ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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NUMERICAL ANALYZING OF BEAM BENDING UNDER DIFFERENT LOADS LOCATING WITH CRACK AND HOLE IN CENTER

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Keywords: Beam, Bending, Crack, Finite element
Ключевые слова: балка, изгиб, трещина, конечный элемент

Յազդիզադե Բ.
Տարբեր բեռների ազդեցության տակ գտնվող, կենտրոնում անցք և ճաք ունեցող հեծանի ծռման թվային վերլուծությունը

Աշխատանքում դիտարկված է հեծանը՝ տարբեր պայմաններով. ճակատային մակերևույթ դուրս եկող տարբեր երկարությամբ ճաքերով հեծան, կլոր անցքով հեծան և նեղ էլիպսների ձև ունեցող հորիզոնական և ուղղահայաց անցքերով հեծան: Հեծանը գտնվում է տարբեր բնույթի նորմալ բեռների ազդեցության տակ: Վերջավոր էլեմենտների մեթոդի օգնությամբ որոշված են մեծագույն նորմալ տեղափոխությունները: Բերված է ստացված արդյունքների համեմատությունը ճաքի, կլոր և էլիպտական անցքերի դեպքերում: Ստացված արդյունքները կարևոր են հակադարձ խնդրի լուծման տեսանկյունից:

Яздизаде Б.
Численный анализ изгиба балки под действием различных нагрузок, имеющей трещину и отверстие в центре

В работе рассмотрена балка с различными условиями: балка с трещинами различных длин, выходящих на лицевую поверхность, балка с круглым отверстием и балка с горизонтальным и вертикальным отверстиями в виде узких эллипсов. Балка находится под действием нормальных нагрузок различного характера. С помощью метода конечных элементов определены максимальные нормальные перемещения. Приводится сравнение полученных результатов для случаев трещины, круглого и эллиптических отверстий. Полученные результаты важны с точки зрения решения обратной задачи.

In this research, a beam with different conditions considered: beam with crack up to edge with the different lengths, beam with circle hole and beam with horizontal and vertical narrow elliptical hole. The beam is under different vertical load position on its surface and for every load position, the maximum vertical displacement calculated with Finite Element approximation. Results are compared with each other when the crack, hole and elliptical hole are inserted. It is important that inverse problem could obtain from these results.

Introduction

Fracture Mechanics or Linear Elastic Fracture Mechanics as we know it, was originated by Wieghardt and Inglis [1]. Both independently showed that cavities and flaws in continuum materials act as stress concentrators which, in the limit of sharp edges (cracks), produce infinite stress at the tip [2].

A fairly thorough description of the approaches for solving the crack problems is made by many researchers [3-6].

These were the first attempts to bring closer the theories of fracture mechanics (FM) and continuum mechanics (CM). About the same time, the Finite Element Method (FEM) and digital computers dashed into the engineering community as a gifted means for quantifying solutions in structural and solid mechanics. Naturally, fracture mechanic researchers implemented their FE methods, while continuum mechanic researchers implemented theirs [7].

The rapid development in computing technologies, especially with respect to increased computational power and data storage capacity, has made numerical simulation of crack closure more and more feasible, provided that finite element (FE) models can be shown to be correct and their limitations and applicability are understood [8]. Through performing a FE analysis, there is also the possibility of checking and refining some fundamental assumptions imposed in analytical methods, e.g., the assumption of infinite plates and simplification of material constitutive relations. Research on investigating problems of crack propagation using the FE method commenced in the early seventies [9-10].

The finite elements method can be easily implemented for beam elements without cracks since the stiffness and generalized geometrical stiffness matrixes of a noncracked beam are already commonly known (for example in [11]). However, the situation essentially changes if the structural elements are transversely cracked.

Defining problem

Consider a beam as shown in pic.1. This beam loaded as illustrated. The beam displacement formula is [12]:

$$EI\frac{\partial^4 y}{\partial x^4} = 0. \tag{1}$$

Where $\boldsymbol{E}$ is module of elasticity, $\boldsymbol{I}$ is moment of inertia, $\boldsymbol{y}$ is vertical displacement of a point in $\boldsymbol{x}$ position.

