ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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INFLUENCE OF SUPERSONIC GAS FLOW ON THE AMPLITUDE OF NON-LINEAR OSCILLATIONS OF RECTANGULAR PLATES

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Բանալի բառեր: Ճկուն սալեր, գազի գերձայնային հոսանք, ամպլիտուդ-արագություն կախվածություն

Բաղդասարյան Գ.Ե., Միկիլյան Մ.Ա., Սաղոյան Ռ.Օ.
Գազի գերձայնային հոսանքի ազդեցությունը ուղղանկյուն սալերի ոչ գծային տատանումների ամպլիտուդի վրա

Դիտարկված է գազի գերձայնային հոսանքով շրջհոսվող իզոտրոպ ուղղանկյուն սալի ոչ գծային տատանումների խնդիրը: Հետազոտությունը կատարված է երկու տիպի ոչ գծայնությունների հաշվառմամբ. աերոառաձգական (քառակուսային և խորանարդային) և երկրաչափական (խորանարդային): Աերոդինամիկական ոչգծայնության (հատկապես նրա քառակուսային ոչգծայնության) հաշվառման շնորհիվ ոչ գծային տատանումների ամպլիտուդի կախվածությունը շրջհոսող գազի արագության փոփոխության որոշակի միջակայքում երկարժեք է: Այդ փաստը ցույց են տալիս աշխատանքում բերված նկարներում երկու ճյուղերի տեսքով, որոնց ստորին ճյուղերը, ամենայն հավանականությամբ, անկայուն են: Անկայուն ճյուղերը բաժանում են երկու հարևան կայուն լուծումների ձգողության տիրույթները: Այստեղից հեշտությամբ ստացվում է գրգռման այն արժեքը, որն անհրաժեշտ է համակարգի մի կայուն ճյուղից մյուսին անցնելու համար: Ցույց է տրված պարամետրի փոփոխության որոշակի այն տիրույթների գոյությունը, որոնց դեպքում հնարավոր չէ գրգռել չմարող ֆլատերային տատանումներ ինչպես մինչկրիտիկական արագություններում, այնպես էլ հետկրիտիկական վիճակներում:

Багдасарян Г.Е., Микилян М.А., Сагоян Р.О.
Влияние сверхзвукового потока на амплитуду нелинейных колебаний прямоугольных пластин

Рассматривается задача нелинейных колебаний изотропной прямоугольной пластинки, обтекаемой сверхзвуковым потоком газа. Исследование проведено с учётом обоих типов нелинейности: аэродинамической (квадратичной и кубической) и геометрической (кубической). Благодаря учёту аэродинамической нелинейности (особенно её несимметричной квадратичной части) установлено, что зависимость амплитуды нелинейных колебаний от скорости обтекающего потока в определённых интервалах изменения скорости является двузначной. Этот факт иллюстрирован на приведённых в тексте фигурах в виде двух ветвей, нижние ветви из которых, по всей вероятности, являются неустойчивыми. Неустойчивые ветви отделяют области тяготения двух соседних устойчивых решений. Отсюда легко находится величина возмущения, необходимого для того, чтобы перебросить систему с одной устойчивой ветви на другую. Показаны существования определённых областей изменения парметра, при которых невозможно возбудить незатухающие флаттерные колебания как при докритических скоростях, так и при послекритической стадии.

The problem of nonlinear oscillations of isotropic rectangular plate in supersonic gas flow is examined. The study was conducted taking into account both types of non-linearity: wind (quadratic and cubic) and geometric (cubic). Due to the aerodynamic nonlinearity (especially its non-symmetric quadratic part) it is established that dependence amplitude-speed is two-valued at the certain intervals of the speed. This fact is illustrated on the figures given in the text in the form of two branches, the lower branches of which, in all probability, are unstable. The unstable branches are separated via the gravitational field of two adjacent sustainable solutions. Thus the perturbation magnitude can be easily found, which is required in order to transfer the system from one stable branch to another. Existence of specific areas of the speed is shown in which undamped flutter type oscillations cannot be excited in both pre-critical speeds and in post-critical stage.

Introduction

The literature is paved by numerous studies on the stability of plates and shells in supersonic gas flow. Significant contributions are reported in the monographs [1-3] and in the review article [4]. The interested reader can consult [1-7] for a linear descriptions of the problem, while in [2,8-15] aspects related to the nonlinear behavior of plates and shells in supersonic flows is discussed. The solution of the linear problem yields the critical value of the flow speed, u_*; at the onset of this critical speed the aeroelastic system loses its stability. The solution of the linear problem can be accomplished using a variety of methods, in many circumstances it can be achieved analytically in an closed form, while if an analytical solution cannot be found, often an approximate one, for example using the Galerkin method, can be reached [2]. Nonlinear panel flutter problems are solved by approximate methods to investigate the dependence of the amplitude of oscillations A on the speed of flowing stream, when the value of flowing speed is in the vicinity of critical flutter speed. In the flutter problems these issues, devoted to the investigation of properties of the function $A(\nu)$, when the plate is flown in both directions with the same speed, is investigated in detail in the works [2,13], where it is shown, that non-linear flutter type oscillations are exist either in pre-critical stage, where $A(\nu)$ is monotone decreasing function, or in post-critical speeds, where $A(\nu)$ is monotone increasing function. Non-linear flutter problems, in account of only geometrical type of non-linearity in the works [14,15] were discussed, also. In the works [8,9] it was shown, that aerodynamic non-linearity (especially its non-symmetric quadratic part) brings to the appearance of new types of dependencies $A(\nu)$ as in pre-critical, as well as in post-critical speeds, which are near to the critical. In the work [10] the influences of geometrical non-linearity on the dependence "amplitude-speed" are investigated in the case of cylindrical panels. It is shown, that dependence of the amplitude of non-linear flutter type oscillations on the value of speed of flowing stream can have multi-value character.

While the aeroelastic behavior in term of non-linear amplitude vs. speed is often the object of discussion, very limited literature deals with the non-linear amplitude vs. frequency. When $u=0$, the relationship between non-linear amplitude and frequency describing the nonlinear vibrations of a plate is classified as "hard" [1], i.e. with increasing non-linear amplitude there is a corresponding increasing in oscillatory frequency. In the work [16] it is shown, that the presence of flowing stream, due to the aeroelastic non-linearity, can be a source of both quantitative and qualitative change of the character of noted monotonically increasing relationship.

In the present work the dependence of amplitude of non-linear oscillations on the value of speed of flowing stream is investigated both in pre- and post- critical stage. the influence of frequency of nonlinear oscillations and geometrical parameters of the plate on the

dependence $A(\nu)$ is studied without the above-mentioned limitations on the value of supersonic flow speed. It is established that the character of the function $A(\nu)$ is changed significantly as quantitatively, as well as qualitatively, depending on the noted parameters. Possible types of the function $A(\nu)$ are shown in the Figures 1-6. Dependencies, illustrated in bottom parts of the Figures 5 and 6, were well-known due to the works [2,13].
Obtained in this study the main results are listed in paragraph 5.

