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THE ASYMPTOTIC SOLUTIONS OF 3D DYNAMIC PROBLEMS FOR ORTHOTROPIC CYLINDRICAL AND TOROIDAL SHELLS

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Key words: asymptotic method, elasticity, shell, vibrations, space problem.
Ключевые слова: асимптотический метод, упругость, оболочка, колебания, трехмерная задача.

Աղալովյան Լ.Ա., Գևորգյան Ռ.Ս., Ղուլղազարյան Լ.Գ.
Օրթոտրոպ գլանային և տորոիդալ թաղանթների համար եռաչափ դինամիկ խնդիրների ասիմպտոտիկ լուծումներ

Ասիմպտոտիկ եղանակը արդյունավետ է բարակապատ մարմինների (հեծաններ, սալեր, թաղանթներ) համար ինչպես ստատիկ այնպես էլ դինամիկ եռաչափ խնդիրները լուծելու համար: Աշխատանքում դիտարկվում են օրթոտրոպ թաղանթների համար հարկադրական տատանումների խնդիրները, երբ դիմային մակերևույթների վրա տրված են տարբեր դասերի եզրային պայմաններ: Ստացված են ընդհանուր ասիմպտոտիկ լուծումները և որպես կիրառություն՝ լուծումներ գլանային և տորոիդալ թաղանթների համար:

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Асимптотические решения трехмерных динамических задач для ортотропных цилиндрических и тороидальных оболочек

Асимптотический метод, развитый в [1-3], эффективен при решении как статических, так и динамических трехмерных задач для тонких тел (балки, пластины, оболочки). В работе рассматриваются задачи о вынужденных колебаниях ортотропных оболочек при различных вариантах граничных условий, заданных на лицевых поверхностях оболочки. Получены общие асимптотические решения и в качестве приложений рассмотрены вынужденные колебания цилиндрических и тороидальных оболочек.

The asymptotic method of solution of singularly perturbed differential equations have been applied for solving three-dimensional dynamic problems of forced vibrations of orthotropic cylindrical and toroidal shells. The obtained generalized asymptotic solution is illustrated on solutions of particular problems.

Introduction

For the last decades for the solution of the problems of elasticity theory (static and dynamic) the asymptotic method of the solution of singularly perturbed differential equations have been successfully applied.

The asymptotic method developed in [1-3] is effective for the solution of as static as well as dynamic three-dimensional problems for thin bodies (beams, plates, shells). Here we consider the problem on forced vibrations of orthotropic shells at various variants of boundary conditions given on the facial surfaces of the shell. A general asymptotic solution is obtained. As supplements, forced vibrations of cylindrical and toroidal shells are considered.

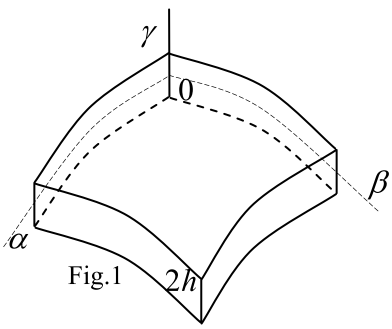

Fig.1

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1. Setting of the problems and basic 3D equations and correlations of elasticity for shells

Consider forced vibrations of orthotropic shell of thickness 2h, occupying the area $D=\{\alpha,\beta,\gamma;\alpha,\beta\in D_0,\ -h\le\gamma\le h\}$, where D_0 is the middle surface, α,β are the curvature lines of the shell middle surface, γ is the rectilinear axis, directed perpendicularly to the middle surface (Fig. 1).

It is required to find the solutions of the three dimensional dynamic problem equations of elasticity theory in D area at the series of the boundary conditions on the facial surfaces $\gamma=\pm h$ and on the lateral surface. In order to diminish and simplify the computations we shall use the components of the nonsymmetric tensor of the stresses τ_{ij}, which are connected with the components of the symmetric tensor σ_{ij} by the formulae [1]

$$\tau_{\alpha\alpha}=\left(1+\frac{\gamma}{R_2}\right)\sigma_{\alpha\alpha},\quad \tau_{\alpha\beta}=\left(1+\frac{\gamma}{R_2}\right)\sigma_{\alpha\beta}\qquad(\alpha,\beta;1,2)$$

$$\tau_{\alpha\gamma}=\left(1+\frac{\gamma}{R_2}\right)\sigma_{\alpha\gamma}\qquad(\alpha,\beta;1,2)\tag{1.1}$$

$$\tau_{\gamma\gamma}=\left(1+\frac{\gamma}{R_1}\right)\left(1+\frac{\gamma}{R_2}\right)\sigma_{\gamma\gamma}$$

The equations of elasticity theory will be written in the form of:
the equations of the movement

$$\frac{1}{AB}\frac{\partial}{\partial\alpha}\left(B\tau_{\alpha\alpha}\right)-k_\beta\tau_{\beta\beta}+\frac{1}{AB}\frac{\partial}{\partial\beta}\left(A\tau_{\beta\alpha}\right)+k_\alpha\tau_{\alpha\beta}+\left(1+\frac{\gamma}{R_1}\right)\frac{\partial\tau_{\alpha\gamma}}{\partial\gamma}+\frac{2\tau_{\alpha\gamma}}{R_1}=$$

$$=\rho\left(1+\frac{\gamma}{R_1}\right)\left(1+\frac{\gamma}{R_2}\right)\frac{\partial^2U}{\partial t^2},\qquad(A,B;\ \alpha,\beta;\ R_1,R_2;\ U,V)$$

$$\frac{\partial\tau_{\gamma\gamma}}{\partial\gamma}-\left(\frac{\tau_{\alpha\alpha}}{R_1}+\frac{\tau_{\beta\beta}}{R_2}\right)+\frac{1}{A}\frac{\partial\tau_{\alpha\gamma}}{\partial\alpha}+\frac{1}{B}\frac{\partial\tau_{\beta\gamma}}{\partial\beta}+k_\beta\tau_{\alpha\gamma}+k_\alpha\tau_{\beta\gamma}=$$

$$=\rho\left(1+\frac{\gamma}{R_1}\right)\left(1+\frac{\gamma}{R_2}\right)\frac{\partial^2W}{\partial t^2}\tag{1.2}$$

$$\left(1+\frac{\gamma}{R_1}\right)\tau_{\alpha\beta}=\left(1+\frac{\gamma}{R_2}\right)\tau_{\beta\alpha}\ \text{(the condition of symmetry)}$$

the correlations of elasticity (Hook's generalized law)

$$\left(1+\frac{\gamma}{R_2}\right)\left(\frac{1}{A}\frac{\partial U}{\partial\alpha}+k_\alpha V+\frac{W}{R_1}\right)=\left(1+\frac{\gamma}{R_1}\right)a_{11}\tau_{\alpha\alpha}+\left(1+\frac{\gamma}{R_2}\right)a_{12}\tau_{\beta\beta}+a_{13}\tau_{\gamma\gamma}$$

$$(A,B;\ \alpha,\beta;\ R_1,R_2;\ U,V;\ a_{11},a_{22};\ a_{13},a_{23})$$

$$\left[1+\gamma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{\gamma^2}{R_1R_2}\right]\frac{\partial W}{\partial\gamma}=\left(1+\frac{\gamma}{R_1}\right)a_{13}\tau_{\alpha\alpha}+\left(1+\frac{\gamma}{R_2}\right)a_{23}\tau_{\beta\beta}+a_{33}\tau_{\gamma\gamma}$$

$$\left(1+\frac{\gamma}{R_1}\right)\left(\frac{1}{B}\frac{\partial U}{\partial\beta}-k_\beta V\right)+\left(1+\frac{\gamma}{R_2}\right)\left(\frac{1}{A}\frac{\partial V}{\partial\alpha}-k_\alpha U\right)=\left(1+\frac{\gamma}{R_1}\right)a_{66}\tau_{\alpha\beta}\tag{1.3}$$

$$\left[1+\gamma\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{\gamma^2}{R_1R_2}\right]\frac{\partial U}{\partial\gamma}-\left(1+\frac{\gamma}{R_2}\right)\frac{U}{R_1}+\frac{1}{A}\left(1+\frac{\gamma}{R_2}\right)\frac{\partial W}{\partial\alpha}=\left(1+\frac{\gamma}{R_1}\right)a_{55}\tau_{\alpha\gamma}$$

$$(A,B;\quad \alpha,\beta;\quad R_1,R_2;\quad U,V;\quad a_{55},a_{44})$$

where $k_\alpha=\dfrac{1}{AB}\dfrac{\partial A}{\partial\beta}, k_\beta=\dfrac{1}{AB}\dfrac{\partial B}{\partial\alpha}$ are the geodesic curvature, A,B are the coefficients of the first quadratic form, R_1,R_2 are the main radiuses of the middle surface curvature, ρ is the density, a_{ij} are constants of elasticity.

