ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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THE INSTABILITY OF SHEAR NORMAL WAVE IN ELASTIC WAVEGUIDE OF WEAKLY INHOMOGENEOUS MATERIAL

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Keywords: inhomogeneous waveguide, instability of normal wave, band of frequency delay.
Բանալի բառեր. անհամասեռ ալիքատար, նորմալ ալիքի անկայունություն, հաճախությունների կասեցման գոտի:
Ключевые слова: неоднородный волновод, неустойчивость нормальной волны, полоса задержки частот.

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Неустойчивость нормальной сдвиговой волны в упругом волноводе из слабо-неоднородного материала

Исследуется влияние продольной, слабой неоднородности упругого материала волновода на распространение нормальной сдвиговой волны при разных механических граничных условиях. Показывается, что при защемлённых гладких поверхностях изотропного упругого слоя возникает асимметричная локализация волновой энергии около срединной плоскости слоя. При механически свободных гладких поверхностях появляются приповерхностные локализации волновой энергии у механически свободных границ волновода, но более интенсивная локализация появляется опять около срединной плоскости слоя. В обоих случаях вследствие воздействия неоднородности материала на нормальную волну, появляются искажения амплитуды и фазовой функции, обусловленные приведёнными коэффициентами (частотами) формообразования. В обоих случаях граничных условий слабая неоднородность материала приводит к появлению частотных зон пропускания или запрещению формированной волны. Оказывается, что при высших формах колебаний возможно возникновение внутреннего резонанса.

Հունանյան Ա.Ա.
Սահքի նորմալ ալիքի անկայունությունը թույլ անհամասեռությամբ նյութից ալիքատարում

Հետազոտվում է ալիքատարի երկայնական, թույլ անհամասեռության ազդեցությունը սահքի նորմալ ալիքի տարածման վրա,, տարբեր եզրային պայմանների դեպքում: Ցույց է տրվում, որ իզոտրոպ առաձգական շերտի ամրակցված հարթ եզրերի դեպքում, միջին մակերևույթի մոտ, առաջանում է ալիքային էներգիայի ասիմետրիկ տեղայնացում: Ալիքատարի մեխանիկորեն ազատ հարթ եզրերի դեպքում, առաջանում են ալիքային էներգիայի մերձմակերևությային տեղայնացումներ: Բայց ալիքային էներգիայի ավելի ինտենսիվ տեղայինացում, նորից տեղի է ունենում միջին մակերևույթի շուրջ: Երկու դեպքում էլ տարածվող ալիքի վրա նյութի անհամասեռության ազդեցության հետևանքով, բերված հաճախության (կամ ալիքի հարմոնիկայի) փոփոխության հետևանքով տեղի է ունենում ալիքի լայնույթի և/կամ փուլի խոտորում: Եզրային պայմանների երկու դեպքում էլ ալիքատարի նյութի թույլ անհամասեռությունը բերում է հաճախությունների արգելանքի կամ թողարկման տիրույթների առաջացման: Պարզվում է, որ բարձր հաճախությունների դեպքում հնարավոր է ներքին ռեզոնանսի առաջացում:

The influence of a weak longitudinal inhomogeneity on normal shear waves under different mechanical boundary conditions is investigated. It is shown, that for clamped smooth surfaces of isotropic elastic layer, occurs asymmetric localization of wave energy near the mid-surface layer. On the other side, for mechanically free smooth surfaces, near-surface localization of wave energy appears near mechanically free surfaces of the waveguide, but more intense localization appears again near the mid-surface layer. In both cases, due to the influence of material inhomogeneity on the normal wave, some distortion of amplitude and phase functions occur due to the change of formation coefficients (of frequencies). In both cases of boundary conditions, the weak inhomogeneity of the material leads to presence of frequency zones of transmission or prohibition of the formed wave. It is shown, that it is possible internal resonance in some higher forms.

Introduction. There are numerous studies on wave propagation in inhomogeneous media. More detail of characteristic phenomena can be found in the monographs [1÷4] etc., as well as analogical phenomena due to inhomogeneity of surface conditions and new physical mechanical properties of material of the waveguide, can be found in some articles of recent years [5÷10] etc. The monographs [11÷14] etc., are devoted to the discussion of structure modeling of inhomogeneous waveguides, as well to the propagation of normal waves in waveguides with longitudinal inhomogeneity of the layer, where the cases of continuous inhomogeneity of the material of the waveguide and the layered periodic structure of the waveguide are considered.

There is a growing range of studies on high-frequency fluctuations and distributions of short-wave signals, due to the advancement of modern technology. They can be used to identify the interaction effects of weak inhomogeneity of the material of the waveguide, as well as the effects of geometric heterogeneity of the surface of the waveguide, with more sensitive signals. Losses of stability of normal propagating high-frequency waves (short wavelength monochromatic signal), whether it is the localization of wave energies, internal resonance, the appearance of forbidden frequency zones or other, have been discussed in many works [15÷19], etc.

The present paper explores the nature of formation of the propagating elastic pure shear normal waves in an isotropic elastic layer with weak, longitudinal inhomogeneity of the material for different mechanical boundary conditions.

