

**DYNAMIC BIMORPH THERMO-PIEZOELECTRIC BENDERS WITH
ARBITRARY SUPPORT LOCATION. PART II: APPLICATION TO
ENERGY HARVESTING-NUMERICAL RESULTS AND DISCUSSIONS**

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Dedicated to the memory of late Professor Vardges Gnuni

Keywords: Thermo-electro-elastic effects, energy harvesting, resonant frequency, piezoceramic, pyroelectric effect

Ключевые слова: термоэлектроупругий эффект, накопитель энергии, резонансная частота, пьезокерамика, пьезоэлектрический эффект

Բանալի բառեր. Ջերմաէլեկտրաառաձգականություն, ռեզոնանսային հաճախություն, պիեզոէլեկտրամիկա, պիեզոէլեկտրիկ

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Динамическая двухслойная термоупругая пластина с произвольно расположенными опорами.

Часть II. Применение к накоплению энергии – численные результаты и обсуждение.

Приводится численный анализ задачи, рассмотренной в предыдущем номере журнала, для частных примеров консольной и шарнирно опертой балок.

Բաղդասարյան Գ.Ե., Հասանյան Ա.Դ., Հասանյան Դ.Զ

Կամայականորեն տեղակայված հենարաններով երկշերտ ջերմաառաձգական դիմամիկական սալ:

Մաս II: Էներգիայի կուտակում – թվային հետազոտություն և քննարկում

Ամսագրի նախորդ համարում դիտարկված խնդրի համար բերված են թվային արդյունքներ մասնավորապես կոնսոլային և հողակապորեն ամրակցված հեծանների համար:

6. Discussions and Numerical Results

The effect of varying the vibrational frequency on the generation of charge is shown in Fig. 2. The parameters s, h, σ, α_T and \tilde{p} are chosen according to Table 1. The regions that peak to infinity represent the vibrational frequency being equal to the natural frequency. In Fig 2a, the support location has a strong effect on the charge coefficient. Fig. 2b,c,d,e show the effect of varying the value of the follower force. When $\lambda > 0$ the follower force is in compression and when $\lambda < 0$, it is in tension. From these plots, λ has the strongest effect on the system when the vibration frequency is below the first natural frequency. When the system is static ($\Omega = 0$), as seen in Fig. 3, by including the follower force, the energy harvesting coefficient can be improved. When the magnitude of the compressive follower force increases, the strongest effect is at higher values of α . For $\alpha = 1.0$, the charge coefficient approaches to infinity monotonically as the compressive follower force approaches λ_{cr} . Contrary to conventional uses of bimorphs, where a compressive follower

force is applied, a tensile follower force is also considered. In Fig. 2d,e, or Fig. 3, for higher values of α , the tensile follower force decreases the energy harvesting coefficient, but for lower values of α , it can improve the energy harvester. For a cantilever plate-layer, the energy harvesting coefficient can be improved to $\tilde{Q}_{gen}^{\mathfrak{g}} = 28$ for $\lambda = -1.6$ from $\tilde{Q}_{gen}^{\mathfrak{g}} = 20$ for $\lambda = 0$. This is a 40% increase of $\tilde{Q}_{gen}^{\mathfrak{g}}$.

An energy harvester can also be improved by increasing the probability of capturing resonance in environments where the vibrational frequency fluctuates. For example, in Fig. 4, the first natural frequency of Fig. 2b,d is considered. In Fig. 4a, a compressive load is applied and the width of the peak, which will be called the bandwidth, is widest for $\alpha = 1$ as compared to $\alpha = 0$ and $\alpha = 0.5$. Conversely, in Fig. 4b, when there is tensile load, the bandwidth is widest for $\alpha = 0$ and $\alpha = 0.5$ as compared to $\alpha = 1$. Because of this, when designing an energy harvester, it can be optimized for environments where the frequency fluctuates. The formulation in this paper can also be a guild to experimenters who try to capture resonance with energy harvesters.