A problem is set: to find the solutions of the system of the equations (1.2), (1.3), satisfying the following boundary conditions on the facial surfaces $\gamma=\pm h$ of the shell:

$$U(-h)=u^-(\alpha,\beta)\exp(i\Omega t),\quad V(-h)=\mathrm{v}^-(\alpha,\beta)\exp(i\Omega t)$$
$$W(-h)=w^-(\alpha,\beta)\exp(i\Omega t) \tag{1.4}$$

$$\tau_{\alpha\gamma}(h)=0,\quad \tau_{\beta\gamma}(h)=0,\quad \tau_{\gamma\gamma}(h)=0 \tag{1.5}$$

or

$$U(h)=0,\quad V(h)=0,\quad W(h)=0 \tag{1.6}$$

are the boundary conditions of the first boundary problem

$$\sigma_{j\gamma}(\alpha,\beta,\pm h,t)=\pm\sigma^{\pm}_{j\gamma}(\alpha,\beta)\exp(i\Omega t);\qquad j=\alpha,\beta,\gamma \tag{1.7}$$

where Ω is the frequency of the outer forcing action.

2. The general integral of the inner problem

In order to find the solutions of the formulated problems in the equations (1.2), (1.3) we pass to dimensionless coordinates and displacements

$$\alpha=R\xi,\quad \beta=R\eta,\quad \gamma=\varepsilon R\zeta=h\zeta,\quad U=Ru,\quad V=R\mathrm{v},\quad W=Rw$$

where R is the characteristic dimension of the shell (the smallest of the radiuses of the curvature and linear dimensions of the middle surface), $\varepsilon=h/R$ is the small parameter. The solutions of the transformed system will be sought in the form of

$$Q_{\alpha\beta}=Q_{jk}(\xi,\eta,\zeta)\exp(i\Omega t)\quad (\alpha,\beta,\gamma);\quad j,k=1,2,3 \tag{2.1}$$

$Q_{\alpha\beta}$ is any of the sought stresses and displacements.

As a result we get a singularly perturbed by small parametre ε system

$$\frac{1}{AB}\frac{\partial}{\partial\xi}(B\tau_{11})-k_\beta R\tau_{22}+\frac{1}{AB}\frac{\partial}{\partial\eta}(A\tau_{21})+k_\alpha R\tau_{12}+\left(\varepsilon^{-1}+r_1\zeta\right)\frac{\partial\tau_{13}}{\partial\zeta}+2r_1\tau_{13}=$$
$$=-\varepsilon^{-2}\Omega_*^2u-(r_1+r_2)\varepsilon^{-1}\zeta\Omega_*^2u-r_1r_2\zeta^2\Omega_*^2u$$
$$(A,B;\quad \alpha,\beta;\quad r_1,r_2;\quad \xi,\eta;\quad u,\mathrm{v};\quad \tau_{11},\tau_{22};\quad \tau_{12},\tau_{21};\quad \tau_{13},\tau_{23})$$
$$\varepsilon^{-1}\frac{\partial\tau_{33}}{\partial\zeta}-\left(r_1\tau_{11}+r_2\tau_{22}\right)+\frac{1}{A}\frac{\partial\tau_{13}}{\partial\xi}+\frac{1}{B}\frac{\partial\tau_{23}}{\partial\eta}+k_\beta R\tau_{13}+k_\alpha R\tau_{23}=$$
$$=-\varepsilon^{-2}\Omega_*^2w-(r_1+r_2)\varepsilon^{-1}\zeta\Omega_*^2w-r_1r_2\zeta^2\Omega_*^2w \tag{2.2}$$

$$\left(1+\varepsilon r_2\zeta\right)\left(\frac{1}{A}\frac{\partial u}{\partial \xi}+k_\alpha R\mathrm{v}+r_1 w\right)=\left(1+\varepsilon r_1\zeta\right)a_{11}\tau_{11}+\left(1+\varepsilon r_2\zeta\right)a_{12}\tau_{22}+a_{13}\tau_{33}$$

$$(A,B;\ \ \alpha,\beta;\ \ r_1,r_2;\ \ \xi,\eta;\ \ u,\mathrm{v};\ \ \tau_{11},\tau_{22};\ \ a_{11},a_{22};\ \ a_{13},a_{23})$$

$$\left[\varepsilon^{-1}+\zeta(r_1+r_2)+\varepsilon\zeta^2 r_1 r_2\right]\frac{\partial w}{\partial \zeta}=\left(1+\varepsilon r_1\zeta\right)a_{13}\tau_{11}+\left(1+\varepsilon r_2\zeta\right)a_{23}\tau_{22}+a_{33}\tau_{33}$$

$$\left(1+\varepsilon r_1\zeta\right)\left(\frac{1}{B}\frac{\partial u}{\partial \eta}-k_\beta R\mathrm{v}\right)+\left(1+\varepsilon r_2\zeta\right)\left(\frac{1}{A}\frac{\partial \mathrm{v}}{\partial \xi}-k_\alpha Ru\right)=\left(1+\varepsilon r_1\zeta\right)a_{66}\tau_{12}$$

$$\left[\varepsilon^{-1}+\zeta(r_1+r_2)+\varepsilon\zeta^2 r_1 r_2\right]\frac{\partial u}{\partial \zeta}-\left(1+\varepsilon r_2\zeta\right)r_1 u+\frac{1}{A}\left(1+\varepsilon r_2\zeta\right)\frac{\partial w}{\partial \xi}=\left(1+\varepsilon r_1\zeta\right)a_{55}\tau_{13}$$

$$(A,B;\ \ r_1,r_2;\ \ \xi,\eta;\ \ u,\mathrm{v};\ \ \tau_{13},\tau_{23};\ \ a_{55},a_{44})$$

$$\left(1+\varepsilon r_1\zeta\right)\tau_{12}=\left(1+\varepsilon r_2\zeta\right)\tau_{21}$$

$$r_1=\frac{R}{R_1},\ r_2=\frac{R}{R_2},\ \ \Omega_*^2=\rho h^2\Omega^2$$

The solution of such systems is combined from the solution of the inner problem (I^{int}) and the solution for the boundary layer I_b [1,4,5]

$$I=I^{\mathrm{int}}+I_b \tag{2.3}$$

The solution of inner problem I^{int} has the form

$$\tau_{jk}^{\mathrm{int}}(\xi,\eta,\zeta)=\varepsilon^{-1+s}\tau_{jk}^{(s)}(\xi,\eta,\zeta),\quad j,k=1,2,3;\quad s=\overline{0,N} \tag{2.4}$$

$$\left(u^{\mathrm{int}}(\xi,\eta,\zeta),\mathrm{v}^{\mathrm{int}}(\xi,\eta,\zeta),w^{\mathrm{int}}(\xi,\eta,\zeta)\right)=\varepsilon^{s}\left(u^{(s)}(\xi,\eta,\zeta),\mathrm{v}^{(s)}(\xi,\eta,\zeta),w^{(s)}(\xi,\eta,\zeta)\right)$$

Substituting (2.4) into (2.2) we get a recurrent system for determining the values $\tau_{jk}^{(s)},u^{(s)},\mathrm{v}^{(s)},w^{(s)}$.

From this system the stresses tensor components can be expressed through the displacements by the formulae

$$\tau_{13}^{(s)}=\frac{1}{a_{55}}\left[\frac{\partial u^{(s)}}{\partial \zeta}-P_u^{(s-1)}\right],\quad \tau_{23}^{(s)}=\frac{1}{a_{44}}\left[\frac{\partial \mathrm{v}^{(s)}}{\partial \zeta}-P_{\mathrm{v}}^{(s-1)}\right]$$

$$\tau_{11}^{(s)}=\frac{1}{\Delta}\left[\Delta_2\frac{\partial w^{(s)}}{\partial \zeta}+\Delta_{23}P_{2\tau}^{(s-1)}+\Delta_1 P_{3\tau}^{(s-1)}-\Delta_2 P_w^{(s-1)}\right] \tag{2.5}$$

$$(11,22,33;\quad \Delta_2,\Delta_3,\Delta_{12};\quad \Delta_{23},\Delta_1,\Delta_2;\quad \Delta_1,\Delta_{13},\Delta_3)$$

$$\tau_{12}^{(s)}=P_{1\tau}^{(s-1)},\qquad \tau_{21}^{(s)}=P_{1\tau}^{(s-1)}-r_2\zeta\tau_{21}^{(s-1)}+r_1\zeta\tau_{12}^{(s-1)}$$

where

$$P_{j\tau}^{(m)}=0,\quad P_{u,\mathrm{v},w}^{(m)}\equiv 0\quad \textit{when}\quad m<0$$

$$P_{1\tau}^{(s-1)}=\frac{1}{a_{66}}\left[\frac{1}{B}\frac{\partial u^{(s-1)}}{\partial \eta}-k_\beta R\mathrm{v}^{(s-1)}+r_1\zeta\left(\frac{1}{B}\frac{\partial u^{(s-2)}}{\partial \eta}-k_\beta R\mathrm{v}^{(s-2)}\right)+\right.$$