1. The Problem Statement. Two model problems on distributions of pure shear, horizontally polarized, elastic normal waves $\vec{U}(x,y,t)=\{0;\ 0;\ \mathrm{w}(x,y,t)\}$ in an isotropic weakly inhomogeneous layer- waveguide are considering.

The shear component of the displacement has the following form

$$\mathrm{w}(x,y,t)=A_0\cdot\exp\left[i(k_0x-\omega_0t)\right] \tag{1.1.1}$$

where A_0 – constant amplitude, k_0 – wave number, and ω_0 – the frequency of normal wave.

It is obvious that in the case of a homogeneous elastic medium, the body wave that is also the surface normal wave is localized throughout the thickness of the layer. The purpose of the examinations of the cases of weak inhomogeneity of the material of waveguide layer is to identify the losses of stability of normal wave in the waveguide for different types of boundary conditions on surfaces of weakly inhomogeneous waveguide

Task 1.1 Longitudinal inhomogeneity of the material and clamped surfaces of the layer-waveguide. Assume normal wave (1.1.1) is distributed in an isotropic, elastic, longitudinally weakly inhomogeneous layer $\{|x|<\infty;\ |y|\le h_0;\ |z|<\infty\}$ with clumped surfaces $y=\pm h_0$.

Then the equation of medium motion has the following form

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}=\rho(x)\frac{\partial^2\mathrm{w}}{\partial t^2}, \tag{1.1.2}$$

where mechanical stresses according to the Hooke's law can be written in the following forms

$$\sigma_{zx}(x,y,t) = G(x)\frac{\partial \mathrm{w}(x,y)}{\partial x}\ ;\ \sigma_{zy}(x,y,t) = G(x)\frac{\partial \mathrm{w}(x,y)}{\partial y}\ , \tag{1.1.3}$$

here $G(x)$ – the shear modulus of the material, which, as the density of the material – $\rho(x)$, for longitudinally weakly inhomogeneous medium are presented in the following forms

$$G(x) = G_0\left[1+\varepsilon_1 \sin(k_1 x) + \delta_1 \cos(k_1 x)\right];$$

$$\rho(x) = \rho_0\left[1+\varepsilon_2 \sin(k_1 x) + \delta_2 \cos(k_1 x)\right]. \tag{1.1.4}$$

Here are taken designations:

$k_1 \triangleq \pi/a$ – the number of inhomogeneity waviness of the material layer,

a – half step of inhomogeneity waviness of the material layer,

$\varepsilon_1;\ \varepsilon_2;\ \delta_1;\ \delta_2$ – small amplitudes of inhomogeneity, which, for weak inhomogeneity of the material satisfy the restriction $\varepsilon_n^2 + \delta_n^2 \ll 1$.

G_0 – the shear modulus and ρ_0 – the density of the corresponding homogeneous material.

We obtain the equation of motion with variable periodic coefficients considering (1.1.3) and (1.1.4)

$$\begin{aligned}&\left[1+\varepsilon_1 \sin(k_1 x) + \delta_1 \cos(k_1 x)\right]\Delta w + k_1\left[\varepsilon_1 \cos(k_1 x) - \delta_1 \sin(k_1 x)\right]\frac{\partial w}{\partial x} = \\ &= c_t^{-2}\left[1+\varepsilon_2 \sin(k_1 x) + \delta_2 \cos(k_1 x)\right]\frac{\partial^2 w}{\partial t^2},\end{aligned} \tag{1.1.5}$$

where $\Delta \triangleq \partial^2/\partial x^2 + \partial^2/\partial y^2$ – the Laplace operator, and $c_{0t}^2 \triangleq G_0/\rho_0$ – the speed of shear normal wave.

On clamped planes $y = \pm h_0$, the boundary conditions have the form

$$\mathrm{w}(x,-h_0,t) = \mathrm{w}(x,+h_0,t) = 0\,. \tag{1.1.6}$$

Then the wave solution of the equation of motion (1.1.5) satisfying the clamped boundary conditions (1.1.6) can be represented in the form of Fourier series

$$\mathrm{w}(x,y,t) = \sum_{n=1}^{\infty} \mathrm{w}_n(x)\cdot \sin(\mu_n y)\cdot e^{i\omega_n t}\ , \tag{1.1.7}$$

where $\mu_n = \pi n/h_0$ – wave number on thickness of waveguide, $n \in \mathbb{N} \triangleq \{1;2;...\}$ – a natural number. It is obvious that under these boundary conditions the zero form does not exist $\mathrm{w}_0(x) \equiv 0$.

