ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
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SHEAR WAVES IN PERIODIC WAVEGUIDE WITH ALTERNATING BOUNDARY CONDITIONS

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Keywords: shear waves, wavequide, Bloch conditions, Brillouin zone.
Ключевые слова: сдвиговые волны, волновод, условия Блоха, зона Брюллена.

Փիլիպոսյան Դ.Գ., Ղազարյան Ռ.Ա., Ղազարյան Կ.Բ.

Սահքի ալիքները խառը եզրային պայմաններով առաձգական պարբերական ալիքատարում

Ուսումնասիրված է սահքի ալիքների տարածման խնդիրը առաձգական պարբերական կառուցվածքով ալիքատարում: Ալիքատարը կազմված է տարբեր եզրային պայմաններով երեք տարբեր համասեռ նյութերից: Ֆլոկեյի տեսության սահմաններում, օգտագործելով անցման մատրիցի գաղափարը, Ֆլոկեյի ալիքային թվի ալիքի հաճախությունից կախվածության որոշման խնդիրը բերված է օրթոգոնալ անցման մատրիցի սեփական արժեքների որոշմանը: Թվային և անալիտիկ մոթոդների հիման վրա բերված է սահքի ալիքների հաճախությունների արգելված գոտիների կառուցվածքի հետազոտությունը:
Ցույց է տրված, որ իրարից տարբեր պարբերական եզրային պայմանների դեպքում հնարավոր է արգելման գոտիների տեղաշարժ՝ դեպի Բրյուլենի գոտու կենտրոն:

Пилипосян Д.Г., Казарян Р.А., Казарян К.Б.
Сдвиговые волны в упругом периодическом волноводе с альтернативнымн граничными условиями

Исследована задача распространения сдвиговых волн в упругом волноводе периодической структуры, состоящей из трёх различных материалов с однородными или альтернативно меняющимися граничными условиями на стенках волновода. В рамках теории Флоке, используя концепцию матрицы перехода, задача определения зависимости волновых чисел Флока от частоты волны сведена к задаче определения собственных чисел ортогональной блок-матрицы. Аналитически и численно проведён анализ дисперсионных соотношений структуры запретных зон частот сдвиговых волн. В случае альтернативных граничных условий задача решена численно. Показано, что в случае альтернативных граничных условий возможно смещение запретных зон частот к середине зоны Брюллена.

The propagation of shear waves in elastic waveguide of periodic structure consisting of three different materials with alternating along the guide walls boundary conditions is investigated. Using the transfer matrix approach the problem is reduced to the solution of a block transfer matrix eigenvalue problem. Bloth the dispersion and the band gap structure analysis have been carried out numerically. It is shown that for alternating boundary conditions along the waveguide walls, by modulating the ratio of the length of the unit cell to the width of the waveguide, the minimum widths of the stop bands can be moved to the middle of the Brillouin zone

The present paper is concerned with the study of Bloch waves in phononic waveguides. Phononic crystals are periodic materials that have a potential to control the propagation of elastic and acoustic waves [1]. In particular they have frequency ranges in which waves cannot propagate, called acoustic band gaps [2, 3]. Interest in this class of materials has been motivated by recent investigations of the electromagnetic analogues called photonic crystals [4]. These materials exhibit band gaps for electromagnetic waves, and are referred to in the popular scientific literature as «invisibility cloaks» [5].

The complete band gap phenomenon plays an important role in designing phononic crystal applications such as elastic wave filters, couplers, and waveguides, especially. The phononic crystal waveguide is an important elementary component to build an acoustic wave circuit. Different methods have been developed for investigating the properties of Bloch waves in waveguides. Using asymptotic methods the influence of the periodically varying thickness of a homogeneous waveguide on the trapped modes and cutoff frequencies has been studied in [6,7]. A modal decomposition approach based on eigenfunction expansion of elastic displacements and stresses, where the eigenfunctions are orthogonal wavefield modes of an infinite homogeneous waveguide, the problem of wave propagation in periodic waveguides has been investigated in [8,9]. This approach has been further applied to study the band gap spectrum and associated Bloch eigenfunction for out of plane and in-plane waves in acoustic waveguides with alternating boundary conditions along the guide walls for [10,11]. In all these papers the unit cell in a periodic waveguide is consist of two different materials. It is interesting to see the effect of an additional element in the unit cell on the properties of the dispersion curves and the band structure.