1. Formulation of the problem of stability

The problem is formulated by considering a thin isotropic rectangular plate of constant thickness h. It is referred to the Cartesian coordinate plane α, β, γ and the coordinate plane α, β coincides with the middle plane of the plate and the coordinate lines α and β are directed along the edges of the plate. A supersonic gas flow with freestream velocity magnitude $\vec{u}$, is aligned with the axis 0α, on one side of the panel only. To investigate the aeroelastic stability of the examined plate the following assumptions are considered:

a) the Kirchhoff's hypothesis on non-deformable normal [18];

b) for the flexible plate the normal displacements are comparable with the thickness of the plate [1];

c) the third-order nonlinear Piston Theory Aerodynamics (PTA) is used when calculating the aerodynamic pressure [19,20].

Based on these assumptions the nonlinear aeroelastic governing equations can be cast as [2]:

$$\frac{1}{Eh}\Delta^2 F + \frac{\partial^2 w}{\partial\alpha^2}\frac{\partial^2 w}{\partial\beta^2} - \left(\frac{\partial^2 w}{\partial\alpha\partial\beta}\right)^2 = 0, \tag{1}$$

$$D\Delta^2 w - \frac{\partial^2 w}{\partial\alpha^2}\frac{\partial^2 F}{\partial\beta^2} - \frac{\partial^2 w}{\partial\beta^2}\frac{\partial^2 F}{\partial\alpha^2} + 2\frac{\partial^2 w}{\partial\alpha\partial\beta}\frac{\partial^2 F}{\partial\alpha\partial\beta} + \rho_0 h\frac{\partial^2 w}{\partial t^2} + \left(\rho_0 h\varepsilon + \frac{æp_\infty}{a_\infty}\right)\frac{\partial w}{\partial t} +$$
$$+æp_\infty\left[M\frac{\partial w}{\partial\alpha} + \frac{æ+1}{4}M^2\left(\frac{\partial w}{\partial\alpha}\right)^2 + \frac{æ+1}{12}M^3\left(\frac{\partial w}{\partial\alpha}\right)^3\right] = 0, \tag{2}$$

Herein

$$D = \frac{Eh^3}{12\left(1-\mu^2\right)}, \quad M = \frac{u}{a_\infty}, \quad a_\infty^2 = \frac{æp_\infty}{\rho_\infty},$$

$w(\alpha,\beta,t)$ – is the out-of-plane plate deflection, M – is the Mach number of undisturbed flow, a_∞ – is the sound speed for the undisturbed gas, æ is the isentropic gas coefficient, μ – is the Poisson's ratio, ρ_0 – is the density of plate's material, p_∞ and ρ_∞ – are the pressure and gas density in the undisturbed state, ε – is the coefficient of linear attenuation, and $F = F(\alpha,\beta,t)$ – is the stress function.

To investigate the issues of stability of the examined system the boundary conditions must be added to the set of Eqs. (1) and (2). A simple-supported edges along the contour of the rectangular plate, implying that the plate is free to move in the plane, will be considered $\left(0 \le \alpha \le a,\ 0 \le \beta \le b\right)$. Consequently, according to [2], the following boundary conditions are used:

for $\alpha = 0$, $\alpha = a$

$$w = 0, \quad M_\alpha = -D\left(\frac{\partial^2 w}{\partial \alpha^2} + \mu \frac{\partial^2 w}{\partial \beta^2}\right) = 0, \tag{3}$$

$$T_\alpha^0 = 0,\ T_{\alpha\beta}^0 = 0 \tag{4}$$

for $\beta = 0$, $\beta = b$

$$w = 0, \quad M_\beta = -D\left(\frac{\partial^2 w}{\partial \beta^2} + \mu \frac{\partial^2 w}{\partial \alpha^2}\right) = 0, \tag{5}$$

$$T_\beta^0 = 0,\ T_{\beta\alpha}^0 = 0, \tag{6}$$

where T_α^0, T_β^0, $T_{\alpha\beta}^0$ – are the average values of the force at the edges of the plate.

2. Reduction to the stability problem, described by the system of ordinary differential equations

An approximate solution of Eq. (2), satisfying conditions (3) and (5) which, let's present in the form [2]

$$w\left(\alpha, \beta, t\right) = f_1(t)\sin\lambda_1\alpha \cdot \sin\mu_1\beta + f_2(t)\sin\lambda_2\alpha \cdot \sin\mu_1\beta \tag{7}$$

$$\left(\lambda_i = \frac{i\pi}{a},\ \mu_k = \frac{k\pi}{b}\right)$$

where $f_i(t)$ – are functions of time t, still to be determined.

Substituting (7) into (1) the linear non-homogeneous system of ordinary differential equations with respect to the function F will be obtained. The Solution of the noted equation, satisfying the boundary conditions (4) and (6) is presented as:

$$F\left(\alpha, \beta, t\right) = \frac{Eh}{4}\left[-\frac{\mu_1^2}{\lambda_1^2} f_1 f_2 \cos\left(\lambda_1\alpha\right) + \frac{\mu_1^2}{8\lambda_1^2} f_1^2 \cos\left(\lambda_2\alpha\right) + \frac{\mu_1^2}{9\lambda_1^2} f_1 f_2 \cos\left(\lambda_3\alpha\right) + \right.$$
$$+ \frac{\mu_1^2}{32\lambda_1^2} f_2^2 \cos\left(\lambda_4\alpha\right) + \frac{9\lambda_1^2\mu_1^2}{\Delta_{\lambda_1\mu_2}} f_1 f_2 \cos\left(\lambda_1\alpha\right)\cos\left(\mu_2\beta\right) -$$
$$\left. - \frac{\lambda_1^2\mu_1^2}{\Delta_{\lambda_3\mu_2}} f_1 f_2 \cos\left(\lambda_3\alpha\right)\cos\left(\mu_2\beta\right) + \left(\frac{\lambda_1^2}{2\mu_1^2} f_2^2 + \frac{\lambda_1^2}{8\mu_1^2} f_1^2\right)\cos\left(\mu_2\beta\right)\right],$$

To calculate the functions $f_i(t)$ Eq. (2) will be used. Substituting Eq. (7) and the already found expression F into Eq. (2), and solving it by using the Bubnov-Galerkin method, the nonlinear system of ordinary differential equations, with respect to unknown functions $x_1 = f_1(t)/h,\ x_2 = f_2(t)/h$, one obtains [2,13]:

$$\begin{aligned}
&\frac{d^2x_1}{d\tau^2} + \chi\frac{dx_1}{d\tau} + x_1 - \frac{2}{3}k\nu x_2 + k\nu^2\left[\alpha_{11}x_1^2 + \alpha_{12}x_2^2 + \right.\\
&\qquad \left. + \nu x_2\left(\beta_{11}x_1^2 + \beta_{12}x_2^2\right)\right] + Qx_1\left(\gamma_{11}x_1^2 + \gamma_{12}x_2^2\right) = 0\\
&\frac{d^2x_2}{d\tau^2} + \chi\frac{dx_2}{d\tau} + \gamma^2 x_2 + \frac{2}{3}k\nu x_1 + k\nu^2\left[\alpha_{21}x_1x_2 + \right.\\
&\qquad \left. + \nu x_1\left(\beta_{21}x_1^2 + \beta_{22}x_2^2\right)\right] + Qx_2\left(\gamma_{21}x_1^2 + \gamma_{22}x_2^2\right) = 0.
\end{aligned} \tag{8}$$

