ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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FREE VIBRATIONS OF THE INFINITE PLATE STRENGTHENED BY THE STIFFENING RIBS

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Key words: vibrations, frequency, stiffening rib, plate, optimal design.
Ключевые слова: колебания, частота, ребро жёсткости, пластинка, оптимальный проект.

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Свободные колебания бесконечной пластинки, усиленной рёбрами жёсткости
Исследуются свободные колебания бесконечной прямоугольной пластинки, усиленной периодически расположенными поперечными одинаковыми рёбрами жёсткости, изготовленными из другого более прочного материала. Определяются оптимальные геометрические параметры конструкции, обеспечивающие при её постоянном весе наибольшее значение низшей частоты свободных колебаний. Для сравнения исследуются также случаи, когда рёбра заменяются равными с ними по весу дополнительными нижним слоем, или одинаковыми наружными слоями пластинки. Показывается, что наибольшего значения низшей частоты свободных колебаний можно достичь в случае усиления пластинки рёбрами жёсткости.

Մուստաֆա Բարակատ
Կոշտության կողերով ուժեղացված անվերջ սալի ազատ տատանումները
Հետազոտվում են անվերջ երկար ուղղանկյուն սալի ազատ տատանումները, երբ սալն ուժեղացված է ավելի ամուր նյութից պատրաստված պարբերաբար դասավորված միատեսակ կոշտության կողերով: Որոշվում են կառուցվածքի օպտիմալ երկրաչափական պարամետրերը, որոնք հաստատուն կշռի դեպքում ապահովում են նրա ազատ տատանումների ստորին հաճախության մեծագույն արժեքը: Համեմատության համար հետզոտվում են նաև այն դեպքերը, երբ կողերը փոխարինվում են միևնույն կշռի սալի լրացուցիչ ստորին շերտով, կամ միատեսակ արտաքին շերտերով: Ցույց է տրվում, որ ստորին հաճախության մեծաույն արժեքին կարելիէ հասնել սալը կողերով ուժեղացման դեպքում:

The natural vibrations of the infinite rectangular plate, strengthened by the periodically located transverse identical stiffening ribs, prepared of another more durable material are investigated. The optimum geometric parameters of construction are determined, which ensure the greatest value of the lowest frequency of the natural vibrations, with its fixed weight. For the comparison also the cases are investigated, when ribs are substituted by equal with them by the weight lower layer, or identical outer layers of plate. It shows that the greatest value of the lowest frequency of the natural vibrations is possible to reach in the case of strengthening the plate by stiffening ribs.

The natural vibrations of the infinitely long plate with a width b, by the thickness h_1, hinge-supported on two opposite sides $y=0$ and $y=b$, strengthened by transverse periodically located at a distance a from each other identical stiffening ribs with the dimensions of the cross section $\alpha h_r \times h_r$, are examined (Fig.1).

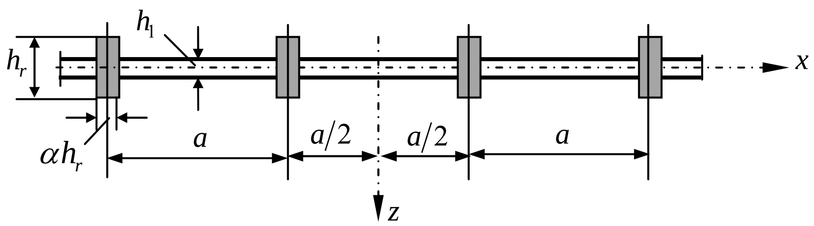

Fig.1

The problem of determining the optimum dimensions of the cross section of ribs and distance between them, which with the retention of the total weight of ribs ensure the greatest value of lowest frequency of the natural vibrations, is posed.

The optimum design on vibrations of the rectangular plates, strengthened by stiffening ribs, is investigated in the works [2] - [4].

In view of the symmetry of construction, the problem of the natural vibrations of the plate of the thickness h_1 in the section $0 \le x \le a/2$, elastic supported on by right the end $x = a/2$ to the stiffening rib is examined.

