ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 65, №2, 2012 Механика

LOCALIZED BUCKLING OF THE SEMI-INFINITE ISOTROPIC PLATE NEAR ELASTICALLY FASTENED EDGE

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Keywords: Localized, Buckling, Isotropic, Semi-infinite plate
Ключевые слова: локализация, выпучивание, неустойчивость, полубесконечная пластинка

Ռեզա Շարիֆիան
Կիսանվերջ իզոտրոպ սալի լոկալիզացված անկայունությունը առաձգականորեն ամրակցված եզրի մոտակայքում

Դիտարկված է կիսանվերջ սալ-շերտի լոկալիզացված անկայունությունը, երբ երկու կիսանվերջ ազատ հենված եզրերում կիրառված է հավասարաչափ բաշխված բեռ, իսկ վերջավոր եզրը առաձգականորեն ամրակցված է: Ստացված է խնդրի բնութագրիչ հավասարումը և անալիտիկորեն դուրս են բերված լոկալիզացված անկայունության գոյության պայմանները: Կրիտիկական բեռի համար ստացված են թվային արժեքներ կախված սալի և հենարանի առաձգական գործակիցներից.

Реза Шарифиан
Локализованная потеря устойчивости полубесконечной изотропной пластинки в окрестности упруго-опертого края

Рассмотрена задача о локализованной потере устойчивости полубесконечной пластинки-полосы, нагруженной по полубеслонечным шарнирно опертым краям и упруго опертой по конечному краю. Выведено характеристическое уравнение задачи и аналитически установлены условия существования локализованной неустойчивости. Получены также численные решения для критической нагрузки в зависимости от упругих характеристик пластинки и опоры.

Localized buckling of a semi-infinite isotropic plate near elastically fastened edge has been investigated. Mathematical model is of structure is provided and characteristic equation of the problem is derived. The existence conditions of localized buckling are derived analytically. For the cases when localized buckling exists numerical solutions and plots for the critical loads are provided.

Introduction

The existence of edge waves along the free edge of a homogeneous and isotropic semi-infinite thin plate, modeled using Kirchhoff theory, was first noted by Konenkov [1] in 1960. Konenkov established that, for isotropic plates, precisely one edge wave solution exists for all values of the two free parameters, namely the bending stiffness and Poisson's ratio. The edge wave speed is found to be proportional to and slightly less than the speed of flexural (one-dimensional) waves on a plate of infinite extent.
Ambartsumyan and Belubekyan [2] in (1994) considered localized bending waves along the edge of a plate using several non-classical plate theories, concluding that Timoshenko–Mindlin plates do not admit localized edge waves. One of the latest developments in the field has been the localized bending waves in an elastic orthotropic plate; by Mkrtchyan [3] in (2003).The analogy between localized vibrations of plates and plate localized non-stability was established in [4]. Further investigations on the late localized non-stability problems were done, for example [5]-[7]. In the present paper the mathematical model and differential equations is presented. The results and conclusions are then reported.

Mathematical Modeling

A semi-infinite plate with two simply supported edges as sketched in Fig.1 is considered. The width of the plate is b and the thickness is $2h$. The Cartesian coordinate system (x, y, z) is chosen so that the plane (xoy) is coincident with the plate middle surface, while z is the coordinate along the thickness; the x axes and y are aligned the edges. The plate in Cartesian coordinates to be defined by a domain:

$0 \le x \le \infty \quad ,0 \le y \le b \quad ,-h \le z \le h$

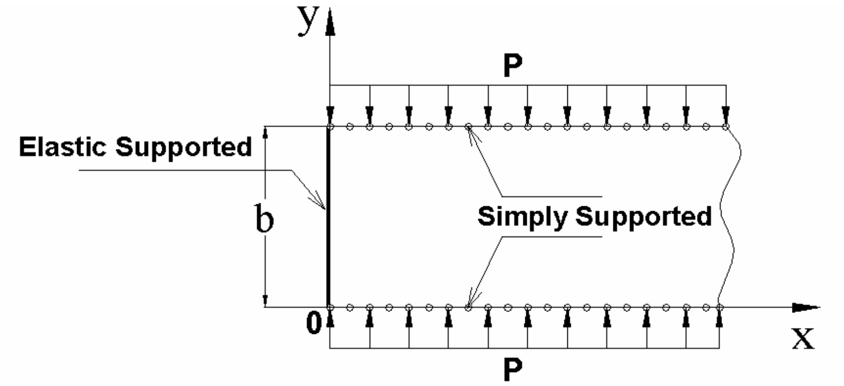

Fig.1 uniformly compressed semi-infinite plate simply supported along the edges $y=0$ and $y=b$

