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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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ELASTIC-SPIN WAVES PROPAGATION IN A PERIODIC FERROMAGNETIC LAYERED MEDIUM

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Key words: spin wave, elastic-spin wave, ferromagnetic periodic structure.
Բանալի բառեր` սպինային ալիք; առաձգա-սպինային ալիք; ֆերոմագնիսական, պարբերական կառուցվածք:
Ключевые слова: спиновые волны; упруго-спиновые волны; ферромагнитная, периодическая структура.

Աթոյան Լ.Հ., Դանոյան Զ.Ն.

Առաձգա-սպինային ալիքների տարածումը պարբերական, ֆերոմագնիսական, շերտավոր կառուցվածքում

Պարբերական ֆերոմագնիս/ոչմագնիս կառուցվածքում հետազոտվում են սպինային և առաձգա-սպինային ալիքների տարածման պարզեցված մոդելներ: Հետազոտվող ալիքները նկարագրելու համար օգտագործվում են մոդելներ, որտեղ հաշվի են առնված փոխանակման փոխազդեցությունը ֆերոմագնիսում, ինչպես նաև մոդելներ հաշվի առնող միջավայրերի առաձգական հատկությունները: Ցույց է տրված, որ առաձգա-սպինային ալիքների հաճախությունների սպեկտրը տրոհվում է ալիքների անցելիության և արգելափակման գոտիների:

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Распространение упруго-спиновых волн в периодической, ферромагнитной, слоистой среде

Исследуются упрощённые модели распространения спиновых и упруго-спиновых волн в периодической среде ферромагнетик/немагнетик. Для описания исследуемых волн используются модели, учитывающие обменные эффекты в ферромагнетике, а также модели, берущие в расчёт упругие свойства структуры. Показано, что весь спектр упруго-спиновых волн разбивается на полосы частот пропускания и запирания.

Simplified models describing the spin and elastic-spin wave propagation in periodic ferromagnetic structure, consisting of ferromagnetic and nonmagnetic layers, are investigated. For a description of the elastic-spin waves are used models that take into account the exchange effects in ferromagnetic as well as models, taking into account the elastic properties of the structure. It is shown that the frequency spectrum of elastic-spin waves is divided into transmission and locking bands.

Introduction. More and more attention of researchers attract the problems of spin waves existence and propagation in artificial periodic constitutive media called metamaterials [1–14]. Years ago such periodic structures were called superlattices. If the structure composed of magnetic layers with different magnetization or magnet/non-magnet layers, then such structures called magnetic metamaterials or magnonic crystals by analogy with photonic crystals. In this paper, some one-dimensional mathematical

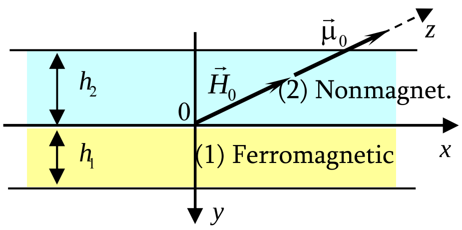

Fig. 1.

models, describing magnetic oscillations, spin and elastic-spin waves in considered structures are analyzed.

1. Problem statement. Suppose that a periodic layered structure is consisting of an infinite series of ferromagnetic and nonmagnetic layers of different thicknesses of h_1, h_2. The structure assigned to the Cartesian coordinate system as shown in Fig.1. It assumed that the anisotropy axis of easy magnetization of ferromagnetic layers are parallel to each other and coincide with the direction of Oz axis. Suppose that the structure is in an external homogeneous constant magnetic field $\vec{H}_0 = (0,0,H_0)$ and the vector of the saturation magnetization per unit volume of ferromagnetic layer $\vec{M}_0 = \rho_1 \vec{\mu}_0$ ($\vec{\mu}_0$ is the ferromagnetic saturation magnetization per unit mass, ρ_1 is the mass density of the material of ferromagnetic) is parallel to $\vec{H}_0$ and both $\vec{M}_0, \vec{\mu}_0$ are directed along the axis of easy magnetization, that is Oz axis. We suppose, that the excitations in the structure does not depend on the Oz coordinate and are characterized in the ferromagnetic layer by the vector of the elastic displacement $\vec{u}_1 = \left(0,0,u_1(x,y,t)\right)$, vector $\vec{\mu} = \left(\mu(x,y,t), \nu(x,y,t), 0\right)$ of the magnetic moment, magneto static potential $\phi_1(x,y,t)$ and in the nonmagnetic layer by the vector of elastic displacement $\vec{u}_2 = \left(0,0,u_2(x,y,t)\right)$ and magneto static potential $\phi_2(x,y,t)$. The excitations of magnetic field intensities in ferromagnetic and nonmagnetic are: $\vec{H}_1 = -\mathrm{grad}\varphi_1, \vec{H}_2 = -\mathrm{grad}\varphi_2$.