The equation of the natural vibrations of plate takes the form:

$$D_1 \Delta^4 w + \rho_1 h_1 \frac{\partial^2 w}{\partial t^2} = 0, \qquad (1)$$

where $D_1 = E_1 h_1^3 / 12(1 - \nu_1^2)$ – the flexural stiffness of plate, ρ_1, E_1, ν_1 – density, the modulus of elasticity and Poisson ratio of the material of plate.

Solution of equation (1), satisfactory to the conditions for hinged support on the long sides of the plate

$$w = 0, \ \frac{\partial^2 w}{\partial y^2} = 0 \ \text{with} \ y = 0, \ y = b, \qquad (2)$$

is assumed in the form:

$$w = (C_{1m} \operatorname{ch} \mu_1 \lambda_m x + C_{2m} \operatorname{sh} \mu_1 \lambda_m x + C_{3m} \cos \mu_2 \lambda_m x + C_{4m} \sin \mu_2 \lambda_m x) \sin \omega_m t \sin \lambda_m y, \qquad (3)$$

where:

$$\lambda_m = \frac{m\pi}{b}, \ \mu_1 = \sqrt{k_m + 1}, \ \mu_2 = \sqrt{k_m - 1}, \ k_m^2 = \omega_m^2 \frac{\rho_1 h_1}{D_1 \lambda_m^4}, \qquad (4)$$

C_{im} $(i = 1, 2, 3, 4)$ – the coefficients, determined from the boundary conditions on the lines $x = 0$, $x = a/2$, ω_m – the natural vibration frequency of plate.

The boundary conditions of the plate in question will depend on the mode of the natural vibrations of construction. The symmetrical and anti-symmetrical modes of the natural vibrations are examined.

a) Symmetrical mode of vibrations.

The cases the local (when ribs are not deformed) and general symmetrical modes of natural vibrations are possible.

–The local mode of vibrations.

In the case of the local symmetrical mode of the natural vibrations of plate boundary conditions on the lines $x = 0$ and $x = a/2$ will be:

$$\frac{\partial w}{\partial x} = 0, \ \frac{\partial^3 w}{\partial x^3} = 0 \ \text{with} \ x = 0, \qquad (5)$$

$$w = 0, \ 2D_1 \frac{\partial^2 w}{\partial x^2} = C_0 \frac{\partial^3 w}{\partial x \partial y^2} \ \text{with} \ x = a/2. \qquad (6)$$

Here C_0 – the torsion stiffness of rib, determined from the formula:

$$C_0 = \frac{E_2}{2(1 + \nu_2)} \alpha h_r^4 \beta, \ \text{where} \ \beta = \alpha^2 \left[\frac{1}{3} - \frac{64}{\pi^5} \alpha \sum_{1,3,\ldots} \frac{1}{n^5} \operatorname{th} \frac{\pi n}{2\alpha} \right],$$

where E_2 and ν_2 – the modulus of elasticity and Poisson ratio of the material of rib.

The satisfaction of conditions (5) and (6) leads to the uniform system of equations relative to coefficients C_{im}, from the condition for existence of nontrivial solution of which the transcendental equation relative to the coefficient k_m is obtained:

$$H_1(k_m)=\mu_1\left(\mu_1 \mathrm{ch}(\mu_1\lambda_m a/2)+\frac{C_0}{2D_1}\lambda_{msh}(\mu_1\lambda_m a/2)\right)\cos(\mu_2\lambda_m a/2)+$$
$$+\mu_2\left(\mu_2\cos(\mu_2\lambda_m a/2)+\frac{C_0}{2D_1}\lambda_m\sin(\mu_2\lambda_m a/2)\right)\mathrm{ch}(\mu_1\lambda_m a/2)=0. \quad (7)$$

– General symmetrical mode of natural vibrations.

Boundary conditions of the plate on the line $x=0$ will be written down in the form (5), and on the line $x=a/2$ – in the form:

$$\frac{\partial w}{\partial x}=0,\ B_r\frac{\partial^4 w}{\partial y^4}+\rho_2 A_r\frac{\partial^2 w}{\partial y^2}=2D_1\frac{\partial^3 w}{\partial x^3}\ \text{with}\ x=a/2, \quad (8)$$

where: $B_r=E_2\,\alpha h_r^4/12$ – the flexural stiffness of the rib, $A_r=\alpha h_r^2$ – the cross-sectional area of the rib, ρ_2 – material density of rib.

