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ON A PROBLEM OF SUPERSONIC PANEL FLUTTER IN THE PRESENCE OF CONCENTRATED INERTIAL MASSES AND MOMENTS

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Key words: stability, elastic rectangular plate, supersonic overrunning, divergence, localized divergence, flutter

Ключевые слова: устойчивость, упругая прямоугольная пластинка, сверхзвуковое обтекание, дивергенция, локализованная дивергенция, флаттер.

Բանալի բառեր. կայունություն, առաձգական ուղղանկյուն սալ, գերձայնային շրջհոսում, դիվերգենցիա, տեղայնացված դիվերգենցիա, ֆլատեր

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О задаче сверхзвукового панельного флаттера при наличии сосредоточенных инерционных масс и моментов

В линейной постановке исследуется динамическое поведение возмущённого движения тонкой упругой прямоугольной пластинки вблизи границ области устойчивости при набегании сверхзвукового потока газа на её свободный край в предположении, что вдоль свободной кромки и шарнирно опёртой противоположной ей кромки, приложены сосредоточенные инерционные массы и моменты поворота соответственно. Найдены критические скорости дивергенции и флаттера. Установлено, что инерционный момент поворота приводит к стабилизации возмущённого движения системы.

Բելուբեկյան Մ.Վ. , Մարտիրոսյան Ս.Ռ.
Գերձայնային պանելային ֆլատերի խնդրի մասին կենտրոնացված իներցիոն զանգվածի և մոմենտի առկայության դեպքում

Դիտարկված է գերձայնային գազի հոսքում ուղղանկյուն սալի կայունության մի խնդիր: Հոսքը ուղղված է ազատ եզրից դեպի հակադիր հոդակապորեն ամրակցված եզրը զուգահեռ մյուս երկու հոդակապորեն ամրակցված եզրերին: Ցույց է տված դիվերգենցիայի և ֆլատերի առաջացման հնարավորությունը: Գտնված են դիվերգենցիայի և ֆլատերի կրիտիկական արագության արժեքները:

By analyzing, as an example, a thin elastic rectangular plate streamlined by supersonic gas flows, we study the loss stability phenomenon of the overrunning of the gas flow at is free edge under the assumption of presence of concentrated inertial masses and moments at the free and hinged edges respectively. For some special casesof a problem of a panel flutter critical velocities of divergence and flutter are found. It is established that inertial moment of rotation leads to the stabilization of perturbed motion of the system.

Introduction. In this paper we study some special cases of the problem of supersonic panel flutter, which the General case for moderate values of the parameters was investigated by an analytically method in [1]. Each of them is of independent interest for the study in terms of identifying new mechanical effects associated with the loss of stability of the system "plate–flow".

The theoretical and numerical methods to study of the divergence and flutter instability of plates and shells devoted a huge amount of works, a General review of which is contained in the monography by Algazin S.D. and Kijko I.A. [2(pp. 210-245)] and the article by Novichkov J.N. [3].

The results can be used in the processing of experimental studies of divergence and panel flutter of the modern supersonic aircraft.

1. Statement of the problem. Considered a thin elastic rectangular plate, in a Cartesian coordinate system $Oxyz$ occupies the area: $0 \le x \le a$, $0 \le y \le b$, $-h \le z \le h$. We choose the Cartesian coordinate system $Oxyz$ so that the Ox and Oy axes lie in the plane of the undisturbed plate, and the Oz axis is perpendicular to the plate and directed to the side of supersonic gas flow streamlining it from one side in the direction of the Ox axis with an undisturbed velocity V. We assume a plane, potential flow. And, also, we assume that the plate is not exposed to the tensil forces in the middle surface. Let the $x = 0$ edge of the plate is free and edges: $x = a$, $y = 0$, $y = b$ are hinged. We assume that the concentrated inertial masses m_c and rotation moments I_c are applied to the $x = 0$ free edge and to the $x = a$ hinged edge respectively [1, 4(p.27, 101), 5].

Under the influence of certain factors, the undisturbed equilibrium state of our plate can be broken down, and it will begin to perform disturbed motion with a deflection $w = (x, y, t)$. The deflection $w = (x, y, t)$ will cause an excess pressure Δp on to the upper streamlined surface of the plate from the side of streamlining gas flow, which is taken into account by the approximate formula of the "piston theory" [6, 7]: $\Delta p = -a_0 \rho_0 V \, \partial w / \partial x$, where a_0 is the sound velocity in the undisturbed gas medium, ρ_0 is the density of undisturbed gas flow. Let us assume that the deflections $w = (x, y, t)$ are small as compared with the thickness $2h$ of the plate.

Find out the conditions under which the possible loss of stability of the undisturbed state of equilibrium of the plate, when the bending of the plate due to the corresponding aerodynamic loads Δp and concentrated inertial masses m_c and rotation moments I_c are applied to the free $x = 0$ edge. Thus, in accordance with the new approach, the influence of the distributed mass of the plate and the resistance forces can be neglected.

Then, under assumption of the validity of the Kirchhoff hypotheses and "piston theory", the small bending vibrations of the points of the plate middle surface about the undisturbed equilibrium state is described by the differential equation [1, 4(245)]

$$D \Delta^2 w + a_0 \rho_0 V \, \partial w / \partial x = 0, \tag{1.1}$$

$\Delta^2 w = \Delta(\Delta w)$, Δw is a Laplace's operator; D is a cylindrical rigidity.

In the accepted assumptions concerning a way of fixing of edges of the plate the boundary conditions can be written in the form [1, 4(101), 5]

$$\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0, \quad \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + (2-\nu) \frac{\partial^2 w}{\partial y^2} \right) = -D^{-1} m_c \frac{\partial^2 w}{\partial t^2}, \quad x = 0; \tag{1.2}$$

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} = -I_c D^{-1} \frac{\partial^3 w}{\partial x \partial t^2}, \quad x = a; \tag{1.3}$$

$$w = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0, \quad y = 0 \text{ and } y = b; \tag{1.4}$$

The problem of the stability of an elastic thin rectangular plate streamlined by a supersonic gas flow, which is described by correlations (1.1)-(1.4), lies in finding the minimal value velocity V_{cr} (i.e. the critical velocity) such that, in the case $V < V_{cr}$, disturbed motion will be stable and, for $V \ge V_{cr}$ – unstable. In other words, is required to determine the values of velocity at which the equation (1.1) with the corresponding boundary conditions (1.2)-(1.4) has the non-trivial solutions.

We see that the analysis of the stability of the plane form of the plate in the potential supersonic flow reduces to a study of the differential equation (1.1) with the corresponding boundary conditions (1.2)-(1.4) for the deflections $w=(x,y,t)$.

