers, i.e. rods without bending stiffness, and more massive bodies, are under consideration by many authors.

§1. Problem statement and system of resolving equations derivation

Let an elastic continuum isotropic sheet representing infinite thin plate of constant thickness h is strengthened by two parallel infinite and finite stringers with different elastic properties and sufficiently small rectangular cross section attached to the $y = a$ and $y = -c$ $(a\,;\,c > 0)$ lines of its upper surface. It is assumed that the stringers are attached to the plate rigidly.

The aim of the paper is to determine the intensity of tangential stresses distribution along contact lines, and normal (axial) stresses arising in elastic stringers, and by that determine the contraction of the contacting bodies (plate-stringers) subjected to concentrated forces $P\delta(x)\delta(y-a)$ and $Q\delta(x-d)\delta(y+c)$ $(d > 0)$, applied to elastic stringers, as well as uniformly distributed tensile horizontal stresses of intensity σ_0, acting on the infinite plate at infinity.
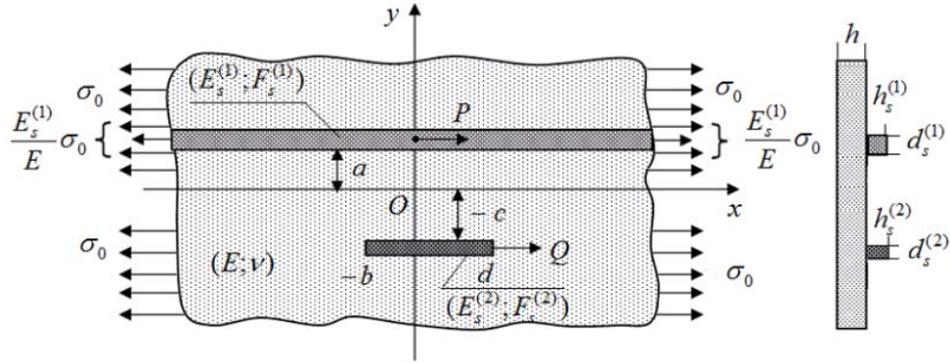
Fig. 1. Diagram of the deforming contact bodies

In the contact problem under consideration, the combined model of uniaxial stress state and contact along a line are accepted with respect to the stringers [1– 9], i.e. it is assumed that the distribution of contact tangential stresses intensity are concentrated along the middle lines of the contact areas. Moreover, it is assumed that the stringers do not resist to bending, i.e. do not have bending stiffness. With respect to the plate the model generalized plane stress state is assumed to be true, due to which the plate is deformed as a plane (see Fig. 1).

Proceeding to derivation of resolving equations for the contact problem let us note, that the elastic stringers are stretched or compressed in horizontal direction being in uniaxial stress state, then according to the aforesaid, the differential equations of equilibrium can be written as follows:

$$\frac{du_s^{(1)}(x;a)}{dx} = -\frac{1}{2E_s^{(1)}F_s^{(1)}} \int_{-\infty}^{\infty} \mathrm{sgn}(s-x)\tau^{(1)}(s)\,ds - \frac{P\partial(x)}{2E_s^{(1)}F_s^{(1)}} + \frac{\sigma_0}{E} \quad (-\infty < x < \infty),$$

(1.1)

$$\frac{du_s^{(2)}(x;-c)}{dx} = -\frac{1}{2E_s^{(2)}F_s^{(2)}} \int_{-b}^{d} \mathrm{sgn}(u-x)\tau^{(2)}(u)\,du + \frac{Q}{2E_s^{(2)}F_s^{(2)}} \quad (-b < x < d),$$ (1.2)

under the following conditions:

$$\frac{du_s^{(1)}(x;a)}{dx}\bigg|_{|x|\to\infty} = \frac{\sigma_0}{E_s^{(1)}}; \frac{du_s^{(2)}(x;-c)}{dx}\bigg|_{x=-b+0} = 0; \frac{du_s^{(2)}(x;-c)}{dx}\bigg|_{x=d-0} = \frac{Q}{E_s^{(2)}F_s^{(2)}},$$

(1.3)

as well as the equilibrium conditions of the stringers:

$$\int_{-\infty}^{\infty} \tau^{(1)}(s)\,ds = P \;\; ; \;\; \int_{-b}^{d} \tau^{(2)}(u)\,du = Q.$$ (1.4)