Note that in other context effect of location of boundary condition on critical buckling load was investigated by V. Ts. Gnuni in [1].

Next, the effect of the nondimensional energy harvester's properties $h, s,$ and ρ on the generation of charge is studied. The remaining material properties $L, \mathfrak{g}, \alpha_T, p_3^{(1)},$ and $d_{31}^{(1)}$ affect $Q_{gen}^{\mathfrak{g}}$ linearly from our formulation of Eqn. (44b) and they will not be considered. The variation of these properties represents changing the material properties while the piezoelectric properties are held constant. When plotting the changes of one of these parameters, the remaining parameters are set to the values of Table 1. In Fig 5, the effect of varying the volume fraction h is shown when $\Omega = 0$. In Fig. 5a, when $\lambda = 0$, the support location has no effect on the generation of charge as h is changed. At $h = 1.3$, the energy harvesting coefficient approaches a maximum value and increasing h to infinity, the energy harvesting coefficient converges to a constant value. These characteristics are also observed when there is a follower force included, as seen in Fig. 5,c,d,e. Similar to the discussion earlier, the charge coefficient is improved in the presence of a compressive follower force when $\alpha = 1$, and it is improved in the presence of a tensile follower force when $\alpha = 0$ and $\alpha = 0.5$.

Fig 6 shows the effect of varying the compliance ratio s on $\tilde{Q}_{gen}^{\mathfrak{g}}$. From these results, it is determined that decreasing the stiffness of the material relative to the piezoelectric stiffness, the energy harvester improves. Increasing the material stiffness to infinity, the value of $\tilde{Q}_{gen}^{\mathfrak{g}}$ converges to a finite value, but decreasing it, $\tilde{Q}_{gen}^{\mathfrak{g}}$ approaches infinity.

Note that the effect of the varying the density ratio σ on $\tilde{Q}_{gen}^{\mathfrak{g}}$. σ only appears as a product of Ω^4 in the expression of p and q . Because of this, the variation of σ will have no effect on $\tilde{Q}_{gen}^{\mathfrak{g}}$, when $\Omega = 0$. Varying the densities of the piezoelectric and the substrate does

change the values of the natural frequencies and consequently values of σ , \tilde{Q}_{gen}^g is infinity.

This is because of resonance effects.