Herein, along with the dimensionless time $\tau = \omega_1 t$, the following notations are considered:

$$\left.\begin{aligned}
&\omega_i^2 = \frac{D}{\rho_0 h}\left(\lambda_i^2 + \mu_1^2\right)^2 \quad (i=1,2),\ \ k = \frac{4æp_\infty}{\rho_0\omega_1^2 h^2},\ \ Q = \frac{h}{16\rho_0\omega_1^2},\\
&\nu = M\frac{h}{a}, \qquad \gamma = \frac{\omega_2}{\omega_1}, \qquad \chi = \frac{2}{\omega_1}\left(\varepsilon + \frac{æp_\infty}{\rho_0 h a_\infty}\right)
\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}
&\alpha_{11} = \frac{2}{9}(æ+1),\ \ \alpha_{12} = \frac{56}{45}(æ+1),\ \ \alpha_{21} = \frac{16}{45}(æ+1),\\
&\beta_{11} = \beta_{21} = \frac{\pi^2}{40}(æ+1),\ \ \beta_{22} = \frac{11\pi^2}{70}(æ+1),\ \ \beta_{12} = -\frac{9\pi^2}{70}(æ+1),\\
&\gamma_{11} = Eh\lambda_1^4\left(1+\varphi^4\right), \gamma_{22} = Eh\lambda_1^4\left(16+\varphi^4\right),\\
&\gamma_{12} = \gamma_{21} = Eh\lambda_1^4\left(4\left(1+\varphi^4\right) + \frac{81\varphi^4}{\left(1+4\varphi^2\right)^2} + \frac{\varphi^4}{\left(9+4\varphi^2\right)^2}\right) \qquad \varphi = ab^{-1}
\end{aligned}\right\} \tag{10}$$

where ω_1 and ω_2 are first and second natural frequencies of the plate, while ν is the reduced speed parameter.

3. Solution of the linear problem

The solution of the nonlinear problem is usually preceded by analysis of the corresponding linear problem, since: a) the critical parameter $\nu = \nu_{cr}$ (hence the critical flow speed $u_{cr} = ah^{-1}\nu_{cr}a_\infty$, e.g. $M_{cr} = ah^{-1}\nu_{cr}$), at which the unperturbed state of the plate becomes unstable with respect to any small perturbation can be found, and b) the critical state

u_{cr} (e.g. M_{cr} or ν_{cr}), is necessary in the investigation of the type of stability of a nonlinear system.
Thus, the linear system of equations obtained linearizing equations (8) has the form

$$\frac{d^2 x_1}{d\tau^2} + \chi \frac{dx_1}{d\tau} + x_1 - \frac{2}{3} k\nu x_2 = 0,$$
$$\frac{d^2 x_2}{d\tau^2} + \chi \frac{dx_2}{d\tau} + \gamma^2 x_2 + \frac{2}{3} k\nu x_1 = 0. \qquad (11)$$

Representing the solution by $x_1 = y_1 e^{\lambda\tau}$, $x_2 = y_2 e^{\lambda\tau}$, the characteristic equation with respect to λ can be cast as:

$$\lambda^4 + 2\chi\lambda^3 + \left(\gamma^2 + 1 + \chi^2\right)\lambda^2 + \chi\left(\gamma^2 + 1\right)\lambda + \gamma^2 + \frac{4}{9} k^2 \nu^2 = 0.$$

The unperturbed form of the plate is stable if the real parts of the roots of the characteristic equation are negative. Consequently, according to Hurwitz's theorem [21], the conditions for stability can be written in the form:

$$\chi > 0, \quad \chi\left(1+\gamma^2\right) > 0, \quad \left(\gamma^2 - 1\right)^2 + 2\chi^2\left(1+\gamma^2\right) - \frac{16}{9} k^2 \nu^2 > 0.$$

The first two inequalities require that the damping (internal and aerodynamic) is positive. From the third inequality it follows that for small values ν, all characteristic roots λ lie in the left half of a complex variable, and the trivial solution $w \equiv 0$ is asymptotically stable with respect to small perturbations. The value of the parameter $\nu = \nu_{cr}$, for which two of the characteristic exponents are purely imaginary, and the remaining lie in the left half-plane, is critical and corresponds to the panel flutter speed in the linear formulation of this problem. Accordingly, the critical flutter speed in the case of the selected buckling form of the plate [2] can be obtained from the third inequality:

$$\nu_{cr} = \frac{3}{4} \frac{\gamma^2 - 1}{k} \sqrt{1 + \frac{2\chi^2\left(\gamma^2 + 1\right)}{\left(\gamma^2 - 1\right)^2}}. \qquad (12)$$

Taking $\nu = \nu_{cr}$ from the characteristic equation Eq. (11a), the critical vibration frequency θ_{cr} in the linear formulation $\left(\lambda_{cr} = \pm i\theta_{cr}\right)$ is

$$\theta_{cr}^2 = \frac{1}{2}\left(\gamma^2 + 1\right). \qquad (13)$$

It is worth noting that Eqs. (12) and (13) can also be found in [2,4] as well as other references, demonstrating that the proposed solution is a very good approximation for both ν_{cr} and θ_{cr}, determined on the basis of the exact solution [4-7,22].

4. Solution of nonlinear problem

The nonlinear problem described by Eqs. (8) is studied next. This system of equations is different from the one used to study the linear stability for flexible plates forced by non-conservative aerodynamic loading, specifically in terms of the aerodynamics, since now quadratic and cubic nonlinear terms are included in the problem formulation. Specifically, in the system of equation (8) asymmetric quadratic nonlinearities of aerodynamic and aeroelastic origin are included along with cubic terms. The quadratic nonlinearities are inherent to the problems of the stability of flexible shells. Therefore, the approximate periodic solution of Eqs. (8) is presented as [10]

$$x_1 = A_1 \cos\theta\tau + B_1 \sin\theta\tau + C_1 + \ldots, \quad x_2 = A_2 \cos\theta\tau + B_2 \sin\theta\tau + C_2 + \ldots \tag{14}$$

Here A_i, B_i, C_i и $\theta = \omega\omega_1^{-1}$ $(i = 1, 2)$ are unknown constants; ω is unknown frequency of nonlinear vibrations and the dots denote high-order harmonic terms which, without loss of generality and accuracy, can safely be discarded. Contrarily to existing solutions, as reported in [2,13], the proposed one, Eqs. (14), includes also the constant terms $C_i \neq 0$, which are used to characterize the quadratic nonlinearities [10,23]. When substituted into Eqs. (8) the constant member and first harmonics $\cos\theta\tau$ and $\sin\theta\tau$ are retained while the terms containing harmonics are neglected. Although straightforwardly obtainable, the system of nonlinear algebraic equations is lengthily and is not presented here. To obtain the approximate solution of this system the following assumptions are made [10]:

a) the damping is small such that $\chi |B_i| << |A_i|$, $|B_i| << |A_i|$;

b) the aeroelastic system reaches a steady oscillatory state with finite amplitude around the equilibrium state, which is infinitesimally different from the unperturbed state, $(|A_i| >> |C_j|; \; j = 1,2)$.