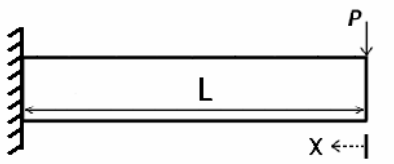
As we know for the beam of Pic.1 solution of above differential equation with such boundary conditions, $\left(dy/dx\right)_{x=L} = 0,\ y_{x=L} = 0$, is $y = \frac{p}{6EI}(2L^3 - 3L^2x + x^3)$.

The maximum displacement (at $x = 0$) is $\frac{PL^3}{3EI}$. These parameters are considered for beam:

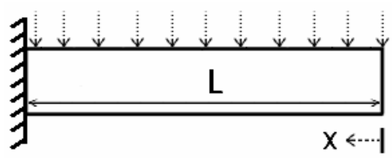
$L = 1.5\ m$, $P = 100\ N$, $b = 0.01\ m$, $h = 0.12\ m$, $E = 2 \times 10^9\ N/m^2$

Where $\boldsymbol{h}$ and $\boldsymbol{b}$ are the beam height and thickness respectively. Maximum displacement for this beam is 0.03906 m.

Maximum displacement calculated from numerical solution (finite element) is 0.03917 m. It is clear that the error of finite element solution is less than 0.3 %.



Pic.1



Pic.2

In pic.2 the load $\boldsymbol{P}$ is applied on different horizontal position $\boldsymbol{x}$ and displacement of the end of beam is calculated for each position. Mention that the displacement due to the slope of the beam must have been calculated for the rest of the beam i.e. $x \times \mathrm{Sin}(\theta_x)$, where θ_x is slope angle of beam at $\boldsymbol{x}$ position where the load is applied and $\theta_x = \left(\frac{dy}{dx}\right)_x = \frac{P(L-x)^2}{2EI}$.

Therefore, we have this formula for general maximum displacement:

$$y_{\max} = \frac{P(L-x)^3}{3EI} + x \times \mathrm{Sin}(\theta_x) = \frac{P(L-x)^3}{3EI} + x \times \mathrm{Sin}\left(\frac{P(L-x)^2}{2EI}\right) \tag{2}$$

The results of different load positions are shown in fig.1:

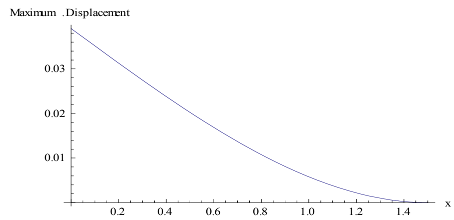
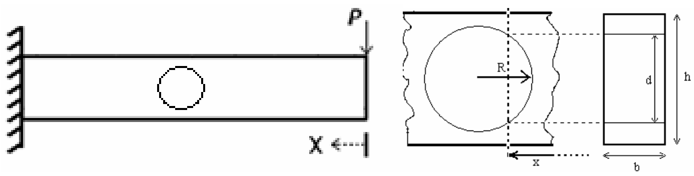


Fig.1
X = Load position distance from end of the beam

Now let solve beam displacement with one hole inside it located in the center as illustrated in pic.3.



Pic.3

Moment of inertia of the beam is not constant around the inserted hole. For calculation of beam displacement in this condition, *Castigliano's theorem* is used. For the beam, show in pic.3 this theorem is as follows [12]:

$$y = \frac{\partial U}{\partial P} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx \tag{3}$$

Where ***U*** is the strain energy and ***M*** is moment.

The moment of inertia of the cross section of the beam ($I^{'}$), at ***x*** position that show in pic.3 is:

$$I^{'} = \frac{bh^3 - b(2\sqrt{R^2-(L/2-x)^2})^3}{12}$$

Where $2\sqrt{R^2 - \left(\frac{L}{2}-x\right)^2} = d$ as shown in pic.3.

Relation (3) is expanded below:

$$y = \int_0^{L/2-R}\frac{12Px}{Ebh^3}xdx + \int_{L/2-R}^{L/2+R}\frac{12Px}{E\left(bh^3 - b(2\sqrt{R^2-(L/2-x)^2})^3\right)}xdx + \int_{L/2+R}^{L}\frac{12Px}{Ebh^3}xdx \tag{4}$$

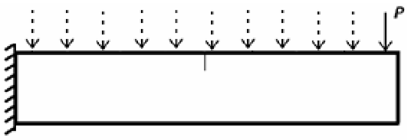
After giving value to variables and calculate the integrals, displacement is equal to 0.03942 m. Displacement calculated from *finite element software* is equal to 0.03997 m. The difference between these two solutions is 1.3%.