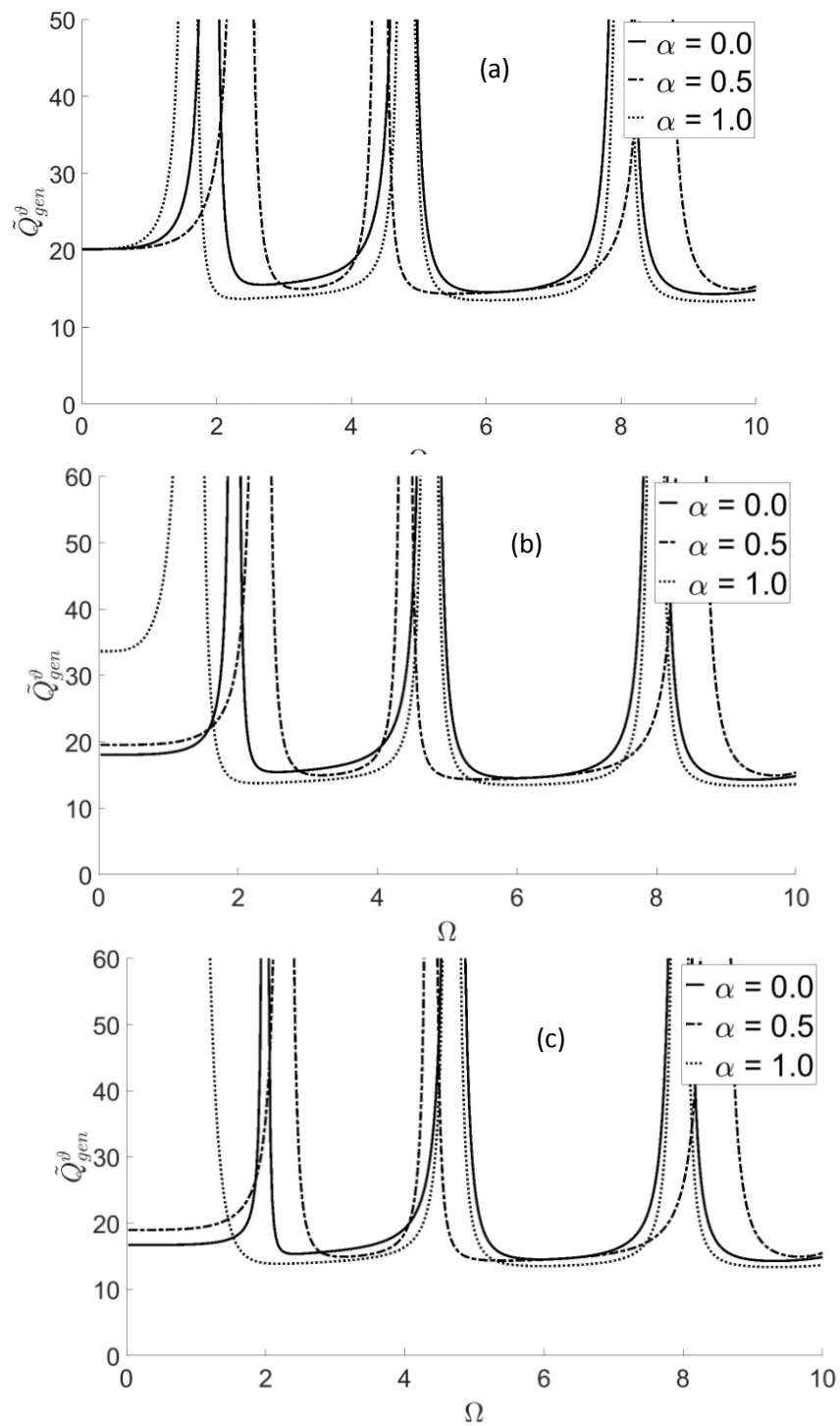
7. Conclusion

An analytical analysis of a bimorph's thermal energy harvesting coefficient was performed. The analysis also takes into account pyroelectric and thermal expansion effects. The most general analytical expression for the energy conversion coefficients are presented for bi-layer. These coefficients were derive for more general situations, when mechanical, electrical, thermal fields are present. We derive coefficients (transformation coefficients) for sensing, actuating and energy harvesting. As a particular case, the analytical expressions for energy harvesting coefficients due to pyroelectric and thermal expansion effects were obtained. The influences of support location, material properties, and a conservative follower force were analyzed in order to optimize the energy harvester. The numerical simulation of the thermal energy harvesting coefficient showed that support location strongly influences the generated charge. The follower force was seen to influence the energy harvester the most when the vibration frequency was below the first resonance frequency. Contrary to convention, where a compressive in plain follower force is applied, a tensile follower force was shown to also improve the energy harvester. In addition, by varying support location and the follower force, the bandwidth, or the range of vibrating frequency over which resonance effects are observed, can be made wider so that the probability of the energy harvester operating at resonance frequency can be increased. This is beneficial for designs which operate in environments where the vibration frequency fluctuates over a certain frequency range.

The effect of volume fraction and compliance ratio was studied for a static system. It was shown that at a certain volume fraction, the energy harvester is optimized. The compliance ratio improves the energy harvester by taking it close to zero. The density ratio does not affect the charge coefficient directly. By changing the densities of the two layers of the bimorph, the natural frequency of the system is changed, and this indirectly influences the charge coefficient. It was also shown that the density ratio has no effect for a static system as expected since it appears as a coefficient of the vibrational frequency in the equilibrium equations.