The purpose of this paper is to investigate shear Bloch waves in a finite-width infinite periodic elastic waveguide with the unit cell of a period d consisting of three different perfectly bonded homogeneous materials (Figure 1). We will investigate the effect of both non alternating and periodically alternating boundary conditions along the guide walls on the structure and properties of band gaps.

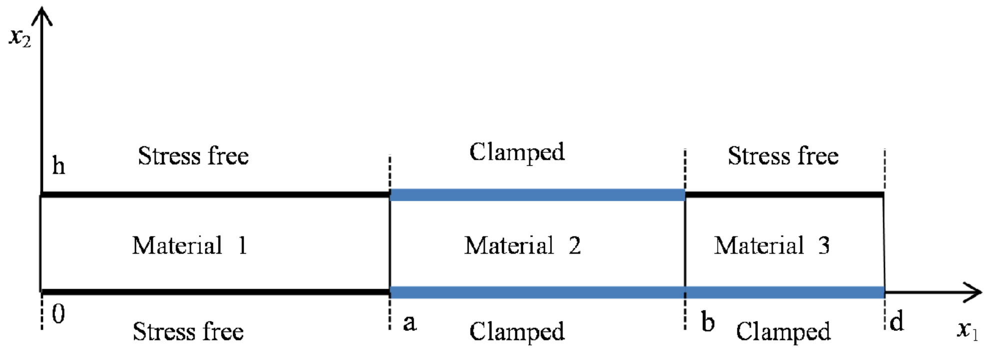

Fig.1. Periodic unit cell consisting of three different elastic materials under alternating boundary conditions along the guide walls

The equation of motion of elastic SH wave propagation in a waveguide is the following

$$\frac{\partial \sigma_{13}^{(s)}}{\partial x_1}+\frac{\partial \sigma_{23}^{(s)}}{\partial x_2}+\rho_s \frac{\partial^2 u^{(s)}}{\partial t^2}=0, \quad s=1,2,3, \tag{1}$$

$$\sigma_{13}^{(s)}=G_s \frac{\partial u^{(s)}\left(x_1, x_2, t\right)}{\partial x_1}, \quad \sigma_{23}^{(s)}=G_s \frac{\partial u^{(s)}\left(x_1, x_2, t\right)}{\partial x_2}, \tag{2}$$

where ρ_s and G_s are the mass density and the rigidity respectively, superscripts s indicates that the functions belong to media s. Harmonic time dependence, $\exp(i\omega t)$ for the all physical variables with ω as wave angular frequency is assumed henceforth.

We will solve the problem of shear wave propagation in the periodic waveguide with the following three types of boundary conditions along the waveguides walls

I. Stress free boundary conditions in sub-domain $(s)=1, \quad x_1 \in (0,a)$

$$\sigma_{23}^{(1)}\left(x_1, h, t\right)=0, \quad \sigma_{23}^{(1)}\left(x_1, 0, t\right)=0,$$

II. Clamped boundary conditions in sub-domain $(s)=2, \quad x_1 \in (a,b)$.

$$u^{(2)}\left(x_1, 0, t\right)=u^{(2)}\left(x_1, h, t\right)=0,$$

III. Stress free on the upper wall and clamped on the lower wall $(s)=3$ in sub-domain $x\in(b,d)$ $\sigma_{yz}^{(3)}\left(x_1,h,t\right)=0,\ u^{(3)}\left(x,0,t\right)=0$.

Introducing dimensionless coordinates $x_1=xh,\ x_2=yh$ and parameters $\omega=\tilde{\omega}h/c_1$ $\alpha_s=c_s/c_1$ $(c_s=\sqrt{\rho_s/G_s})$ the solutions of equation (1) can be written as follows

$$u^s\left(x,y\right)=\sum_n\left[C_n^{(s)}\exp(iq_n^{(s)}x)+D_n^{(s)}\exp(-iq_n^{(s)}x)\right]\psi_n^{(s)}\left(y\right), \tag{3}$$

where

$q_n^{(s)}\neq 0$, $C_n^{(s)},D_n^{(s)}$ are constants and $\psi_n^{(1)}\left(y\right)=\cos\left(p_n^{(1)}y\right)$,