2. **General solution of the problem**. For finding the general solution of the problem of stability of the plate (1.1)-(1.4), we will reduce it to a problem on eigenvalues for the ordinary differential equation.

We try to find the General solution to the boundary-value problem defined by equation (1.1) and by the boundary (1.2) - (1.4) in the form of harmonic vibrations [9]

$$w(x,y,t)=\sum_{n=1}^{\infty} C_n \exp(\mu_n px+\lambda t)\cdot\sin(\mu_n y)\,,\ \mu_n=\pi n b^{-1}, \tag{2.1}$$

then, in accordance with the expression (2.1), the considered problem of the panel flutter (1.1)–(1.4) is reduced to the following boundary value problem on eigenvalues λ of nonselfadjoint operator for the ordinary differential equations on the forms of vibrations $f_n(x)=C_n\exp(\mu_n px)$, where C_n are the arbitraries constants, which are not equal to zero simultaneously; n is the half–waves number along of side b; p – are the roots of the characteristic equation

$$(p^2-1)^2+\alpha_n^3 p=0\,,\ \alpha_n^3=a_0\rho_0 VD^{-1}\mu_n^{-3}\,,\ \alpha_n^3>0\,, \tag{2.2}$$

corresponding to differential equation (1.1). The characteristic equation (2.2) has two negative root $p_1<0$, $p_2<0$ and a pair of complex-conjugate roots $p_{3,4}=\alpha\pm i\beta$ with a positive real part $\alpha>0$. The roots of the equation (2.2) are determined by the following expressions [10]:

$$p_{1,2}=-0.5\sqrt{2(q+1)}\pm\sqrt{\sqrt{q^2-1}-0.5(q-1)}\,, \tag{2.3}$$

$$p_{3,4}=0.5\sqrt{2(q+1)}\pm i\sqrt{\sqrt{q^2-1}+0.5(q-1)}\,, \tag{2.4}$$

$$q>1\,. \tag{2.5}$$

Here q is the only real root of the cubic equation

$$8\cdot(1+q)^2(q-1)=\alpha_n^6,\ \alpha_n^3=a_0\rho_0 VD^{-1}\mu_n^{-3},\ \mu_n=\pi n b^{-1}. \tag{2.6}$$

From the relations (2.6) it is easy to obtain the expressions of the dependence of the velocity V of gas flow on the system parameters

$$V=2\sqrt{2(q-1)}\cdot(q+1)\cdot\pi^3 n^3\gamma^3 D(a_0\rho_0 a^3)^{-1}. \tag{2.7}$$

As well as,

$$V=2\sqrt{2(q-1)}\cdot(q+1)\cdot\pi^3 n^3 D(a_0\rho_0 b^3)^{-1}. \tag{2.8}$$

Here γ is a relation of the width a of the plate to its length b

$$\gamma=ab^{-1}\,. \tag{2.9}$$

Then, the General solution (2.1) of the equation (1.1), due to ratios (2.3) and (2.4), can be written as

$$w(x,y,t)=\sum_{n=1}^{\infty}\sum_{k=1}^{4}C_{nk}\cdot\exp(\mu_n p_k x+\lambda t)\cdot\sin(\mu_n y)\,. \tag{2.10}$$

Substituting the General solution (2.10) the differential equation (1.1) in the boundary conditions (1.2)-(1.4), we obtain a homogeneous system of algebraic equations of the fourth order relatively the arbitraries constants: C_{nk}. Equate to zero the determinant of this system of equations - characteristic determinant leads to the dispersion equation [1]

$$\chi_n\delta_n A_0\lambda^4+(\chi_n A_1+\delta_n A_2)\lambda^2+A_3=0\,. \tag{2.11}$$

And introducing the notation

$$k_n=\chi_n\cdot\delta_n^{-1}\,,\ \chi_n>0\,,\ \delta_n>0\,, \tag{2.12}$$

we can rewrite characteristic equation (2.11) in the form

$$A_0\lambda^4+(k_nA_1+A_2)\chi_n^{-1}\lambda^2+\chi_n^{-1}\delta_n^{-1}A_3=0\,,\ \gamma\in(0,\infty)\,,\chi_n>0\,,\ \delta_n>0\,. \tag{2.13}$$

Here

$$\delta_n=m_cD^{-1}b^3(\pi n)^{-3}\ \text{and}\ \ \chi_n=I_cD^{-1}b(\pi n)^{-1} \tag{2.14}$$

are the reduced values of the concentrated masses m_c and inertial moments of rotation I_c, applied to the free $x=0$ and hinged $x=a$ edges of the plate, respectively;

$$\begin{aligned}A_0=A_0(q,n,\gamma)==2\sqrt{2(q+1)}\cdot\{(1-\exp(-2\sqrt{2(q+1)}\pi n\gamma)\cdot B_1B_2+\\ +\exp(-\sqrt{2(q+1)}\cdot\pi n\gamma)\cdot\\ \Big[(\sqrt{2(q-1)}-\sqrt{2(q+1)})\cdot B_1\cdot\mathrm{ch}(\pi n\gamma B_1)\cdot\sin(\pi n\gamma B_2)-\\ -(\sqrt{2(q-1)}+\sqrt{2(q+1)})\cdot B_2\mathrm{sh}(\pi n\gamma B_1)\cdot\cos(\pi n\gamma B_2)\Big]\Big\}\,;\end{aligned} \tag{2.15}$$

$$\begin{aligned}A_1=A_1(q,n,\gamma)=2\Big\{\Big[2(q+1)(q-\sqrt{(q^2-1)}-\nu)-(1-\nu)^2\Big]B_1B_2+\\ +\Big[2(q+1)(q+\sqrt{(q^2-1)}-\nu)-(1-\nu)^2\Big]B_1B_2\exp(-2\sqrt{2(q+1)}\pi n\gamma)+\\ +2\Big[(q+1)\sqrt{(q^2-1)}\cdot(\sqrt{2(q-1)}+\sqrt{2(q+1)})sh(\pi n\gamma B_1)+\\ +(4q^2+2q-1+2q\nu+\nu^2)B_1\mathrm{ch}(\pi n\gamma B_1)\Big]B_2\cos(\pi n\gamma B_2)\cdot\\ \cdot\exp(-\sqrt{2(q+1)}\cdot\pi n\gamma)+\\ +\Big[((2q^2+3q-1)-2(3q^2+3q-2)\nu+(3q+1)\nu^2)sh(\pi n\gamma B_1)+\\ +2(q+1)\cdot\sqrt{(q^2-1)}\cdot(\sqrt{2(q+1)}-\sqrt{2(q-1)})B_1\,ch(\pi n\gamma B_1)\Big]\cdot\\ \cdot\sin(\pi n\gamma B_2)\cdot\exp(-\sqrt{2(q+1)}\cdot\pi n\gamma)\Big\}\,;\end{aligned} \tag{2.16}$$