In formulas (1.1) ‾ (1.4), $u_s^{(1)}(x;a)$ and $u_s^{(2)}(x;-c)$ are the horizontal displacements of the stringers at $y = a$ and $y = -c$; lines, $\tau^{(1)}(x) = d_s^{(1)}\tau^{(1)}(x;a)$, $\tau^{(1)}(x;a)$ is the intensity of unknown contact tangential stresses, arising under the elastic infinite stringer at $y = a$ line, $\tau^{(2)}(x) = d_s^{(2)}\tau^{(2)}(x;-c)$, $\tau^{(2)}(x;-c)$ is the intensity of unknown

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contact tangential stresses, arising under the elastic infinite stringer at $y=-c$ line, $E_s^{(k)}$ $(k=1,2)$ are the elastic moduli, $F_s^{(k)}=d_s^{(k)}h_s^{(k)}$ are the areas of cross sections, $h_s^{(k)}$ and $d_s^{(k)}$ are the height and width of the stringers, respectively. P and Q are the intensities of axial concentrated forces applied to the stringers at $(0;a)$ and $(d;-c)$ points, respectively, E is the elastic modulus of the infinite plate.

From the other hand, for horizontal strain of the plate subjected to contact tangential stresses $\tau^{(1)}(x)$ $(-\infty < x < \infty)$ and $\tau^{(2)}(x)$ $(-b \le x \le d)$, at $y=a$ and $y=-c$, and to uniformly distributed horizontal tensile stresses of constant intensity σ_0, acting at the infinity of the plate, respectively, is given by

$$hl\frac{du^{(1)}(x;a)}{dx}=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\tau^{(1)}(s)}{s-x}ds+\frac{1}{\pi}\int_{-b}^{d}K(u-x)\tau^{(2)}(u)du+\frac{hl}{E}\sigma_0 \quad (-\infty < x < \infty),$$

(1.5)

$$hl\frac{du^{(2)}(x;-c)}{dx}=\frac{1}{\pi}\int_{-b}^{d}\frac{\tau^{(2)}(u)}{u-x}du+\frac{1}{\pi}\int_{-\infty}^{\infty}K(s-x)\tau^{(1)}(s)ds+\frac{hl}{E}\sigma_0 \quad (-\infty < x < \infty),$$

(1.6)

$$K(t)=\frac{t}{t^2+(a+c)^2}-\frac{2A(a+c)^2 t}{\left[t^2+(a+c)^2\right]^2}; \quad l=\frac{4E}{(3-\nu)(1+\nu)}; \quad A=\frac{1+\nu}{3-\nu},$$

(1.7)

$u^{(1)}(x;a)$ and $u^{(2)}(x;-c)$ are the horizontal displacements of the infinite plate at $y=a$ and $y=-c$; lines, respectively, ν is the Poisson ratio of the plate.

The contact conditions take the formulas

$$\frac{du_s^{(1)}(x;a)}{dx}=\frac{du^{(1)}(x;a)}{dx} \quad (-\infty < x < \infty), \quad \frac{du_s^{(2)}(x;-c)}{dx}=\frac{du^{(2)}(x;-c)}{dx}$$

(1.8)

With respect to the distributions of contact tangential stresses $\tau^{(1)}(x)$ $(-\infty < x < \infty)$ and $\tau^{(2)}(x)$ $(-b \le x \le d)$, we derive the following system of singular integral equations of the first kind:

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\left[\frac{1}{s-x}+\frac{\lambda}{2}\pi\,\mathrm{sgn}(s-x)\right]\tau^{(1)}(s)ds\frac{1}{\pi}\int_{-b}^{d}K(u-x)\tau^{(2)}(u)du=-\frac{\lambda}{2}P\,\mathrm{sgn}\,x, \quad (1.9)$$

$$(-\infty < x < \infty),$$

$$\frac{1}{\pi}\int_{-b}^{d}\left[\frac{1}{u-x}+\frac{\lambda_1}{2}\pi\,\mathrm{sgn}(u-x)\right]\tau^{(2)}(u)du\frac{1}{\pi}\int_{-\infty}^{\infty}K(s-x)\tau^{(1)}(s)ds=\frac{\lambda_1}{2}Q-\frac{hl}{E}\sigma_0,$$

$$(-b < x < d),$$

(1.10)

where $\lambda = \dfrac{hl}{E_s^{(1)}F_s^{(1)}}$; $\lambda_1 = \dfrac{hl}{E_s^{(2)}F_s^{(2)}}$.