$$\left.+\frac{1}{A}\frac{\partial \mathrm{v}^{(s-1)}}{\partial \xi}-k_\alpha Ru^{(s-1)}+r_2\zeta\left(\frac{1}{A}\frac{\partial \mathrm{v}^{(s-2)}}{\partial \xi}-k_\alpha Ru^{(s-2)}\right)-r_1\zeta a_{66}\tau_{12}^{(s-1)}\right]$$

$$P_{2\tau}^{(s-1)} = \frac{1}{A}\frac{\partial u^{(s-1)}}{\partial \xi} + k_\alpha R \mathrm{v}^{(s-1)} + r_1 w^{(s-1)} + r_2\zeta\left(\frac{1}{A}\frac{\partial u^{(s-2)}}{\partial \xi} + k_\alpha R \mathrm{v}^{(s-2)} + r_1 w^{(s-2)}\right) -$$

$$-r_1\zeta a_{11}\tau_{11}^{(s-1)} - r_2\zeta a_{12}\tau_{22}^{(s-1)}$$

$$(2\tau, 3\tau;\quad A, B;\quad \alpha,\beta;\quad r_1, r_2;\quad \xi,\eta;\quad u, \mathrm{v};\quad \tau_{11}, \tau_{22};\quad a_{11}, a_{22})$$

$$P_{4\tau}^{(s-1)} = r_1\tau_{11}^{(s-1)} + r_2\tau_{22}^{(s-1)} - \frac{1}{A}\frac{\partial \tau_{13}^{(s-1)}}{\partial \xi} - \frac{1}{B}\frac{\partial \tau_{23}^{(s-1)}}{\partial \eta} - k_\beta R\tau_{13}^{(s-1)} - k_\alpha R\tau_{23}^{(s-1)} - \qquad (2.6)$$

$$-(r_1 + r_2)\zeta\Omega_*^2 w^{(s-1)} - r_1 r_2\zeta^2\Omega_*^2 w^{(s-2)}$$

$$P_{5\tau}^{(s-1)} = -\frac{1}{AB}\frac{\partial}{\partial \eta}\left(A\tau_{22}^{(s-1)}\right) + k_\alpha R\tau_{11}^{(s-1)} - \frac{1}{AB}\frac{\partial}{\partial \xi}\left(B\tau_{12}^{(s-1)}\right) - k_\beta R\tau_{21}^{(s-1)} - r_2\zeta\frac{\partial \tau_{23}^{(s-1)}}{\partial \zeta} -$$

$$-2r_2\tau_{23}^{(s-1)} - (r_1 + r_2)\zeta\Omega_*^2 \mathrm{v}^{(s-1)} - r_1 r_2\zeta^2\Omega_*^2 \mathrm{v}^{(s-2)}$$

$$(5\tau, 6\tau;\quad A, B;\quad u, \mathrm{v};\quad \alpha,\beta;\quad r_2, r_1;\quad \xi,\eta;\quad \tau_{11}, \tau_{22};\quad \tau_{12}, \tau_{21};\quad \tau_{23}, \tau_{13})$$

$$P_u^{(s-1)} = -\zeta(r_1 + r_2)\frac{\partial u^{(s-1)}}{\partial \zeta} - \zeta^2 r_1 r_2\frac{\partial u^{(s-2)}}{\partial \zeta} + r_1 u^{(s-1)} + \zeta r_1 r_2 u^{(s-2)} - \frac{1}{A}\frac{\partial w^{(s-1)}}{\partial \xi} -$$

$$-\frac{r_2\zeta}{A}\frac{\partial w^{(s-2)}}{\partial \xi} + r_1\zeta a_{55}\tau_{13}^{(s-1)} \qquad (u, \mathrm{v};\quad A, B;\ r_1, r_2;\ \xi,\eta;\quad \tau_{13}, \tau_{23};\ a_{55}, a_{44})$$

$$P_w^{(s-1)} = -\zeta(r_1 + r_2)\frac{\partial w^{(s-1)}}{\partial \zeta} - \zeta^2 r_1 r_2\frac{\partial w^{(s-2)}}{\partial \zeta} + r_1\zeta a_{13}\tau_{11}^{(s-1)} + r_2\zeta a_{23}\tau_{22}^{(s-1)}$$

$$\Delta_1 = a_{13}a_{23} - a_{33}a_{12},\ \Delta_2 = a_{12}a_{23} - a_{22}a_{13},\ \Delta_3 = a_{13}a_{12} - a_{11}a_{23}$$

$$\Delta_{ij} = a_{ii}a_{jj} - a_{ij}^2,\quad i, j = 1,2,3;\ \Delta = a_{11}\Delta_{23} + a_{13}\Delta_2 + a_{12}\Delta_1$$

The displacement vector components are determined from the equations

$$\frac{\partial^2 u^{(s)}}{\partial \zeta^2} + a_{55}\Omega_*^2 u^{(s)} = a_{55}P_{6\tau}^{(s-1)} + \frac{\partial P_u^{(s-1)}}{\partial \zeta}, \qquad (u, \mathrm{v};\quad a_{55}, a_{44};\ 6\tau, 5\tau)$$

$$\frac{\partial^2 w^{(s)}}{\partial \zeta^2} + \frac{\Delta}{\Delta_{12}}\Omega_*^2 w^{(s)} = F_w^{(s-1)} \qquad (2.7)$$

$$F_w^{(s-1)} = \frac{1}{\Delta_{12}}\left[\Delta P_{4\tau}^{(s-1)} - \Delta_2\frac{\partial P_{2\tau}^{(s-1)}}{\partial \zeta} - \Delta_3\frac{\partial P_{3\tau}^{(s-1)}}{\partial \zeta} + \Delta_{12}\frac{\partial P_w^{(s-1)}}{\partial \zeta}\right]$$

The equations (2.7) have the solutions

$$u^{(s)}(\xi,\eta,\zeta) = C_1^{(s)}(\xi,\eta)\sin\delta^u\zeta + C_2^{(s)}(\xi,\eta)\cos\delta^u\zeta + \overline{u}^{(s)}(\xi,\eta,\zeta) \qquad (2.8)$$

$$\mathrm{v}^{(s)}(\xi,\eta,\zeta) = C_3^{(s)}(\xi,\eta)\sin\delta^{\mathrm{v}}\zeta + C_4^{(s)}(\xi,\eta)\cos\delta^{\mathrm{v}}\zeta + \overline{\mathrm{v}}^{(s)}(\xi,\eta,\zeta)$$

$$w^{(s)}(\xi,\eta,\zeta) = C_5^{(s)}(\xi,\eta)\sin\delta^w\zeta + C_6^{(s)}(\xi,\eta)\cos\delta^w\zeta + \overline{w}^{(s)}(\xi,\eta,\zeta)$$

$$\delta^u = \sqrt{a_{55}}\,\Omega_*,\ \delta^{\mathrm{v}} = \sqrt{a_{44}}\,\Omega_*,\ \delta^w = \sqrt{\frac{\Delta}{\Delta_{12}}}\,\Omega_*$$

$\overline{u}^{(s)}$, $\overline{\mathrm{v}}^{(s)}, \overline{w}^{(s)}$ are the private solutions of the equations (2.7).

According to (2.3) the boundary conditions (1.4)-(1.6) have the form

$$\begin{aligned}
&\tau_{13}^{(s)}(\zeta=1)=-\overline{\tau}_{13b}^{(s)}(\zeta=1)\\
&\tau_{23}^{(s)}(\zeta=1)=-\overline{\tau}_{23b}^{(s)}(\zeta=1)\\
&\tau_{33}^{(s)}(\zeta=1)=-\overline{\tau}_{33b}^{(s)}(\zeta=1)
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
&u^{(s)}(\zeta=1)=-\overline{u}_{b}^{(s)}(\zeta=1)\\
&\mathrm{v}^{(s)}(\zeta=1)=-\overline{\mathrm{v}}_{b}^{(s)}(\zeta=1)\\
&w^{(s)}(\zeta=1)=-\overline{w}_{b}^{(s)}(\zeta=1)
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
&u^{(s)}(\zeta=-1)=u^{-(s)}(\xi,\eta)\\
&\mathrm{v}^{(s)}(\zeta=-1)=\mathrm{v}^{-(s)}(\xi,\eta)\\
&w^{(s)}(\zeta=-1)=w^{-(s)}(\xi,\eta)\\
&u^{-(0)}=u^{-}/R,\quad u^{-(s)}=-\overline{u}_{b}^{(s)}(\zeta=-1),\ \ s\neq 0\quad (u,\mathrm{v},w)\ .
\end{aligned} \tag{2.11}$$