The equation of mechanical motion of the medium, the Landau-Lifshits equation of magnetization moment motion and the quasi-static Maxwell's equations for the magnetic field in the ferromagnetic layer [1, 2, 3, 9] are:

$$\left.\begin{aligned}
&\frac{\partial^2 u_1}{\partial t^2} = s_1^2 \Delta u_1 + M_0 f \left(\frac{\partial \mu}{\partial x} + \frac{\partial \nu}{\partial y}\right) \\
&\frac{\partial \mu}{\partial t} = \omega_M \left(\rho_1^{-1} \frac{\partial \varphi_1}{\partial y} + \hat{b}\nu + \bar{b}\mu_0 \frac{\partial u_1}{\partial y} - \bar{\lambda}\Delta\nu\right) \\
&\frac{\partial \nu}{\partial t} = -\omega_M \left(\rho_1^{-1} \frac{\partial \varphi_1}{\partial x} + \hat{b}\mu + \bar{b}\mu_0 \frac{\partial u_1}{\partial x} - \bar{\lambda}\Delta\mu\right) \\
&\Delta\varphi_1 = \rho_1 \left(\frac{\partial \mu}{\partial x} + \frac{\partial \nu}{\partial y}\right)
\end{aligned}\right\} \tag{1.1}$$

Here $s_1 = \sqrt{G_1 / \rho_1}$ is the velocity of the elastic wave, G_1 is the shear modulus, $\omega_M = \gamma M_0$, γ is the gyromagnetic constant, f is the coefficient of piezomagnetism, $\hat{b} = b + H_0 / M_0, \bar{b} = b + f$, b is the constant of magnetic anisotropy, Δ is the two-dimensional Laplace operator, $\bar{\lambda}$ is the coefficient of the exchange interaction. The exchange interaction should be taken into account when different magnetic layers are in contact, or if the nonmagnetic spacer is very thin $(d_2 \leq 30 \overset{0}{A})$. The equation of motion and the magneto-static equation in the nonmagnetic layer are:

$$\frac{\partial^2 u_2}{\partial t^2} = s_2^2 \Delta u_2, \ \Delta \varphi_2 = 0 \tag{1.2}$$

$s_2 = \sqrt{G_2 / \rho_2}$ is the velocity of the elastic wave, G_2 is the shear modulus, ρ_2 is the density of the nonmagnetic.

Contact and boundary conditions for displacements, stresses and magnetic potentials are:

$$\begin{aligned}
& u_1(0) = u_2(0); \ \varphi_1(0) = \varphi_2(0); \\
& \sigma_{23}^{(1)}(0) = \sigma_{23}^{(2)}(0); \frac{\partial \varphi_1(0)}{\partial y} - \rho_1 \nu(0) = \frac{\partial \varphi_2(0)}{\partial y}; \\
& u_1(h_1) = \lambda u_2(-h_2); \ \varphi_1(h_1) = \lambda \varphi_2(-h_2); \\
& \sigma_{23}^{(1)}(h_1) = \lambda \sigma_{23}^{(2)}(-h_2); \frac{\partial \varphi_1(h_1)}{\partial y} - \rho_1 \nu(h_1) = \lambda \frac{\partial \varphi_2(-h_2)}{\partial y}; \\
& \sigma_{23}^{(1)} = \rho_1 s_1^2 \frac{\partial u_1}{\partial y} + \mu_0 \bar{b} \rho_1 \nu; \sigma_{23}^{(2)} = \rho_2 s_2^2 \frac{\partial u_2}{\partial y};
\end{aligned} \tag{1.3}$$

$\lambda -$ is the Floquet constant. It should be noted, that if exchange interaction is taken into account, then we need to add to relations (1.3) the condition for magnetization density $\vec{\mu}$:

$$\mu(0) = \nu(0) = 0 \ \text{or} \ \frac{\partial \mu(0)}{\partial y} = \frac{\partial \nu(0)}{\partial y} = 0 \tag{1.4}$$

Let us consider the simplified one-dimensional mathematical models with the aim of understanding the nature of elastic-spin waves in periodic structures.