Thus, under assumptions made, the solution of the problem is reduced to system of singular integral equations of the first kind with moving singularity (1.9) and (1.10) under integral constraints (1.4).

2. In order to solve the resolving system constrained by (1.4), we apply Fourier transform. Then, using the convolution formula we obtain the following functional equation with respect to Fourier transform of unknown function $\tau^{(1)}(x)$ $(-\infty < x < \infty)$:

$$\left[\lambda + |\sigma|\right]\overline{\tau}^{(1)}(\sigma) = \lambda P + H\left(|\sigma|\right)\varphi(\sigma) \qquad (-\infty < \sigma < \infty). \tag{2.1}$$

$$\overline{\tau}^{(1)}(\sigma) = F\left[\tau^{(1)}(x)\right] = \int_{-\infty}^{\infty} \tau^{(1)}(s)e^{i\sigma s}ds; \quad \varphi(\sigma) = \int_{-b}^{d} \tau^{(2)}(u)e^{i\sigma u}du, \tag{2.2}$$

$$H\left(|\sigma|\right) = \left[A(a+c)\sigma^2 - |\sigma|\right]e^{-(a+c)|\sigma|}.$$

Let us note, that the solution of (2.1) must satisfy (1.4), which will be transformed to $\overline{\tau}^{(1)}(0) = P$ and $\varphi(0) = Q$. Solving (2.1) with respect to $\overline{\tau}^{(1)}(\sigma)$ we will arrive at

$$\overline{\tau}^{(1)}(\sigma) = \frac{\lambda P}{\lambda + |\sigma|} + \frac{\varphi(\sigma)}{\lambda + |\sigma|}H\left(|\sigma|\right) \qquad (-\infty < \sigma < \infty). \tag{2.3}$$

It is important to determine also the Fourier transform of normal (axial) stresses $\sigma_x^{(1)}(x;a)$ $(-\infty < x < \infty)$, which arise in infinite stringer:

$$\overline{\sigma}_x^{(1)}(\sigma;a) = -\frac{P}{F_s^{(1)}}\frac{i\,\mathrm{sgn}\,\sigma}{\lambda + |\sigma|} + \frac{\varphi(\sigma)}{F_s^{(1)}}\frac{i\,\mathrm{sgn}\,\sigma}{\lambda + |\sigma|}H_0\left(|\sigma|\right) + \frac{E_s^{(1)}}{E}2\pi\sigma_0\delta(\sigma), \tag{2.4}$$

$$(-\infty < \sigma < \infty),$$

in which

$$H_0\left(|\sigma|\right) = \left[A(a+c)|\sigma|-1\right]e^{-(a+c)|\sigma|}; \quad \overline{\sigma}_x^{(1)}(\sigma;a) = F\left[\sigma_x^{(1)}(x;a)\right]. \tag{2.5}$$

(2.1) and (2.4) are derived taking into account the following relations [3,10 – 13]:

$$F\left[\frac{t}{t^2 + y^2}\right] = \frac{i\pi\,\mathrm{sgn}\,\sigma}{e^{|\sigma y|}}; \quad F\left[\frac{t}{\left(t^2 + y^2\right)^2}\right] = \frac{i\pi\sigma}{2|y|e^{|\sigma y|}}, \tag{2.6}$$

$$F\left[\mathrm{sgn}\,t\right] = \frac{2i}{\sigma}; \quad F[1] = 2\pi\delta(\sigma) \qquad (-\infty < \sigma; \, y; \, t < \infty),$$

If we apply Fourier inverse transform to (2.3) and (2.4), between unknown functions $\tau^{(1)}(x)$, $\sigma_x^{(1)}(x;a)$ $(-\infty < x < \infty)$ and $\tau^{(2)}(x)$ $\tau^{(2)}(x)$ we will derive the following functional relation:

$$\tau^{(1)}(x) = \frac{\lambda}{\pi} P H_c(x) + \frac{1}{\pi} \int_{-b}^{d} H_{oc}(u-x) \tau^{(2)}(u) du$$

$$(-\infty < x < \infty),$$ (2.7)