Because the bending caused by shear stress is neglected in analytical solution, perhaps this percentage is occurred. Anyway this different is acceptable for continuing the procedure in different case with finite element method only.

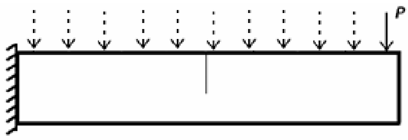
Best element chooses for finite element analysis are explain in [13, 14].

First Problem:

Consider a beam as shown in pic.4. This beam loaded with different vertical loads on its surface then calculating the Maximum vertical displacement at the end of it. This beam has a vertical crack up to edge inside it that the length of it is 33% of the beam height. These calculations do again with 83% crack length, inside the beam as shown in pic.5.



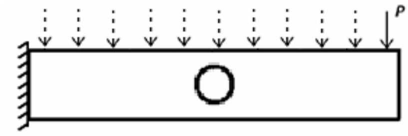
Pic.4



Pic.5

Second Problem:

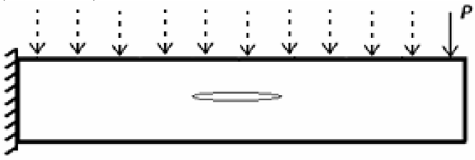
As like as previous beam all procedures do again with a circular hole instead of crack as shown in pic.6. Recall that for first load position the displacement calculated analytically from equation (4).



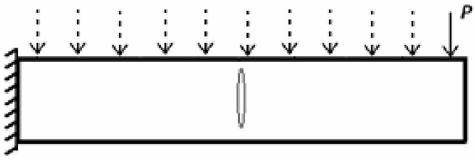
Pic.6

Third Problem:

The same as Second Problem applying horizontal and vertical elliptical hole instead of circular hole as shown in pic.7 and pic.8. The radiuses of elliptical holes are (0.04 m) and (0.001 m).



Pic.7



Pic.8

Displacement for all condition is calculated. Comparisons of results give some important rule for detecting some condition on inverse problems.

Results

<u>*First Problem:*</u>

Displacement graph for crack with length of 33% of beam height is shown in fig.2 and fig.3 in column and line view respectively. Note that crack position is at distance of 0.65 m from beam end. Each column denote the displacement of free end of the beam and data below the column is denote to the distance of the load that applied from free end of the beam.

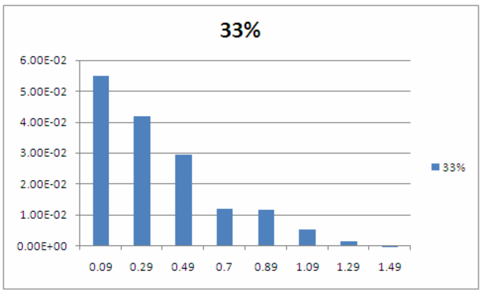


Fig.2. *Maximum displacement of beam with crack inside (33% crack length)*

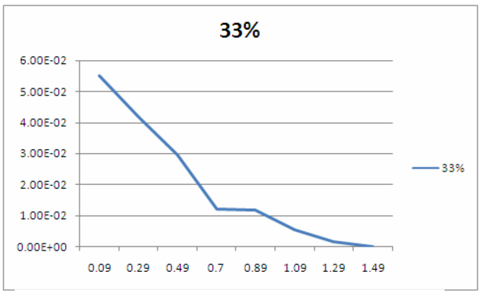


Fig.3. *Maximum displacement of beam with crack inside (33% crack length)*

Displacement graph for crack with length of 83% of beam height is shown in fig.4 and fig.5

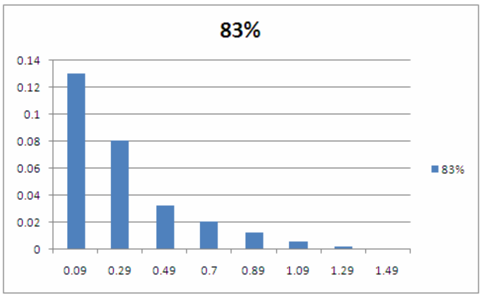


Fig.4. *Max displacement of beam with crack inside (83% crack length)*

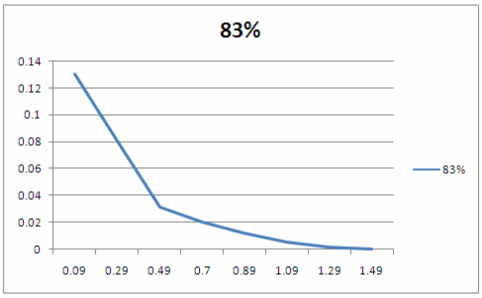


Fig.5. *Max displacement of beam with crack inside (83% crack length*

Second Problem:

Displacement graph for hole is shown in figure.6 and fig.7 in column and line view respectively. Each column denote the displacement of free end of the beam and data below the column is denote to the distance of the load that applied from free end of the beam.