2. Consider the model when the ferromagnetic layer is non-deformable (u_1 = 0) and exchange interactions neglected ($\bar{\lambda}$=0). It is obvious that this model can describe only oscillations, not waves. From (1.1) it follows equation for the normal component of the magnetization ν :

$$\frac{\partial^2 \nu}{\partial y^2} + r^2 \nu = f(t) \tag{2.1}$$

Here $r = \omega_M \Omega_{SV}, \Omega_{SV} = \sqrt{\hat{b}(\hat{b}+1)}$, $f(t)$ is an arbitrary function of time. In order to identify the character of oscillatory process, assume that the right side of (2.1) $f(t) = A\cos\omega t, A$ is a constant. The solution of (2.1) is as follows:

$$\nu = \frac{A}{r^2 - \omega^2} \cos \omega t \tag{2.2}$$

From (2.2) it follows that the frequency $\omega = r$ is the frequency of resonance. Now we can state that the oscillatory process has a resonance character for this case.

3. Accounting of exchange interaction $(\bar{\lambda} \neq 0)$ in non-deformable $(u_1 = 0)$ ferromagnetic layer. To the problems of propagation of spin waves in non-deformable media are devoted works [3-5, 7, 8] and many others. The equations in ferromagnetic layer are:

$$\begin{aligned}
&\frac{\partial \mu}{\partial t} = \omega_M \left(\rho_1^{-1} \frac{\partial \varphi_1}{\partial y} + \hat{b}\nu - \bar{\lambda} \frac{\partial^2 \nu}{\partial y^2} \right) \\
&\frac{\partial \nu}{\partial t} = -\omega_M \left(\hat{b}\mu - \bar{\lambda} \frac{\partial^2 \mu}{\partial y^2} \right) \\
&\frac{\partial^2 \varphi_1}{\partial y^2} = \rho_1 \frac{\partial \nu}{\partial y}
\end{aligned} \tag{3.1}$$

The solutions we seek in the form:

$$(\mu, \nu, \varphi_1) = (M, N, \Phi_1) e^{i(qy - \omega t)} \tag{3.2}$$

here M, N, Φ_1 are constants. Substituting (3.2) in the system (3.1) and using the non-zero solutions existence condition, we come to the following dispersion equation:

$$q^2 [\omega^2 - \omega_M^2 (\hat{b} + \bar{\lambda} q^2)(1 + \hat{b} + \bar{\lambda} q^2)] = 0 \tag{3.3}$$

From (3.3) follows that q^2 can have three values:

$$q^2 = \{-\Omega_{DE} + \sqrt{\frac{1}{4} + \Omega^2}; -\Omega_{DE} - \sqrt{\frac{1}{4} + \Omega^2}; 0\}, \qquad (3.4)$$

where $\Omega_{DE} = \hat{b} + \frac{1}{2}$, $\Omega = \frac{\omega}{\omega_M}, \omega_M = \gamma M_0$. Volume homogeneous wave exists only when q^2 has a positive value. It means that, when the following inequality is true:

$$q^2 = -\Omega_{DE} + \sqrt{\frac{1}{4} + \Omega^2} > 0$$

then the wave process exists. Thus, we conclude that the model, when it is taken into account the exchange interaction, even for non-deformable ferromagnetic, describes the spin wave process.

4. The ferromagnetic layer is elastic $(u_1 \neq 0)$, exchange interaction is neglected $(\overline{\lambda} = 0)$. In addition to these conditions for the model farther simplification, we neglect the magnetic potential also. To the problems of propagation of elastic-spin waves are devoted the works [1, 2, 6, 9, 12]. The equations in ferromagnetic layer are:

$$\frac{\partial^2 u_1}{\partial t^2} = s_1^2 \frac{\partial^2 u_1}{\partial y^2} + M_0 f \frac{\partial \nu}{\partial y}$$

$$\frac{\partial \mu}{\partial t} = \omega_M \hat{b} \nu + \omega_M \overline{b} \mu_0 \frac{\partial u_1}{\partial y} \qquad (4.1)$$

$$\frac{\partial \nu}{\partial t} = -\omega_M \hat{b} \mu$$

The general solutions of (4.1) we seek in the form:

$$(\mu, \nu, u_1) = (M(y), N(y), U_1(\mathrm{y}))\, \mathrm{e}^{-i\omega t} \qquad (4.2)$$