$$\sigma_x^{(1)}(x;a) = -\frac{P}{\pi F_s^{(1)}} H_s(x) - \frac{1}{\pi F_s^{(1)}} \int_{-b}^{d} H_{os}(u-x) \tau^{(2)}(u) du + \frac{E_s^{(1)}}{E} \sigma_0$$ (2.8)

$$(-\infty < x < \infty),$$

providing the qualitative and quantitative picture of the interaction of the stringers. Here the following notations are introduced:

$$\begin{bmatrix} H_c(x) \\ H_s(x) \end{bmatrix} = \int_0^{\infty} \begin{bmatrix} \cos(\sigma x) \\ \sin(\sigma x) \end{bmatrix} \frac{d\sigma}{\lambda + \sigma}; \quad \begin{bmatrix} H_{oc}(x) \\ H_{os}(x) \end{bmatrix} = \int_0^{\infty} \begin{bmatrix} H(\sigma)\cos(\sigma x) \\ H_o(\sigma)\sin(\sigma x) \end{bmatrix} \frac{d\sigma}{\lambda + \sigma}.$$ (2.9)

Thus, the distribution of contact tangential stresses of intensity $\tau^{(1)}(x)$ $(-\infty < x < \infty)$ and normal (axial) stresses $\sigma_x^{(1)}(x;a)$ $(-\infty < x < \infty)$, arising in the infinite stringer, consequently are expressed in terms of intensity $\tau^{(2)}(x)$ $(-b \le x \le d)$ of contact tangential intensity in the elastic finite stringer by formulas (2.7) and (2.8).

3. For derivation of the distribution of contact tangential stresses $\tau^{(2)}(x)$ $(-b \le x \le d)$ let us substitute the expression of $\tau^{(1)}(x)$ $(-\infty < x < \infty)$ given by (2.7) into (1.10). Then, we will arrive at the following singular integral equation of the first kind:

$$\frac{1}{\pi} \int_{-b}^{d} \left[\frac{1}{u-x} + \frac{\lambda_1}{2} \pi \operatorname{sgn}(u-x) + M(u;x) \right] \tau^{(2)}(u) du = \frac{\lambda_1}{2} Q - \frac{hl}{E} \sigma_0 - \frac{\lambda}{\pi} P N(x),$$ (3.1)

$$(-b < x < d),$$

Here the following notations are introduced:

$$M(u;x) = \frac{1}{\pi} \int_{-\infty}^{\infty} K(s-x) H_{oc}(u-s) ds; \quad N(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} K(s-x) H_c(s) ds.$$ (3.2)

Note, that the solution of (3.1) must satisfy the first constraint in (1.4).

Taking into account that near the contact line end-points the intensities of the contact tangential stresses have singularity of the square root power of integrable order, let us represent the solution of (3.1) under (1.4(b)) in the form of expansion into series with respect to Chebyshev polynomials of the first kind

$$\tau^{(2)}(u) = \frac{1}{\sqrt{1 - g^2(u)}} \sum_{n=0}^{\infty} X_n T_n[g(u)]; \quad g(u) = \frac{2u+b-d}{b+d}; \quad |g(u)| < 1,$$ (3.3)

where $T_n(x) = \cos(n \arccos x)$ $\left(|x| \le 1; \ n = \overline{0;\infty}\right)$ – are the Chebyshev polynomials of the first kind, and the unknown coefficients X_n $\left(n = \overline{0;\infty}\right)$ must be determined.

Substituting expansion (3.3) of the function $\tau^{(2)}(u)$ into (3.1), and using the following spectral relations [6–8]:

$$\frac{1}{\pi} \int_{-b}^{d} \frac{1}{u-x} \frac{T_n[g(u)]}{\sqrt{1-g^2(u)}} du = \begin{cases} 0; & n = 0, \\ U_{n-1}[g(x)]; & \left(n = \overline{1;\infty}\right), \end{cases} \quad (-b < x < d),$$

$$\frac{1}{\pi} \int_{-b}^{d} \sqrt{1-g^2(u)}\, U_{n-1}[g(u)] U_{m-1}[g(u)]\, du = \begin{cases} 0; & n \ne m, \\ \dfrac{b+d}{4}; & n = m, \end{cases} \quad \left(n;m = \overline{1;\infty}\right),$$