Table 1: Material properties used in the numerical simulation

Material Prop	PZT-5A	Aluminum
Compliance ($10^{-12}m^2/N$)	16.4	14.3
Density (kg/m^3)	7750	2700
Height (mm)	0.75	0.5
Thermal expansion ($10^{-6} K^{-1}$)	2	22
Pyroelectric coefficient ($10^{-6} Cm^{-2}/K$)	238	-
Piezoelectric coefficient ($10^{-12} C/N$)	-171	-
Dielectric permittivity ($10^{-12} F/m$)	15051.8	-



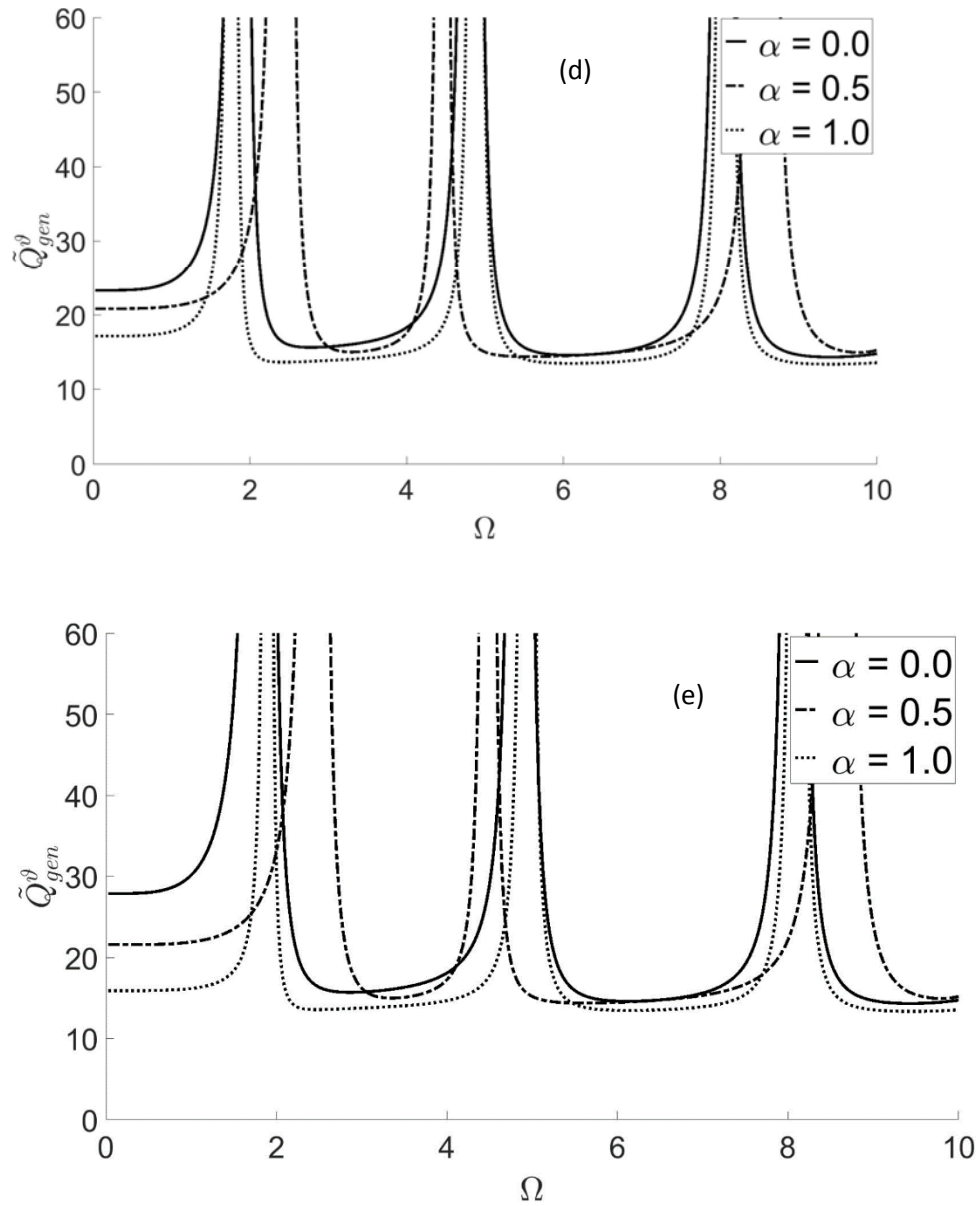


Figure 2. Non-dimensional charge coefficient \tilde{Q}_{gen}^{θ} vs. non-dimensional frequency Ω for follower force (a) $\lambda = 0$, (b) $\lambda = 0.8$, (c) $\lambda = 1.6$, (d) $\lambda = -0.8$ and (e) $\lambda = -1.6$

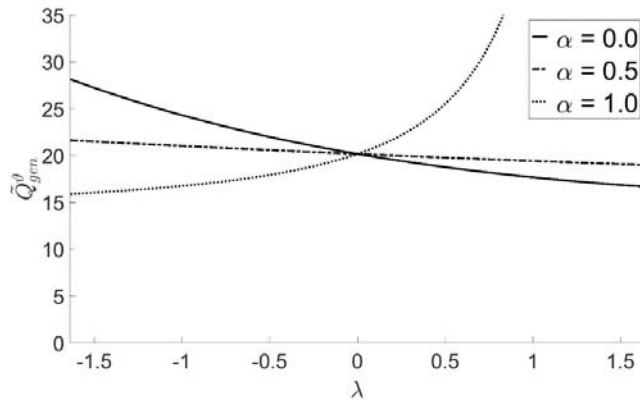


Figure 3. Non-dimensional charge coefficient \tilde{Q}_{gen}^0 vs. non-dimensional follower force λ for a static system $\Omega = 0$.