Substituting (4.2) in (4.1) we come to the equations:

$$-\omega^2 U_1 = s_1^2 U_{1yy} + M_0 f N_y$$

$$i\omega M = \omega_M \hat{b} N + \omega_M \overline{b} \mu_0 U_{1y} \qquad (4.3)$$

$$i\omega N = -\omega_M \hat{b} M$$

Expressing M, N through U_1 from (4.3) we obtain the following equation:

$$U_{1yy} + q^2 U_1 = 0. \qquad (4.4)$$

This is the equation of harmonic oscillations of displacements amplitude in ferromagnetic. Above are used the following notations:

$$q=\frac{\omega}{\sqrt{s_1^2+\alpha}},\alpha=\frac{M_0 f\bar{b}\omega_M^2\mu_0}{\omega^2-\omega_M^2\hat{b}^2}$$

The solution of (4.4) is as follows:

$$U_1(y)=C_1\sin qy+C_2\cos qy; \tag{4.5}$$

C_1, C_2 are unknown constants. Using (4.5), we find the solutions of the system (4.1):

$$\begin{aligned} &u_1(y,t)=(C_1\sin qy+C_2\cos qy)e^{-i\omega t};\\ &\nu(y,t)=\bar{\alpha}q(C_1\cos qy-C_2\sin qy)e^{-i\omega t};\\ &\mu(y,t)=-\frac{i\omega\bar{\alpha}q}{\omega_M\hat{b}}(C_1\cos qy-C_2\sin qy)e^{-i\omega t}; \end{aligned} \tag{4.6}$$

Here we use following notation: $\bar{\alpha}=\frac{\bar{b}\hat{b}\omega_M^2\mu_0}{\omega^2-\omega_M^2\hat{b}^2}$. In analogous way, we find the displacements of nonmagnetic layer from (1.2):

$$u_2(y,t)=(C_3\cos py+C_4\sin py)e^{i\omega t},\quad p=\omega/s_2 \tag{4.7}$$

C_3, C_4 are constants. As we can see from (4.6) and (4.7) the solutions of the problem depend on spatial coordinate also, i.e. that there is a wave process in the structure. Thus, despite the fact that we did not take into account neither the exchange interaction, nor the magnetic potential, but owing to elasticity, this model is suitable for describing the elastic-spin wave propagation.

Let us examine them more detailed. Substituting solutions (4.6) and (4.7) in the boundary and contact conditions (1.3) we obtain the system of equations for amplitudes. The solvability condition of this system gives the following dispersion equation:

$$\lambda^2+D\lambda+1=0 \tag{4.8}$$

where:

$$D=\frac{sd\sin ph_2-m\sin qh_1+h\cos ph_2+g\cos qh_1}{s\sin qh_1-h\cos qh_1};$$

$$h=\rho_1 q(s_1^2+\mu_0\bar{b}\bar{\alpha})\cos qh_1;d=\frac{\rho_1 q(s_1^2+\mu_0\bar{b}\bar{\alpha})\cos qh_1}{\rho_2 s_2^2 p};$$

$$m=\rho_2 s_2^2 p\sin ph_2;s=-\rho_1 q(s_1^2+\mu_0\bar{b}\bar{\alpha})\sin qh_1;g=\rho_2 s_2^2 pd\cos ph_2;$$

Substituting $\lambda=e^{ika}$ $(a=h_1+h_2)$, k cross-wave number, averaged over the period of the structure, called Bloch's wave number) into (4.8) we obtain typical for periodic structures dispersion equation:

$$\cos ka=\cos qh_1\cos ph_2-\frac{1}{2}\left[\frac{z_1}{z_2}+\frac{z_2}{z_1}\right]\sin qh_1\sin ph_2, \tag{4.9}$$

here $z_1 = \dfrac{\rho_1(s_1^2 + \mu_0\bar{b}\bar{\alpha})}{\sqrt{s_1^2 + \alpha}}$, $z_2 = \rho_2 s_2$ are impedances of the layers.

From (4.9) follows that the frequency spectrum of elastic-spin waves divided into bands of wave transmission and stopping. Let us consider some special cases:

a) Impedances of two layers are equal $z_1 = z_2$. The dispersion equation takes the form:

$$ka = \omega\left(\frac{h_1}{\sqrt{s_1^2 + \alpha}} + \frac{h_2}{s_2}\right) \tag{4.10}$$

If the thicknesses of the layers are equal, then the Bloch's wave number is as follows $k = \dfrac{q+p}{2}$.