The plate is uniformly compressed along the edges $y = 0$ and $y = b$ with a constant load P. The stability equation of a rectangular isotropic plate compressed along the edges $y = 0$ and $y = b$ by a load P can be written as [8]:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + P\frac{\partial^2 w}{\partial y^2} = 0\,, D = \frac{2Eh^3}{3(1-\nu^2)} \tag{1}$$

where w, D, E and ν define the deflection, the flexural stiffness, the Young's modulus and the Poisson's ratio of the plate, respectively.
The boundary conditions on the simply supported edges at $y=0$, $y=b$ are:

$$w = 0 \text{ and } \frac{\partial^2 w}{\partial y^2} = 0 \text{ at } y=0,\, b \tag{2}$$

We consider the edge $x=0$ with elastic support and it can be expressed as [9]

$$M_x = 0,\ \ N_x - C_1 w = 0 \qquad at \quad x = 0 \tag{3}$$

where M_x is the bending moment and N_x is the generalized cutting force. Taking into account expressions for moments and forces, boundary conditions of elastic support take the form:

$$\begin{cases} \dfrac{\partial^2 w}{\partial x^2} + \nu \dfrac{\partial^2 w}{\partial y^2} = 0 \\ \dfrac{\partial w}{\partial x}\left[\dfrac{\partial^2 w}{\partial x^2} + (2-\nu)\dfrac{\partial^2 w}{\partial y^2}\right] - \gamma w = 0 \end{cases} \qquad x = 0 \tag{4}$$

where $\gamma = C_1 D^{-1}$ (5)

Schematically these boundary condition are represented in Fig. 2.

Fig. 2 Schematically elastic supported boundary condition at *x=0*

One additional boundary condition is needed. If the plate is semi-infinite, the localization condition prescribes attenuation as $x \to \infty$, hence an additional constraint is

$$\lim_{x \to \infty} w = 0 \tag{6}$$

If the suggested problem has a solution, then a localized buckling exists near the edge of plate *x=0*.
The solution of equation (1), satisfying to boundary conditions (2) can be represented as follows:

$$w = \sum_{n=1}^{\infty} g_n(x) \sin \lambda_n y, \quad \text{where } \lambda_n = n\pi / b \tag{7}$$

Eq.(7) and Eq.(1) yield to the following linear ordinary differential equation and the function $g_n(x)$ can be determined by solving the ordinary differential equation

$$g_n^{IV} - 2\lambda_n^2 g_n^{II} + \lambda_n^4 \left(1 - \eta_n^2\right) g_n = 0 \tag{8}$$

where $\eta_n^2 = \dfrac{P}{D\lambda_n^2}$ (9)

According to Eq.(4) the functions $g_n(x)$ should satisfy to following boundary conditions:

$$\begin{cases} g_n^{II} - \lambda_n^2 g_n = 0 \\ g_n^{III} - (2-\nu)\lambda_n^2 g_n^{I} - \gamma g_n = 0 \end{cases} \qquad x = 0 \tag{10}$$

and the attenuation condition (6) is reduced to

$$\lim_{x \to \infty} g_n(x) = 0 \tag{11}$$

The solution (8) can be represented as

$$g_n = Ae^{-\lambda_n px} \tag{12}$$

Substation of Eq.(12) into Eq.(5) yields to characteristic equation

$$p^4 - 2p^2 + 1 - \eta_n^2 = 0 \tag{13}$$

From Eq.(13) follows that solution (12), satisfying to condition (11), will be:

$$g_n = A_n e^{-p_1\lambda_n x} + B_n e^{-p_2\lambda_n x} \tag{14}$$

where $p_1 = \sqrt{1+\eta_n}$, $p_2 = \sqrt{1-\eta_n}$ (15)

and it is necessary that the following condition would be satisfied:

$$0 < \eta_n < 1 \tag{16}$$

The requirement that solution (14) must satisfy to conditions (10) yields to following system of homogeneous algebraic equations with respect to unknown constants A_n and B_n .

$$\left.\begin{aligned} &(p_1{}^2 - \nu)A_n + (p_2{}^2 - \nu)B_n = 0 \\ &[p_1(p_1{}^2 - 2 + \nu) + \gamma\lambda_n^{-3}]A_n + [p_2(p_2{}^2 - 2 + \nu) + \gamma\lambda_n^{-3}]B_n = 0 \end{aligned}\right\} \tag{17}$$

Equating the determinant of system (17) to zero yields to an equation for critical buckling load of the plate:

$$\mathrm{K}(\eta_n) \equiv (p_1^2 - \nu)[p_2(p_2{}^2 - 2 + \nu) + \gamma\lambda_n^{-3}] - (p_2^2 - \nu)[p_1(p_1{}^2 - 2 + \nu) + \gamma\lambda_n^{-3}] = 0 \tag{18}$$

When the equation (18) has roots satisfying to condition (16), then localized buckling takes place.