The representation of the solution in form (1.1.7), leads the equation of motion (1.1.5) to infinite system of ordinary differential equations with periodic coefficients with respect to amplitude functions of each succession of the n-th wave form

$$\left[\mathrm{w}_n''(x) + \mu_n^2\left(\eta_n^2 - 1\right)\mathrm{w}_n(x)\right] +$$

$$+\varepsilon_1 \sin(k_1 x)\left[\mathrm{w}_n''(x) - (k_1\delta_1/\varepsilon_1)\,\mathrm{w}_n'(x) + \mu_n^2\left(\varepsilon_{21}\eta_n^2 - 1\right)\mathrm{w}_n(x)\right] +$$

$$+\delta_1\cos(k_1x)\left[\mathrm{w}_n''(x)+(k_1\varepsilon_1/\delta_1)\,\mathrm{w}_n'(x)+\mu_n^2(\delta_{21}\eta_n^2-1)\mathrm{w}_n(x)\right]=0, \qquad (1.1.8)$$

here $\eta_n^2 \triangleq \omega_n^2/(c_{0t}^2\mu_n^2)$ – given phase speed of the n-th wave form.

It is obvious that due to the inhomogeneity of the material the process is represented by the interaction of three related normal wave modes characterized in equations (1.1.8) relations, given in square brackets. Since the interaction is due to inhomogeneity functions $\varepsilon_1\sin(k_1x)$ and $\delta_1\cos(k_1x)$ from (1.1.4), the solution of (1.1.8) with variable periodic coefficients is natural to look for, in general, in terms of expansion by given function of inhomogeneity, based on the fact of the features

$$\mathrm{w}_n(x)=a_{0n}+\sum_{m=1}^{\infty}\gamma^m\cdot\left(a_{mn}\cos(k_mx)+b_{mn}\sin(k_mx)\right);\quad n;m\in\mathbb{N}. \qquad (1.1.9)$$

where $k_m \triangleq mk_1=(m\pi/a)$ – wave number in the direction of wave propagation corresponding to the m-th harmonic of the wave, and $\gamma \triangleq \max\left\{\sqrt{\varepsilon_i^2+\delta_i^2}\right\}$, $i=1;2$ is small parameter which characterizes weak inhomogeneity of the material.

We obtain the recurrent infinite system of homogeneous algebraic equations for the constant amplitudes $\{a_{mn};\ b_{mn}\}$ generated by the interaction of the propagating normal wave modes (wave signal) and a longitudinal weak inhomogeneity of the material, substituting the relations (1.1.9) in equations (1.1.8)

$$\mu_n^2\left(\eta_n^2-1\right)a_{0n}+\mu_n^2\gamma\left[\left(\eta_n^2-\varepsilon_{12}\right)(\varepsilon_2/\gamma)\sin(k_1x)+\left(\eta_n^2-\delta_{12}\right)(\delta_2/\gamma)\cos(k_1x)\right]a_{0n}+$$

$$+\sum_{m=1}^{\infty}\gamma^m\left[\mu_n^2\left(\eta_n^2-1\right)-k_m^2\right]\cdot\left[\sin(k_mx)b_{mn}+\cos(k_mx)a_{mn}\right]+ \qquad (1.1.10)$$

$$+\sum_{m=1}^{\infty}\gamma^m\cdot\left[\left(\mu_n^2\left(\varepsilon_{21}\eta_n^2-1\right)-k_m^2\right)\sin(k_mx)-(k_1\delta_1/\varepsilon_1)\,k_m\cos(k_mx)\right]\varepsilon_1\sin(k_1x)b_{mn}+$$

$$+\sum_{m=1}^{\infty}\gamma^m\cdot\left[\left(\mu_n^2(\delta_{21}\eta_n^2-1)-k_m^2\right)\sin(k_mx)+(k_1\varepsilon_1/\delta_1)\,k_m\cos(k_mx)\right]\delta_1\cos(k_1x)b_{mn}+$$

$$+\sum_{m=1}^{\infty}\gamma^m\cdot\left[\left(\mu_n^2\left(\varepsilon_{21}\eta_n^2-1\right)-k_m^2\right)\cos(k_mx)+(k_1\delta_1/\varepsilon_1)\,k_m\sin(k_mx)\right]\varepsilon_1\sin(k_1x)a_{mn}+$$

$$+\sum_{m=1}^{\infty}\gamma^m\cdot\left[\left(\mu_n^2(\delta_{21}\eta_n^2-1)-k_m^2\right)\cos(k_mx)-(k_1\varepsilon_1/\delta_1)\,k_m\sin(k_mx)\right]\delta_1\cos(k_1x)a_{mn}=0$$

In the resulting relations appear characterizing interaction of independent normal harmonics of coefficients $\nu_j;\ \alpha_j;\ \beta_j$ on the distributions of wave signal in the layer with weak longitudinal inhomogeneity (1.1.4)

$$\nu_m=\mu_n^2\left(\eta_n^2-1\right)-k_m^2; \qquad (1.1.11)$$

$$\alpha_m=\mu_n^2\left(\varepsilon_2\eta_n^2-\varepsilon_1\right)-\varepsilon_1k_m^2; \qquad (1.1.12)$$

$$\beta_m=\mu_n^2\left(\delta_2\eta_n^2-\delta_1\right)-\delta_1k_m^2. \qquad (1.1.13)$$

The solution in the first approximation will have the following form, considering the fact that in the zero approximation $\gamma^0 = 1$ and $k_0 = 0$ are corresponding to the normal form on axis $0x$ in the case of homogeneous medium

$$\mathrm{w}_{0n}(x,y,t) = \sum_{n=1}^{\infty} a_{0n} \sin(\mu_{0n} y) \cdot e^{i\omega_{0n} t} . \tag{1.1.14}$$

From which, respectively, it follows that in zero approximation the weakly inhomogeneous layer allows only one group of discrete frequencies $\omega_{0n} = c_{0t}\left(\pi n/h_0\right)$ for propagating shear wave with appropriate numbers of formations $\mu_n = \pi n/h_0$.