b) Let us consider the structure when instead of ferromagnetic layer is nonmagnetic one. In (4.9) we must take $\alpha = \bar{\alpha} = 0$, after that we come to equation:

$$\cos ka = \cos qh_1 \cos ph_2 - \frac{1}{2}\left[\frac{\rho_1 s_1}{\rho_2 s_2} + \frac{\rho_2 s_2}{\rho_1 s_1}\right]\sin qh_1 \sin ph_2\,. \tag{4.11}$$

This is the known dispersion equation for layered periodic elastic nonmagnetic structure with two different impedances. To investigate the interaction between elastic and magnetic excitations we return to the system (4.1). The solutions of the system we seek in the following form:

$$(\mu, \nu, u_1) = (M, N, U_1)e^{iqy}e^{i\omega t} \tag{4.12}$$

Substituting (4.12) in (4.1), after some transformations we obtain the relations between the magnetic and elastic waves amplitudes:

$$U_1 = \frac{iM_0 fq}{q^2 s_1^2 - \omega^2}N;\; M = \frac{\omega\omega_M \bar{b}\mu_0 q}{\omega^2 - \omega_M^2 \hat{b}^2}U_1;\; \Phi_1 = \frac{iM_0 fq\rho_1\bar{\alpha}}{q^2 s_1^2 - \omega^2}N. \tag{4.13}$$

As we see from (4.13), the relationship is resonant. The resonance frequency is in the region of ultrasonic and hypersonic waves ($\approx 10^{10}\dfrac{1}{\sec}$). This phenomenon known as magneto-acoustic resonance, which used in practice to construct ultra-acoustic or hyper-acoustic generators and other devices.

c) Let us consider the case when the thickness of ferromagnetic layer $h_1 \to 0$. Dispersion equation takes the form:

$$\cos ka = \cos ph_2 - \frac{1}{2}\left[\frac{\rho_1(s_1^2 + \mu_0\bar{b}\bar{\alpha})}{\rho_2 s_2\sqrt{s_1^2 + \alpha}} + \frac{\rho_2 s_2\sqrt{s_1^2 + \alpha}}{\rho_1(s_1^2 + \mu_0\bar{b}\bar{\alpha})}\right]qh_1 \sin ph_2\,.$$

We would like to make a remark, which may be of practical value. After solving the problem (4.1) and finding functions μ, ν, we can calculate magnetic potential φ_1, satisfying the boundary conditions, from the equation:

$$\frac{\partial^2 \varphi_1}{\partial y^2} = \rho_1 \frac{\partial \nu}{\partial y}.$$

Here are the results of some numerical experiments:

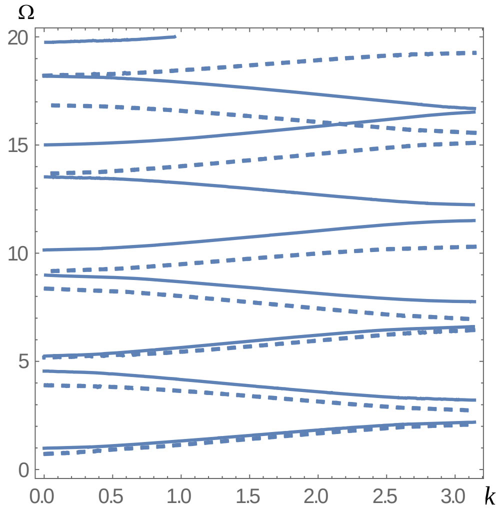

Fig.2. Dispersion curves for elastic-spin waves in the structure ferromagnetic/nonmagnetic. Frequency spectrum divided into wave pass-bands and stop-bands (gaps). The solid curve corresponds to the particular case when $h_1 < h_2$, the dashed one corresponds to the case when $h_1 > h_2$, $\Omega = \omega / \omega_M$.

From the Fig. 2 we can conclude that by changing the ratio of the layer thicknesses it is possible to ensure the desired frequency appeared in either the pass-band or the stop-band of the wave's frequency spectrum. It may find application in the construction of magnetic filters and other devices of spintronics.

5. Concluding remarks. The investigation of the simplified one-dimensional models, describing the propagation of elastic-spin waves perpendicular to the surfaces in magnetically ordered periodic media bring us to conclusion, that to describe the mentioned waves we must take into account exchange effects, or elasticity of media. If we neglect both of them, the considered model can describe only the oscillatory process. The frequency spectrum of elastic–spin waves in magnetic periodic structure divided into frequency bands of wave transmission and stopping. It finds application in practice.

We emphasize an important property of magnonic crystals, by changing the external magnetic field we can change the properties of crystals, this means that we can control the wave process. Photonic crystals do not possess this property.

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