The satisfaction of conditions (5) and (8) leads to the uniform system of equations relative to coefficients C_{im}, from where transcendental equation relative to the coefficient k_m is obtained:

$$H_2(k_m)=\mu_2\sin(\mu_2\lambda_m a/2)\left[(B_r\lambda_m^2-\rho_2 A_r)\mathrm{ch}(\mu_1\lambda_m a/2)-2D_1\lambda_m\mu_1^3\mathrm{sh}(\mu_1\lambda_m a/2)\right]+$$
$$+\mu_1\mathrm{sh}(\mu_1\lambda_m a/2)\left[(B_r\lambda_m^2-\rho_2 A_r)\cos(\mu_2\lambda_m a/2)-2D_1\lambda_m\mu_2^3\sin(\mu_2\lambda_m a/2)\right]=0. \quad (9)$$

b) The anti-symmetrical mode of vibrations.

In this case the boundary conditions of plate will be written down in the form:

$$w=0,\ \frac{\partial^2 w}{\partial x^2}=0\ \text{with}\ x=0, \quad (10)$$

$$\frac{\partial w}{\partial x}=0,\ B_r\frac{\partial^4 w}{\partial y^4}+\rho_2 A_r\frac{\partial^2 w}{\partial y^2}=2D_1\frac{\partial^3 w}{\partial x^3}\ \text{with}\ x=a/2. \quad (11)$$

The satisfaction of conditions (10) and (11) leads to the uniform system of equations relative to coefficients C_{im}, and the value of the coefficient k_m will be determined from the transcendental equation:

$$H_3(k_m)=\mu_2\cos(\mu_2\lambda_m a/2)\left[(B_r\lambda_m^2-\rho_2 A_r)\mathrm{sh}(\mu_1\lambda_m a/2)-2D_1\lambda_m\mu_1^3\mathrm{ch}(\mu_1\lambda_m a/2)\right]-$$
$$-\mu_1\mathrm{ch}(\mu_1\lambda_m a/2)\left[(B_r\lambda_m^2-\rho_2 A_r)\sin(\mu_2\lambda_m a/2)+2D_1\lambda_m\mu_2^3\cos(\mu_2\lambda_m a/2)\right]=0. \quad (12)$$

Thus of the value of the coefficient k_m for the assigned geometric and physic mechanical parameters of construction will be determined from the equations (7), (9) and (12). Moreover from the first values of the coefficients k_m obtained from these equations should be selected smallest, to which will correspond the form of the natural vibrations of construction.

The value of the natural vibration frequency of plate will be determined from the expression (4), from where

$$\omega_m^2=k_m^2\frac{D_1\lambda_m^4}{\rho_1 h_1}. \quad (13)$$

By control of the parameters h_r, α and a, it is possible to determine their optimum values, which ensure, with constant weight of stiffening ribs, the greatest value of the lowest natural vibration frequency.

The condition of the weight constancy of ribs will be determined from the condition of the equality of the weight of ribs to the weight of the continuous additional layer of plate by thickness h_2, from where

$$h_r = \sqrt{\frac{h_2 a}{\alpha}}. \tag{14}$$

The determination of the optimum geometric parameters of construction is reduced to the following problem of nonlinear programming:

To find:

$$\max_{\bar{x}} \min_{m} \omega_m, \quad \bar{x} = \{\alpha, h_r, a\}, \tag{15}$$

with the limitations:

$$H_i(k_m) = 0, \ (i = 1,2,3), \ h_r = \sqrt{\frac{h_2 a}{\alpha}} \tag{16}$$

$$h_1 \le h_r \le 0.2b, \ 0.2 < \alpha < 5, \tag{17}$$

The limitations in the form of equalities (16) correspond to equations (7), (9), (12) and to condition (14), while the limitations in the form of inequalities (17) are caused by the limits of applicability for of the stiffening ribs of the classical theory of beams.

Problem (15) - (17) is solved by the method of the deformed polyhedron in combination with the method of straight search [5], from where the greatest value of the lowest natural vibration frequency ω and the corresponding values of the parameters h_r, α and a are determined.