According to these assumptions, and neglecting the degrees above the first and any of the products of B_1, B_2, C_1 and C_2, the nonlinear system can be represented by a subsystem of nonlinear equations including:

Two equations obtained by equating to zero the zero order terms:

$$C_1 - \frac{2}{3}k\nu C_2 + \frac{1}{2}k\nu^2\left(\alpha_{11}A_1^2 + \alpha_{12}A_2^2\right) + k\nu^3 A_2\left(\beta_{11}A_1C_1 + \beta_{12}A_2C_2\right) +$$

$$+\frac{1}{2}k\nu^3 C_2\left(\beta_{11}A_1^2 + \beta_{12}A_2^2\right) + QA_1\left(\gamma_{11}A_1C_1 + \gamma_{12}A_2C_2\right) +$$

$$+\frac{1}{2}QC_1\left(\gamma_{11}A_1^2 + \gamma_{12}A_2^2\right) = 0,$$

$$\gamma^2 C_2 + \frac{2}{3}k\nu C_1 + \frac{1}{2}k\nu^2\alpha_{21}A_1A_2 + k\nu^3 A_1\left(\beta_{21}A_1C_1 + \beta_{22}A_2C_2\right) +$$

$$+\frac{1}{2}k\nu^3 C_1\left(\beta_{21}A_1^2 + \beta_{22}A_2^2\right) + QA_2\left(\gamma_{21}A_1C_1 + \gamma_{22}A_2C_2\right) +$$

$$+\frac{1}{2}QC_2\left(\gamma_{21}A_1^2 + \gamma_{22}A_2^2\right) = 0;$$

Two equations obtained by equating to zero the coefficients of $\cos\theta\tau$:

$$\left(1-\theta^2\right)A_1 + \chi\theta B_1 - \frac{2}{3}k\nu A_2 + 2k\nu^2\left(\alpha_{11}A_1C_1 + \alpha_{12}A_2C_2\right) +$$

$$+\frac{3}{4}k\nu^3 A_2\left(\beta_{11}A_1^2 + \beta_{12}A_2^2\right) + \frac{3}{4}QA_1\left(\gamma_{11}A_1^2 + \gamma_{12}A_2^2\right) = 0,$$

$$\left(\gamma^2-\theta^2\right)A_2 + \chi\theta B_2 + \frac{2}{3}k\nu A_1 + \alpha_{21}k\nu^2\left(A_1C_2 + A_2C_1\right) +$$

$$+\frac{3}{4}k\nu^3 A_1\left(\beta_{21}A_1^2 + \beta_{22}A_2^2\right) + \frac{3}{4}QA_2\left(\gamma_{21}A_1^2 + \gamma_{22}A_2^2\right) = 0;$$

Two equations obtained by equating to zero the coefficients of $\sin\theta\tau$:

$$\left(1-\theta^2\right)B_1 - \frac{2}{3}k\nu B_2 - \chi\theta A_1 + \frac{1}{2}k\nu^3\beta_{11}A_1A_2B_1 +$$

$$+\frac{1}{4}k\nu^3\left(\beta_{11}A_1^2 + 3\beta_{12}A_2^2\right)B_2 + \frac{1}{4}Q\left(3\gamma_{11}A_1^2 + \gamma_{12}A_2^2\right)B_1 +$$

$$+\frac{1}{2}Q\gamma_{12}A_1A_2B_2 = 0,$$

$$\left(\gamma^2-\theta^2\right)B_2 - \chi\theta A_2 + \frac{2}{3}k\nu B_1 + \frac{1}{4}k\nu^3\left(3\beta_{21}A_1^2 + \beta_{22}A_2^2\right)B_1 +$$

$$+\frac{1}{2}k\nu^3\beta_{22}A_1A_2B_2 + \frac{1}{2}Q\gamma_{21}A_1A_2B_1 +$$

$$+\frac{1}{4}Q\left(\gamma_{21}A_1^2 + 3\gamma_{22}A_2^2\right)B_2 = 0.$$

It should be noted that the third subsystem takes into account damping terms. Assuming that the damping is small $(\chi \approx 0)$, by performing a linearization of this equations, one obtains: $B_1 \approx 0,\ \ B_2 \approx 0$ for $\chi \approx 0$

Using the first subsystem let's express C_1 and C_2 trough A_1 and A_2 (see (16)). From the second subsystem the amplitudes of oscillations of the examined aeroelastic system, A_1 and A_2, are computed as function of the parameters θ and ν. Then for $\chi \approx 0$ it has the form:

$$\begin{aligned} &A_1(1-\theta^2) - \frac{2}{3}k\nu A_2 + 2k\nu^2\alpha_{11}A_1C_1 + 2k\nu^2\alpha_{12}A_2C_2 + \\ &\qquad + \frac{3}{4}k\nu^3 A_2(\beta_{11}A_1^2 + \beta_{12}A_2^2) + \frac{3}{4}QA_1(\gamma_{11}A_1^2 + \gamma_{12}A_2^2) = 0, \\ &A_2(\gamma^2-\theta^2) + \frac{2}{3}k\nu A_1 + k\nu^2\alpha_{21}(A_1C_2 + A_2C_1) + \\ &\qquad + \frac{3}{4}k\nu^3 A_1(\beta_{21}A_1^2 + \beta_{22}A_2^2) + \frac{3}{4}QA_2(\gamma_{21}A_1^2 + \gamma_{22}A_2^2) = 0. \end{aligned} \tag{15}$$

Herein

$$\begin{aligned} C_1 &= -\frac{k\nu^2}{2\Delta}\left[(\alpha_{11}A_1^2 + \alpha_{12}A_2^2)\Delta_2 - \alpha_{21}A_1A_2\Delta_4\right] \\ C_2 &= -\frac{k\nu^2}{2\Delta}\left[\alpha_{21}A_1A_2\Delta_1 - \left(\alpha_{11}A_1^2 + \alpha_{12}A_2^2\right)\Delta_3\right] \end{aligned} \tag{16}$$

where

$$\Delta_1 = 1 + \frac{3}{2}Q\gamma_{11}A_1^2 + \frac{1}{2}Q\gamma_{12}A_2^2 + k\nu^3\beta_{11}A_1A_2,$$

$$\Delta_2 = \gamma^2 + k\nu^3\beta_{22}A_1A_2 + \frac{3}{2}Q\gamma_{22}A_2^2 + \frac{1}{2}Q\gamma_{21}A_1^2,$$

$$\Delta_3 = \frac{2}{3}k\nu + \frac{3}{2}k\nu^3\beta_{21}A_1^2 + \frac{1}{2}k\nu^3\beta_{22}A_2^2 + Q\gamma_{21}A_1A_2,$$

$$\Delta_4 = -\frac{2}{3}k\nu + \frac{3}{2}k\nu^3\beta_{12}A_2^2 + \frac{1}{2}k\nu^3\beta_{11}A_1^2 + Q\gamma_{12}A_1A_2,\ \Delta = \Delta_1\Delta_2 - \Delta_3\Delta_4.$$

In the particular case when ν is in close proximity of ν_{cr} [2], it follows $A_1 \approx -A_2$. This can be demonstrated from the third subsystem after the linearization which leads to:

$$(1-\theta^2)B_1 - \frac{2}{3}k\nu B_2 - \chi\theta A_1 = 0,\ (\gamma^2-\theta^2)B_2 + \frac{2}{3}k\nu B_1 - \chi\theta A_2 = 0. \tag{17}$$

Taking into account that at the flutter boundary $\nu = \nu_{cr}$ и $\theta = \theta_{cr}$, from (12), (13) and (17) we have

$$\chi\theta_{cr}A_1 = \frac{1-\gamma^2}{2}(B_1+B_2),\ -\chi\theta_{cr}A_2 = \frac{1-\gamma^2}{2}(B_1+B_2);$$

As a result, as expected, $A_1 \approx -A_2$, if ν is in the close proximity of ν_{cr} and θ tends to θ_{cr}. It should be noted that studies conducted in the [2,8,9] are based on an approximate equality $A_1 \approx -A_2$.