$\psi_n^{(2)}\left(y\right)=\sqrt{2}\sin\left(p_n^{(2)}y\right),\ \psi_n^{(3)}\left(y\right)=\sqrt{2}\sin\left(p_n^{(3)}y\right)$

$$p_n^{(1)}=\pi\left(n-1\right),\quad p_n^{(2)}=\pi n,\quad p_n^{(3)}=\pi(2n-1)/2, \tag{4}$$

$$q_n^{(s)}=\sqrt{\frac{\tilde{\omega}^2}{\alpha_s^2}-\left(p_n^{(s)}\right)^2}. \tag{5}$$

To use the matrix based approach [8] the solution of (1)-(2) within each homogeneous material of the guide can be rewritten in the following vector form

$$\boldsymbol{U}^{(s)}\left(x,y\right)=\begin{pmatrix}u^{(s)}(x,y)\\ \dfrac{G_s}{G_1}\dfrac{\partial u^{(s)}(x,y)}{\partial x}\end{pmatrix}=\sum_n^N\begin{pmatrix}a_n^{(s)}(x)\psi_n^{(s)}\left(y\right)\\ iq_n^{(s)}\gamma^{(s)}b_n^{(s)}(x)\psi_n^{(s)}\left(y\right)\end{pmatrix}, \tag{6}$$

where

$$a_n^{(s)}(x)=C_n^{(s)}\exp(iq_n^{(s)}x)+D_n^{(s)}\exp(-iq_n^{(s)}x), \tag{7}$$

$$b_n^{(s)}(x)=C_n^{(s)}\exp(iq_n^{(s)}x)-D_n^{(s)}\exp(-iq_n^{(s)}x), \tag{8}$$

$\gamma^{(s)}=G_s/G_1,\quad y\in\left(0,h\right)$.

Since the interface and Bloch conditions can be applied to each component of the vector $\boldsymbol{U}\left(x,y\right)$ they can be rewritten as follows:

$$\boldsymbol{U}^{(1)}\left(\beta_1,y\right)=\boldsymbol{U}^{(2)}\left(\beta_1,y\right), \tag{9}$$

$$\boldsymbol{U}^{(2)}\left(\beta_2,y\right)=\boldsymbol{U}^{(3)}\left(\beta_2,y\right), \tag{10}$$

$$\boldsymbol{U}^{(1)}\left(0,y\right)=\lambda\boldsymbol{U}^{(3)}\left(\beta_3,y\right), \tag{11}$$

where $\lambda=\exp\left(ikd\right)$, k is the Bloch number, $\beta_1=a/h$, $\beta_2=b/h$, and $\beta_3=d/h$.

Multiplying both sides of condition (9) by a single mode $\psi_m^{(2)}(y)$ and (10) and (11) by $\psi_m^{(3)}(y)$, integrating over the thickness of the waveguide, and introducing the vectors

$\boldsymbol{A}^{(s)}\left(x\right)=\left(a_1^{(s)}(x),a_2^{(s)}(x)....a_N^{(s)}(x),b_1^{(s)}(x),b_2^{(s)}(x)....b_N^{(s)}(x)\right)^T$,

conditions (9)-(11) can be written in the following block matrix form

$$\hat{M}_{12}\boldsymbol{A}^{(1)}(\beta_1)=\hat{M}_{22}\boldsymbol{A}^{(2)}(\beta_1), \tag{12}$$

$$\hat{M}_{23}\boldsymbol{A}^{(2)}(\beta_2)=\hat{M}_{33}\boldsymbol{A}^{(3)}(\beta_2), \tag{13}$$

$$\lambda\hat{M}_{13}\boldsymbol{A}^{(1)}(0)=\hat{M}_{33}\boldsymbol{A}^{(3)}(\beta_3). \tag{14}$$

In (12)-(14) $\hat{M}_{sj}$ are the following block matrices

$$\hat{M}_{12}=\begin{pmatrix}\hat{L}_{12} & 0\\ 0 & \hat{Q}_1\hat{L}_{12}\end{pmatrix},\ \hat{M}_{13}=\begin{pmatrix}\hat{L}_{13} & 0\\ 0 & \hat{Q}_1\hat{L}_{13}\end{pmatrix},\hat{M}_{23}=\begin{pmatrix}\hat{L}_{23} & 0\\ 0 & \hat{Q}_2\hat{L}_{23}\end{pmatrix},$$

$$\hat{M}_{22}=\begin{pmatrix}\hat{I} & 0\\ 0 & \hat{Q}_2\end{pmatrix},\ \hat{M}_{33}=\begin{pmatrix}\hat{I} & 0\\ 0 & \hat{Q}_3\end{pmatrix},$$