(3.4)

where $U_{n-1}(x) = \dfrac{\sin(n \arccos x)}{\sin(\arccos x)}$ $\left(|x| \le 1; \ n = \overline{0;\infty}\right)$ – are the Chebyshev polynomials of the second kind, for derivation of the coefficients X_n $\left(n = \overline{1;\infty}\right)$ following the traditional method [6–8], we will obtain the following quasi-completely regular infinite system of linear algebraic equations:

$$X_m + \sum_{n=1}^{\infty} H_{nm} X_n = \alpha_m \qquad \left(m = \overline{1;\infty}\right).$$

(3.5)

The kernels and the free term of (3.5) are defined as follows:

$$H_{nm} = H_{nm}^{(1)} + H_{nm}^{(2)}; \quad \alpha_m = \alpha_m^{(1)} + \alpha_m^{(2)} - X_0 H_{0m},$$

$$\alpha_m^{(1)} = \begin{cases} 0; & m \ne 1, \\ \dfrac{\lambda_1}{2} Q - \dfrac{hl}{E}\sigma_0; & m = 1, \end{cases} \quad \alpha_m^{(2)} = -\frac{4\lambda P}{\pi^2(b+d)} \int_{-b}^{d} \sqrt{1-g^2(x)}\, U_{m-1}[g(x)] N(x)\, dx,$$

$$H_{nm}^{(1)} = -\frac{\lambda_1(b+d)}{\pi} \begin{cases} 0; & |m-n| = 1, \\ \dfrac{2m\left[1+(-1)^{m+n}\right]}{\left[(m+n)^2-1\right]\left[(m-n)^2-1\right]}; & |m-n| \ne 1, \end{cases}$$

(3.6)

$$H_{nm}^{(2)} = \frac{4}{\pi^2(b+d)} \int_{-b}^{d}\int_{-b}^{d} \frac{\sqrt{1-g^2(x)}}{\sqrt{1-g^2(u)}} T_n[g(u)] U_{m-1}[g(x)] M(u;x)\, du\, dx$$

$\left(n;m = \overline{1;\infty}\right).$

Let us note, that it is characteristic to system (3.5), that the coefficient X_0 is not explicitly involved in its left hand side, therefore the rest coefficients X_n $\left(n = \overline{1;\infty}\right)$ will be

linearly dependent on X_0. At that, the unknown X_0 is determined from equilibrium condition (1.4(b)) as follows:

$$X_0 = \frac{2Q}{\pi(b+d)}.$$ (3.7)

The normal (axial) stresses arising in the finite stringer are evaluated according to the following formula:

$$\sigma_x^{(2)}(x;-c) = \frac{Q}{\pi F_S^{(2)}}\left[\pi - \arccos g(x)\right] - \frac{b+d}{2F_S^{(2)}}\sqrt{1-g^2(x)}\sum_{n=1}^{\infty}\frac{1}{n}X_n U_{n-1}\left[g(x)\right],$$ (3.8)

$$(-b \le x \le d).$$

Note, that after evaluation of (3.3), the normal (axial) stresses arising in infinite stringer are given by (2.8).

4. We investigate the behavior of intensity of the contact tangential stresses $\tau^{(1)}(x)$ $(-\infty < x < \infty)$ and normal stresses $\sigma_x^{(1)}(x;a)$ $(-\infty < x < \infty)$ – arising in the infinite stringer, which characterize their behavior near and far from the concentrated force P. acting point. Let us first derive asymptotic formulas for functions $\tau^{(1)}(x)$ and $\sigma_x^{(1)}(x;a)$ as $|x| \to 0$. Since as $|\sigma| \to \infty$ the following asymptotic representations take place [10]:

$$\frac{\lambda}{\lambda+|\sigma|} = \sum_{n=0}^{\infty}\left(\left|\frac{\sigma}{\lambda}\right|\right)^{-2n-1} - \sum_{n=0}^{\infty}\left(\frac{\sigma}{\lambda}\right)^{-2n-2},$$ (4.1)

$$\frac{i\operatorname{sgn}\sigma}{\lambda+|\sigma|} = \frac{i}{\lambda}\sum_{n=0}^{\infty}\left(\frac{\sigma}{\lambda}\right)^{-2n-1} - \frac{i\operatorname{sgn}\sigma}{\lambda}\sum_{n=0}^{\infty}\left(\frac{\sigma}{\lambda}\right)^{-2n-2}.$$ (4.2)