$$\begin{aligned}A_2=A_2(q,n,\gamma)=4(q+1)\cdot\Big[(1+\exp(-2\sqrt{2(q+1)}\pi n\gamma)\cdot B_1B_2-\\ -2B_1B_2\mathrm{ch}(\pi n\gamma B_1)\cos(\pi n\gamma B_2)\cdot\exp(-\sqrt{2(q+1)}\cdot\pi n\gamma)+\end{aligned} \tag{2.17}$$

$$+3(q-1)\mathrm{sh}(\pi n\gamma B_1)\sin(\pi n\gamma B_2)\cdot\exp(-\sqrt{2(q+1)}\cdot\pi n\gamma)\big];$$

$$A_3=A_3(q,n,\gamma,\nu)=2\sqrt{2(q+1)}\left\{\left[2(q+1)(q-\sqrt{q^2-1}-\nu)-(1-\nu)^2\right]B_1B_2-\right. \tag{2.18}$$

$$-\left[2(q+1)(q+\sqrt{q^2-1}-\nu)-(1-\nu)^2\right]\cdot B_1B_2\exp(-2\sqrt{2(q+1)}\pi n\gamma)+$$

$$+\{[(4q^2+2q-1)\sqrt{2(q-1)}-(2q^2-4q+1)\sqrt{2(q+1)}-$$

$$-2((2q-1)\sqrt{2(q+1)}-q\sqrt{2(q-1)})\nu+$$

$$+(\sqrt{2(q+1)}+\sqrt{2(q-1)})\nu^2]\mathrm{sh}(\pi n\gamma B_1)+$$

$$+4(q+1)\sqrt{q^2-1}\cdot B_1\,\mathrm{ch}(\pi n\gamma B_1)\}B_2\cos(\pi n\gamma B_2)\exp(-\sqrt{2(q+1)}\pi n\gamma)+$$

$$+[-B_1\cdot((4q^2+2q-1)\sqrt{2(q-1)}+(2q^2-4q+1)\sqrt{2(q+1)}+$$

$$+2((2q-1)\sqrt{2(q+1)}+q\sqrt{2(q-1)})\nu-(\sqrt{2(q+1)}-\sqrt{2(q-1)})\nu^2)\mathrm{ch}(\pi n\gamma B_1)-$$

$$-6(q^2-1)\sqrt{(q^2-1)}\mathrm{sh}(\pi n\gamma B_1)]\sin(\pi n\gamma B_2)\exp(-\sqrt{2(q+1)}\pi n\gamma)\};$$

$$B_1(q)=\sqrt{\sqrt{q^2-1}-0.5(q-1)},\ B_2(q)=\sqrt{\sqrt{q^2-1}+0.5(q-1)}, \tag{2.19}$$

It follows that [1]

$$A_0=A_0(q,n,\gamma)>0,\ A_2=A_2(q,n,\gamma)>0, \tag{2.20}$$

for all numbers $q>1$, $n\geq 1$, $\gamma>0$ of parameters.

The system “plate–flow”, described by the relations (1.1) – (1.4) is asymptotically stable if all eigenvalues λ of the boundary value problem for ordinary differential equation have negative real parts, and unstable if at least one eigenvalue λ is on the right side of the complex plane.

The critical velocity V_{cr} that characterizes the transition from stability to instability of the disturbed motion of the system “plate-flow” is determined by the condition of equality to zero of the real part of one or more of the eigenvalues.

This article discusses four particular cases of the original problem of stability of (1.1)–(1.4) studied in [1] at moderate values of its “essential” parameters.

In [1] conducted a decomposition of the space of the “essential” parameters $M=\{\gamma$, k_n, ν, q, $n\}$ of the problem of stability (1.1)–(1.4) on the stability region M_0 and the regions of instability M_1, M_2 and M_3 in which, respectively, either all roots of the characteristic equation are in the left part of the complex plane, or among the roots there is one positive root or has two positive roots, or a pair of complex–conjugate roots with positive real part. The behavior of the system “plate–flow” near the borders of the region of stability M_0 is investigated. The critical velocity of divergence of the panel and the critical velocity of localized divergence in the vicinity of the free edge of the plate, as well as, the critical velocity of panel flutter are found. It is shown that, depending on the relation between of the system parameters, the critical flutter velocity can be both less and greater than the critical velocity of divergence.

3.1. Considered the case where on the free edge $x=0$ of the plate are applied the concentrated inertial masses m_c and inertial rotation moments I_c on the hinged edge $x=a$ are absent ($k_n=0$).

In this case the characteristic equation (2.11) can be written in the form

$$\delta_n A_2 \lambda^2 + A_3 = 0 . \tag{3.1}$$

Here A_2 and A_3 are determined by the expressions (2.17) and (2.18) respectively.

The roots of the equation (3.1) is equal to

$$\lambda_{1,2} = \pm\sqrt{-A_3 \cdot \left(\delta_n \cdot A_2\right)^{-1}} . \tag{3.2}$$

At $\delta_n > 0$ because of the conditions (2.20), the region of stability $M_0 \in M$ of the disturbed motion of the system "plate–flow" will be determined by the inequality

$$A_3 > 0 . \tag{3.3}$$

It is obviously, that under the condition (3.3) the equation (3.1) has a pair $\lambda_{1,2} = \pm i\omega$ of purely imaginary roots. This means, that the rectangular plate performs harmonic oscillations about the undisturbed equilibrium state.

The boundary of the region of stability $M_0 \in M$ of the disturbed motion of the system "plate–flow" in the space of its parameters M is the hypersurface

$$A_3 = 0 , \tag{3.4}$$

where the characteristic equation (3.1) has a zero root $\lambda_0 = 0$ of multiplicity 2. This means, that the system perturbed motion loses static stability, i.e. there is a divergence of panel.

From the condition (2.20) and the method of partitioning the parameters space M into the regions of the stability and the instability of the disturbed motion of the system, it follows that this particular case corresponds to the only region of instability M_1 defined by the correlation

$$A_3 < 0 . \tag{3.5}$$

Here the characteristic equation (3.1) has two real roots of different signs: $\lambda_1 < 0$, $\lambda_2 > 0$. This means that one of the two own motions of the plate is increasing exponentially (the deflections will increase over time according to the exponential law).

Substituting the first root $q_{cr.div} = q_{cr.div}(n,\gamma,\nu)$ of the equation (3.4) in the formula (2.7), we obtain the $V_{cr.div} = V_{cr.div}(n,\gamma,\nu)$ critical divergence velocity, which delimits the stability region M_0 and the static instability (divergence) region M_1 of the disturbed motion of the system "plate-flow".