In the first approximation $m = 1$ from the solution (1.1.9) will have

$$\mathrm{w}_1(x,y,t) = \sum_{n=1}^{\infty} \left[a_{0n} + \gamma a_{1n} \cos(k_1 x) + \gamma b_{1n} \sin(k_1 x)\right] \sin(\mu_{1n} y) \cdot e^{i\omega_0 t} . \tag{1.1.15}$$

The wave number μ_{1n} and amplitudes of first approximation will find from three related infinite system of following equations

$$\mu_n^2\left[\left(\eta_n^2 - 1\right) + \varepsilon_2\left(\eta_n^2 - \varepsilon_{12}\right)\sin(k_1 x) + \delta_2\left(\eta_n^2 - \delta_{12}\right)\cos(k_1 x)\right] a_{0n} = 0 ; \tag{1.1.16}$$

$$\left[\mu_n^2\left(\eta_n^2 - 1\right) - k_1^2\right] \cdot b_{1n} \cdot \sin(k_1 x) = 0 ; \tag{1.1.17}$$

$$\left[\mu_n^2\left(\eta_n^2 - 1\right) - k_1^2\right] \cdot a_{1n} \cdot \cos(k_1 x) = 0 . \tag{1.1.18}$$

The wave number of formation of the first approximation obtain from the condition of existence of non-trivial solutions of the system (1.1.16)÷(1.1.18)

$$\mu_{1n}(k_1 x) = \left(\pi n/h_0\right) \sqrt{\frac{1 + \varepsilon_2 \sin(k_1 x) + \delta_2 \cos(k_1 x)}{1 + \varepsilon_1 \sin(k_1 x) + \delta_1 \cos(k_1 x)}} . \tag{1.1.19}$$

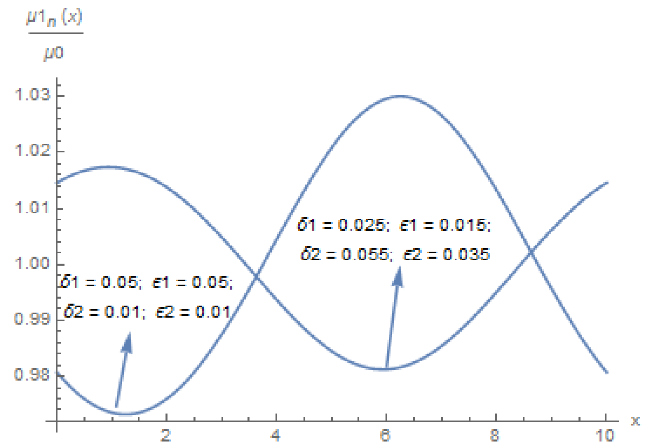


Fig. 1.1. The forms of change of formation coefficient (or frequency) on distributions of normal wave signal

It is obvious that the quantities under square root sign are positively defined (in the case of weak inhomogeneity of the material, when $\varepsilon_n^2 + \delta_n^2 \ll 1$).Therefore, forbidden frequency zones in the first approximation do not arise.

From (1.1.19) we see that the coefficient of formation (or phase function) $\mu_{1n} = \mu_{0n} \cdot f(x)$ is already variable because of the inhomogeneity of the material (Fig. 1.1). Fig. 1.1 also shows that at relatively large compared to the density, stiffness coefficients, when $\varepsilon_1 > \varepsilon_2$ and $\delta_1 > \delta_2$, and at relatively large compared to the stiffness, density coefficients, when $\varepsilon_1 < \varepsilon_2$ and $\delta_1 < \delta_2$, the changes of the oscillation frequencies are different, while remaining periodic.

From coincidence of harmonics, amplitudes $\{a_{1n}\}$ and $\{b_{1n}\}$ of first approximation expressed through the amplitudes of wave signal $\{a_{0n}\}$, will find from (1.1.16)÷(1.1.18)

$$b_{1n} = \frac{\mu_{1n}^2\left(\varepsilon_2\left(\pi n/h_0\right)^2 - \varepsilon_1\right)}{\left(\left(\pi n/h_0\right)^2 - \left(\pi/a\right)^2\right) - \mu_{1n}^2} a_{0n}; \quad a_{1n} = \frac{\mu_{1n}^2\left(\delta_2\left(\pi n/h_0\right)^2 - \delta_1\right)}{\left(\left(\pi n/h_0\right)^2 - \left(\pi/a\right)^2\right) - \mu_{1n}^2} a_{0n} . \tag{1.1.20}$$