This system (15) is solved numerically for the following initial set of parameters: $E = 7.3\cdot10^{10}\,\text{N/m}^2$; $\mu = 0.34$; $\rho_0 = 2.79\cdot10^3\,\text{kg/m}^3$ (Duralumin), while the flow properties used are $æ = 1.4$; $\rho_\infty = 1.29 kg/m^3$; $a_\infty = 340.29 m/s$ (air). The dependency of the amplitude A of steady oscillations at point $(a/2, b/2, 0)$ for which $A = A_1$ on the parameter ν, characterizing the flowing speed for several h/a, a/b and θ.

Numerical calculations, having done in [8,9], show that the relation h/a has significant influence (as qualitative, as well as quantitative) on the character of the dependence "amplitude-speed". Therefore, the cases of thick and thin plates will be considered separately.

4.1. Influence of supersonic flow on the character of dependence "amplitude-speed" in the case of sufficiently thick plates

The results of numerical solution of the system (15) for $a = 70h,\ b = 5a$ and several fixed values of θ, representing dependence of the amplitude of flutter type oscillations on the speed parameter ν, are brought in the Table 1 and on plotted on its basis Figures 1-3. The dependence $A(\nu)$ for several a/b and fixed θ is brought on the Table 2.

The most interesting result of these calculations is the following: limit cycle oscillations are possible as in pre-critical speeds of flowing stream $(\nu < \nu_{Cr})$, as well as in post-critical stage $(\nu > \nu_{Cr})$. A similar result in a qualitative sense, was obtained in [2.13], in which it is established, that for the certain parameters of the problem either hard type of oscillations $(\nu > \nu_{Cr})$, or only soft type excitations $(\nu < \nu_{Cr})$ occur. The reason for the noted discrepancy is the accounting of the wind type quadratic nonlinearity and the rejection of the assumption $A_1 \approx -A_2$.

Table 1 show that for the chosen parameter b/a the change of the frequency θ has as qualitative, as well as quantitative influence on the dependence "amplitude-speed". Namely:

- If $\theta \in (0;1]$, such speed value ν_{cr} exists, for which if $\nu < \nu_{cr}$ then generation of limit cycle oscillations is impossible. For $\nu \geq \nu_{cr}$ dependence of the amplitude of oscillations on the speed of flowing stream (the function $A(\nu)$) is a two-value one, which tends to zero with the increasing ν (Fig.1).

Table 1. Influence of the parameter θ on the dependence "amplitude-speed".

$\frac{\nu}{\nu_G}$ \ θ	0.5	1	1.1	1.5	2	4
0.2	-	-	0.387	1.558	2.500	5.693 0.036
0.24	-	-	0.097	1.522	2.480	5.687 0.044
0.25	-	-	0	1.512	2.475	5.685 0.047
0.568	-	-	-	1.446 0	2.511	5.519 0.137
0.57	-	-	-	1.451 0.100	2.513	5.516 0.138
0.7	-	-	-	2.221 0.625	2.754	4.561 0.155
0.722	-	1.526 1.477	1.829 1.165	2.293 0.622	2.721	4.175 0.152
0.77	1.582 1.466	1.997 0.997	2.054 0.922	2.265 0.602	2.500	3.165 0.147
0.8	1.811 1.126	1.976 0.892	2.009 0.837	2.140 0.585	2.286 0	2.532 0.142
0.9	1.526 0.792	1.550 0.685	1.555 0.656	1.567 0.518	1.562 0.273	1.027 0.130
1	1.154 0.629	1.147 0.561	1.144 0.542	1.125 0.454	1.078 0.312	0.267 0.187
1.4	0.477 0.335	0.460 0.310	0.455 0.303	0.429 0.271	0.382 0.224	-
2.6	0.120 0.105	0.113 0.099	0.111 0.097	0.102 0.088	0.087 0.074	-

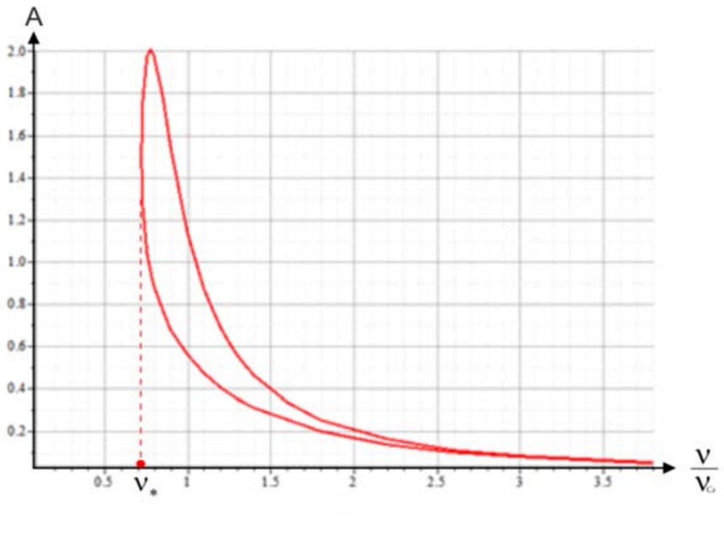

Fig.1. Plot of the function $A(\nu)$ for $\theta = 1$.

- If $\theta \in (1;\theta_1)$ (the value of θ_1 depends on the geometry of the plate), then a segment $[\nu_*, \nu^*]$ of the parameter ν exists, generation of limit cycle oscillations is impossible (Fig.2). Out of this segment for $\nu \le \nu_*$ the function $A(\nu)$ is a unique-value and monotone decreasing, and for $\nu \ge \nu^*$ the function $A(\nu)$ is a two-value one and qualitatively analogous to the function plotted in the Fig.1. With the increasing θ the length of the segment $[\nu_*, \nu^*]$ decreases and equals to zero at the certain value θ.