Substituting the solution (2.8) into (2.5) and satisfying the conditions (2.9), (2.11) we determine the value $C_i^{(s)}$ and the solution of the inner problem

$$u^{\mathrm{int}(s)}=\frac{\Phi_u^{(s)}(\zeta=1)\sin\delta^u(1+\zeta)-(\overline{u}^{(s)}(\zeta=-1)-u^{-(s)}(\zeta=-1))\cos\delta^u(1-\zeta)}{\cos 2\delta^u}+\overline{u}^{(s)}$$

$$(u,\mathrm{v},w) \tag{2.12}$$

where

$$\Phi_u^{(s)}(\zeta=1)=\frac{1}{\delta^u}\left[P_u^{(s-1)}(\zeta=1)-\frac{\partial\overline{u}^{(s)}(\zeta=1)}{\partial\zeta}-a_{55}\overline{\tau}_{13b}^{(s)}(\zeta=1)\right] \tag{2.13}$$

$$(u,\mathrm{v};13,23;a_{55},a_{44})$$

$$\Phi_w^{(s)}(\zeta=1)=\frac{\Delta_{12}P_w^{(s-1)}(\zeta=1)-\Delta_2P_{2\tau}^{(s-1)}(\zeta=1)-\Delta_3P_{3\tau}^{(s-1)}(\zeta=1)-\Delta\overline{\tau}_{33b}^{(s)}(\zeta=1)}{\delta^w\Delta_{12}}-$$

$$-\frac{1}{\delta^w}\frac{\partial\overline{w}^{(s)}(\zeta=1)}{\partial\zeta}$$

The solution (2.12) will be finite, if

$$\cos 2\delta^u\neq 0\quad (u,\mathrm{v},w) \tag{2.14}$$

The values of the frequency Ω, at which $\cos 2\delta^u=0\ \ (u,\mathrm{v},w)$, coincide with the main values of the frequencies of the free vibrations [6] resonance takes place.

The conditions (1.4), (1.6) ((2.10), (2.11)) correspond to the solution

$$u^{\mathrm{int}(s)}=\frac{(u^{-(s)}(\zeta=-1)-\overline{u}^{(s)}(\zeta=-1))\sin\delta^u(1-\zeta)-(\overline{u}_b^{(s)}(\zeta=1)+\overline{u}^{(s)}(\zeta=1))\sin\delta^u(1+\zeta)}{\sin 2\delta^u}+\overline{u}^{(s)}$$

$$(u,\mathrm{v},w) \tag{2.15}$$

which will be finite, if Ω is not the frequency of the free vibrations, i.e. $\sin 2\delta^u\neq 0\ \ (u,\mathrm{v},w)$.

3. Forced vibrations of shells in the boundary layer zone

The solution of the inner problem which is determined by the formulae (2.1), (2.4), (2.5), (2.12), (2.15) in general case will not satisfy the boundary conditions on the lateral surfaces (end-walls) of the shell. For this it is necessary to have another solution as well. Such solution is the solution for the boundary layer-solution, which satisfies trivial conditions on the facial surfaces $\gamma = \pm h$ and quickly decreases when removing from the lateral (end-wall) surface into the inside the shell. In order to build this solution near the lateral surface $\alpha = \alpha_0$, we pass to the dimensionless displacement vector components

$$u = U / R, \quad \mathrm{v} = V / R, \ w = W / R \tag{3.1}$$

and new independent variables

$$\alpha - \alpha_0 = h\xi_1, \ \beta = R\eta, \ \gamma = h\zeta \tag{3.2}$$

Then expanding all the geometrical parameters entering the equations (1.2), (1.3) into Taylor series by variable ξ_1, the solution of the transformed equations (1.2), (1.3) will be sought in the form of (2.1), (2.4) having written index "b" (from the word boundary) to all the sought values. The stresses tensor components succeed to be expressed through the displacements:

$$\tau_{23b}^{(s)} = \frac{1}{a_{44}}\left[\frac{\partial \mathrm{v}_b^{(s)}}{\partial \zeta} - R_{4\tau}^{(s-1)}\right], \ \tau_{12b}^{(s)} = \frac{1}{a_{66}}\left[A_0 \frac{\partial \mathrm{v}_b^{(s)}}{\partial \xi_1} - R_{6\tau}^{(s-1)}\right] \tag{3.3}$$

$$\tau_{12b}^{(s)} - \tau_{21b}^{(s)} = R_{7\tau}^{(s-1)} \ , \ A_0 = A(\alpha = \alpha_0)$$

$$\tau_{13b}^{(s)} = \frac{1}{a_{55}}\left[A_0 \frac{\partial w_b^{(s)}}{\partial \xi_1} + \frac{\partial u_b^{(s)}}{\partial \zeta} - R_{5\tau}^{(s-1)}\right]$$

$$\tau_{11b}^{(s)} = \frac{1}{\Delta}\left[\left(A_0 \frac{\partial u_b^{(s)}}{\partial \xi_1} - R_u^{(s-1)}\right)\Delta_{23} + R_{\mathrm{v}}^{(s-1)}\Delta_1 + \left(\frac{\partial w_b^{(s)}}{\partial \zeta} - R_w^{(s-1)}\right)\Delta_2\right] \tag{3.4}$$

$$(11b, 22b, 33b; \ \ \Delta_{23}, \Delta_1, \Delta_2; \ \ \Delta_1, \Delta_{13}, \Delta_3; \ \ \Delta_2, \Delta_3, \Delta_{12})$$

The displacements vector components are determined from the equations:

$$\frac{1}{a_{66}} A_0^2 \frac{\partial^2 \mathrm{v}_b^{(s)}}{\partial \xi_1^2} + \frac{1}{a_{44}} \frac{\partial^2 \mathrm{v}_b^{(s)}}{\partial \zeta^2} + \Omega_*^2 \mathrm{v}_b^{(s)} = T_{\mathrm{v}}^{(s-1)} \tag{3.5}$$

$$\begin{gathered} \frac{\Delta_{23}}{\Delta} A_0^2 \frac{\partial^2 u_b^{(s)}}{\partial \xi_1^2} + A_0\left(\frac{\Delta_2}{\Delta} + \frac{1}{a_{55}}\right)\frac{\partial^2 w_b^{(s)}}{\partial \xi_1 \partial \zeta} + \frac{1}{a_{55}} \frac{\partial^2 u_b^{(s)}}{\partial \zeta^2} + \Omega_*^2 u_b^{(s)} = T_u^{(s-1)} \\ a_{55} A_0^2 \frac{\partial^2 w_b^{(s)}}{\partial \xi_1^2} + A_0\left(\frac{\Delta_2}{\Delta} + \frac{1}{a_{55}}\right)\frac{\partial^2 u_b^{(s)}}{\partial \xi_1 \partial \zeta} + \frac{\Delta}{\Delta_{12}} \frac{\partial^2 w_b^{(s)}}{\partial \zeta^2} + \Omega_*^2 w_b^{(s)} = T_w^{(s-1)} \end{gathered} \tag{3.6}$$

where $R_{i\tau}^{(s-1)}, T_{u,\mathrm{v},w}^{(s-1)}$ are well-known values, $Q^{(m)} \equiv 0$ at $m < 0$. The antiplane boundary layer (boundary torsion) is determined by the equations (3.5) and correction (3.3), and the plane boundary layer is determined by (3.6), (3.4).

For the applications the approximation $s = 0$ is of great importance. Then the right parts of the equations (3.5), (3.6) are equal to null. It is necessary to find the damping solutions of these equations, satisfying the conditions

$$\tau_{23b}^{(0)} = 0 \ \ at \ \ \zeta = 1; \ \ \mathrm{v}_b^{(0)} = 0 \ \ at \ \ \zeta = -1 \tag{3.7}$$

$$\tau_{13b}^{(0)} = \tau_{33b}^{(0)} = 0 \ \ at \ \ \zeta = 1 \ ; \ u_b^{(0)} = w_b^{(0)} = 0 \ \ at \ \ \zeta = -1 \tag{3.8}$$

The solution of the problem (3.5), (3.7) is

$$\mathrm{v}_b^{(0)}(\xi_1, \eta, \zeta) = \exp(-\lambda_a \xi_1) C^{(0)}(\eta) \mathrm{v}_{b0}^{(0)}(\zeta) \tag{3.9}$$

where λ is the root of the equation

$$\cos 2\sqrt{a_{44}(\Omega_*^2 + \frac{A_0^2}{a_{66}}\lambda_a^2)} = 0 \tag{3.10}$$

i.e.

$$\lambda_{an} = \pm\sqrt{\frac{a_{66}}{A_0^2}\left(\frac{\pi^2(2n+1)^2}{16a_{44}} - \Omega_*^2\right)}, \quad \mathrm{v}_{b0n}^{(0)}(\zeta) = \cos\frac{\pi}{4}(2n+1)(1-\zeta) \tag{3.11}$$

The functions $\{\mathrm{v}_{b0n}^{(0)}\}$ compose an orthogonal system on the interval $[-1;1]$.