From (1.1.20) it is seen that the amplitude distortion compared with the distortion of the phase function, is quadratic. Here also find the number of resonant harmonics, when $a_{1n} \to \infty$ and/or $b_{1n} \to \infty$, where occurs internal resonance (Fig.1.3)

$$n = \left(h_0/a\right)\sqrt{\frac{1+\gamma_1 \sin(k_1 x + \varphi_1)}{\gamma_\vartriangle \sin(k_1 x + \varphi_\vartriangle)}} . \tag{1.1.21}$$

Here are taken the following designations

$$\gamma_1 \triangleq \sqrt{\varepsilon_1{}^2 + \delta_1{}^2} ;$$

$$\varphi_\vartriangle \triangleq \arccos\frac{\left(\varepsilon_1 - \varepsilon_2\right)}{\sqrt{\left(\varepsilon_1-\varepsilon_2\right)^2+\left(\delta_1-\delta_2\right)^2}} = \arcsin\frac{\left(\delta_1 - \delta_2\right)}{\sqrt{\left(\varepsilon_1-\varepsilon_2\right)^2+\left(\delta_1-\delta_2\right)^2}} ;$$

$$\gamma_\vartriangle \triangleq \sqrt{\left(\varepsilon_1-\varepsilon_2\right)^2+\left(\delta_1-\delta_2\right)^2} ; \quad \varphi_1 \triangleq \arccos\frac{\varepsilon_1}{\sqrt{\varepsilon_1{}^2+\delta_1{}^2}} = \arcsin\frac{\delta_1}{\sqrt{\varepsilon_1{}^2+\delta_1{}^2}} .$$

It is easy to get the instability zones of harmonics from (1.1.21) (when the quantities under square root sign have not a positive value. (Fig. 1.2))

$$a\left(2m-1-\varphi_\vartriangle/\pi\right) \le x \le a\left(2m-\varphi_\vartriangle/\pi\right)$$

$$m = 0;1;2;\ldots \tag{1.1.22}$$

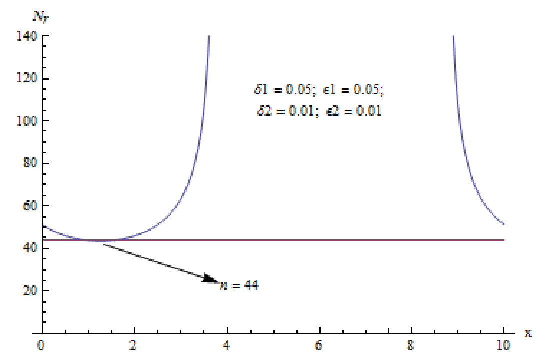
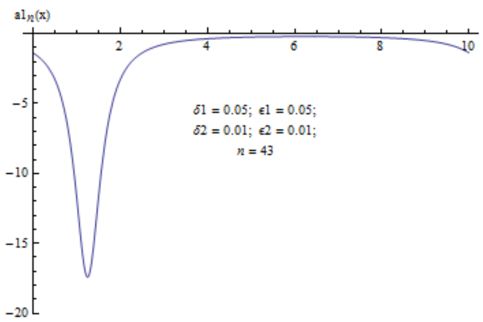
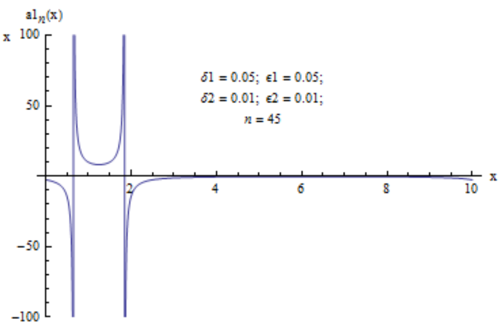


Fig.1.2. The line of resonance by harmonic $x_r \rightleftarrows N_r$ and by cuts

Whence it follows that in some cases of medium inhomogeneity, the quantities under square root sign can be negative and then the corresponding harmonics lose stability and will be represented by exponential functions $\exp\left[\pm\mu_{1n}(\varepsilon_i;\delta_i;a/h_0)\cdot y\right]$.

From Fig. 1.2, in each section we can find the numbers of resonant forms. It is seen that starting from a certain numbers of harmonics, resonant forms periodically exist at certain intervals, by choosing the characteristics of inhomogeneity of the material, Fig. 1.3 (see formulas (1.1.23) and (1.1.24)).





a) b)

Fig. 1.3. The character of changes of amplitudes (a) and (b) for certain material inhomogeneity characteristics, before and after the occurrence of the resonance

And also find the number of resonant harmonics

$$N_r = (h_0/a)\sqrt{\frac{1+\gamma_1 \sin(k_1 x_r + \varphi_1)}{\gamma_\vartriangle \sin(k_1 x_r + \varphi_\vartriangle)}}\,, \tag{1.1.23}$$

and the respective values x_r of the intervals of definition

$$a(2m+1-\varphi_\vartriangle/\pi) > x_r > a(2m-\varphi_\vartriangle/\pi); \quad m = 0;1;2;... \tag{1.1.24}$$

In the second approximation when $m = 2$, the solution will have the following form

$$\mathrm{w}_2(x,y,t) = \mathrm{w}_1(x) + \gamma^2 \sum_{n=1}^{\infty} \left[a_{2n}\cos(k_2 x) + b_{2n}\sin(k_2 x)\right]\sin(\mu_{2n} y)\cdot e^{i\omega_0 t}$$