At the $V \geq V_{cr.div}$ velocities there is a "soft" transition trough point $\lambda_0 = 0$ in the right part of the complex plane of the eigenvalues λ of problem (1.1)-(1.4), causing the smooth changing the nature of disturbed motion of the system from harmonic vibrations to a monotonically increasing aperiodic motion. This changes the dynamic behavior of plates: in the plate, performing harmonic oscillations, there is stresses, leading to changes in the surface shape of the plate. The surface of the plate "buckles" with limited velocity of “buckling”. As monotonous “buckling” of the plate has no oscillatory nature, it can be considered as quasi-static process, i.e. there is a divergence.

Numerical calculations have been performed for different values of the parameters of the problem, showed the following. And $n=1$ with fixed values of remaining parameters the critical velocity of divergence reaches a minimum value.

For all the value $\gamma \in (0,2)$ we can say, that when the velocity $V \geq V_{cr.div.}$ is the phenomenon of divergence observed. The value reduced critical velocity of divergence $V_{cr.div.} \cdot D^{-1}(a_0\rho_0 a^3)$ depends on Poisson's ratio ν and the parameter $\gamma = ab^{-1}$ is the relationship of the sides a and b of the rectangular plate: it is less in plates from materials with the largeness of the Poisson's ratio ν, and with increase in parameter γ the reduced divergence critical velocity grows (see table 1).

Table 1.

γ \ ν	0.125	0.25	0.33	0.375	0.5
0.01	$0.345 \cdot 10^{-2}$	$0.297 \cdot 10^{-2}$	$0.268 \cdot 10^{-2}$	$0.243 \cdot 10^{-2}$	$0.197 \cdot 10^{-2}$
0.1	0.352	0.306	0.273	0.240	0.197
0.2	1.511	1.290	1.163	1.063	0.882
0.3	3.650	3.324	2.912	2.721	2.619
0.4	7.789	6.758	5.985	5.507	4.478
0.5	14.945	13.503	11.078	10.778	9.056
0.6	26.284	21.790	19.146	18.608	13.889
0.7	45.587	37.826	31.267	29.552	25.011
0.9	473.50	96.90	78.72	70.05	53.15
1.0	520.29	157.17	114.21	101.74	72.91
1.1	562.28	243.30	168.02	143.73	100.70
1.2	608.75	323.02	225.85	194.62	135.24
1.3	699.07	401.61	287.15	252.49	172.13
1.4	811.70	495.72	364.73	315.35	214.99
1.5	975.85	595.22	448.61	380.36	273.35
1.6	1166.20	704.47	544.44	470.73	326.25
1.8	1695.90	992.18	762.25	695.12	440.54
2.0	2598.09	1382.02	1045.62	953.53	604.31

It is easy to show that equation (3.4) in the limiting case, where $\gamma \to 0$ $(b \to \infty)$ when $q > 1$ and all ν identically equal to zero. Hence, in this limiting case the undisturbed form of equilibrium of the plate is statically unstable. And when values of $\gamma \geq 2$ equation (3.4) can be reduced to the simplified form

$$2(q+1)\cdot(q-\sqrt{q^2-1}-\nu)-(1-\nu)^2=0\,,\ \ \gamma \in [2,\infty)\,. \tag{3.6}$$

Equation (3.6) exactly coincides with the dispersion equation obtained in the work [10] the study of phenomenon localized divergence arising in the vicinity of the free edge of the elastic semi-infinite plate-strip, streamlined by a supersonic gas flow in the direction from the free edge to the supported edge along the semi-infinite hinged edges. The reduced critical velocity of localized divergence $V_{loc.div.} \cdot D^{-1}(a_0\rho_0 b^3)$ depends only on the Poisson's ratio ν : it is less in plates from materials with the largeness of the Poisson's ratio.

In table 2 for several values of Poisson's ratio values are given reduced critical velocities of localized divergence $V_{loc.div.} \cdot D^{-1}(a_0\rho_0 b^3)$ of rectangular plate observed in the values that accuracy coincide with the values obtained in the work [10].

Thus for all the value $\gamma \in [2,\infty)$, we can say that when the velocity $V \geq V_{\text{loc}.div}(\nu)$ is the phenomenon of localized divergence in the vicinity of the free edge $x = 0$ of our plate observed, which is in good agreement with the results of numerical analysis (table 2). At the values of velocities $V \geq V_{\text{loc}.div}(\nu)$ the vicinity of the free edge $x = 0$ of the plate is «buckling». As in this case a parameter $q = q_{loc.div}$ is determined from the simplified equation (3.6), we can say, that for values $\gamma \in [2,\infty)$ found approximate expression (3.6), making it easy to find the reduced critical velocities $V_{loc.div.} \cdot D^{-1}(a_0\rho_0 b^3)$ of the localized divergence, substituting these values $q_{loc.div.}$ in expression (2.8).

Table 2.

ν	0.125	0.25	0.33	0.375	0.5
$V_{loc.div} \cdot D^{-1}(a_0\rho_0 b^3)$	324.761	173.371	130.702	120.741	77.398

From expression (3.2) and conditions (3.4) follows, that static loss of stability in the form of divergence for values of and localized divergence takes place only, and dynamic loss of stability is absent.

In a conclusion we will mark that for values of $\gamma \in (0,2)$ at the $V \geq V_{cr.div}$ of gas flow velocities (Table 1) there is the divergence panel, resulting in a "buckling" of the plate. For values of $\gamma \in [2,\infty)$ at the $V \geq V_{\text{loc}.div}(\nu)$ of gas flow velocities (Table 2) there is the divergence phenomenon localized in the vicinity of the free edge of the rectangular plate, in which the "buckling" just strip along the vicinity of the free edge of the plate. And the presence of the concentrated masses m_c on a free edge $x = 0$ of the plate does not result in dynamic instability, i.e. the panel flutter is absent.

3.2. Considered the case where on the hinged edge $x = a$ of the plate are applied the inertial rotation moments I_c and concentrated inertial masses m_c on the free edge $x = 0$ are absent ($k_n = \infty$).

In this case the characteristic equation (2.11) can be written in the form

$$\chi_n A_1 \lambda^2 + A_3 = 0 \tag{3.7}$$

Here A_1 and A_3 are determined by the expressions (2.16) and (2.18) respectively.