$\hat{I}$ is the identity matrix, and L_{sk} and Q_s are $N\times N$ matrices with the following elements

$$L_{sk}^{nm}=\int_0^1\psi_n^{(s)}\psi_m^{(k)}dy,\ s<k,\quad Q_s^{mn}=i\gamma^{(s)}q_n^{(s)}\delta_{mn}, \tag{15}$$

where δ_{mn} is the Kronecker delta operator. We also need the transfer matrix within a homogeneous material [7]

$$\hat{T}^{(s)}(x',x)=\begin{pmatrix}\hat{C}^{(s)}(x',x) & i\hat{S}^{(s)}(x',x)\\ i\hat{S}^{(s)}(x',x) & \hat{C}^{(s)}(x',x)\end{pmatrix}, \tag{16}$$

where $\hat{C}^{(s)}(x',x)$, $\hat{S}^{(s)}(x',x)$ are matrices with entries $\cos\left(q_n^{(s)}(x'-x)\right)$, $\sin\left(q_n^{(s)}(x'-x)\right)$.

First by using the transfer matrix we compute $\boldsymbol{A}^{(3)}(\beta_3)$ in terms of $\boldsymbol{A}^{(3)}(\beta_2)$ within material 3

$$\boldsymbol{A}^{(3)}(\beta_3)=\hat{T}^{(3)}(\beta_3,\beta_2)\boldsymbol{A}^{(3)}(\beta_2). \tag{17}$$

It follows from (13) that the interface transmission condition at $x=\beta_2$ can be written in the form

$$\boldsymbol{A}^{(3)}(\beta_2)=\left(\hat{M}_{33}\right)^{-1}\hat{M}_{23}\boldsymbol{A}^{(2)}(\beta_2), \tag{18}$$

where $\left(\hat{M}_{33}\right)^{-1}$ is the inverse matrix of $\hat{M}_{33}$. Further applying this procedure again

$$\boldsymbol{A}^{(2)}(\beta_2)=\hat{T}^{(2)}(\beta_2,\beta_1)\boldsymbol{A}^{(2)}(\beta_1),$$

$$\boldsymbol{A}^{(2)}(\beta_1)=\left(\hat{M}_{22}\right)^{-1}\hat{M}_{12}\boldsymbol{A}^{(1)}(\beta_1),$$

$$\boldsymbol{A}^{(1)}(\beta_1)=\hat{T}^{(1)}(\beta_1,0)\boldsymbol{A}^{(1)}(0),$$

and substituting these into (18) we will obtain the following relationship between $\boldsymbol{A}^{(3)}(\beta_3)$ and $\boldsymbol{A}^{(1)}(0)$

$$\boldsymbol{A}^{(3)}(\beta_3)=\hat{T}^{(3)}(\beta_3,\beta_2)\left(\hat{M}_{33}\right)^{-1}\hat{M}_{23}\hat{T}^{(2)}(\beta_2,\beta_1)\left(\hat{M}_{22}\right)^{-1}\hat{M}_{12}\hat{T}^{(1)}(\beta_1,0)\boldsymbol{A}^{(1)}(0) \tag{19}$$

Finally after applying the Bloch-Floquet boundary conditions (14) we arrive at the following matrix eigenvalue problem for $\boldsymbol{A}^{(1)}(0)$

$$\left(\hat{M} - \lambda \hat{I}\right)\boldsymbol{A}^{(1)}(0) = 0, \tag{20}$$

where

$$\hat{M} = \left(\hat{M}_{13}\right)^{-1}\hat{M}_{33}\hat{T}^{(3)}(\beta_3,\beta_2)\left(\hat{M}_{33}\right)^{-1}\hat{M}_{23}\hat{T}^{(2)}(\beta_2,\beta_1)\left(\hat{M}_{22}\right)^{-1}\hat{M}_{12}\hat{T}^{(1)}(\beta_1,0) \tag{21}$$

is the transfer matrix.