The plane boundary layer is the solution of the problem (3.6), (3.8). It has the form

$$\begin{aligned} u_b^{(0)}(\xi_1,\eta,\zeta) &= K_b^{(0)}(\eta)\exp(-\lambda_p\xi_1 + k\zeta) \\ w_b^{(0)}(\xi_1,\eta,\zeta) &= LK_b^{(0)}(\eta)\exp(-\lambda_p\xi_1 + k\zeta) \end{aligned} \tag{3.12}$$

where k_i are the roots of the characteristic equation

$$B_2k^4 + (\lambda_p^2B_3 + B_5)k^2 + \lambda_p^4B_1 + \lambda_p^2B_4 + \Omega_*^4 = 0 \tag{3.13}$$

$$B_1 = \frac{\Delta_{23}}{\Delta a_{55}}A_0^4,\ B_2 = \frac{\Delta_{12}}{\Delta a_{55}},\ B_3 = \left(\frac{\Delta_{23}\Delta_{12} - \Delta_2^2}{\Delta^2} - 2\frac{\Delta_2}{\Delta a_{55}}\right)A_0^2$$

$$B_4 = \left(\frac{\Delta_{23}}{\Delta} + \frac{1}{a_{55}}\right)A_0^2\Omega_*^2,\ B_5 = \left(\frac{\Delta_{12}}{\Delta} + \frac{1}{a_{55}}\right)\Omega_*^2$$

Multiplier L_i corresponds to each k_i

$$L_i = \frac{1}{(\Delta + \Delta_2a_{55})\lambda_pk_i}(\Delta_{23}a_{55}\lambda_p^2A_0^2 + \Delta k_i^2 + \Delta a_{55}\Omega_*^2) \tag{3.14}$$

Using (3.4), (3.12) satisfying conditions (3.8), we obtain a system of homogeneous algebraic equations, for the existence of the nonzero solution it is necessary the determinant of the system to be equal to zero, which can be given by the equation for determining λ_p :

$$\sum_{(1,2,3,4)}(-1)^1S_1\left[Q_2(L_3 - L_4) + Q_3(L_4 - L_2) + Q_4(L_2 - L_3)\right] = 0 \tag{3.15}$$

$$S_i = \left(\Delta_{12}k_iL_i - \Delta_2\lambda_pA_0\right)\exp(2k_i)$$

$$Q_i = \left(k_i - \lambda_pA_0L_i\right)\exp(2k_i),\ \ i = 1,2,3,4$$

The roots of the equation (3.15) are complex, we are interested in the roots with $\mathrm{Re}\lambda_p > 0$. Some of the first values λ and λ_p for the shells from glassplastics 2:1 are brought in Table 1.

When removing from the lateral surface $\alpha = \alpha_0$ into the inside the shell, the values of the antiplane boundary layer damp as $\exp(-\lambda_{an}\xi_1)$, and the values of the plane boundary layer damp as $\exp(-\lambda_{pn}\xi_1)$. From Table 1 follows, that it is possible to be restricted to five-six first boundary functions, as the functions with big numbers will decrease very quickly.

From the brought formulae, by the formal exchange ξ_1 into $\xi_1 = \dfrac{\alpha_1 - \alpha_0}{h} - \xi = \dfrac{\alpha_1 - \alpha}{h}$, $\alpha \in [\alpha_0, \alpha_1]$ the data for the boundary layer near $\alpha = \alpha_1$ can be obtained.

Table 1

$\Omega_* = 1200$	λ_{an}			
	0.709406	6.3865	10.6442	14.9019
	3.54803	7.80573	12.0634	16.3211
	4.96726	9.22496	13.4826	17.7403
	λ_{pn}			
	0.262736	3.02606 +0.492501 I	6.53197	8.99958
	1.00957 +0.529569 I	4.16682	6.8507 +0.0707007 I	9.46818
	1.93879	4.63984	7.05556	10.6242 +0.488721 I
	2.45353	5.03027 +0.4013 I	8.62144 +0.392575 I	11.1858

4. Conjugation of the inner problem and boundary layer solutions

The general solution of the formulated problems has the form

$$I = I^{\text{int}} + I_b^I + I_b^{II} \tag{4.1}$$

where I^{int} is the solution of the inner problem, I_b^I is the solution of the boundary layer at $\alpha = \alpha_0$, I_b^{II} at $\alpha = \alpha_1$.

When solving singularly perturbed problems it is considered that it is possible to neglect I_b^{II} when the conditions at $\alpha = \alpha_0$ are satisfied and vice versa. It puts restrictions on the tangential dimension of the shell. It is necessary that

$$1 + \exp\left(-\frac{\alpha_1 - \alpha_0}{h}\lambda_{a1}\right) \approx 1, \qquad 1 + \exp\left(-\frac{\alpha_1 - \alpha_0}{h}\operatorname{Re}\lambda_{p1}\right) \approx 1 \tag{4.2}$$

We shall consider the conditions (4.2) satisfied.

Consider the procedure of the conjugations of the inner problem and boundary layer solutions, using the boundary conditions 3D of the problem on the lateral surface. Let at $\alpha = \alpha_0$ the conditions of rigid fastening be given

$$u(\xi = 0) = 0, \quad \mathrm{v}(\xi = 0) = 0, \quad w(\xi = 0) = 0 \tag{4.3}$$

or the conditions of free edge

$$\tau_{11}(\xi = 0) = 0, \quad \tau_{12}(\xi = 0) = 0, \quad \tau_{13}(\xi = 0) = 0 \tag{4.4}$$

The general solution may be represented in the form of

$$\begin{aligned} \mathrm{v}^{(s)} &= \mathrm{v}^{\text{int}(s)} + \exp(-\lambda_{an}\xi_1) C_{1n}^{(s)}(\eta) \mathrm{v}_{b0n}^{(0)}(\zeta) + \overline{\mathrm{v}}_{bn}^{(s)}(\xi_1, \eta, \zeta) \\ u^{(s)} &= u^{\text{int}(s)} + A_{1n}^{(s)}(\eta) \operatorname{Re} u_{bn}^{(0)} + A_{2n}^{(s)}(\eta) \operatorname{Im} u_{bn}^{(0)} + \overline{u}_{bn}^{(s)}(\xi_1, \eta, \zeta) \quad (u, w) \end{aligned} \tag{4.5}$$

$$\tau_{ij}^{(s)} = \tau_{ij}^{\text{int}(s)} + \tau_{ijb}^{(s)} \qquad i, j = 1, 2, 3; \quad n = \overline{0, N}$$

In case of the conditions (4.3) the satisfaction of the second condition brings to the correlation

$$C_{1n}^{(s)}(\eta)\mathrm{v}_{b0n}^{(0)}(\zeta)=-\mathrm{v}^{\mathrm{int}(s)}-\overline{\mathrm{v}}_{bn}^{(s)}(\xi=0,\eta,\zeta) \tag{4.6}$$

from where

$$C_{1n}^{(s)}(\eta)=\int\limits_{-1}^{1}\left(-\mathrm{v}^{\mathrm{int}(s)}-\overline{\mathrm{v}}_{bn}^{(s)}(\xi=0,\eta,\zeta)\right)\cos\frac{\pi(2n+1)(1-\zeta)}{4}d\zeta$$

The satisfaction of the rest two conditions (4.3) brings to an algebraic system

$$\begin{aligned}&A_{1n}^{(s)}(\eta)\operatorname{Re}u_{bn}^{(0)}+A_{2n}^{(s)}(\eta)\operatorname{Im}u_{bn}^{(0)}=\\&=-u^{\mathrm{int}(s)}(0,\eta,\zeta)-\overline{u}_{bn}^{(s)}(\xi_1=0)\quad(u,w)\qquad n=\overline{1,N}\end{aligned} \tag{4.7}$$

From where $A_{1n}^{(s)}(\eta)$ and $A_{2n}^{(s)}(\eta)$ are determined by collocation method or by the method of least squares.

By the analogous way the conditions (4.4) and other variants of conditions on the lateral surface are satisfied.