Obtained value of the lowest natural vibration frequency of the plate, strengthened by stiffening ribs is compared with values, obtained for the cases, when the ribs of plate are substituted equal to them by the weight by additional lower layer with thickness h_2, or two outer layers with thickness $h_2/2$, symmetrically located relative to the middle plane of plate (Fig.2).

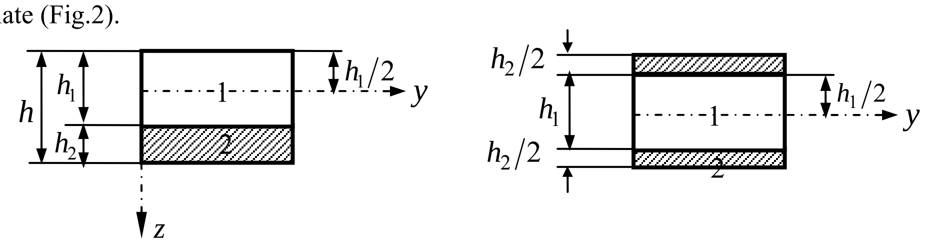

Fig.2

In the case of two-layered plate for determining the natural vibration frequencies the expression is obtained

$$\omega_m = \frac{\pi^2 m^2}{b^2} \sqrt{\frac{D^*}{\rho_1 h_1 + \rho_2 h_2}}, \tag{18}$$

where:

$$D^* = D - \frac{K^2}{C}, \tag{19}$$

D, C and K – correspondingly the flexion stiffness, tension stiffness and mutual influence of flexion and tension stiffness of two-layered plate.

Assuming, that the coordinate plane $x0y$ coincides with the middle plane of upper layer, the expressions for the stiffness of the plate in question, accordingly [1], are obtained:

$$C = \frac{E_1}{1-\nu_1^2} h_1 + \frac{E_2}{1-\nu_2^2} h_2 ,\ K = \frac{1}{2}\frac{E_2}{1-\nu_2^2} h_2 \left(h_1 + h_2\right),$$

$$D = \frac{1}{12}\left[\frac{E_1}{1-\nu_1^2} h_1^3 + \frac{E_2}{1-\nu_2^2} h_2 \left(4h_2^2 + 3h_1^2 + 6h_2 h_1\right)\right],$$

In the case of three-layered plate the natural vibration frequencies will be determined according to the formula:

$$\omega_m = \frac{\pi^2 m^2}{b^2}\sqrt{\frac{D_0}{\rho_1 h_1 + \rho_2 h_2}}\ , \qquad (20)$$

where

$$D_0 = \frac{2}{3}\left[\frac{E_1}{1-\nu_1^2}\left(\frac{h_1}{2}\right)^3 + \frac{E_2}{1-\nu_2^2}\left(\left(\frac{h_1}{2} + h_2\right)^3 - \left(\frac{h_1}{2}\right)^3\right)\right].$$

Thus, from the solution of problem (15) - (17) and by the formulas (18), (20) the values of the lowest natural vibration frequency are determined for the appropriate cases of strengthening the plate. Comparative analysis of the numerical results of calculation with the given values E_1, E_2, ν_1, ν_2, ρ_1, ρ_2, h_1, h_2 will make it possible to reveal the project, with which the great value of the lowest natural vibration frequency of construction is obtained.

The plate with the parameters $E_2 = 7E_1$, $\nu_1 = \nu_2 = \nu = 0.25$, $\rho_2 = 3.2\rho_1$, $h_2 = 0.2h_1$ is examined as a numerical example.

For the different values of the reduced thickness $\overline{h}_1 = h_1/b$ the values of the reduced lowest natural vibration frequency $\overline{\omega} = \omega\sqrt{b^2\rho_1/E_1}$ are determined for the cases of strengthening by stiffening ribs $\overline{\omega}_I$, by the additional lower layer $\overline{\omega}_{II}$ and by outer layers $\overline{\omega}_{III}$. The results of calculation are given in the table. Also are there given the corresponding optimum values of the parameters of ribbed plate $\overline{h}_r = h_r/b$, α, $\overline{a} = a/b$.

