ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
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THE LOCALIZED INSTABILITY OF THE ELASTIC PLATE-STRIP STREAMLINED BY SUPERSONIC GAS FLOW

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Key words: the localized divergence instability, plate-strip, critical velocity, supersonic gas flow
Ключевые слова: локализованная дивергентная неустойчивость, пластина–полоса, критическая скорость, сверхзвуковой поток газа

The paper is devoted to the analysis of the divergence localized instability of the elastic plate–strip model in a supersonic gas flow. The critical velocity of the gas flow is found, which reduce to the divergence localized instability arising in the vicinity of free edge of a plate–strip.

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Локализованная неустойчивость упругой пластины–полосы, обтекаемой сверхзвуковым потоком газа

В предлагаемой работе исследуется локализованная дивергентная неустойчивость, возникающая в окрестности свободного края полубесконечной пластины – полосы при обтекании её сверхзвуковым потоком газа.

Բելուբեկյան Մ.Վ., Մարտիրոսյան Ս.Ռ.

Առաձգական սալի տեղայնացված անկայունությունը գազի գերձայնային հոսանքում

Ներկայացված աշխատանքում ուսումնասիրված է գերձայնային գազի հոսքում շրջհոսող առաձգական սալի դիվերգենտ անկայունությունը, որը առաջանում է սալի ազատ եզրի շրջակայքում:

The consideration of the problems of the thin elastic plates stability when of the plate behavior is connected with influence of a supersonic stream of gas flowing around it, has the important applied and theoretical value. The question on elastic stability inevitably arises at a design stage and designing of any flying machine for flight safety. The theoretical researches of these problems allow to reveal various kinds of loss the stability caused by character of deformations. In one cases the static deformations, and in others – the dynamic which result, accordingly, to the static loss of stability – a divergence instability and to the dynamic loss of stability – a flatter instability [1, 2] take place. In [3] the extensive literature devoted to research of the divergence and flatter instability were brought.

In early studies of vibrations and stability of the cantilever plate which is flowed around by a supersonic stream of gas, losses of stability of both kinds have been found out: the divergence and flatter [1, 4]. It has appeared that value of critical velocity of the stream, leading to divergence instability, is essential less values of critical velocity of the stream leading to flatter instability [1, 4].

As it is known [5], the wave possessing properties of the “Rayleigh” type waves can extend along free edge of the thin elastic semi-infinite plate-strip making bending vibrations. By analogy with the localized bending vibrations the effect of the localized instability of a semi-infinite plate-strip in the vicinity of the free edge compressed on semi-infinite to supported edges [6] is investigated. The following analogy – the localized divergence instability arising in the vicinity of free edge of a semi-infinite plate – strips at a flow its supersonic stream of gas which is investigated in offered work. It is shown that at a flow the localized instability arises a supersonic stream of gas along semi-infinite the supported edges of a semi-infinite plate – strips in a vicinity of its free edge. The value of critical velocity of a flowing around stream of gas is found, at which achievement the localized instability is observed. Thus value of critical velocity is less in plates-strips from materials with the large of the Poisson's ratio.

Note that critical velocity of divergence it is essential to a cantilever plate less of a flatter critical velocity [1]. It is supposed, as in case of a flow of a semi-infinite plate – strips a supersonic stream of the gas running along semi-infinite of supported edges, critical velocity of divergence will be less of a flatter critical velocity. Therefore, the problem of stability of a plate – strip, which are flowed around by a supersonic stream of gas, in static statement is of interest.

1. The Statement of the problem. We will consider a thin elastic semi-infinite plate – strip, which is streamlined by supersonic gas flow. Let plate–strip occupies the $0 \le x \prec \infty$, $0 \le y \le b$, $-h \le z \le h$ area in the Cartesian coordinate system $Oxyz$. The Cartesian coordinate system $Oxyz$ is chosen so that the axis Ox and Oy lie in the plane of the unperturbed plate – strip. The plate–strip on a side is flowed by supersonic gas flow with undisturbed velocity V in the Ox direction.