5. Forced vibrations of an orthotropic cylindrical shell

For an orthotropic cylindrical shell

$$r_1=0,\ r_2=1,\ A=B=1,\ k_\alpha=k_\beta=0 \tag{5.1}$$

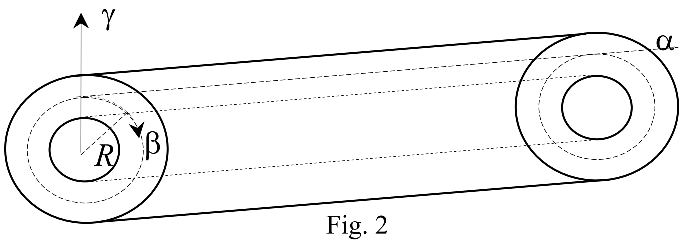

Fig. 2

Under the boundary conditions (1.4), (1.5) the solution is determined by the formulae (2.12), yet

$$\Phi_u^{(s)}(\zeta=1)=\frac{1}{\delta^u}\left[-\zeta\left(\frac{\partial u^{(s-1)}}{\partial\zeta}+\frac{\partial w^{(s-2)}}{\partial\xi}\right)-\frac{\partial w^{(s-1)}}{\partial\xi}-\frac{\partial\overline{u}^{(s)}}{\partial\zeta}-a_{55}\overline{\tau}_{13b}^{(s)}\right]_{\zeta=1} \tag{5.2}$$

$$\Phi_{\mathrm{v}}^{(s)}(\zeta=1)=\frac{1}{\delta^{\mathrm{v}}}\left[-\zeta\frac{\partial\mathrm{v}^{(s-1)}}{\partial\zeta}+\mathrm{v}^{(s-1)}-\frac{\partial w^{(s-1)}}{\partial\eta}+\zeta a_{44}\tau_{23}^{(s-1)}-\frac{\partial\overline{\mathrm{v}}^{(s)}}{\partial\zeta}-a_{44}\overline{\tau}_{23b}^{(s)}\right]_{\zeta=1}$$

$$\begin{aligned}\Phi_w^{(s)}(\zeta=1)=&\frac{1}{\delta^w\Delta_{12}}[\Delta_{12}\zeta\frac{\partial w^{(s-1)}}{\partial\zeta}-\Delta_2\frac{\partial u^{(s-1)}}{\partial\xi}-\Delta_2\zeta\frac{\partial u^{(s-2)}}{\partial\xi}-\\&-\Delta_3\frac{\partial\mathrm{v}^{(s-1)}}{\partial\eta}-\Delta_3 w^{(s-1)}-\Delta\overline{\tau}_{33b}^{(s)}]_{\zeta=1}-\frac{1}{\delta^w}\frac{\partial\overline{w}^{(s)}(\zeta=1)}{\partial\zeta}\end{aligned}$$

In case $u^-(\xi,\eta)=u^-=\mathrm{const},\ \mathrm{v}^-(\xi,\eta)=\mathrm{v}^-=\mathrm{const},\ w^-(\xi,\eta)=w^-=\mathrm{const}$, if we are restricted by the first two approaches, we get the solution

$$U^{\mathrm{int}}=\left(\frac{u^-\cos(1-\zeta)\delta^u}{\cos2\delta^u}+\frac{h}{2\delta^u\cos2\delta^u}\left(\left(\frac{u^-}{R\cos2\delta^u}-2a_{55}\overline{\tau}_{13b}^{(1)}(\zeta=1)\right)\sin\delta^u(1+\zeta)-\right.\right.$$

$$-\delta^u \cos\delta^u (1-\zeta)\left(\frac{u^-(1+\zeta)}{R}-2u^{-(1)}(\zeta=-1)\right)\Bigg)\exp(i\Omega t)$$

$$V^{\text{int}}=\left(\frac{\mathrm{v}^- \cos(1-\zeta)\delta^{\mathrm{v}}}{\cos 2\delta^{\mathrm{v}}}+\frac{h}{2\delta^{\mathrm{v}}\cos 2\delta^{\mathrm{v}}}\left(\left(\frac{3\mathrm{v}^-}{R\cos 2\delta^{\mathrm{v}}}-2a_{44}\overline{\tau}^{(1)}_{23b}(\zeta=1)\right)\sin\delta^{\mathrm{v}}(1+\zeta)-\right.\right.$$

$$\left.\left.-\delta^{\mathrm{v}}\cos\delta^{\mathrm{v}}(1-\zeta)\left(\frac{\mathrm{v}^-(1+\zeta)}{R}-2\mathrm{v}^{-(1)}(\zeta=-1)\right)\right)\right)\exp(i\Omega t) \qquad (5.3)$$

$$W^{\text{int}}=\left(\frac{w^- \cos(1-\zeta)\delta^w}{\cos 2\delta^w}+\frac{h}{2\delta^w\Delta_{12}\cos 2\delta^w}\times\right.$$

$$\times\left(\left(\frac{w^-}{R\cos 2\delta^w}\left(\Delta_{12}-2\Delta_3\right)-2\Delta\overline{\tau}^{(1)}_{33b}(\zeta=1)\right)\sin\delta^u(1+\zeta)-\right.$$

$$\left.\left.-\delta^w\Delta_{12}\cos\delta^w(1-\zeta)\left(\frac{w^-(1+\zeta)}{R}-2w^{-(1)}(\zeta=-1)\right)\right)\right)\exp(i\Omega t)$$

The stresses will be determined by the formulae (2.5).

Under the boundary conditions (1.4), (1.6) we have

$$U^{\text{int}}=\left(\frac{u^- \sin(1-\zeta)\delta^u}{\sin 2\delta^u}+\frac{h}{\sin 2\delta^u}\left(\left(u^{-(1)}(\zeta=-1)-\frac{u^-(1+\zeta)}{2R}\right)\sin\delta^u(1-\zeta)-\right.\right.$$

$$\left.\left.-\overline{u}^{(1)}_b(\zeta=1)\sin\delta^u(1+\zeta)\right)\right)\exp(i\Omega t) \qquad (U,V,W;u,\mathrm{v},w) \qquad (5.4)$$

The asymptotic solution for shells comparing with the one for plates has a number of differences: if the functions entering the boundary conditions are polynomials, the iteration process for the plates breaks on the definite approximation and mathematically exact solution for a layer is obtained. And for the shells, as it follows from the formulae (5.3), (5.4), the iteration process doesn't break, therefore the solution will be asymptotic, i.e. the exactness will be approximately of the first rejected member of the series. For the plates in the dynamic problems the boundary layer doesn't influence on the solution of the inner problem, for the shells it influences (beginning from $s \geq 1$).

From the formulae (2.12), (2.15), (5.3), (5.4) it follows that in the shells two types of shear and longitudinal vibrations arise, they are independent for the initial approximation, and taking into account the following approximations they inter influence.

6. Forced vibrations of the toroidal shell

Consider an orthotropic toroidal shell in a toroidal system of coordinates $\{\theta,\varphi,\gamma : |\theta| \leq \pi, 0 \leq \varphi \leq 2\pi, |\gamma| \leq h\}, h << r\}$ (Fig.3).

Let on the inner surface $\gamma = -h$ normal, harmonical in time loading act:

$$\overline{\sigma}_{\gamma\gamma}(\theta,\varphi,\gamma=-h,t)=\sigma(\theta,\varphi)\sin\omega t,\ \ \overline{\sigma}_{j\gamma}(\theta,\varphi,\gamma=-h,t)=0,\ \ j=\theta,\varphi \qquad (6.1)$$

and the outer surface is free:

$$\overline{\sigma}_{j\gamma}(\theta,\varphi,\gamma=h,t)=0,\quad j=\theta,\varphi,\gamma \qquad (6.2)$$

or rigidly fastened:

$$\overline{u}_j(\theta,\varphi,\gamma=h,t)=0, \quad j=\theta,\varphi,\gamma \tag{6.3}$$

Consider a close toroidal shell, by virtue of which here the boundary layer doesn't exist. It is required to find stress-strain state of the shell.

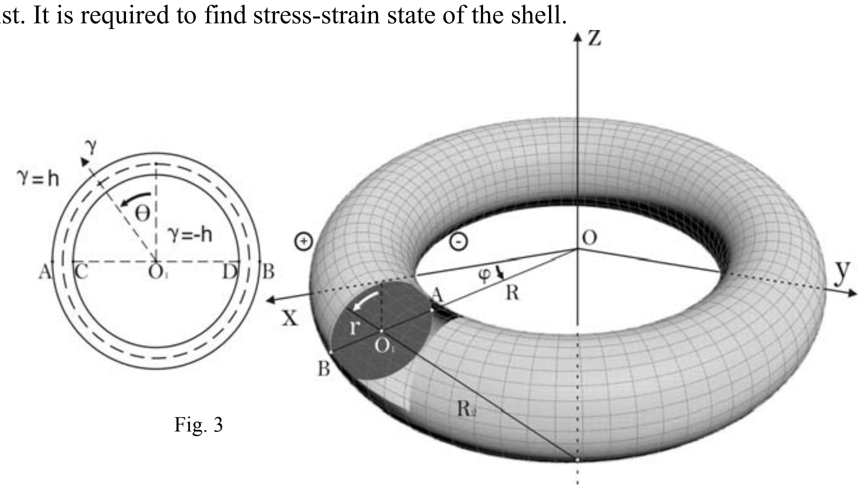

Fig. 3

For the considered shell $\alpha=\theta,\beta=\varphi$, and

$$A=r, B=R+r\sin\theta, R_1=r, R_2=(R+r\sin\theta)/\sin\theta, k_\alpha=0,$$

$$k_\beta=\cos\theta/(R+r\sin\theta) \tag{6.4}$$

For the solution of the set boundary value problem all the required values will be sought in the form of

$$\overline{Q}(\theta,\varphi,\gamma,t)=Q(\theta,\varphi,\gamma)\sin\omega t, \quad \overline{Q}=\left\{\overline{\sigma}_{ij},\overline{u}_j\right\} \tag{6.5}$$

and nonsymmetric tensor of stresses τ_{ij} by formula (1.1) will be applied.