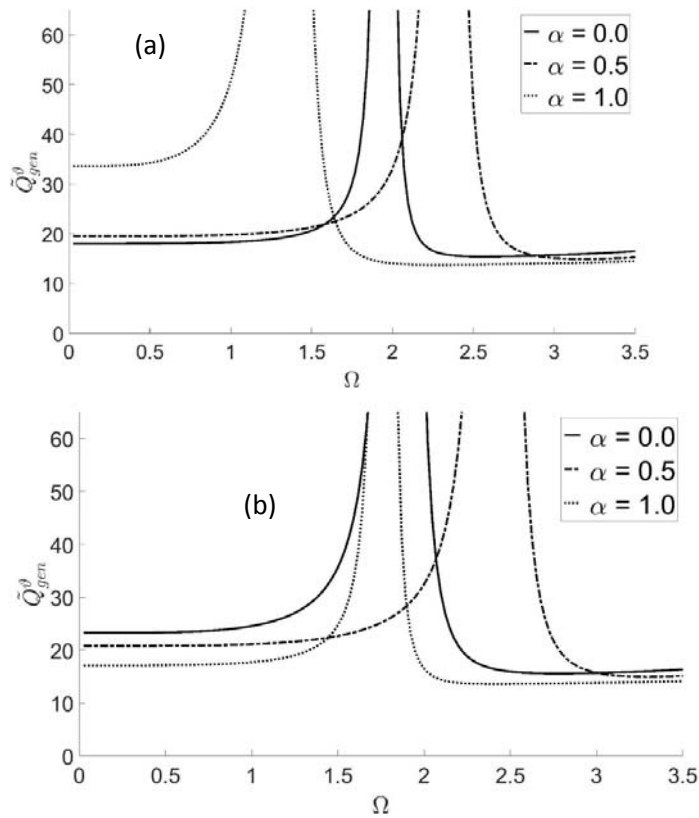
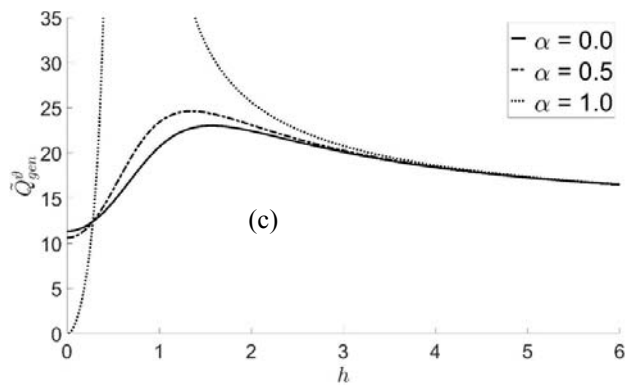
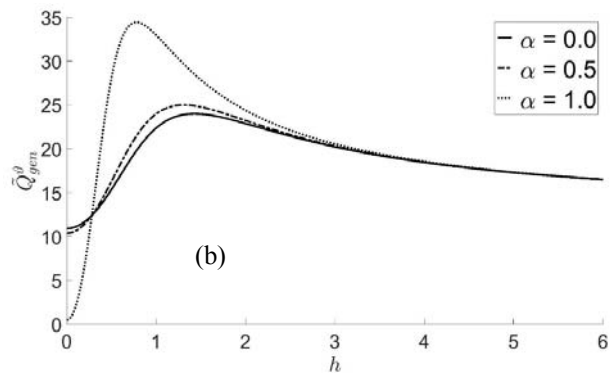
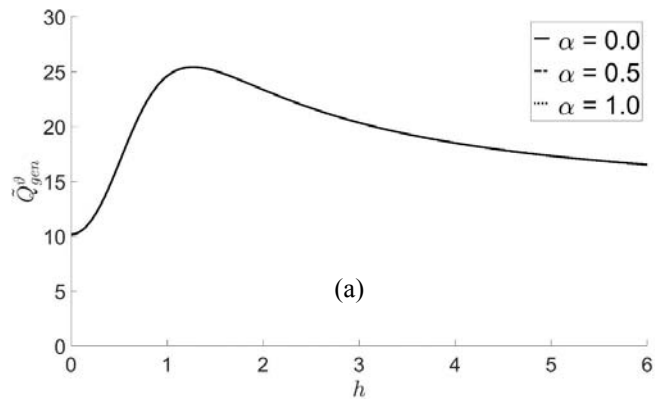


Figure 4. Display of the first natural frequency's bandwidth of Fig. 2b,d for follower force (a) $\lambda = 0.8$ and (b) $\lambda = -0.8$.