Let the plate-strip along the edge $x = 0$ is free and the semi–infinite edges $y = 0$ and $y = b$ are supported.

Under the influence of any reasons not indignant equilibrium state of a plate-strip can be broken and the plate-strip will start to make bending vibrations with a $w = w(x, y, t)$ deflection. The deflection $w = w(x, y, t)$ will cause superfluous pressure Δp upon the top streamline surface of a plate-strip of from outside flowing around stream of gas which is considered by the approached formula of "the piston theory" [7]: $\Delta p = -a_0 \rho_0 V \dfrac{\partial w}{\partial x}$, ρ_0 – is density of undisturbed gas flow, a_0 – is the sound velocity in undisturbed gaseous medium.

The small bending vibrations of points of a median surface of a thin elastic plate–strip under the assumption of Kirchhoff's hypothesis and "the piston theory" validity satisfy to the equation [1, 4]

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + a_0 \rho_0 \left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right) + 2\rho h \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1.1)$$

And to corresponding boundary conditions

$$\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = 0, \quad \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + (2-\nu)\frac{\partial^2 w}{\partial y^2}\right) = 0, \quad x = 0; \qquad (1.2)$$

$$w = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0, \quad y = 0 \text{ and } y = b. \qquad (1.3)$$

Here ρ is the density of the plate–strip material; *D* is the stiffness of bending; ν is the Poisson's ratio.

The problem of stability of the supported plate–strip, streamlined by supersonic gas flow, is to define such a least value of the flow velocity V_{cr}, so that if $V < V_{cr}$ the perturbed motion will be stable, and if $V > V_{cr}$ – unstable.

With a view of simplification we will suppose that the distributed mass of a plate-strip and the resistance forces are negligible. Then the initial equation (1.1) rewrites in the following form:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+a_0\rho_0 V\frac{\partial w}{\partial x}=0. \qquad (1.4)$$

It is required to find the solution of the equation (1.4) satisfying to boundary conditions (1.2), (1.3) and to an attenuation condition on infinity [5,6]

$$\lim_{x\to\infty} w=0 \qquad (1.5)$$

The common solution of the equation (1.4) satisfying to boundary conditions (1.2) and (1.3) is represented in a form

$$w=\sum_{n=1}^{\infty} C_n \exp(\lambda_n px)\cdot\sin(\lambda_n y),\ \ \lambda_n=\pi nb^{-1}. \qquad (1.6)$$

Substituting expression (1.6) for w in the equation (1.4), the following characteristic equation is obtained form

$$(p^2-1)^2+\alpha_n^3 p=0,\ \alpha_n^3=a_0\rho_0 VD^{-1}\lambda_n^{-3},\ \alpha_n^3>0. \qquad (1.7)$$

It is obvious that the characteristic equation (1.7) has two negative real roots $p_1<0$, $p_2<0$, and two complex $p_{3,4}=\alpha\pm i\beta$ roots with a positive real part $\alpha>0$. It means that for a semi-infinite plate-strip with free edge $x=0$ and the supported semi-infinite edges $y=0$ and $y=b$ the solution (1.6), satisfying to a condition (1.5), exists, and it is possible to present it in a form

$$w=\sum_{n=1}^{\infty}(C_{n1}\exp(\lambda_n p_1 x)+C_{n2}\exp(\lambda_n p_2 x))\cdot\sin(\lambda_n y),\ \ \lambda_n=\pi nb^{-1}. \qquad (1.8)$$

C_{n1}, C_{n2} are arbitrary constants: ($C_{n1}^2+C_{n2}^2\neq 0$).

Such approach is similar to a method of the solution of the surface waves, of the localized bending vibrations and of the localized instability [5, 6].

Note that in the absence of a flow $\alpha_n^3=0$ ($V=0$) from a parity (1.7) obviously follows that

$$p_1=p_2=-1,\quad p_3=p_4=1,\ \ \alpha_n^3=0. \qquad (1.9)$$

2. The solution in the form of expression (1.8) should satisfy to the boundary conditions (1.2) corresponding to absence at $x=0$ of the bending moment and the generalized crosscutting force.