In the equations of motion (1.2) and correlations of elasticity (1.3) we pass to dimensionless coordinates and displacements by formulae

$$\xi=\theta,\ \eta=\varphi,\ \zeta=\gamma/h=\varepsilon^{-1}\gamma/r,\ u_\theta=u/r$$
$$u_\varphi=v/r,\ u_\gamma=w/r,\ \varepsilon=h/r,\ h<<r \tag{6.6}$$

As a result we get a singularly perturbed by small parametre ε system.

The solution of this system will be sought in the form of

$$\tau_{ij}(\theta,\varphi,\gamma)=\varepsilon^{-1+s}\tau_{ij}(\xi,\eta,\zeta),\ i,j=\theta,\varphi,\gamma$$
$$(u_\theta,u_\varphi,u_\gamma)=\varepsilon^s(u^{(s)},v^{(s)},w^{(s)}),\ s=\overline{0,N} \tag{6.7}$$

Substituting (6.7) into this system and equalizing in each equation the coefficients at the same degrees ε, all the stresses can be expressed through the displacements by formulae

$$\tau_{\theta\theta}^{(s)}=b_{11}e_{\theta\theta}^{(s)}+b_{12}e_{\beta\beta}^{(s)}+b_{13}e_{\gamma\gamma}^{(s)},$$

$$\tau_{\beta\beta}^{(s)}=b_{12}e_{\theta\theta}^{(s)}+b_{22}e_{\varphi\varphi}^{(s)}+b_{23}e_{\gamma\gamma}^{(s)},$$

$$\tau_{\gamma\gamma}^{(s)}=b_{13}e_{\theta\theta}^{(s)}+b_{23}e_{\varphi\varphi}^{(s)}+b_{33}e_{\gamma\gamma}^{(s)},$$

$$\tau_{\varphi\gamma}^{(s)}=b_{44}e_{\varphi\gamma}^{(s)},\quad \tau_{\alpha\gamma}^{(s)}=b_{55}e_{\theta\gamma}^{(s)},\quad \tau_{\theta\varphi}^{(s)}=b_{66}e_{\theta\varphi}^{(s)}$$

$$e_{\theta\theta}^{(s)} = e_{\theta\theta*}^{(s-1)}, \quad e_{\varphi\varphi}^{(s)} = e_{\varphi\varphi*}^{(s-1)}, \quad e_{\gamma\gamma}^{(s)} = \frac{\partial w^{(s)}}{\partial \zeta} + e_{\gamma\gamma*}^{(s-1)}, \tag{6.8}$$

$$e_{\varphi\gamma}^{(s)} = \frac{\partial v^{(s)}}{\partial \zeta} + e_{\varphi\gamma*}^{(s-1)}, \quad e_{\theta\gamma}^{(s)} = \frac{\partial u^{(s)}}{\partial \zeta} + e_{\theta\gamma*}^{(s-1)}, \quad e_{\theta\varphi}^{(s)} = e_{\theta\varphi*}^{(s-1)}$$

The displacements are determined from the equations

$$b_{33}\frac{\partial^2 w^{(s)}}{\partial \zeta^2} + \rho\omega^2 h^2 w^{(s)} = R_w^{(s-1)}$$

$$b_{55}\frac{\partial^2 u^{(s)}}{\partial \zeta^2} + \rho\omega^2 h^2 u^{(s)} = R_u^{(s-1)} \tag{6.9}$$

$$b_{44}\frac{\partial^2 v^{(s)}}{\partial \zeta^2} + \rho\omega^2 h^2 v^{(s)} = R_v^{(s-1)}$$

where

$$R_w^{(s-1)} = \tau_{\theta\theta}^{(s-1)} + \frac{r\sin\xi}{B}\tau_{\varphi\varphi}^{(s-1)} - \frac{\partial \tau_{\theta\gamma}^{(s-1)}}{\partial \xi} - \frac{r}{B}\frac{\partial \tau_{\varphi\gamma}^{(s-1)}}{\partial \eta} - \frac{r\cos\xi}{B}\tau_{\theta\gamma}^{(s-1)} -$$

$$-\rho\omega^2 h^2 L(w^{(s-1)}) - \frac{\partial}{\partial \zeta}(b_{13}e_{\theta\theta*}^{(s-1)} + b_{23}e_{\varphi\varphi*}^{(s-1)} + b_{33}e_{\gamma\gamma*}^{(s-1)})$$

$$R_u^{(s-1)} = -\frac{1}{B}\frac{\partial}{\partial \xi}(B\tau_{\theta\theta}^{(s-1)}) + \frac{r\cos\xi}{B}\tau_{\varphi\varphi}^{(s-1)} - \frac{r}{B}\frac{\partial \tau_{\varphi\theta}^{(s-1)}}{\partial \eta} - \tag{6.10}$$

$$-\zeta\frac{\partial \tau_{\theta\gamma}^{(s-1)}}{\partial \zeta} - \rho\omega^2 h^2 L(u^{(s-1)}) - b_{55}\frac{\partial}{\partial \zeta}e_{\theta\gamma*}^{(s-1)} - 2\tau_{\theta\gamma}^{(s-1)}$$

$$R_v^{(s-1)} = -\frac{\partial \tau_{\varphi\varphi}^{(s-1)}}{\partial \xi} - \frac{1}{B}\frac{\partial}{\partial \eta}(B\tau_{\theta\varphi}^{(s-1)}) - \frac{r\cos\xi}{B}\tau_{\varphi\theta}^{(s-1)} -$$

$$-\zeta\frac{r\cos\xi}{B}\frac{\partial \tau_{\varphi\gamma}^{(s-1)}}{\partial \zeta} - \rho\omega^2 h^2 L(u^{(s-1)}) - b_{44}\frac{\partial}{\partial \zeta}e_{\varphi\gamma*}^{(s-1)} - \frac{2r\cos\xi}{B}\tau_{\varphi\gamma}^{(s-1)}$$

$$e_{\theta\theta*}^{(s-1)} = \frac{\partial u^{(s-1)}}{\partial \xi} + w^{(s-1)} + \zeta\frac{r\sin\xi}{B}\left(\frac{\partial u^{(s-2)}}{\partial \xi} + w^{(s-2)}\right) - \zeta(a_{11}\tau_{\theta\theta}^{(s-1)} + a_{12}\frac{r\sin\xi}{B}\tau_{\varphi\varphi}^{(s-1)})$$

$$e_{\varphi\varphi*}^{(s-1)} = \frac{r}{B}\left(\frac{\partial v^{(s-1)}}{\partial \eta} + \sin\xi w^{(s-1)} + \cos\xi u^{(s-1)}\right) +$$

$$+\zeta\frac{r}{B}\left(\frac{\partial v^{(s-2)}}{\partial \eta} + \sin\xi w^{(s-2)} + \cos\xi u^{(s-2)}\right) - \zeta\left(a_{12}\tau_{\theta\theta}^{(s-1)} + a_{22}\frac{r\sin\xi}{B}\tau_{\varphi\varphi}^{(s-1)}\right)$$

$$e_{\gamma\gamma*}^{(s-1)} = L\left(\frac{\partial w^{(s-1)}}{\partial \zeta}\right) - \zeta(a_{13}\tau_{\theta\theta}^{(s-1)} + a_{23}\frac{r\sin\xi}{B}\tau_{\varphi\varphi}^{(s-1)})$$

$$e_{\varphi\gamma*}^{(s-1)} = L\left(\frac{\partial v^{(s-1)}}{\partial \zeta}\right) - \frac{r\sin\xi}{B}\left(v^{(s-1)} + \zeta v^{(s-2)} + a_{44}\zeta\tau_{\varphi\gamma}^{(s-1)}\right) +$$

$$+\frac{r}{B}\left(\frac{\partial w^{(s-1)}}{\partial\eta}+\zeta\frac{\partial w^{(s-2)}}{\partial\eta}\right)$$

$$e_{\theta\gamma*}^{(s-1)}=L\left(\frac{\partial u^{(s-1)}}{\partial\zeta}\right)-u^{(s-1)}-\zeta\frac{r\sin\xi}{B}\left(u^{(s-2)}+a_{55}\tau_{\theta\gamma}^{(s-1)}\right)+$$

$$+\frac{\partial w^{(s-1)}}{\partial\eta}+\zeta\frac{r\sin\xi}{B}\frac{\partial w^{(s-2)}}{\partial\eta}$$

$$e_{\theta\varphi*}^{(s-1)}=\frac{r}{B}\left(\frac{\partial u^{(s-1)}}{\partial\eta}-\cos\xi v^{(s-1)}\right)+\zeta\frac{r}{B}\left(\frac{\partial u^{(s-2)}}{\partial\eta}-\cos\xi v^{(s-2)}\right)+$$

$$+\frac{\partial v^{(s-1)}}{\partial\xi}+\zeta\frac{r\sin\xi}{B}\frac{\partial v^{(s-2)}}{\partial\xi}-a_{66}\zeta\tau_{\theta\varphi}^{(s-1)}$$

$$L(Q^{(s-1)})=\zeta(r_1+r_2)Q^{(s-1)}+\zeta r_1r_2Q^{(s-2)},\ r_1=r/R_1=1,\ \ r_2=r/R_2$$