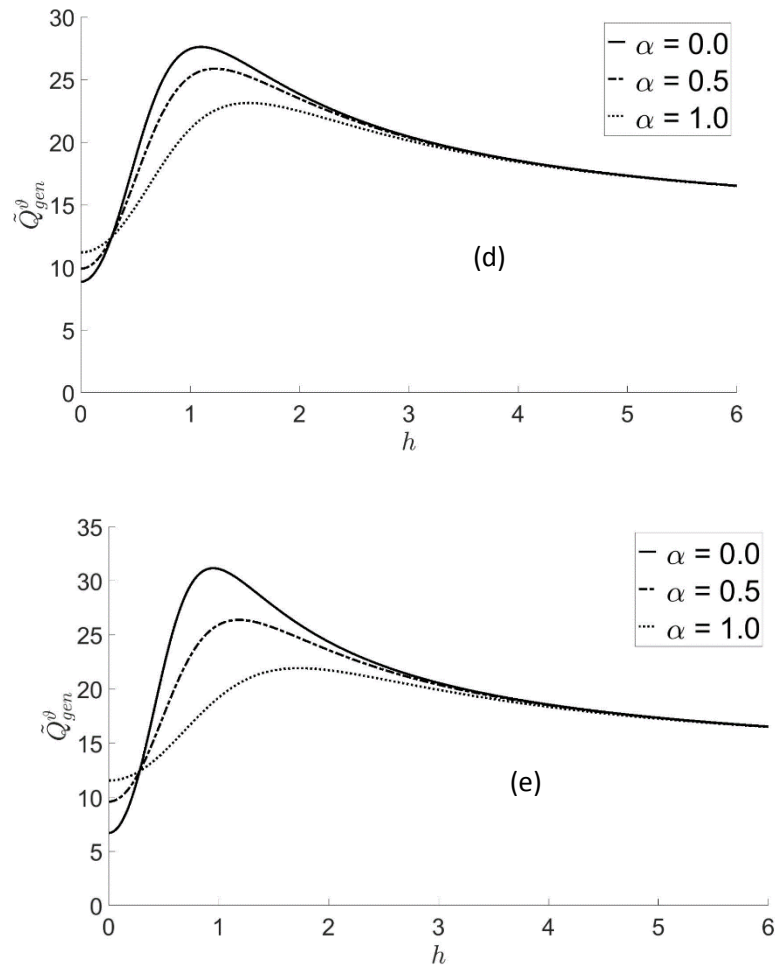
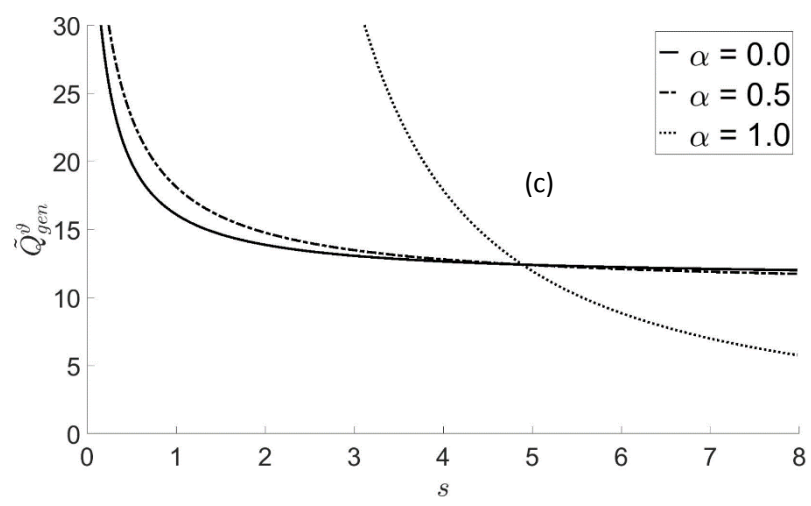