The roots of the equation (3.7) is equal to

$$\lambda_{1,2} = \pm\sqrt{-A_3 \cdot (\chi_n \cdot A_1)^{-1}}\ . \tag{3.8}$$

At $\chi_n > 0$ because of the conditions (2.20), the region of stability $M_0 \in M$ of the disturbed motion of the system “plate–flow” will be determined by the inequalities

$A_1 > 0$, $A_3 > 0$. (3.9)

It is obviously, that under the condition (3.9) the equation (3.7) has a pair $\lambda_{1,2} = \pm i\omega$ of purely imaginary roots. This means, that the rectangular plate performs harmonic oscillations about the undisturbed equilibrium state.

The boundaries of the region of stability $M_0 \in M$ are the hypersurfaces

$A_1 = 0$, (3.10)

$A_3 = 0$. (3.11)

On the hypersurface (3.10) the characteristic equation (3.7) has two roots equal to infinity, i.e. $\lambda_{1,2} = \pm\infty$. And on the hypersurface (3.11) the characteristic equation (3.7) has a zero root $\lambda_0 = 0$ of multiplicity 2.

In this case, the region of instability M_1 consists of two subregions M_{11} and M_{12} which are determined by the relations respectively

$A_1 > 0$, $A_3 < 0$; (3.12)

$A_1 < 0$, $A_3 > 0$. (3.13)

It is obviously, that in both subregions M_{11} and M_{12} the characteristic equation (3.7) has two real roots of the different signs, namely: $\lambda_1 < 0$, $\lambda_2 > 0$. This means that one of the two disturbed motions of the system "plate-flow" is increasing exponentially.

On the boundary of the stability region M_0

$A_1 > 0$, $A_3 = 0$, (3.14)

the disturbed motion of the system loses of static stability: there is a divergence of panel.

Substituting the first root $q_{cr.div} = q_{cr.div}(n, \gamma, \nu)$ of the equation (3.11) in the formula (2.7), we obtain the $V_{cr.div}$ critical divergence velocity, which delimits the stability region M_0 and the static instability (divergence) region M_{11} of the disturbed motion of a rectangular plate. At $V \geq V_{cr.div}$ velocities of gas flow the roots $\lambda_{1,2} = \pm i\omega$ of the characteristic equation (3.7) of a "soft" transition through the point $\lambda_0 = 0$, respectively, to the left and to right parts of the complex plane of the eigenvalues λ and remain so, at least, when values of the velocity of the gas flow V close to the critical value $V_{cr.div}$, purely real: $\lambda_1 < 0$, $\lambda_2 > 0$. This changes the dynamic behavior of plates: in the plate, performing harmonic oscillations, there is stresses, leading to changes in the surface shape of the plate. The surface of the plate "buckles" with limited velocity of "buckling". As monotonous "buckling" of the plate has no oscillatory nature, it can be considered as quasi-static process, i.e. there is a divergence.

Numerical studies have shown that the transition across the border (3.14) from the region M_0 in a subregion M_{11} is possible only if values $\gamma \in (0, 0.83)$ of parameter. Because of identity of equations (3.4) and (3.14), $V_{cr\,div}$ equal to the corresponding critical divergence velocities, are shown in table 1. Thus the reduced critical divergence velocity $V_{cr\,div} \cdot D^{-1}(a_0\rho_0 a^3)$ depends on the Poisson's ratio ν and parameter γ: it is less in plates from materials with the largeness of the Poisson's ratio ν and with increase in parameter γ the reduce divergence critical velocity grows (see table 1).

On the boundary of the stability region M_0

$$A_1 = 0, \; A_3 > 0 \tag{3.15}$$

the disturbed motion of the system loses of the dynamic stability: there is a “dynamic buckling”, which can be mistaken for “ panel flutter” [12 (c.719), 13]. The “flutter”critical velocities $V_{cr.fl.}$ delimited the region M_0 of stability and the subregion M_{12} of the instability of system disturbed motion are determined by substituting the first root $q_{cr.fl} = q_{cr.fl}(n,\gamma,\nu)$ of equation (3.10) in the expression (2.7). When the velocity of gas flow $V \geq V_{cr.fl.}$ there is a transition across the boundary (3.15), which takes place only for $\gamma \in (0.83,1.5]$ values: the eigenvalues $\lambda_{1,2} = \pm i\omega$ transition through the infinitely distant point $\lambda = \infty$, respectively, on the left and on the right parts of the complex plane and remain so, at least, when values of the velocity of the gas flow V close to the critical value $V_{cr.fl.}$, real: $\lambda_1 < 0$, $\lambda_2 > 0$. There is an abrupt (“instant”) change in the character of the system disturbed motion from sustainable to unsustainable [5]. In the plate arise stresses, leading to an abrupt (“instant”) to change its form: so-called “dynamic buckling”, in which the plate “bulge” infinite speed “buckling” [12(p. 719]. This process is not oscillatory as well as divergence. However, despite the discrepancies existing in the scientific literature [4 (p. 63), 5, 12(p. 719), 13], it is conditionally possible to consider as “quasi-oscillatory” process, i.e. as the panel flutter, usually leading to the destruction of the plate [13]. Table 3 presents the several values of the reduced flutter critical velocity $V_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ are found by substitution of the first root $q_{fl} = q_{cr.fl}(n,\gamma,\nu)$ of the equation (3.10) for $n = 1$ and some $\gamma \in (0.83,1.5]$ and ν in formula (2.7).

Table 3.

γ \ ν	0.125	0.25	0.33	0.375	0.5
0.9	109.68	85.44	73.56	66.57	53.15
1.0	191.38	126.24	105.64	96.09	72.91
1.1	492.51	185.72	146.83	131.12	97.05
1.2	591.77	274.98	206.46	178.48	126.00
1.3	673.97	378.02	257.51	242.36	166.22
1.4	802.32	483.93	352.53	302.71	207.61
1.5	936.02	595.22	448.61	380.12	273.35

From the data of table 3 it follows that the flutter critical velocity is less than in plates made of materials with a large Poisson's ratio ν, and with increasing γ it grows.

On the boundary of the instability region M_{11}

$$A_1 = 0, \; A_3 < 0 \tag{3.16}$$

at the velocities $V \geq \tilde{V}_{cr.fl.}$ of gas flow for all $\gamma \in (0, 0.83)$ the eigenvalues $\lambda_1 < 0$, $\lambda_2 > 0$ moving through an infinitely remote points $\lambda_{1,2} = \pm\infty$ on the imaginary axis of the complex plane and remain so, at least, when values of the velocity V of the gas flow close to the critical value $\tilde{V}_{cr.fl.}$ of the pure imaginary: $\lambda_{1,2} = \pm i\omega$. During this transition if the plate is not destroyed, the perturbed motion of the system "plate–flow" becomes stable [5, 13]. Table 4 presents the several values of the reduced flutter critical velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ for some values $\gamma \in (0, 0.83)$ and Poisson's ratio ν.