Then, after application of Fourier inverse integral transform, taking into account the properties of Fourier integrals, for $\tau^{(1)}(x)$ and $\sigma_x^{(1)}(x;a)$ we will derive the following asymptotic formulas as $|x| \to 0$:

$$\tau^{(1)}(x) = \frac{\lambda P}{\pi}\sum_{n=0}^{\infty}(-1)^n\left[\frac{\pi|\lambda x|^{2n+1}}{2(2n+1)!} + \frac{(\lambda x)^{2n}}{(2n)!}\left(\psi(2n+1) + \ln\frac{1}{|\lambda x|}\right)\right],$$ (4.3)

$$\sigma_x^{(1)}(x;a) = \frac{P}{\pi F_s^{(1)}}\sum_{n=0}^{\infty}(-1)^n\left[-\frac{\pi(\lambda x)^{2n}}{2(2n)!}\operatorname{sgn}(\lambda x) + \frac{(\lambda x)^{2n+1}}{(2n+1)!}\left(\psi(2n+2) + \ln\frac{1}{|\lambda x|}\right)\right].$$ (4.4)

From representations (4.3) and (4.4) it follows that the function $\tau^{(1)}(x)$ has logarithmic singularity as $|x| \to 0$, and the function $\sigma_x^{(1)}(x;a)$ has finite discontinuity as $|x| \to 0$. Both phenomena are due to the concentrated force with intensity P.

Let us now reveal the behavior of the functions $\tau^{(1)}(x)$ and $\sigma_x^{(1)}(x;a)$ as $|x| \to \infty$.

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Taking into account that as $|\sigma| \to 0$ the following asymptotic representations take place [10]:

$$\frac{\lambda}{\lambda+|\sigma|} = \sum_{n=0}^{\infty} \left(\frac{\sigma}{\lambda}\right)^{2n} - \sum_{n=0}^{\infty} \left(\left|\frac{\sigma}{\lambda}\right|\right)^{2n+1}, \tag{4.5}$$

$$\frac{i\,\mathrm{sgn}\,\sigma}{\lambda+|\sigma|} = \frac{i\,\mathrm{sgn}\,\sigma}{\lambda} \sum_{n=0}^{\infty} \left(\frac{\sigma}{\lambda}\right)^{2n} - \frac{i}{\lambda} \sum_{n=0}^{\infty} \left(\frac{\sigma}{\lambda}\right)^{2n+1}, \tag{4.6}$$

After application of Fourier generalized inverse transform, taking into account the properties of the Fourier integrals, for the functions $\tau^{(1)}(x)$ and $\sigma_x^{(1)}(x;a)$ we will derive the following asymptotic expansions as $|x| \to \infty$:

$$\tau^{(1)}(x) = \frac{\lambda P}{\pi} \sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)!}{(\lambda x)^{2n+2}}, \tag{4.7}$$

$$\sigma_x^{(1)}(x;a) = -\frac{P}{\pi F_s^{(1)}} \sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{(\lambda x)^{2n+1}} + \frac{E_s^{(1)}}{E}\sigma_0 . \tag{4.8}$$

It is evident from (4.7) and (4.8), that as $|x| \to \infty$ $\tau^{(1)}(x) = O(x^{-2})$;
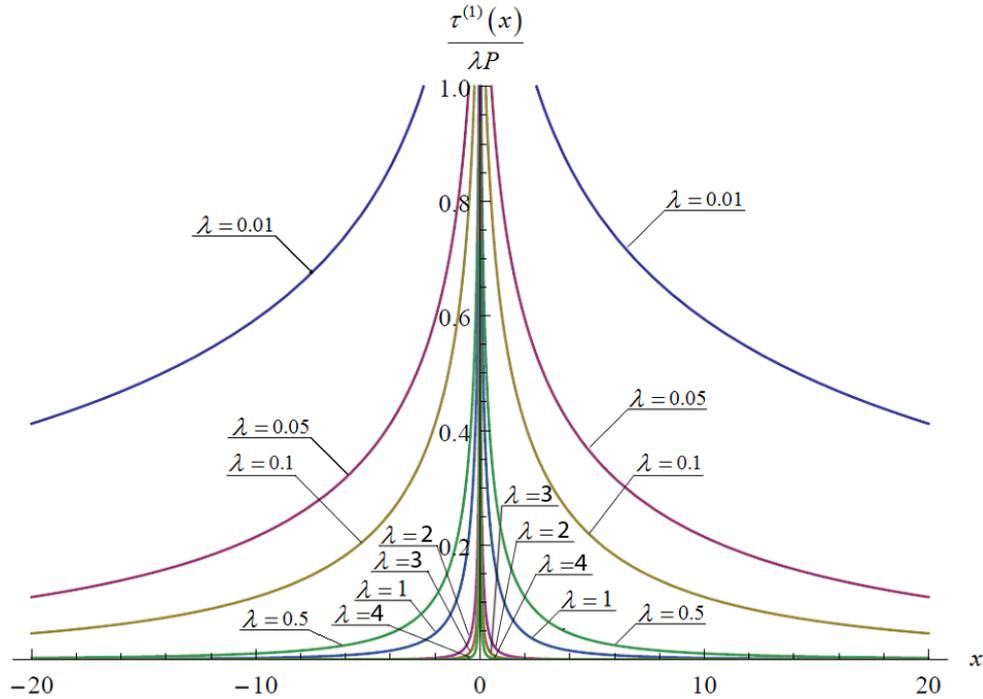
Fig.2. Contact tangential stresses under the infinite elastic stringer