For homogeneous boundary conditions on the guide walls, L_{sk} in (15) become identity matrices, the propagating modes separate from each other [7] and, after writing $\lambda = \exp(\imath k\beta)$, each gives rise to the following dispersion equation

$$\cos(k_n d) = A_n \cos\left((\beta_3 - \beta_2)q_n^{(3)}\right) - (R_1 + R_2)\sin\left((\beta_3 - \beta_2)q_n^{(3)}\right), \tag{22}$$

$$A_n = \cos\left(\beta_1 q_n^{(1)}\right)\cos\left(\beta_2 q_n^{(2)}\right) - \frac{\left(G^{(1)^2} q_n^{(1)^2} + G^{(2)^2} q_n^{(2)^2}\right)}{2 q_n^{(1)} q_n^{(2)} G^{(1)} G^{(2)}}\sin\left(\beta_1 q_n^{(1)}\right)\sin\left(\beta_2 q_n^{(2)}\right) \tag{23}$$

$$R_{1,2} = \frac{G^{(1,2)} q^{(1,2)} + G^{(3)} q^{(3)}}{G^{(1,2)} G^{(3)} q^{(1,2)}}\sin\left(\beta_{(1,2)} q^{(1,2)}\right)\cos\left(\beta_{(2,1)} q^{(2,1)}\right), \tag{24}$$

where the first values in subscripts and superscripts in (24) correspond to R_1 and the second values to R_2.

Numerical results

The structure of wave propagation depends on the ratio of the length of the unit cell to the height of the waveguide β / h, the reduced wave number $k\beta$, the filling fraction, and differences between the elastic properties of three materials. The Bloch parameter k is a phase shift across the Brillouin zone, and hence the graphs are $2\pi/d$ periodic and even. Therefore only values $0 \le k \le \pi/d$ will be shown on graphs.

It follows from (5) that for non alternating boundary conditions when the lower and upper walls are clamped there exist cut off frequencies in the acoustic region for each material below which waves do not propagate. Below the lowest of these three frequencies no propagation will be possible creating a total stop band. In the case of non alternating boundary conditions when the waveguide walls are traction free, the propagating solutions start at $\omega = 0$.

Figure 2 shows the band structure, calculated using the dispersion equation (22), of the first and second modes for two periodic waveguides of the same geometry but consisting of different materials. Both waveguides have homogeneous boundary conditions along the guide walls. Dashed lines correspond to the first mode and the bold lines correspond to the second mode for traction free boundaries (which is the same as the first mode for the clamped boundaries). The two graphs clearly demonstrate that the band gaps are larger for a waveguide with bigger differences between the impedances of the constituent materials.

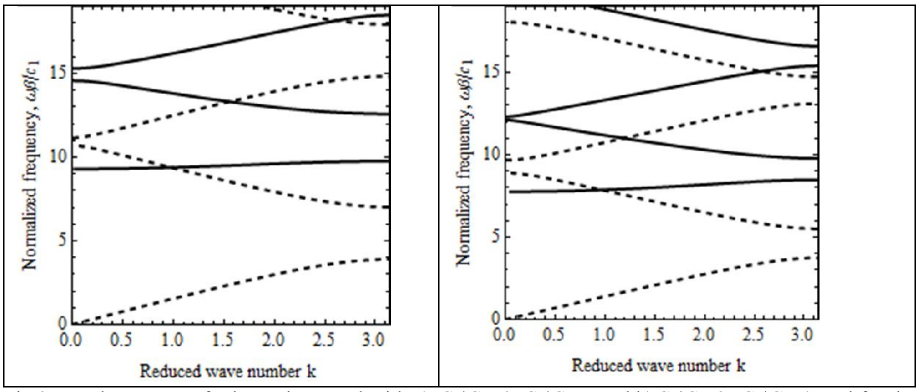

Fig.2. Band structure of phononic crystal with a) G_2/G_1=6, G_3/G_1=7 and b)G_2/G_1=3, G_3/G_1=4, and for the traction free waveguide, β/h=0.8. Dashed lines solid lines and show the band structure for the first and second modes for traction free boundary conditions on the guide wall (Solid lines describe the band structure of the first mode for clamped boundaries as well).

It follows from (5) that for non alternating boundary conditions when the lower and upper walls are clamped there exist cut off frequencies in the acoustic region for each material below which waves do not propagate. Below the lowest of these three frequencies no propagation will be possible creating a total stop band. In the case of non alternating boundary conditions when the waveguide walls are traction free, the propagating solutions start at $\omega = 0$.