Numerical results showed the following. The flutter critical velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ is less than in plates made of materials with a large Poisson's ratio ν, and with increasing γ it grows for all values $\gamma \in (0.01, 0.83)$, and for all $\gamma \in (0, 0.01]$ the flutter critical velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ does not depend on the parameters γ, ν and it equal to $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3) = 6.3$ (tabl. 4).

Table 4.

γ \ ν	0.125	0.25	0.33	0.375	0.5
0.01	6.33	6.33	6.33	6.33	6.33
0.1	6.72	6.69	6.63	6.58	6.56
0.2	8.16	7.62	7.49	7.25	6.87
0.3	10.76	9.94	9.51	9.20	8.40
0.4	15.14	13.62	12.66	11.97	10.59
0.5	22.04	19.35	17.44	16.61	13.78
0.6	32.02	25.31	24.82	22.82	18.61
0.7	46.68	38.82	33.67	31.52	27.15

Numerical results showed the following. The flutter critical velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ is less than in plates made of materials with a large Poisson's ratio ν, and with increasing γ it grows for all values $\gamma \in (0.01, 0.83)$, and for all $\gamma \in (0, 0.01]$ the flutter critical velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ does not depend on the parameters γ, ν and it equal to $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3) = 6.3\,3$ (tabl. 4). Note that in monography [4] it is shown that in the problem of panel flutter of a console, the divergence critical velocity equal to 6.33 and the flutter critical velocity – 124.4. Comparison of these results with the results of this work, it follows that the flutter critical velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ equal to the divergence critical velocity and about twenty times less than the flutter critical velocity which are found in the work [4].

It is easy to show that the limit of the ratio A_3 to A_1 is equal to 1 for all values $\gamma \in (1.5, \infty]$: $\lim A_3 \cdot A_1^{-1} = 1$. And this in accordance with the expression (3.8) means that the characteristic performances λ of the system "plate–flow" for all $\gamma \in (1.5, \infty]$ are purely imaginaries numbers $\lambda_{1,2} = \pm i\omega$ i.e. the system perturbed motion is stable. The plate makes harmonic oscillations about the unperturbed equilibrium state. Thus, applied to the edge $x = a$ the inertial rotation moments lead to stabilization of the system disturbed motion "plate–flow" for all $\gamma \in (1.5, \infty]$.

3.3. Consider the case in which $a \gg b$.

It is easy to show that in this case the characteristic equation (2.11) is transformed to the following

$$\tilde{\chi}_n \tilde{\delta}_n a_{01} \lambda^4 + (a_{11}\tilde{\chi}_n + a_{21}\tilde{\delta}_n)\lambda^2 + a_{31} = 0. \tag{3.17}$$

Here

$$a_{01} = \sqrt{2(q+1)}, \; a_{11} = 2(q+1)\cdot(q - \sqrt{q^2-1} - \nu) - (1-\nu)^2, \; a_{21} = 2(q+1), \tag{3.18}$$

$$a_{31} = \sqrt{2(q+1)} \cdot [2(q+1)\cdot(q - \sqrt{q^2-1} - \nu) - (1-\nu)^2]; \tag{3.19}$$

$$\tilde{\chi}_n = I_c\, b \cdot (\pi n D)^{-1}, \; \tilde{\delta}_n = m_c D^{-1} b^3 (\pi n)^{-3}, \; \tilde{\chi}_n > 0, \; \tilde{\delta}_n > 0. \tag{3.20}$$

For all $q > 1$ it follows that

$$a_{11}\tilde{\chi}_n + a_{21}\tilde{\delta}_n > 0, \; \mathit{\Delta} = (a_{11}\tilde{\chi}_n + a_{21}\tilde{\delta}_n)^2 - 4\tilde{\chi}_n\tilde{\delta}_n a_{01}a_{31} = (\tilde{\chi}_n a_{11} - \tilde{\delta}_n a_{12})^2 \geq 0. \tag{3.21}$$

Here $\mathit{\Delta}$ is the discriminant of the biquadratic equation (3.17).

In accordance with the conditions (3.21), the stability region M_0 defined by the correlation

$$a_{31} > 0. \tag{3.22}$$

Under this condition equation (3.17) has two pairs of purely imaginary roots $\lambda_{1,2} = \pm i\omega_1$, $\lambda_{3,4} = \pm i\omega_2$: the rectangular plate performs harmonic oscillations about the unperturbed equilibrium state. And the region of instability M_1 by the correlation $a_{31} < 0$ is determined. It follows, that in the region M_1 of the characteristic equation (3.17) has a pair of purely imaginary roots $\lambda_{1,2} = \pm i\omega$ and two real roots $\lambda_3 < 0$, $\lambda_4 > 0$. This means that one of the two proper motions of the plate is dampened, and the other the movement of plates is unlimited deviation exponentially from the equilibrium state.

The boundary of the stability region M_0 is a hypersurface

$$a_{31} = 0. \tag{3.23}$$

Or, in accordance with the expression (3.19), is

$$2(q+1)\cdot(q - \sqrt{q^2-1} - \nu) - (1-\nu)^2 = 0. \tag{3.24}$$

where the characteristic equation (3.17) has a zero root $\lambda_0 = 0$ of multiplicity 2 and a pair of pure imaginary roots are equal to $\lambda_{1,2} = \pm i\sqrt{2(q+1)\cdot\tilde{\chi}_n^{-1}}$ according to the expressions (3.18).

The condition (3.24) determines the loss of stability of the disturbed motion of the system "plate–flow" in the form of a localized divergence in the vicinity of the free edge $x = 0$ of the plate [10].

The critical velocities $V_{loc.div}$ of the localized divergence that delimites the stability region M_0 and the region of instability M_1 of the system perturbed motion are determined by substituting the first root $q_{loc.div} = q_{loc.div}(\nu)$ of equation (3.24) in expression (2.8). It follows that the reduce critical velocity $V_{loc.div} \cdot D^{-1}(a_0\rho_0 b^3)$ of the localized divergence depends on the parameter n and Poisson's ratio ν: when a fixed value of parameter n the critical velocity is less than in plates made of materials with a large Poisson's ratio, and when the fixed of parameter ν it reaches the lowest value when $n = 1$ (tabl.2).

Thus, in the case in which $a \gg b$ the system "plate-flow" loses stability in a localized divergence in the vicinity of the free edge $x = 0$ of the plate at all the velocities $V \geq V_{loc.div}$ of the gas flow. The critical velocity $V_{loc.div}$ of localized divergence does not depend on the coefficients $\tilde{\chi}_n$ and $\tilde{\tilde{\delta}}_n$. The presence of the inertial moment I_c ($\tilde{\chi}_n \neq 0$) of rotation on the hinged edge $x = a$ leads to the stabilization when the inertial mass m_c ($\tilde{\tilde{\delta}}_n = 0$) on the free edge $x = 0$ is absent.