Numerical analysis is performed and main characteristics of the contact problem are investigated. In Figures 2 and 3, the contact tangential stresses and the normal stresses arising in infinite stringer are captured in its finite part, symmetric with respect to the concentrated force application point. It can be seen, that the increase of parameter λ in the range $\lambda = 0.01$, $\lambda = 0.05$, $\lambda = 0.1$, $\lambda = 0.5$, $\lambda = 1$, $\lambda = 2$, $\lambda = 3$, $\lambda = 4$ which is inversely proportional to the stringer elastic modulus, leads to decrease of both stresses.
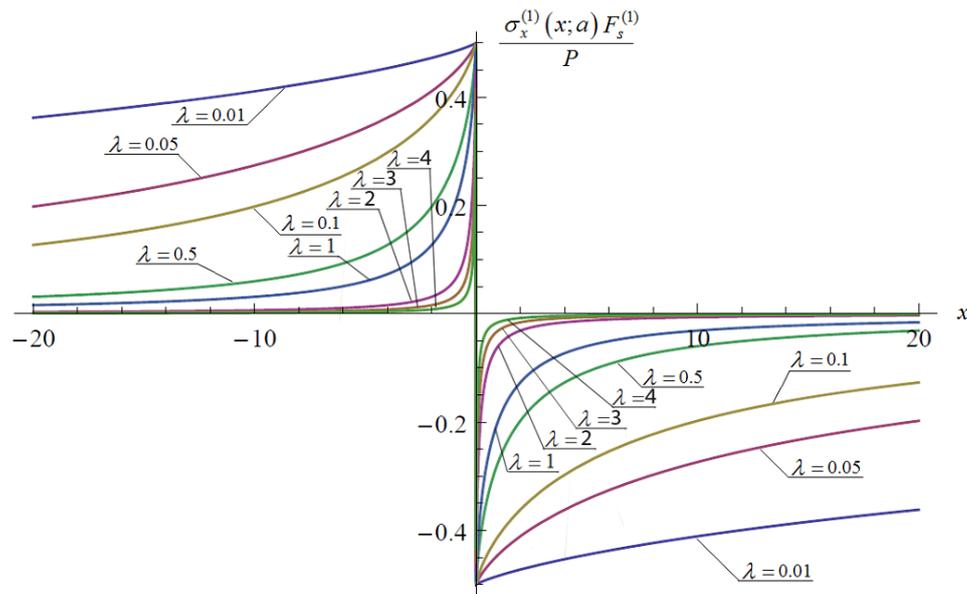
Fig. 3. Normal stresses arising in the infinite elastic stringer

Conclusion

Using Fourier generalized integral transform the closed form solution of the problem is derived in terms of expansion into infinite series with respect to Chebyshev orthogonal polynomials. The implementation of the solution requires solution of infinite system of linear algebraic equations, the quasi-completely regularity of which is established. Consideration of particular cases showed the consistency of the solution with solutions of corresponding problems evaluated earlier.

Numerical analysis revealed the main characteristics of the stress state of the contacting bodies: the contact tangential stresses between the plate and the infinite stringer, as well as the normal stresses arising in the infinite stringer. Under fixed geometrical configuration of the infinite stringer, it was established, that with decrease of its elastic modulus leads to decrease of both tangential and normal stresses.

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