Considering (1.1.15), (1.1.19) and (1.1.20), from (1.1.10) for relatively constants a_{0n}, a_{2n} and b_{2n} amplitudes we have three infinite systems of homogeneous arithmetic equations. The formation number in second approximation will find from the condition of existence of non-trivial solution

$$\mu_n = (n\pi/h_0)\sqrt{\mathrm{N}(\varepsilon_i;\delta_i;k_1 x)/\mathrm{M}(\varepsilon_i;\delta_i;k_1 x)}\,, \tag{1.1.25}$$

where have been taken the following designations

$$\mathrm{N}(\varepsilon_i;\delta_i;k_1 x) \triangleq \left[\begin{array}{l} 1+\dfrac{\varepsilon_2 n^2 a^2 + (n^2a^2 - h_0^2)\beta_{1n}}{n^2a^2}\sin(k_1 x) + \\ \dfrac{1}{2}(\varepsilon_2\beta_{1n} + \delta_2\alpha_{1n}) + \dfrac{\delta_2 n^2a^2 + (n^2a^2 - h_0^2)\alpha_{1n}}{n^2a^2}\cos(k_1 x) + \\ + \dfrac{n^2a^2(\delta_2\beta_{1n} + \varepsilon_2\alpha_{1n}) - 2h_0^2(\delta_1\beta_{1n} + \varepsilon_1\alpha_{1n})}{2n^2a^2}\sin(2k_1 x) + \\ + \dfrac{2h_0^2(\varepsilon_1\beta_{1n} - \delta_1\alpha_{1n}) - n^2a^2(\varepsilon_2\beta_{1n} - \delta_2\alpha_{1n})}{2n^2a^2}\cos(2k_1 x) \end{array}\right]; \tag{1.1.26}$$

$$\mathrm{M}\left(\varepsilon_i;\delta_i;k_1x\right)\triangleq\frac{1}{2}\begin{bmatrix}\left(\varepsilon_1\beta_{1n}+\delta_1\alpha_{1n}\right)+\\ +2\left(1+\left(\varepsilon_1-\beta_{1n}\right)\sin(k_1x)+\left(\delta_1-\alpha_{1n}\right)\cos(k_1x)\right)+\\ +\left(\delta_1\alpha_{1n}-\varepsilon_1\beta_{1n}\right)\cos(2k_1x)-\left(\delta_1\beta_{1n}+\varepsilon_1\alpha_{1n}\right)\sin(2k_1x)\end{bmatrix}.$$

Amplitudes a_{2n} and b_{2n} will find from coincidence of harmonics

$$a_{2n}=\alpha_2\left(\varepsilon_i;\delta_i;\left(na/h_0\right)\right)a_{0n};\qquad b_{2n}=\beta_2\left(\varepsilon_i;\delta_i;\left(na/h_0\right)\right)a_{0n};$$

$$a_{2n}=\frac{A_{21}B_1-A_{11}B_2}{A_{21}A_{12}+A_{11}A_{22}}a_{0n};\quad b_{2n}=\frac{A_{22}B_1+A_{12}B_2}{A_{11}A_{22}+A_{12}A_{21}}a_{0n};$$

$$A_{11}\triangleq\left\{\varepsilon_1k_2k_1\cos(k_1x)-\delta_1k_2k_1\sin(k_1x)\right\};$$

$$A_{12}\triangleq\begin{Bmatrix}\left[\left(\left(n\pi/h_0\right)^2-\mu_n^2\right)-k_2^2\right]+\varepsilon_1\left(\left(\varepsilon_{21}\left(n\pi/h_0\right)^2-\mu_n^2\right)-k_2^2\right)\sin(k_1x)+\\ +\delta_1\left((\delta_{21}\left(n\pi/h_0\right)^2-\mu_n^2)-k_2^2\right)\end{Bmatrix};$$

$$B_1\triangleq-\frac{1}{2}\left\{\alpha_1\delta_1\left[\mu_n^2(\delta_{21}\eta_n^2-1)-2k_1^2\right]-\beta_1\varepsilon_1\left[\mu_n^2\left(\varepsilon_{21}\eta_n^2-1\right)-2k_1^2\right]\right\};$$

$$A_{21}\triangleq\begin{Bmatrix}\left[\mu_n^2\left(\eta_n^2-1\right)-k_2^2\right]+\delta_1\left(\mu_n^2(\delta_{21}\eta_n^2-1)-k_2^2\right)\cos(k_1x)-\\ -\varepsilon_1\left(\mu_n^2\left(\varepsilon_{21}\eta_n^2-1\right)-k_2^2\right)\sin(k_1x)\end{Bmatrix};$$

$$A_{22}\triangleq\left\{\delta_1k_2k_1\sin(k_1x)+\varepsilon_1k_2k_1\cos(k_1x)\right\};$$

$$B_2\triangleq-\frac{1}{2}\left\{\beta_1\delta_1\left[\mu_n^2(\delta_{21}\eta_n^2-1)-2k_1^2\right]+\alpha_1\varepsilon_1\left[\mu_n^2\left(\varepsilon_{21}\eta_n^2-1\right)-2k_1^2\right]\right\}.$$