2.4. Let us consider the case corresponding to the condition $a \ll b$.

Numerical studies of the characteristic equation (2.11) has shown that its solution corresponding to the occasion, meet the condition

$$q \gg 1 . \qquad (3.25)$$

Then, introducing the notation

$$r = \sqrt{2q} \cdot \pi n \gamma , \qquad (3.26)$$

the characteristic equation (2.11) and expression (2.7) can be written, respectively, as

$$a_{02}\tilde{\tilde{\chi}}\tilde{\tilde{\delta}}\lambda^4 + (a_{12}\tilde{\tilde{\chi}} + a_{22}\tilde{\tilde{\delta}})\lambda^2 + a_{32} = 0 , \qquad (3.27)$$

$$V = r^3 D(a_0\rho_0 a^3)^{-1} . \qquad (3.28)$$

Here

$$a_{02} = \mathrm{sh}(r) - 2\mathrm{sh}(r/2) \cdot \cos(\sqrt{3}r/2) ; \qquad (3.29)$$

$$a_{12} = [1/2 \cdot \exp(-r) + \exp(r/2)\cos(\sqrt{3}r/2)] \cdot r^3 ; \qquad (3.30)$$

$$a_{22} = [\mathrm{ch}(r) - \exp(r/2) \cdot \sin(\pi/6 - \sqrt{3}r/2) - \exp(-r/2) \cdot \sin(\pi/6 + \sqrt{3}r/2)] \cdot r \qquad (3.31)$$

$$a_{32} = [-1/2 \cdot \exp(-r) + \exp(r/2) \cdot \sin(\pi/6 - \sqrt{3}r/2)] \cdot r^4 ; \qquad (3.32)$$

$$\gamma = ab^{-1} ; \ \tilde{\tilde{\chi}} = I_c a D^{-1} ; \ \tilde{\tilde{\delta}} = m_c a^3 D^{-1} . \qquad (3.33)$$

From expressions (3.29) and (3.31) it is obvious that

$$a_{02} > 0 , \ a_{22} > 0 \ \text{at all} \ r > 0 . \qquad (3.34)$$

It can be shown that in the absence of flow plates $V = 0$ or $r = 0$ the characteristic equation (3.27) describes by the correlation

$$\tilde{\tilde{\chi}} \cdot \tilde{\tilde{\delta}} \cdot \lambda^4 + 3 \cdot (\tilde{\tilde{\chi}} + \tilde{\tilde{\delta}}) \cdot \lambda^2 = 0 . \qquad (3.35)$$

At all values of $\tilde{\tilde{\chi}} \in (0, \infty)$, $\tilde{\tilde{\delta}} \in (0, \infty)$ the equation (3.35) has a pair of purely imaginary roots and the zero root $\lambda_0 = 0$ of multiplicity 2. This means that when the gas flow velocities $V \geq V^{(1)}_{cr.div.} = 0$ then the perturbed motion of the system loses stability in the form of divergence: the plate "buckles".

Note that the correlations (3.27), (3.35) and (3.28) are identical with the corresponding equations describing the characteristic equation and the formula for calculating the gas flow velocity to the problem of stability of a streamlined a supersonic flow of gas, an elongated plate $0 \leq x \leq a$, $0 \leq y < \infty$ with a free edge $x = 0$ under the same assumptions.

Therefore, the behavior of the disturbed motion of the system "rectangular plate – flow" in this case is the same as in the case of a system "elongated plate – flow".

In accordance with the first of the inequalities (3.34), the stability region M_0 defined by the correlations

$$a_{12}\tilde{\tilde{\chi}} + a_{22}\tilde{\tilde{\delta}} > 0, \quad a_{32} > 0, \quad \Delta > 0. \tag{3.36}$$

Here

$$\Delta = (a_{12}\tilde{\tilde{\chi}} + a_{22}\tilde{\tilde{\delta}})^2 - 4\tilde{\tilde{\chi}}\tilde{\tilde{\delta}}a_{02}a_{32} \tag{3.37}$$

is the discriminant of the biquadratic equation (3.27).

And the instability regions M_1, M_2, M_3 will be determined, respectively, by the correlations: $a_{32} < 0$, $\Delta > 0$; $a_{12}\tilde{\chi} + a_{22}\tilde{\delta} < 0$, $a_{32} > 0$, $\Delta > 0$; $a_{32} > 0$, $\Delta < 0$.

The boundaries of the stability region M_0 of the condition $a_{12}\tilde{\tilde{\chi}}_n + a_{22}\tilde{\tilde{\delta}}_n > 0$ are the hypersurfaces

$$a_{32} = 0, \tag{3.38}$$

$$\Delta = 0. \tag{3.39}$$

On the hypersurfacies (3.38) and (3.39) the characteristic equation (3.27) has a zero root $\lambda_0 = 0$ of multiplicity 2, and a pair of the purely imaginary roots $\lambda_{1,2} = \pm i\omega$ respectively.

On the boundary of the stability region M_0 of the

$$a_{12}\tilde{\tilde{\chi}} + a_{22}\tilde{\tilde{\delta}} > 0, \quad \Delta > 0, \quad a_{32} = 0, \tag{3.40}$$

the perturbed motion of the system loses the static stability: there is a divergence of the panel. The critical divergence velocities $V_{cr.div}$ are determined by substituting the roots $r_{cr.div}$ of equation (3.38) into the expression (3.28).

On the boundary of the stability region M_0 of the

$$a_{12}\tilde{\tilde{\chi}} + a_{22}\tilde{\tilde{\delta}} > 0, \quad a_{32} > 0, \quad \Delta = 0, \tag{3.41}$$

and on the boundaryof the static instability region M_2 of the

$$a_{12}\tilde{\tilde{\chi}} + a_{22}\tilde{\tilde{\delta}} < 0, \quad a_{32} > 0, \quad \Delta = 0, \tag{3.42}$$

the system perturbed motion loses its dynamic stability: there is a panel flutter. The critical flutter velocities $V_{cr.fl}$ and $\tilde{V}_{cr.fl}$, respectively, delimited of the regions M_0, M_3 and of the regions M_2, M_3 are determined by substituting the roots of equation (3.39) into the expression (3.28). According to the correlations (3.28) and (3.37) the reduced critical flutter velocities $V_{cr.fl} \cdot D^{-1}(a_0 \rho_0 a^3)$ and $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0 \rho_0 a^3)$ depend on the parameter $\tilde{\tilde{k}} = \tilde{\tilde{\chi}} \cdot \tilde{\tilde{\delta}}^{-1}$.