Substituting expression (1.8) in boundary conditions (1.2), the following homogeneous system of the algebraic equations with respect to the arbitrary constants C_{n1} and C_{n2} ($C_{n1}^2+C_{n2}^2\neq 0$) is obtained:

$$\begin{cases}(p_1^2-\nu)C_{n1}+(p_2^2-\nu)C_{n2}=0\\ p_1(p_1^2-2+\nu)C_{n1}+p_2(p_2^2-2+\nu)C_{n2}=0\end{cases} \qquad (2.1)$$

Equating zero a determinant of system of the equations (2.1), we will receive the dispersive equation

$$K(p_1,p_2)=(p_2-p_1)[(p_1p_2+1)^2-\nu(p_1+p_2)^2-(\nu-1)^2]=0. \qquad (2.2)$$

Here p_1 and p_2 are the negative real roots ($p_1<0$, $p_2<0$) of the equation (1.7).

According to a condition (1.9), the equation (2.2) at $\alpha_n^3 > 0$ identical to the equation

$$K_1(p_1, p_2) = (p_1 p_2 + 1)^2 - \nu(p_1 + p_2)^2 - (\nu - 1)^2 = 0 . \qquad (2.3)$$

It is easy to show that roots of the characteristic equation (1.7) are defined by expressions

$$p_{1,2} = -\frac{A}{4} \pm \sqrt{\frac{A^2}{16} - q_1 + \frac{\alpha_n^3}{A}} , \; p_1 < 0, \;\; p_2 < 0 ; \qquad (2.4)$$

$$p_{3,4} = \frac{A}{4} \pm \sqrt{\frac{A^2}{16} - q_1 - \frac{\alpha_n^3}{A}} , \quad p_{3,4} = \alpha \pm i\beta , \; \alpha > 0 .$$

Here

$$A = 2\sqrt{2(1+q_1)} \; , \; q_1 > 1 ; \qquad (2.5)$$

q_1 is an unique real root of the cubic equation

$$q^3 + q^2 - q - 1 - \frac{\alpha_n^6}{8} = 0 . \qquad (2.6)$$

Really, according to known algorithm of a finding of the solution, offered by Ferrari [8] the characteristic equation (1.7), being the algebraic equation of the fourth degree to equivalently following two quadratics equation:

$$p^2 + 0.5Ap + (q_1 - \alpha_n^3 A^{-1}) = 0 , \qquad (2.7)$$

$$p^2 - 0.5Ap + (q_1 + \alpha_n^3 A^{-1}) = 0 , \qquad (2.8)$$

Here A it is defined by expression (2.5), and q_1 is the real root of the cubic equation (2.6). The equation (2.6) we will copy in a kind

$$8 \cdot (1+q)^2 (q-1) = \alpha_n^6 . \; (2.9)$$

From expression (2.9) and positively of its discriminate $Q = \alpha_n^6 (\frac{1}{27} + \frac{\alpha_n^6}{256})$ follows that under a condition $(q-1) > 0$ the cubic equation (2.6) has one real root q_1 ($q_1 > 1$) and two complex roots. And under a condition $(q-1) < 0$ the equation (2.6) has no solution. It means that the characteristic equation (1.7) has two negative real roots p_1, p_2 and two complex roots $p_{3,4} = \alpha \pm i\beta$ with the positive real part, satisfying to quadratics (2.7), (2.8) accordingly.

Considering the parity (2.9) from the equation (2.7) it we have

$$p_1 + p_2 = -\sqrt{2(1+q_1)} \;\; p_1 \cdot p_2 = q_1 - \sqrt{q_1^2 - 1} , \; q_1 > 1 . \qquad (2.10)$$

Substituting expressions (2.10) in the parity (2.3) we receive the equation

$$L(q_1) = 2(q_1 + 1) \cdot (q_1 - \sqrt{q_1^2 - 1} - \nu) - (1 - \nu)^2 = 0 , \qquad (2.11)$$

whence the values q_1 corresponding to various values of the Poisson's ratio ν are found easily. Further, substituting values q_1 in a parity (2.9), for various values of the Poisson's ratio ν, according to a designation (1.7), we receive the corresponding values of critical velocity V_{cr} of a stream. At values $V \geq V_{cr}$ in a vicinity of free edge $x = 0$ of a plate-strip the phenomenon of the localized instability is observed.