The wave solution in second approximation will have the following form

$$\mathrm{w}_2(x,y,t)=\mathrm{w}_1(x,y,t)+\gamma^2\sum_{n=1}^{\infty}a_{0n}\begin{bmatrix}1+\beta_2\left(\varepsilon_i;\delta_i;\left(na/h_0\right)\right)\sin(k_2x)\\ +\alpha_2\left(\varepsilon_i;\delta_i;\left(na/h_0\right)\right)\cos(k_2x)\end{bmatrix}\sin(\mu_{2n}y)e^{i\omega_0t}\quad(1.1.27)$$

Find the forbidden frequency zone from the obtained relations, (the number of harmonics, for which occur the inequality)

$$\left|-b_n\pm\sqrt{b_n^2-c_n}\right|>1,\qquad(1.1.28)$$

where have taken the following designations

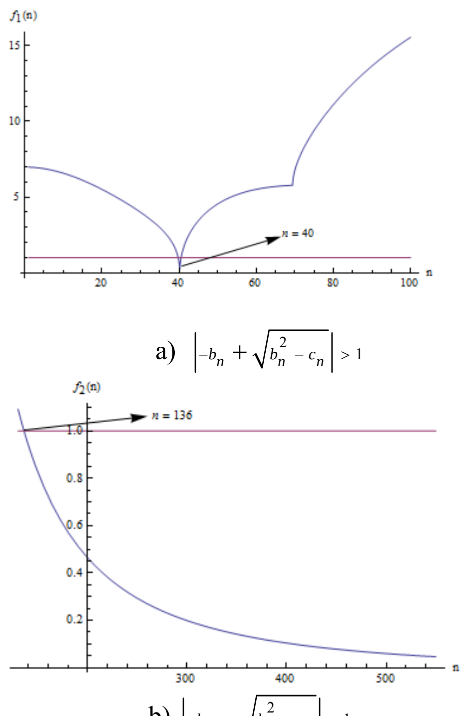
$$c_n\triangleq\frac{\left[\left(na/4h_0\right)^2-1\right]^2-\left[\left(\left(na/4h_0\right)^2\left(\varepsilon_2-\varepsilon_1\right)-\varepsilon_1\right)\right]^2-\delta_1^2/4}{\gamma_1^2/4+\left[\left(\left(na/4h_0\right)^2(\delta_2-\delta_1)-\delta_1\right)\right]^2+\left[\left(\left(na/4h_0\right)^2\left(\varepsilon_2-\varepsilon_1\right)-\varepsilon_1\right)\right]^2}.\quad(1.1.29)$$

The zones of instability of the harmonics are easily obtained from relations (1.1.25) and (1.1.26) (when the quantities under square root sign can be negative), whence it follows that in some cases of medium inhomogeneity, the quantities under square root sign can be negative and then the corresponding harmonics lose stability and will be represented by exponential functions $\exp\left[\pm\mu_{2n}(\varepsilon_i;\delta_i;a/h_0)\cdot y\right]$.

Numerical analysis of the obtained amplitude-phase distortion will be given along with the case of mechanically free boundary conditions of the waveguide. The zones of forbidden frequencies for different characterizing parameters of inhomogeneity of the material are given in Fig 1.4**.** In one case the forbidden frequency occurs for a limited number of harmonics n_i where $i\ \hat{I}\ \{m_1, m_1+1, ...k\}$, but in the other case there are an unlimited number of harmonics $i \geq m_2$.

Task 1.2 Longitudinal inhomogeneity of the material and mechanically free surfaces of the layer-waveguide. Assume the normal wave is propagating in isotropic, elastic, longitudinally weak inhomogeneous layer with mechanically free surfaces $y = \pm h_0$. The weak inhomogeneity has the form set in (1.1.4)

$$\left.\frac{\partial \mathrm{w}(x,y,t)}{\partial y}\right|_{y=-h_0} = \left.\frac{\partial \mathrm{w}(x,y,t)}{\partial y}\right|_{y=+h_0} = 0. \tag{1.2.1}$$



a) $\left|-b_n + \sqrt{b_n^2 - c_n}\right| > 1$

b) $\left|-b_n - \sqrt{b_n^2 - c_n}\right| > 1$

Fig. 1.4. The zones of forbidden frequencies for certain material inhomogeneity characteristics

Proceeding analogously to the case of clamped surfaces, the wave solution of the equations of motion satisfying the boundary conditions for mechanically free surfaces (1.2.1) can be represented in the form

$$\mathrm{w}(x,y,t) = \sum_{n=0}^{\infty} \mathrm{w}_n(x)\cdot\cos(\mu_n y)\cdot e^{i\omega_n t} \tag{1.2.2}$$

where again $\mathrm{w}_n(x)$ is shown in the (1.1.9) and consequently the character of amplitude-phase distortion on the propagation of wave signal will be the same as in the case of clamped surfaces of layer.