For the waveguide with three different constituent materials and non alternating clamped boundary conditions along the guide walls (the horizontal lines show the cut-off frequencies in the three constituent materials) Figure 3 shows wave trapping for the lowest mode. Wave trapping occurs when the waves exponentially decay in one material and propagate in another. This happens when at least two of three materials in the waveguide have different cut-off frequencies. The nature of the trapping is not different from a waveguide described in detail in [11] and is affected by both the difference in acoustic impedances and the wavelength in the sense [6].

In Figure 3a the wave exponentially decays in the first material and propagates in the two others. In Figure 3b the wave exponentially decays in two materials and propagates only in the last one. So by choosing the material properties and the height to length ratio of the waveguide it is possible to control wave trapping in the waveguide with three constituent materials in the unit cell.

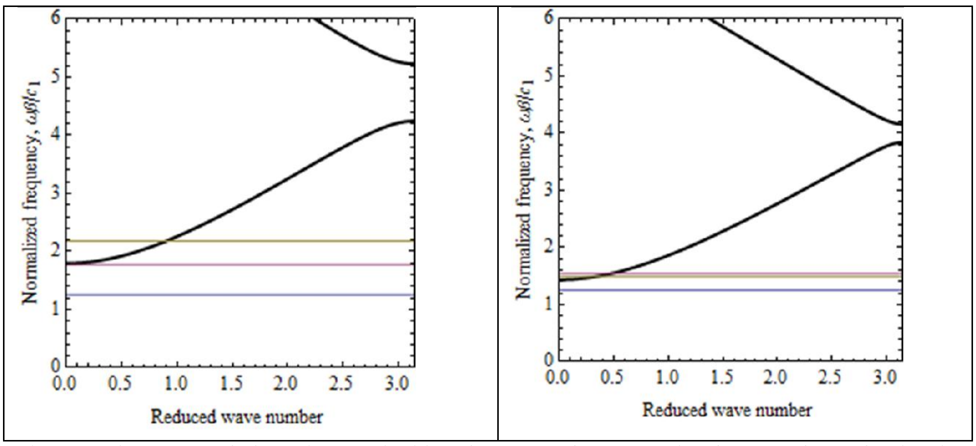

Fig.3. Band structure of phononic crystal with G_2/G_1=1.5, G_3/G_1=1.4, and G_2/G_1=2, G_3/G_1=3 for the clamped waveguide,β/h=0.4. Solid lines and dashed lines show the band structure for n=1 and n=2 modes.

For a homogeneous waveguide made from one constituent material but alternating clamped boundary conditions along the guide walls, the eigenvalue problem (20) has to be solved numerically. In this case the band structure shows that even in a homogeneous guide alternating boundary conditions can created stop bands, i.e. frequency regions where waves do not propagate [7]. Figure 4 shows the dispersion diagrams for different values of the cell length to height ratio. For $\beta / h = 0.3$ there is only a zero frequency cut off and no other band gaps since here the ratio of acoustic impedances is unity and the effect of mixed boundary conditions is not strong. As the cell length increases $(\beta / h = 1.3)$ the modes start mixing (Fig.4b), the zero frequency cut offs become larger and stop band gaps appear with a clear minima within the Brillouin zone, an unusual feature for one-dimensional periodic structures. This feature is more prominent here compared to homogeneous waveguides with only two constituent materials in the unit cell [7]. As the ratio β / h increases the lower mode bounding the band gap becomes nearly flat with no propagating energy (Fig.4b) and this is associated with modes trapped in the layers with Dirichlet boundary condition above and below.

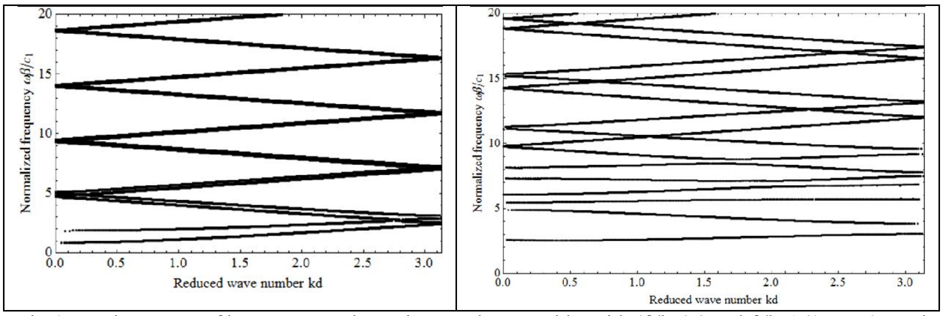

Fig.4. Band structure of homogenous phononic crystal waveguide, with (β/h=0.3 and β/h=1.3), σ_{yz}=0 on the lower and upper walls in the first material, u=0on the lower and upper walls in the second material and with u=0 on the lower and σ_{yz}=0 on the upper wall in third material