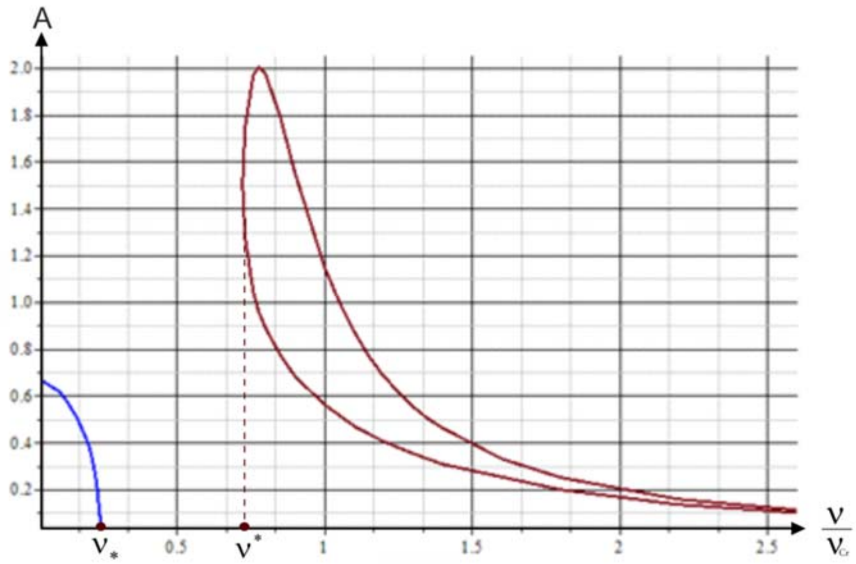

Fig.2. Plot of the function $A(\nu)$ при $\theta = 1.1$

- If $\theta_1 \le \theta < \theta_2$ (the segment $[\theta_1;\theta_2)$ can be changed depending on the plate's geometry) the plot of the function $A(\nu)$ is brought in the Fig.3. Such speed value ν_* exists, for which if $\nu < \nu_*$ the function $A(\nu)$ is a unique-value, while for $\nu \ge \nu_*$ it is a two-value one, which branches have maximum. After maximum points the amplitude is monotone decreases and tends to zero. With the increasing θ the value of ν_* increases.

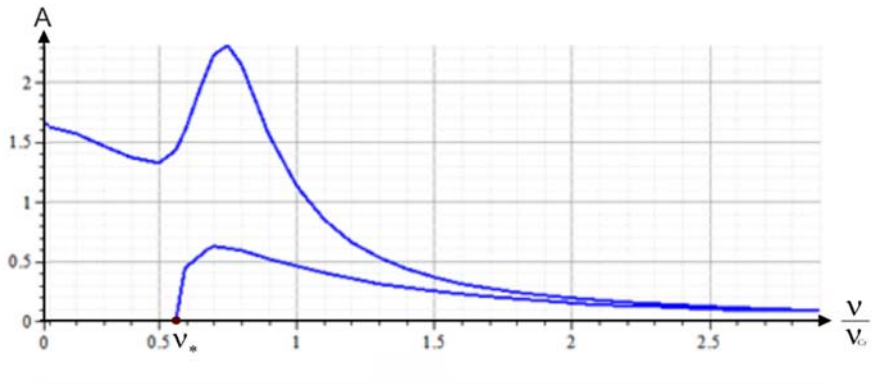

Fig.3. Plot of the function $A(\nu)$ for $\theta = 1.5$.

Let's note that here and in the future for the certain segments of frequency of oscillations the own ν_* and ν^* exist, which are brought in the corresponding figures. For the fixed geometry the change of the parameter θ has quantitative influence on the dependence "amplitude-speed", also. Table 1 shows that increment of the parameter θ brings to the change of the amplitude (the amplitude decreases on the lower branch, and increases – on the upper branch).

Let's study now the influence of the relation b/a on the dependence "amplitude-speed" for $\theta = 2$. The results of numerical calculations of the function $A(\nu)$ are brought on the Table 2.

Table 2. Influence of b/a on the dependence "amplitude-speed"

$\frac{\nu}{\nu_{Cr}}$ / b/a	0.2	0.5	0.75	0.82	0.85	1	1.5	2	3	5	10	15
5	2.5007	2.4245	2.6159	2.1320 0.0425	1.9028 01948	1.0781 0.3119	0.3183 0.2013	0.1569 0.1218	0.0640 0.0564	0.0221 0.0207	0.0054 0.0052	0.0024 0.0023
3	2.6659	2.6069	2.5436	1.9822	1.7125 0.0174	0.9845 0.2815	0.2947 0.1893	0.1459 0.1142	0.0597 0.0528	0.0206 0.0193	0.0051 0.0049	0.0022 0.0021
2	2.9415	2.9076	2.3273	1.6955	1.4818	0.8272 0.2243	0.2530 0.1677	0.1262 0.1005	0.0518 0.0461	0.0179 0.0168	0.0044 0.0042	0.0019 0.0018
1	3.4585	3.4119	0.9213	0.6144	0.5312	0.2957 0.0696	0.0909 0.0766	0.0450 0.0409	0.0183 0.0174	0.0063 0.0061	0.00154 0.00150	0.00068 0.00067

Table 2 shows, that this influence has only qualitative character. In particular, for $\theta = 2$ the Fig.3 is true. Both ν_* and corresponding values of the amplitude decrease with the increasing b/a.

4.2. Influence of supersonic stream on the behavior of non-linear oscillations for thick and sufficiently thin plates

Here numerical analysis is done for several data of parameters θ and b/a for the fixed h/a. The results of numerical solutions are brought in the tables 3-5. Having discussed the brought tables one can note, that the dependence $A(\nu)$ in the case of thin plates, in addition to the already known and brought in the Fig. 1 and 2, which were plotted in the case of thick plates, here new behaviors take place and on its basis Fig. 4-7 are plotted.
As table 3 shows in the case of thin plates for fixed b/a influence of the parameter θ on the dependence "amplitude-speed" has as qualitative, as well as quantitative character.

Table 3. Influence of frequency θ of oscillations on the dependence "amplitude-speed" for h/a=1/120,b/a=2

θ \ v/v_{Cr}	0.1	0.2	0.4	0.9	0.93	1.75	2.72	3.4	3.9	4	4.5	4.6	4.9	5
0.5	-	-	-	-	-	-	-	-	-	-	-	-	0.423 0.397	0.405 0.371
1	-	-	-	-	-	-	-	-	-	-	-	0.483 0.448	0.418 0.363	0.397 0.342
1.1	0.7361	0.5510	0	-	-	-	-	-	-	-	0.503 0.475	0.484 0.435	0.415 0.355	0.394 0.334
2	2.9679	2.9373	2.8077	1.5606 0	1.1244 0.8247	-	-	-	0.6708 0.5810	0.641 0.519	0.465 0.345	0.436 0.322	0.359 0.267	0.337 0.252
3	4.8593	4.8480	4.8023	4.5341 0.1363	4.5096 0.1864	2.6417 2.2734	-	0.7935 0.6475	0.5381 0.3387	0.491 0.306	0.312 0.195	0.286 0.179	0.223 0.143	0.205 0.132
4	6.6573 0.0366	6.6519 0.0738	6.6310 0.1522	6.5142 0.3939	6.5042 0.4111	6.0503 0.9831	1.9286 1.5904	-	-	-	-	-	-	-

Moreover:

- dependence $A(\nu)$ for $\theta \in (0;1]$ has the same character as in the case of sufficiently tick plates (Fig.1);
- depending on the geometrical parameters of the plate the certain such value of frequency $\theta_1 > 1$ exists, that if $\theta \in (1;\theta_1)$ the dependence $A(\nu)$ is identical to the dependence, brought in the Fig.2 and plotted in the case of sufficiently tick plates;

- in an analogous way, such segment $[\theta_1, \theta_2]$ of frequency exists, that for $\theta_1 \le \theta < \theta_2$ (this segment changes with the geometry of the plate), the function $A(\nu)$ has a plot, brought in the Fig.4, for more evidence which is drawn for b/a=5. In this case such certain values $\nu_*, \nu^*, \bar{\nu}$ of speed parameter ν exist, that if $\nu \in [\nu_*, \nu^*]$ then generation of limit cycle oscillations is impossible. Moreover if $\nu > \nu^*$ and $\nu \in (\bar{\nu}, \nu_*)$ the function $A(\nu)$ is a two-value, and for $\nu < \bar{\nu}$ it is a unique-value function. With the increasing frequency of oscillations: a) length of the segment $[\nu_*, \nu^*]$ decreases; b) on the left side of ν_* the amplitude increases, and on the right side of ν^* it decreases. Let's note, that if $\theta = \theta_{cr}$, then $\bar{\nu} = 1$.