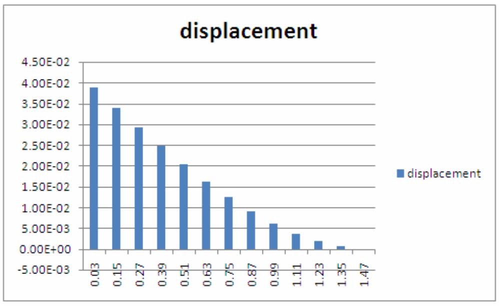


Fig.6. *Maximum displacement of beam with hole inside*

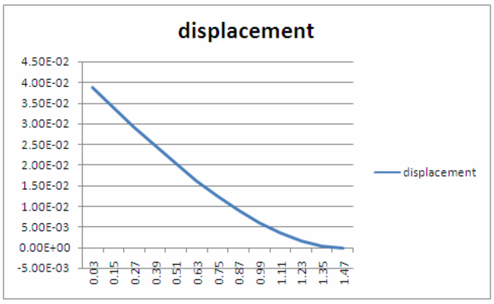


Fig.7. *Maximum displacement of beam with hole inside*

Third Problem:

Displacement graph for horizontal elliptical hole is shown in fig.8 and fig.9 in column and line view respectively and also for vertical elliptical hole is shown in fig.10 and fig.11 Each column denote the displacement of free end of the beam and data below the column is denote to the distance of the load that applied from free end of the beam.

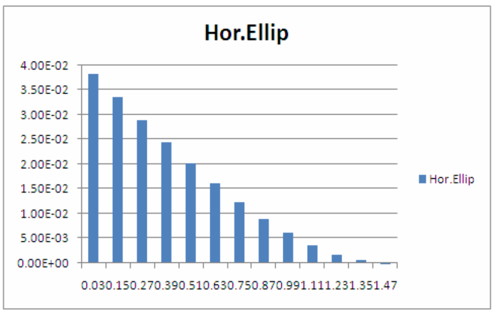


Fig.8. *Maximum displacement of beam with horizontal elliptical hole inside*

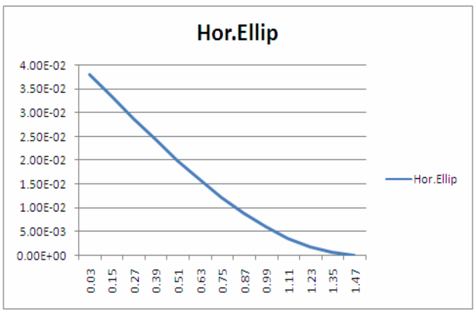


Fig.9. *Maximum displacement of beam with horizontal elliptical hole inside*

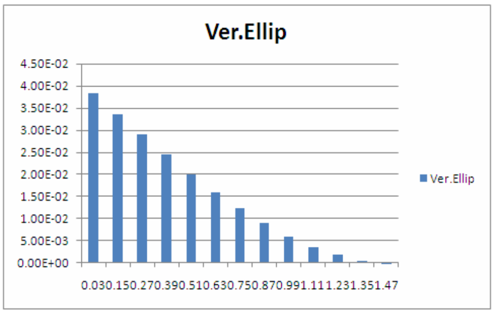


Fig.10. *Maximum displacement of beam with vertical elliptical hole inside*

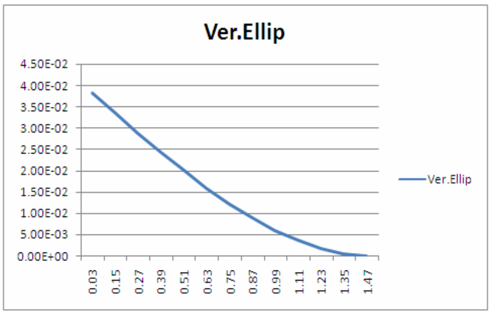


Fig. 11. *Maximum displacement of beam with vertical elliptical hole inside*

Conclusion:

In fig.3 and fig.5 there is some changing at range of displacement value after the position of crack that enables us to distinguish the crack on the beam. It means that we can divide the graph in two parts, before and after the crack. Before the crack, changing in displacement is same as the beam without crack and approximation of simple beam show in table 2 can be used. In second part that load applied after crack range of changing displacement in 2 and 3 approximation degree is shown in table 1. This is trustworthy mention that after analyzing beam displacement in laboratorial test if there is some crack up to edge, we can distinguish it by compare the graphs that obtain from test with these approximation relation in table.1, and also examine the length of the crack.

Table 1. *3rd and 2ed order approximation for second part of graphs in figure.3 and 5*

33% length	$y = -0.0003x^3 + 0.0027x^2 - 0.0217x + 0.1032$	$y = 0.0006x^2 - 0.015x + 0.0694$
83% length	$y = 0.0017x^3 - 0.0141x^2 - 0.0168x + 0.261$	$y = 0.0007x^2 - 0.0521x + 0.1819$

In fig.12 and 13, displacement range is shown for simple beam, circle hole, vertical and horizontal elliptical hole together. The figure imply that before the hole the range of beam displacement is the same for all and after the hole less displacement is belong to simple beam without any hole, then horizontal elliptical hole, vertical elliptical hole and circle hole respectively.

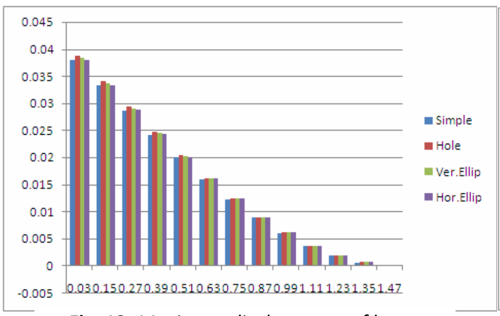


Fig. 12. *Maximum displacement of beam compared in all conditions (column view)*

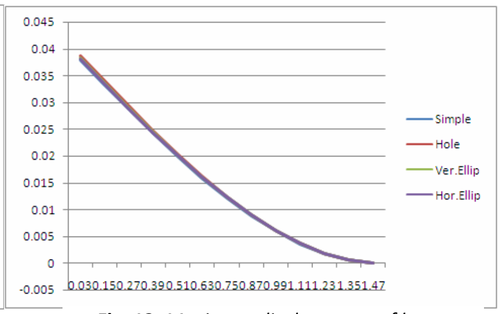


Fig. 13. *Maximum displacement of beam compared in all conditions (line view)*

Table 2 shows 4th and 3rd order approximation of graphs of fig.13. Comparison of these polynomials one can conclude that there is no any obvious distinction between simple beam and horizontal elliptical hole, which mean that with laboratorial test we cannot distinguish horizontal elliptical hole. However, in other case it can be possible if there is a precise test then with comparison of graphs that obtain from the test with these 4th and 3rd order polynomials carefully, we can detect the hole, location and the kind of it, in the beam.

Table 2. *4th and 3rd order approximation for graphs in fig.1, 7, 9 and 11*

	4th order approximation	3rd order approximation
simple beam	$y = 9E\text{-}10x^4 + 1E\text{-}05x^3 - 2E\text{-}05x^2 - 0.0047x + 0.0426$	$y = 1E\text{-}05x^3 - 2E\text{-}05x^2 - 0.0047x + 0.0426$
hole	$y = -2E\text{-}07x^4 + 2E\text{-}05x^3 - 7E\text{-}05x^2 - 0.0047x + 0.0435$	$y = 1E\text{-}05x^3 - 2E\text{-}05x^2 - 0.0048x + 0.0436$
ver.ellips	$y = -7E\text{-}08x^4 + 1E\text{-}05x^3 - 4E\text{-}05x^2 - 0.0047x + 0.0431$	$y = 1E\text{-}05x^3 - 2E\text{-}05x^2 - 0.0048x + 0.0431$
hor.ellips	$y = 1E\text{-}09x^4 + 1E\text{-}05x^3 - 2E\text{-}05x^2 - 0.0047x + 0.0427$	$y = 1E\text{-}05x^3 - 2E\text{-}05x^2 - 0.0047x + 0.0427$

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