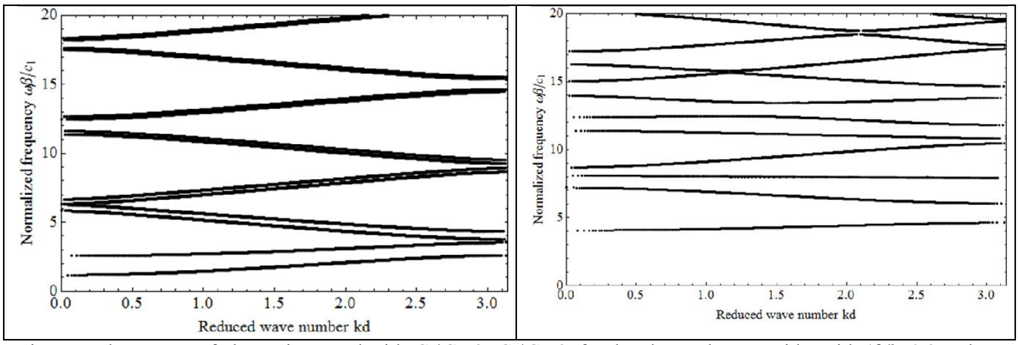

Fig5. Band structure of phononic crystal with G_2/G_1=2, G_3/G_1=3, for the clamped waveguide, with (β/h=0.3 and β/h=1.3),σ_{yz}=0 on the lower and upper walls in the first material, $u = 0$ on the lower and upper walls in the second material and with u=0 on the lower and σ_{yz}=0 on the upper wall in third material

The eigenvalue problem (20) has also been used to carry out calculations for band gap structure for a phononic crystal waveguide consisting of three different materials and having alternating boundary conditions along the guide walls (traction-free on the lower and upper walls $(\sigma_{yz} = 0)$, in the first material, displacement-clamped on the lower and upper walls (u=0), in the second material, and traction-free $\left(\sigma_{yz}=0\right)$ on the upper wall, with displacement-clamped on the lower wall in the third material). Here unlike the homogeneous waveguide with alternating boundary conditions (Fig.4a) there are frequency band gaps as well as a zero frequency cut-off for short cell lengths $(\beta/h=0.3)$. As the cell length increases $\left(\beta/h=1.3\right)$ here again the zero frequency cut offs become larger and stop band gaps appear with a clear minima within the Brillouin zone. This feature is better defined in Figure 5b where the waveguide has three constituent materials in the unit cell and alternating boundaries on the guide walls.

Conclusion

The propagation of elastic SH waves in a quasi one dimensional periodic waveguide has been considered in this paper. Using the orthogonality relationship the transfer matrix method is applied to solve the problem for homogeneous and mixed boundary conditions along the waveguide walls.

The results show that for homogeneous clamped boundary conditions along the guide walls by choosing the material properties and the height to length ratio of the waveguide it is possible to control wave trapping in the waveguide with three constituent materials in the unit cell. If the waveguide is homogeneous then only alternating boundary conditions along the guide walls can create forbidden frequency regions. Moreover stop band gaps appear with a clear minima within the Brillouin zone which is an unusual feature for one-dimensional periodic structures. This feature is more prominent here compared to homogeneous waveguides with only two constituent materials in the unit cell [11].

For alternating boundary conditions along the guide walls the spectrum depends very much on the conditions on the waveguide walls and the parameter characterizing the ratio of the unit cell length to the waveguide height. By modulating this parameter it is possible to move the extrema of the band gaps well within the Brillouin zone. These gaps are considerably larger than in the case of non homogeneous waveguide with alternating boundary conditions. Trapped modes and zero frequency band gaps also have been obtained and discussed.

For a periodic waveguide with homogeneous boundary conditions on the waveguide walls the modal solutions decouple and the analytical expression for the dispersion equation is obtained.

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