Table. Values of the lowest natural vibration frequencies for the different variants of strengthening the plate

$\overline{h}_1$	$\overline{\omega}_I$	$\overline{\omega}_{II}$	$\overline{\omega}_{III}$	$\overline{h}_r$	α	$\overline{a}$
0.008	0.063	0.035	0.045	0.057	0.2	0.4
0.010	0.075	0.044	0.056	0.063	0.2	0.4
0.015	0.104	0.066	0.084	0.087	0.2	0.5
0.020	0.131	0.087	0.113	0.100	0.2	0.5

As the results of calculation show, the greatest values of the lowest natural vibration frequency are obtained during the optimum strengthening of plate by stiffening ribs. In this case an increase in the lowest natural vibration frequency in comparison with the plate, strengthened by outer layers, is substantial for comparatively thin plates. So with $\overline{h}_1 = 0.008$ it composes 40%, and with $\overline{h}_1 = 0.02$ – 13%.

It should be noted that strengthening plate by the symmetrically located outer layers in comparison with the case of two-layered plate is more effective, since it leads to an increase in the lowest natural vibration frequency to 30%.

In Fig.3 on the line $\bar{y} = y/b = 0.5$ and along the entire surface of plate in the section $-0.5 \le \bar{x} = x/a \le 0.5$ for $\bar{h}_1 = 0.01$ the diagrams of the reduced deflections $\bar{w} = w/h_1$ of the ribbed plate are shown, which correspond to its optimum project (Fig.3).

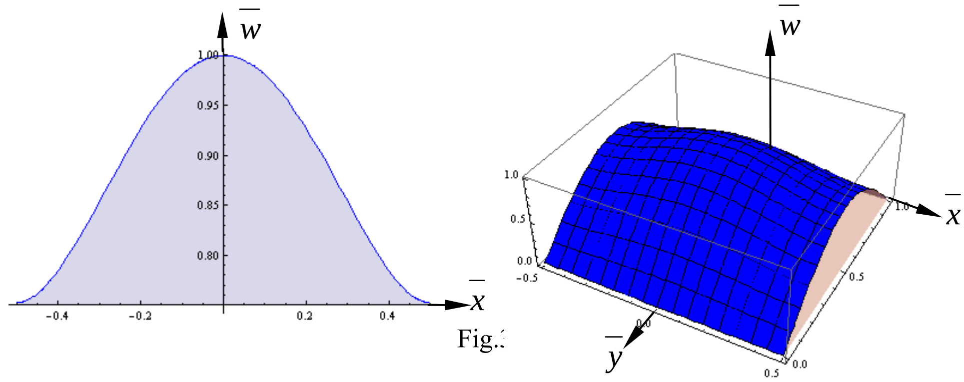

Fig.:

As can be seen from the graphs the optimum design of construction corresponds to the symmetrical general mode of vibrations.

Referencys:

1. Ambartsumyan S.A. The theory of anisotropic plates. M.: Nauka, 1987. 360p.
2. Asatryan V.M., Barakat M.S. Natural vibrations of laminar or strengthened by the oblique located stiffening ribs of the rectangular plate. Bulletin Yerevan state university of architecture and construction (Proceedings of 2-nd Int. Conf. “Architecture and construction – contemporary problems”), 2010, v. 2, № (15/2), pp. 213-218.
3. Belubekyan E.V., Poghosyan A.G. Design of ribbed plate from the composite material of the greatest lowest natural vibration frequency. //Proceedings of NAS of Armenia. Mechanics. 1994. V.47. №5-6. PP.24-29.
4. Belubekyan E.V., Poghosyan A.G. Optimization of the rectangular plate of composite material, reinforced with transverse rigid ribs by the criterion of maximum of the first characteristic oscillation frequency. NAS of Armenia. Institute of Mechanics Problems of mechanics of deformable solid body. Yerevan, 2007, pp. 86-91.
5. Pogosyan A.G. The application of a method of the deformed polyhedron to the solution of the problem of the optimum design of the rectangular compressed plate, strengthened on the edges by stiffening ribs, with the given value of the first natural vibration frequency.// Proceedings of NAS of Armenia. Mechanics. 1998. V.51. №3. PP.28-33.

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