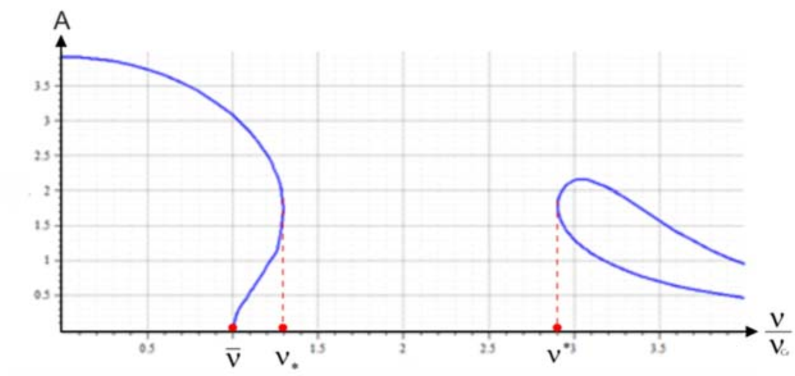

Fig.4. Dependence "amplitude-speed" for $h/a = 1/120,\ b/a = 5,\ \theta = \theta_{cr} \approx 2.84$

The results of numerical calculations of $A(\nu)$ in the case of sufficiently thin plates $(h/a = 1/300,\ b/a = 3)$ for several values of the parameter θ are brought in the Table 4. Table 4 shows, that for the fixed b/a the influence of the parameter θ on the behavior of the function $A(\nu)$ is presented as follows:

- If $\theta \in (0;1]$, then it is impossible to generate limit cycle oscillations in the sufficiently thin plates;
- If $\theta \in (1, \theta_1)$ (with the varying geometry θ_1 varies), then such speed value ν_* exists, that if $\nu > \nu_*$ then it is impossible to generate limit cycle oscillations, and for $\nu \le \nu_*$ the dependence of the amplitude on the flowing speed (the function $A(\nu)$) is a unique-value and monotone decreasing one (Fig.5), moreover

$A(\nu_*)=0$. Let's note, that the mentioned behavior of the function $A(\nu)$ is explored and studied in [2];

Table 4. Influence of the parameter θ on the function $A(\nu)$

θ \ $\frac{\nu}{\nu_{Cr}}$	0.1	0.2	0.26	0.5	0.8	0.85	1	1.1	1.248	1.5	1.59	2.2
0.5	-		-	-	-	-	-	-	-	-	-	-
1.1	0.660	0.446	0	-	-	-	-	-	-	-	-	-
1.9	2.515	2.473	2.433	2.1345	0.7617	0	-	-	-	-	-	-
2.71	3.937	3.917	3.898	3.7717	3.4605	3.3855	3.0976 0	2.8219 0.5721	1.8213 1.6641	-	-	-
3.2	4.755	4.741	4.728	4.6446	4.4501	4.4061 0	4.2490 0.2672	4.1191 0.4320	3.8759 0.7111	3.1738 1.4671	2.401 2.260	-
4	6.061 0.027	6.053 0.054	6.046 0.072	5.9991 0.1523	5.8939 0.2897	5.8710 0.3177	5.7920 0.4120	5.7303 0.4845	5.6237 0.6078	5.3924 0.8713	5.291 0.986	2.437 3.953

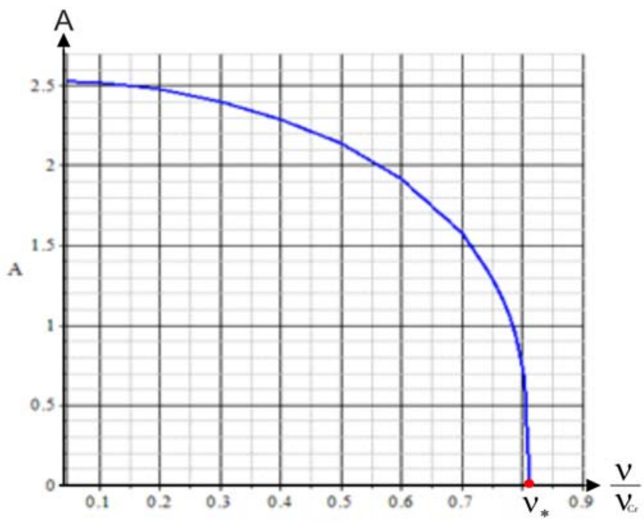

Fig.5. Plot of the function $A(\nu)$ for $\theta=1.9$

- If $\theta \in [\theta_1, \theta_2)$ (this segment changes with the geometry of the plate), then the plot of the function $A(\nu)$ is brought in the Fig.6. For clarity and diversity the Fig.6 is drawn for $h/a = 1/300,\ b/a = 3$. In this case such certain values ν_* and $\bar{\nu}$ of the parameter ν exist, that: a) if $\nu \geq \nu_*$ then it is impossible to generate limit cycle oscillations; b) if $\nu \in (\bar{\nu}, \nu_*)$, then the function $A(\nu)$ is a two-value one; c) if $\nu < \bar{\nu}$, then it is a unique-value, monotone decreasing function.

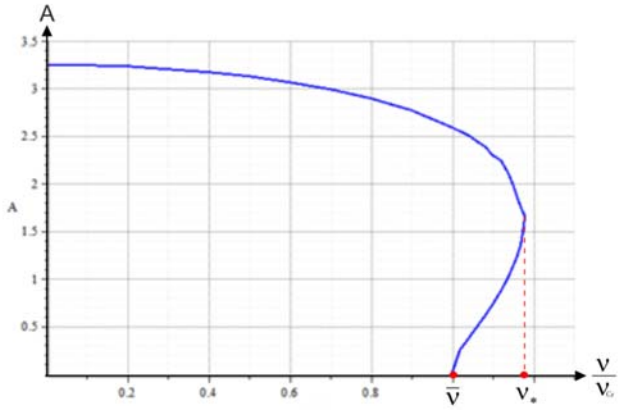

Fig.6. Dependence "amplitude-speed" for $h/a = 1/120,\ b/a = 1,\ \theta = \theta_{cr} \approx 1.9$

For the fixed relations h/a and θ the influence of the parameter b/a has only quantitative character as in the case of relatively tick plates, as well as in the case of sufficiently thin plates. In particular, when $h/a = 1/150$ and $\theta = 2$ the function $A(\nu)$ is presented via the Fig.6, and with the increasing b/a the length of the segment $[\bar{\nu}, \nu_*]$ decreases, moving to the left, moreover the values of the amplitude are decreased (Table 5). Calculations show, also, that for large values of θ as in the case of thick plates, as well as in the case of thin plates the dependence take place, brought in the Fig.7, which indicates, that such certain ν^* exists, that if $0 < \nu \leq \nu^*$, then the function $A(\nu)$ is a two-value one, and out of this segment it is impossible to generate limit cycle oscillations (Fig.7).