The equation (18), after some transforms can be reduced to

$$\mathrm{K}(\eta_n) \equiv (p_2 - p_1)\mathrm{K}_1(\eta) = 0 \tag{19}$$

where

$$\mathrm{K}_1(\eta_n) \equiv p_1^2 p_2^2 + 2(1-\nu)p_1 p_2 - \nu^2 - \gamma\lambda_n^{-3}(p_1 + p_2) \tag{20}$$

Equation (19) has a root $\eta_n = 0$, if $p_2 - p_1 = 0$. It is obvious that the root $\eta_n = 0$ corresponds to the trivial solution $w = 0$. Consequently, the critical value of load is defined by equation

$$\mathrm{K}_1(\eta_n) = 0 \tag{21}$$

In particular case $\gamma = 0$, equations (21) coincides with equation of critical load for the problem for plate with free edges [6].

When the equation (21) has a root satisfying to condition (16), then the plate buckles. The shape of buckling is such, that buckling is localized near edge $x = 0$.

For the function $\mathrm{K}_1(\eta_n) = 0$ following evaluations are valid:

$$\begin{aligned} &\mathrm{K}_1(0) = (3+\nu)(1-\nu) - 2\gamma\lambda_n^{-3}, \\ &\mathrm{K}_1(1) = -\nu^2 - \gamma\lambda_n^{-3}\sqrt{2} < 0 \end{aligned} \tag{22}$$

In the case $\gamma = 0$ (free edge) $K_1(0) > 0$ and the equation (21) has a single root satisfying to (16) [6]. When γ grows, $\gamma > 0$ and reaching the value $K_1(0) \le 0$ the equation will not possess such root.
That is, under

$$\gamma \ge 0.5(3+\nu)(1-\nu)\lambda_n^3 \tag{23}$$

Table 1. Change of the critical load parameter η_1 with change of $\gamma\lambda_1^{-3}$ for $\nu = 0.3$

$\gamma\lambda_1^{-3}$	η_1
0.00	0.9981
0.05	0.9940
0.1	0.9878
0.15	0.9795
0.20	0.9691
0.22	0.9644
0.225	0.9632
0.230	0.9620
0.231	0.9617

Also some plots of $K_1(\eta_1)$ are provided according to Equation (21) for values of $\gamma\lambda_1^{-3} =$ 0.00, 0.05, 0.10, 0.15, 0.20, 0.22, 0.225, 0.230, 0.231.

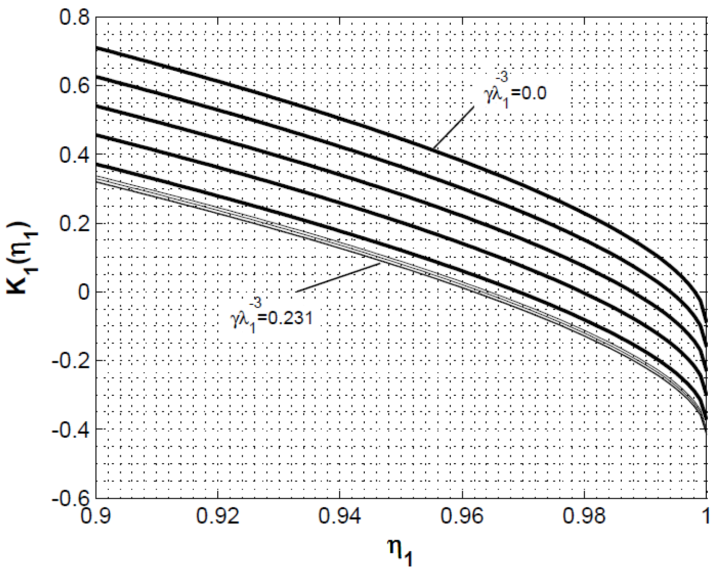

Fig. 3. Plots of $K_1(\eta_1)$ for $\gamma\lambda_1^{-3} =$ 0.00, 0.05, 0.10, 0.15, 0.20, 0.22, 0.225, 0.230, 0.231

No localized buckling exists. Note, that if under $n=1$ the inequality (23) is valid, then it is valid also for arbitrary n . Particularly if

$$0.5(3+\nu)(1-\nu)\lambda_1 \leq \gamma < 0.5(3+\nu)(1-\nu)\lambda_2 \tag{24}$$

Then buckling with shape $n=1$ is impossible, but other shapes of buckling $n \geq 2$ are possible. (Buckling is non-localized).

Taking into account expression for γ, from (23) follows the condition of absence of localized buckling:

$$\frac{C_1}{E} \geq \frac{\pi^3(3+\nu)h^3}{3(1+\nu)b^3} \tag{25}$$

Acknowledgement:
I would like to express thanks to my research supervisor, Dr. Vagharshak Belubekyan for his suggestions and help on this work.

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Поступила в редакцию 30.01.2012