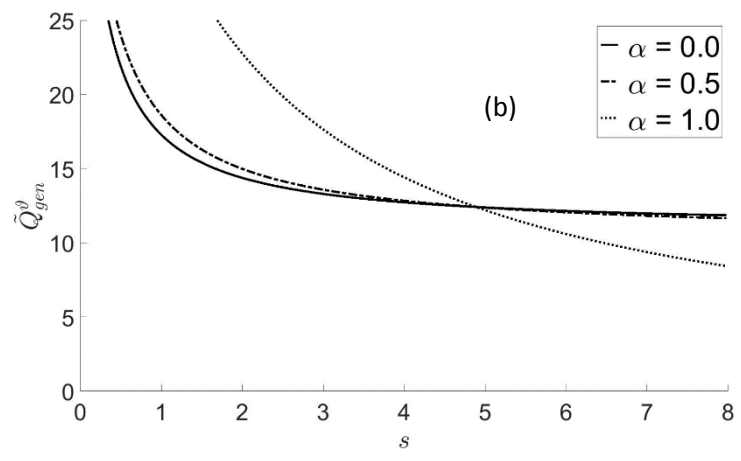
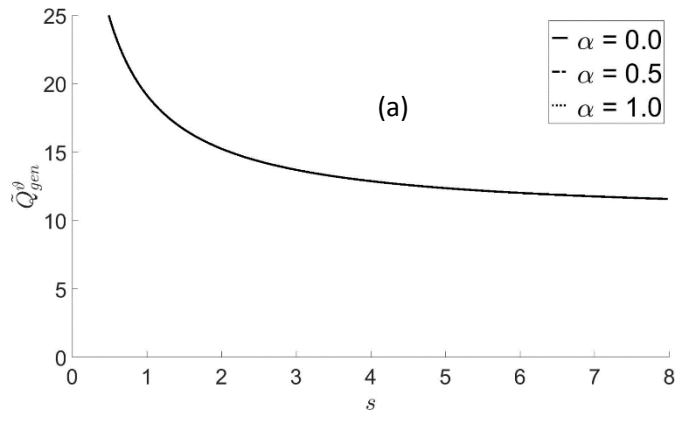