In table 1 for several values of the Poisson's ratio $\nu \in [0, 0.5]$ corresponding values of critical velocity V_{cr} of a stream are bringing.

Table 1

ν	0	0.125	0.25	0.375	0.5
$V_{cr} \cdot (a_0\rho_0 b^3)(\pi^3 n^3 D)^{-1}$	80.0	10.6	5.6	3.9	2.5

From the data bringing in table 1, it is visible that value of critical velocity V_{cr} of a stream is less in plates-strips from materials with the large of the Poisson's ratio ν.

Note that in stability early studies of the cantilever plate–strip ($0 \le x \le a$, $-\infty \le y \le \infty$) in the assumption of movement in supersonic gas flow in a direction from rigid edge $x = 0$ to the free edge $x = a$, losses of stability of a divergence kind. Here the value of the critical velocity of flow, leading to divergence instability, reaches the minimum value $V_{cr.div.} \approx 6.33D(a_0\rho_0 a^3)^{-1}$ [4]. Later the approached values of critical velocity of divergence and flatter ($V_{cr.div.} \approx 6.33D(a_0\rho_0 a^3)^{-1}, V_{cr.fl.} \approx 122.7D(a_0\rho_0 a^3)^{-1}$) of the console plate-strip streamlined of the supersonic gas flow in a direction from free edge $x = 0$ to rigid fixing edge $x = a$ [1] have been received. Thus, unlike critical velocity of the localized instability, the critical velocity of divergence and flatter do not depend on the Poisson's ratio ν.

3. Let the plate-strip edge $x = 0$ is fasteners by the next ways: rigid fixing, rigid sliding contact and hinge joint. We investigate possibility of occurrence of the phenomenon of the localized instability in the vicinity of the edge $x = 0$ at these ways of its fastening.
It is easy to show that in cases in which the edge $x = 0$ rigidly fixing or has rigid sliding contact, or has hinge joint, in a vicinity of the fastened edge $x = 0$ of a streamline plate-strip the phenomenon of the localized instability is not observed.
Substituting the common solution of the equation (1.4) in the form of expression (1.8) in boundary conditions

$$w = 0\,, \frac{\partial w}{\partial x} = 0\,, x = 0\,; \frac{\partial w}{\partial x} = 0\,, \frac{\partial^3 w}{\partial x^3} = 0\,, x = 0\,; w = 0\,, \frac{\partial^2 w}{\partial x^2} = 0\,, x = 0\,; \qquad (3.1)$$

fastenings corresponding to these ways accordingly, we receive homogeneous systems of the algebraic equations concerning arbitrary constants C_{n1}, C_{n2} ($C_{n1}^2 + C_{n2}^2 \ne 0$). Equating to zero of determinants of these systems, we receive accordingly the dispersive equations described by following parities:

$$p_2 - p_1 = 0\,,\ p_1 p_2 (p_2^2 - p_1^2) = 0\,,\ p_2^2 - p_1^2 = 0\,. \qquad (3.2)$$

According to a condition (1.9) from this it follows that the systems of the equations corresponding to these ways of fastening, have only the trivial solution: $C_{n1} = C_{n2} = 0$. Hence, at the above-stated ways of supporting of an edge $x = 0$ of a plate-strip the phenomenon of the localized instability in its vicinity is not observed.

Thus, at a flow the supersonic stream of gas along of a semi-infinite plate-strip the phenomenon of the localized instability in the vicinity of the free edge $x = 0$ is observed. And in cases in which the edge $x = 0$ is supported in the various ways, this phenomenon is not observed.
It is easy to show that in the case when velocity directed from rigidly fixed edge to the free edge the phenomenon of divergence instability is not observed.

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