The numerical investigations showed the following.

For all $\tilde{\tilde{k}} \in [0, \infty]$ at the velocities $V \geq V_{cr.div}^{(1)} = 0$ of the gas flow is a loss of static stability of the system disturbed motion, i.e. divergence: in the plate undergoing harmonic oscillations, there is tension, leading to change its shape: the plate "bulge" with limited velocity "buckling". In accordance with these values $\tilde{\tilde{k}} \in [0, 0.06)$ is possible only the loss of stability of the disturbed motion of the system in the form of divergence. In this case, the transitions from the region of stability M_0 in the divergence instability region M_1 alternate: when the velocities $V \geq 76.22 \cdot D(a_0 \rho_0 a^3)^{-1}$ of the gas flow the perturbed motion of the system, being statically unstable, becomes stable, and at the velocities $V \geq V_{cr.div}^{(2)} \approx 483.73 \cdot D(a_0 \rho_0 a^3)^{-1}$ of the gas flow again loses static stability.

For values $\tilde{\tilde{k}} \in [0.06, 0.3)$ we have the loss of stability of both types: as the divergence of the panel, and panel flutter. Originally statically unstable perturbed motion of the system at velocities $V \geq 76.22 D(a_0 \rho_0 a^3)^{-1}$ of the gas flow becomes stable. But when the velocities $V \geq V_{cr.fl}$ we have the "soft" transition from the region M_0 of stability in the region M_3 of the dynamic instability: the harmonic vibrations of the plate gradually transformed into self-oscillations, i.e. the flutter oscillations.

Table 5 presents the values of the reduced critical flutter velocities $V_{cr.fl} \cdot D^{-1}(a_0 \rho_0 a^3)$ with an accuracy of the order of 10^{-3} for several values of the parameter $\tilde{\tilde{k}} \in [0.06, 0.3)$.

Table 5.

$\tilde{\tilde{k}}$	0.06	0.08	0.1	0.2	0.25
$V_{cr.fl} \cdot D^{-1}(a_0 \rho_0 a^3)$	125	98.61	89.31	76.76	76.34

In this case, the reduced critical flutter velocity $V_{cr.fl} \cdot D^{-1}(a_0 \rho_0 a^3)$ decreases with the growth $\tilde{\tilde{k}} \in [0.06, 0.3)$ (table. 5).

At velocities $V \geq \tilde{V}_{cr.fl} > V_{cr.fl}$ of gas flow is a "soft" transition from the region of static instability M_2 to the region M_3 of the dynamic instability. We can say that the phenomenon of the buckled panel flutter is observed. There is as well as a "smooth" transition to the flutter oscillations in addition to the monotonous "buckling" of the plate that does not have an oscillatory character.

Table 6 presents the values of the reduced critical flutter velocities $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0 \rho_0 a^3)$ with an accuracy of the order of 10^{-3} for some values of the parameter $\tilde{\tilde{k}} \in [0.3, \infty)$. As can be seen from table 6, the critical flutter velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ grows with the parameter $\tilde{\tilde{k}} = \tilde{\tilde{\chi}} \cdot \tilde{\tilde{\delta}}^{-1}$.

It means that when the values of $\tilde{\tilde{k}} \in [0.3, \infty)$ the inertial moment of rotation I_c applied to the hinged edge $x = a$ of the plate leads to the stabilization.

However, the reduced critical flutter velocity $\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$ is approximately equal to 6.33 when the value of parameter $\tilde{\tilde{k}} = \infty$ that by an order of magnitude less than the critical flutter velocity at $\tilde{\tilde{k}} \in [0.3, \infty)$ (tabl. 4, 6).

Table 6.

$\tilde{\tilde{k}}$	0.3	0.4	0.5	0.8	1.0
$\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$	74.61	78.40	79.50	86.35	91.12
$\tilde{\tilde{k}}$	1.2	1.5	2	5	10
$\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$	96.07	100.54	105.15	122.76	132.65
$\tilde{\tilde{k}}$	20	50	100	1000	10000
$\tilde{V}_{cr.fl} \cdot D^{-1}(a_0\rho_0 a^3)$	140.61	147.19	151.42	157.46	160.10

Of the identity of the dispersion equation (3.27) and the dispersion equation obtained in [8] in studying the problem of panel flutter of a plate elongated in the assumption that inertial mass and rotation moments applied simultaneously to the free edge $x = 0$ and to the opposite hinged edge $x = a$ do not exist, should identity in the behavior of the disturbed motion of the system "elongated plate plate–flow" these problems.

Thus, in the case when $a \ll b$ the behavior of the disturbed motion of the system "rectangular plate-flow", similar to the behavior of the disturbed motion of the system "elongated plate-flow" ($0 \le x \le a, 0 \le y < \infty$). Namely, when the velocity of the gas flow is absent ($V = 0$), the system perturbed motion is statically unstable. In the flow ($V \ne 0$) the behavior of the system perturbed motion depends on the value of the ratio of relative values of concentrated inertial moments I_c and masses m_c are applied, respectively, to the hinged edge $x = a$ and free edge $x = 0$ of the plate.

Conclusion. Using an analytically method, investigated by special cases of the problem of panel flutter, where the General case is studied in [1]. On the partition of the space of the "essential" parameters of the system "plate–flow" in regions of the stability and instability is performed. The boundaries of the region of stability are investigated. The boundaries of the divergence of panel, localized divergence and panel flutter are determined. We found the "dangerous" of the boundaries of the stability region in the sense of terminology work N.N. Bautin [14]. You move through them arises the phenomenon of panel flutter, leading to a loss of strength and occurrence of fatigue cracks in the material of the plate. For different values of the problem parameters was found the critical velocity of divergence, localized divergence and flutter. In problems of panel flutter in a linear formulation, as a rule, the critical velocity of divergence less than the flutter critical velocity [2-4, 8, 9, 12].

As well as in [1], in this work, we obtained unexpected results. It turned out, that depending on the relation between the parameters of the problem the flutter critical velocity can be both less and greater than the divergence critical velocity. A number of new mechanical effects are revealed. In particular, shows the stabilizing role of the inertial moment of rotation, applied on the hinged edge of the plate. And also, from a comparison of the obtained results with the results of [8], it was found that the effect of the inertial moment of rotation on the behavior of the disturbed motion of the system "elongated plate–flow" does not depend on its place of application: for hinged edge, or free edge of the plate.
These results can be used for the preliminary quantitative analysis of the problem panel flutter in the nonlinear statement [4, 12, 15, 16].

ЛИТЕРАТУРА

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