Figure 5. Non-dimensional charge coefficient \tilde{Q}_{gen}^{β} vs. volume fraction h for a static case $\Omega = 0$ at support locations $\alpha = 0$ (green), $\alpha = 0.5$ (black), $\alpha = 1.0$ (red) for follower force (a) $\lambda = 0$, (b) $\lambda = 0.8$, (c) $\lambda = 1.6$, (b) $\lambda = -0.8$ and (e) $\lambda = -1.6$.



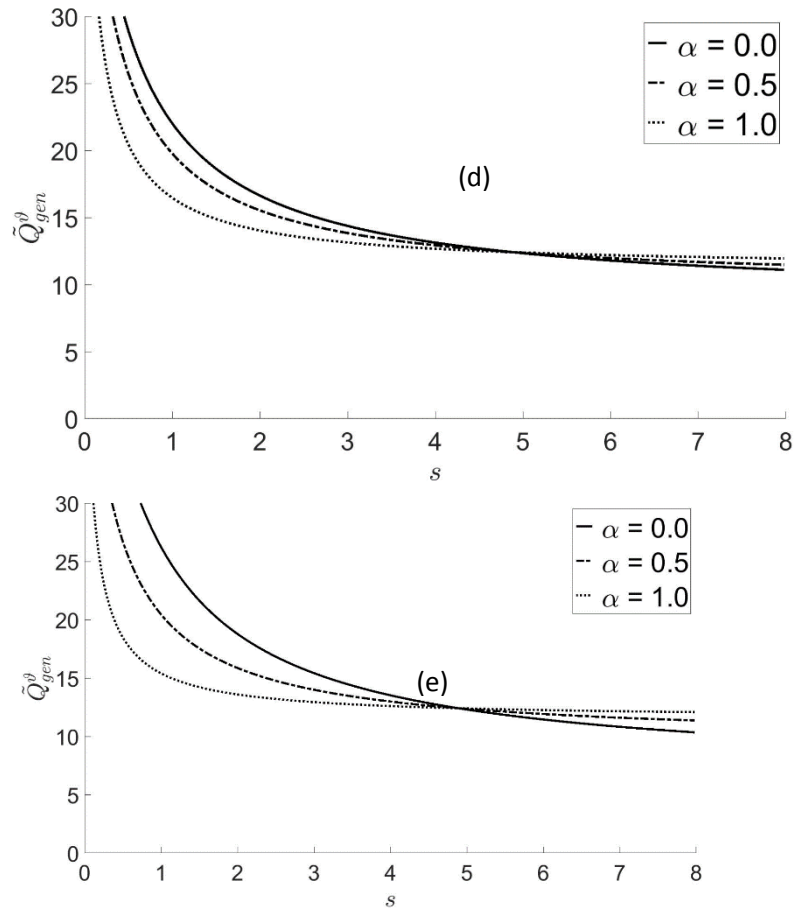


Figure 6. Non-dimensional charge coefficient \tilde{Q}_{gen}^{θ} vs. compliance ratio s at a static case state ($\Omega = 0$), for follower force (a) $\lambda = 0$, (b) $\lambda = 0.8$, (c) $\lambda = 1.6$, (d) $\lambda = -0.8$ and (e) $\lambda = -1.6$

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