Unlike the case of the waveguide with clamped surfaces, in this case the solution of the zero approximation is obtained in the form

$$\mathrm{w}_0(x,y,t) = a_{00} + \sum_{n=1}^{\infty} a_{0n}\cos(\mu_{0n} y)\cdot e^{i\omega_{0n} t} \tag{1.2.3}$$

where $a_{00} = \mathrm{w}_0(x, \pm h_0, t)$ – the values of shear strain on surfaces.

Considering the fact, that the nature of the change in direction of propagation of the

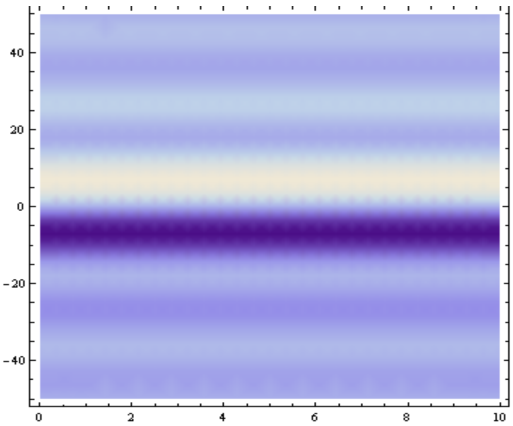
wave signal is again characterized by the equation (1.1.8), the wave field in the waveguide in the case of the mechanically free surfaces in the following approximations are obtained so:
a) in the first approximation, the solution is obtained in the form accounting the material inhomogeneity

$$w_1(x,y,t) = a_{00} + \gamma \sum_{n=1}^{\infty} \left[a_{1n} \cos(k_1 x) + b_{1n} \sin(k_1 x) \right] \cos(\mu_{1n} y) \cdot e^{i\omega_0 t}, \qquad (1.2.4)$$

where determining wave characteristics are the followings: μ_{1n} - wave formation number, and the amplitudes of harmonics a_{1n} and b_{1n} are described in relations (1.1.19) and (1.1.20) accordingly.
b) in the second approximation, the wave field will have the following form

$$w_2(x,y,t) = w_1(x) + \gamma^2 \sum_{n=1}^{\infty} a_{0n} \begin{bmatrix} \alpha_2\left(\varepsilon_i;\delta_i;\left(na/h_0\right)\right)\cos(k_2 x) + \\ +\beta_2\left(\varepsilon_i;\delta_i;\left(na/h_0\right)\right)\sin(k_2 x) \end{bmatrix} \cos(\mu_{2n} y) e^{i\omega_0 t}, \qquad (1.2.5)$$

where determining wave characteristics are the followings: μ_{2n} – wave formation number, and the amplitudes of harmonics a_{2n} and b_{2n} are described in relations (1.1.25), (1.1.26) and (1.1.28), (1.1.29) accordingly.


a)) the levels of the wave surface when the surfaces of the waveguide are clamped

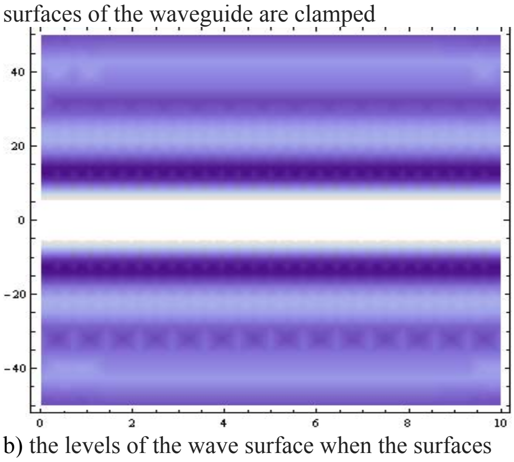

b) the levels of the wave surface when the surfaces of the waveguide are mechanically free

Fig. 2.1

2. Comparative analysis of the obtained results. As shown above, the frequency characteristics at different boundary conditions on the smooth surfaces of the waveguide of weakly inhomogeneous material are identical and because of the inhomogeneity are changed identically. The weak inhomogeneity leads to a distortion of the formation coefficients (Fig. 1.1) as in nature as in value. Formation coefficients $\mu_{1n}(k_1 x)$ are already changing periodically from the value $\mu_{0n} = \left(\pi n/h_0\right)$. At the clamped surfaces of the layer, as in the case of a homogeneous medium does not exist the first harmonic with a constant amplitude. Depending on the characteristics of the inhomogeneity of the material, at certain frequencies $n = N_r$ of the wave signal in certain sections $x = x_r$ occurs internal resonance (Fig. 1.3). The weak inhomogeneity of the material of the waveguide may lead to filtration of specific frequencies of the normal wave (Fig. 1.4).

In figures 2.1 (a) and (b) the levels of wave surfaces for different boundary conditions are given. It is obvious that the wave surface generally preserves the leveled character, existing in the case of homogeneous medium: preserves the symmetry (or asymmetry) through the thickness of the waveguide, but are distorted in the direction of wave propagation. On the lines of level changes jagged deviations are clearly appeared, characterized by the inhomogeneity of the material of the waveguide. At specific frequencies of the wave signal, the interaction of the signal and inhomogeneity leads to parametric resonance.

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