Table 5. Influence of the relation b/a on the dependence “amplitude-speed” for h/a=1/150 и $\theta = 2$

b/a \ $\frac{v}{v_{Cr}}$	0.2	0.3	0.5	0.8	0.8188	0.8195	0.8497	0.854	0.902	0.925	0.98	1	1.275
5	2.4958	2.4230	2.1636	0.9981	0.5336 0	0.4173 0.3249	-						
3	2.6612	2.5954	2.3622	1.4533	1.3142	1.3083	0.9064 0	0.7392 0.4521	-		-	-	-
2	2.9373	2.8846	2.6993	2.0522	1.9775	1.9745	1.8325	1.8094	1.4565 0	0.9617 0.8894	-		
1	3.4570	3.4368	3.3693	3.1796	3.1629	3.1622	3.1337	3.1295	3.0792	3.0528	2.9688 0	2.9545 0.1360	1.9281 1.8105

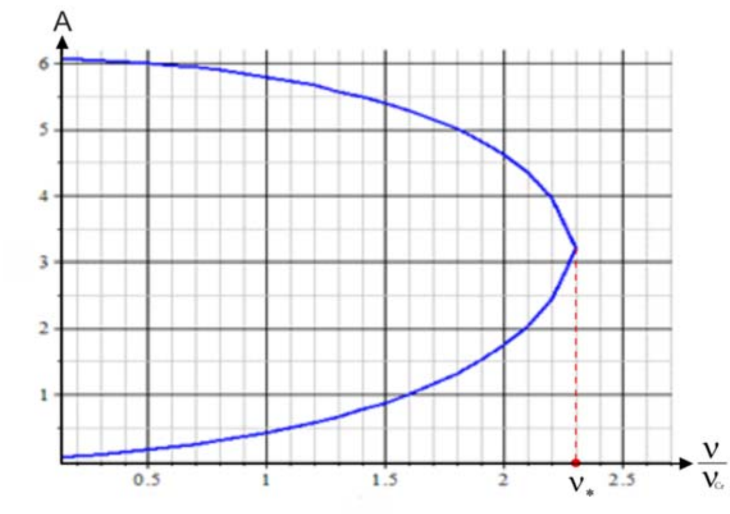

Fig.7. Plot of the function $A(\nu)$ for h/a=1/300, b/a=3, θ=4

In this paper the influence of the relation h/a on the dependence $A(\nu)$ is investigated, also, for the fixed b/a and appropriate critical frequencies. The results of numerical calculations are brought in the Table 6. The noted table is composed of three parts, in which the values of $A(\nu)$ are brought for the fixed b / a and appropriate critical frequencies, which, as it is known (see equation (13)) independent of h / a .

Table 6. Influence of h/a on the dependence “amplitude-speed”.

b/a=5, θ=2.836													
h/a \ ν / ν_{Cr}	0.2	0.5	0.8	1	1.1	1.24	1.29	1.5	2	2.91	3.8	4.4	5
1/300	3.8821	3.7297	3.4070	3.0376 0	2.7616 0.5612	1.9817 1.4143	-	-	-	-	-	-	-
1/120	3.8821	3.7307	3.4169	3.0751 0	2.8372 0.5284	2.3286 1.1123	1.9325 1.5205	-	-	1.8611 1.6941	1.1491 0.5302	0.6363 0.3362	0.3862 0.2295
1/70	3.8837	3.8152	2.4655	0.9210 0	0.6209 0.1567	0.3959 0.1611	0.3448 0.1549	0.2116 0.1241	0.0949 0.0722	0.0387 0.0340	0.0215 0.0198	0.0157 0.0147	0.0121 0.0113
b/a=2, θ=2.505													
h/a \ ν / ν_{Cr}	0.2	0.5	1	1.1	1.2	1.2449	1.2834	2	3.54	3.8	4	4.5	5
1/300	3.9252	3.7932	3.1653 0	2.8983 0.5951	2.4508 1.1474	1.8431 1.7962	-	-	-	-	-	-	-
1/120	3.9252	3.7952	3.2032 0	2.9701 0.5582	2.6272 0.9941	2.3837 1.2683	1.8640 1.8111	-	0.7415 0.7311	0.6863 0.4923	0.5927 0.4039	0.4036 0.2703	0.2818 0.1944
1/70	3.9274	3.8934	0.6856 0	0.4780 0.1521	0.3528 0.1566	0.3124 0.1521	0.2832 0.1469	0.0827 0.0661	0.0226 0.0208	0.0195 0.0187	0.0174 0.0163	0.0136 0.0128	0.0109 0.0104
b/a=1, θ=1.904													
h/a \ ν / ν_{Cr}	0.2	0.5	1	1.1	1.15	1.178	1.187	2	3.5	4	4.5	4.9	5
1/300	3.2304	3.1276	2.5956 0	2.3211 0.7364	2.1285 1.1527	1.7012 1.5717	-	-	-	-	-	-	-
1/120	3.2304	3.1285	2.6155 0	2.3584 0.6932	2.1383 1.0377	1.9080 1.3432	1.7242 1.5524	-	-	-	-	0.1340 0.1329	0.1301 0.1217
1/70	3.2321	3.1855	0.3459 0	0.2587 0.1423	0.2274 0.1424	0.2124 0.1395	0.2079 0.1384	0.0567 0.0506	0.0169 0.0160	0.0128 0.0122	0.0101 0.0096	0.0084 0.0081	0.0081 0.0078

Table 6 shows, that the relation h / a has as qualitative, as well as quantitative influence on the function $A(\nu)$. With the decreasing h / a the behavior of examined dependence varies as follows: in the beginning it is similar to the dependence shown in Figure 3, which is plotted in the case of sufficiently thick plates. Having decreased h / a, the plot of the function $A(\nu)$ is changed, and becomes identical to the plot, constructed in the case of plates of medium thickness (Fig.4). A further decrease of the relative thickness brings to the change of the plot of the function $A(\nu)$ and becomes similar to the Fig.6, which corresponds to the case of sufficiently thin plates.

5. Main results

In conclusion, let's present in our opinion most important some new results obtained in this study. They are the result of the influence of the flowing supersonic stream on the character of nonlinear oscillations of examined aeroelastic system and can be addressed as follows.

- Due to the aerodynamic non-linearity (especially its non-symmetrical quadratic part) it is established, that dependence $A(\nu)$, in the certain segments of speed parameter ν, is a two-value one. This fact is illustrated in the figures in the form of two branches, the lower branches of which, probably, are unstable. Unstable branches separate the areas of two neighboring stable solutions. Thence it is easy to find the magnitude of disturbance required to transfer the system from one stable branch to another;
- Existence of certain areas of change ν is shown at which it is impossible to excite flutter type limit cycle oscillations as in pre-critical speeds, as well as in post-critical stage;
- Results, obtained in this paper can be the basis for formulation and investigation of problems of optimal control for the magnitude of the amplitude of flutter type oscillations via the appropriate choice of geometrical parameters of the plate.

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