The solutions of the equations (6.9) are

$$\begin{aligned}u^{(s)}&=M_u^{(s)}\sin\lambda_1\zeta+N_u^{(s)}\cos\lambda_1\zeta+I_u^{(s)}(\zeta),\quad \lambda_1=\omega h\sqrt{a_{55}\rho}\\ v^{(s)}&=M_v^{(s)}\sin\lambda_2\zeta+N_v^{(s)}\cos\lambda_2\zeta+I_v^{(s)}(\zeta),\quad \lambda_2=\omega h\sqrt{a_{44}\rho}\\ w^{(s)}&=M_w^{(s)}\sin\lambda_3\zeta+N_w^{(s)}\cos\lambda_3\zeta+I_w^{(s)}(\zeta),\ \ \lambda_3=\omega h\sqrt{\rho/b_{33}}\end{aligned}\qquad(6.11)$$

where $I_u^{(s)}(\zeta)$, $I_v^{(s)}(\zeta)$, $I_w^{(s)}(\zeta)$ are private solutions of the equations (6.9)

$$I_u^{(s)}(\zeta)=\frac{1}{\lambda_1}\int_0^\zeta\Phi_u^{(s)}(\tau)\sin\lambda_1(\zeta-\tau)d\tau\qquad(u,v,w;1,2,3)$$

$$\Phi_w^{(s)}(\zeta)=\gamma_{33}\frac{\partial T^{(s)}}{\partial\zeta}\Big/b_{33}+R_w^{(s-1)}(\zeta)/b_{33},$$

$$\Phi_u^{(s)}=R_u^{(s-1)}/b_{55}\qquad(u,v;5,4)\qquad(6.12)$$

$$b_{jj}=(a_{kk}a_{ll}-a_{kl}^2)/\Delta\quad j,k,l=1,2,3,\quad b_{jk}=(a_{jl}a_{kl}-a_{jk}a_{ll})/\Delta,\quad j\neq k\neq l$$

$$b_{55}=1/a_{55},\ b_{44}=1/a_{44}$$

$$\Delta=2a_{12}a_{13}a_{23}+a_{11}a_{22}a_{33}-a_{11}a_{23}^2-a_{22}a_{13}^2-a_{33}a_{12}^2,\quad b_{44}=\frac{1}{a_{44}}\quad(4,5,6)$$

For each s the solution (6.11) contains six unknown functions, which are uniquely determined from six conditions (6.1), (6.2) or (6.1), (6.3).

Satisfying the conditions (6.1), (6.2) we have

$$M_w^{(s)}=\left[\sigma^{(s)}-I_\gamma^{(s)}(\zeta=-1)-I_\gamma^{(s)}(\zeta=1)\right]\Big/\left(2\lambda_3b_{33}\cos\lambda_3\right)$$

$$N_w^{(s)}=\left[\sigma^{(s)}-I_\gamma^{(s)}(\zeta=-1)+I_\gamma^{(s)}(\zeta=1)\right]\Big/(2\lambda_3b_{33}\sin\lambda_3)\ \ ,\ \sin2\lambda_3\neq0\qquad(6.13)$$

$$(\alpha,\beta,\gamma;u,v,w;0,0,\sigma;1,2,3;55,44,33)$$

$$\sigma^{(0)}=\varepsilon\sigma,\ \ \sigma^{(1)}=\varepsilon^2(1+\frac{r\sin\xi}{B})\sigma,\ \ \sigma^{(2)}=\varepsilon^3\frac{r\sin\xi}{B}\sigma,\ \ \sigma^{(s)}=0,\ \ s>2$$

$$I_{\alpha}^{(s)}(\zeta)=\frac{1}{\lambda_1}\int_0^{\zeta}\Phi_u^{(s)}(\tau)\cos\lambda_1(\zeta-\tau)d\tau \qquad (\alpha,\beta,\gamma;u,v,w;1,2,3)$$

If the frequency value of the outer action ω coincides with the main value of the free vibrations frequency, a resonance will take place, when even if one of the correlations is fulfilled

$$\sin 2\lambda_j=0 \quad j=1,2,3$$

$$\lambda_{3\kappa}=\frac{\pi\kappa}{2} \Rightarrow \varpi_{\gamma rez.}=\frac{\pi k}{2h}\sqrt{\frac{b_{33}}{\rho}}$$

$$\lambda_{1\kappa}=\frac{\pi\kappa}{2} \Rightarrow \varpi_{\theta rez.}=\frac{\pi k}{2h}\sqrt{\frac{b_{55}}{\rho}}=\frac{\pi k}{2h}\sqrt{\frac{G_{13}}{\rho}} \tag{6.14}$$

$$\lambda_{2\kappa}=\frac{\pi\kappa}{2} \Rightarrow \varpi_{\varphi rez.}=\frac{\pi k}{2h}\sqrt{\frac{b_{44}}{\rho}}=\frac{\pi k}{2h}\sqrt{\frac{G_{23}}{\rho}}$$

G_{13},G_{23} are shear modules

The conditions (6.1), (6.3) correspond to the solution

$$M_w^{(s)}=\left[\left(\sigma^{(s)}-I_{\gamma}^{(s)}(\zeta=-1)\right)\cos\lambda_3+b_{33}\lambda_{33}I_w^{(s)}(\zeta=1)\sin\lambda_3\right]\Big/(b_{33}\lambda_3\cos 2\lambda_3)$$
$$N_w^{(s)}=\left[\left(I_{\gamma}^{(s)}(\zeta=-1)-\sigma^{(s)}\right)\sin\lambda_3-b_{33}\lambda_3 I_w^{(s)}(\zeta=1)\cos\lambda_3\right]\Big/(b_{33}\lambda_3\cos 2\lambda_3) \tag{6.15}$$

$$\cos 2\lambda_3\neq 0 \qquad (\theta,\varphi,\gamma;u,v,w;0,0,\sigma;1,2,3;55,44,33)$$

A resonance arise, when $\cos 2\lambda_j=0 \quad j=1,2,3$, which correspond to the frequencies

$$\varpi_{rez}^{I}=\frac{\pi(2k-1)}{4h}\sqrt{\frac{b_{55}}{\rho}}=\frac{\pi(2k-1)}{4h}\sqrt{\frac{G_{13}}{\rho}}$$

$$\varpi_{rez}^{II}=\frac{\pi(2k-1)}{4h}\sqrt{\frac{G_{23}}{\rho}} \tag{6.16}$$

$$\varpi_{rez}^{III}=\frac{\pi(2k-1)}{4h}\sqrt{\frac{b_{33}}{\rho}}$$

7. The solutions of private problems on forced vibrations of the toroidal shell

Let $\sigma(\theta,\varphi)=\sigma=\text{const}$. After the first step of iteration in the problem (6.1), (6.2) we have

$$\overline{\tau}_{\theta\theta}=\sigma\frac{b_{13}\sin\lambda_3(1-\zeta)}{b_{33}\sin 2\lambda_3}\sin\omega t, \quad \overline{\tau}_{\varphi\varphi}=\sigma\frac{b_{23}\sin\lambda_3(1-\zeta)}{b_{33}\sin 2\lambda_3}\sin\omega t$$

$$\overline{\tau}_{\gamma\gamma}=\frac{\sigma\sin\lambda_3(1-\zeta)}{\sin 2\lambda_3}\sin\omega t, \quad \overline{\tau}_{\varphi\gamma}=\overline{\tau}_{\theta\gamma}=\overline{\tau}_{\theta\varphi}=0 \tag{7.1}$$

$$\overline{w}=\sigma\frac{\cos\lambda_3(1-\zeta)}{b_{33}\lambda_3\sin 2\lambda_3}\sin\omega t, \quad \overline{u}=\overline{v}=0$$

In the problem (6.1), (6.3) at $\sigma = \text{const}$ we have

$$\bar{\tau}_{\theta\theta} = \sigma\frac{b_{13}\cos\lambda_3(\zeta-1)}{b_{33}\cos 2\lambda_3}\sin\omega t,\quad \bar{\tau}_{\varphi\varphi} = \sigma\frac{b_{23}\cos\lambda_3(\zeta-1)}{b_{33}\cos 2\lambda_3}\sin\omega t$$

$$\bar{\tau}_{\gamma\gamma} = \frac{\sigma\cos\lambda_3(\zeta-1)}{\cos 2\lambda_3}\sin\omega t,\quad \bar{\tau}_{\varphi\gamma} = \bar{\tau}_{\theta\gamma} = \bar{\tau}_{\theta\varphi} = 0 \tag{7.2}$$

$$\bar{w} = \sigma\frac{\sin\lambda_3(\zeta-1)}{b_{33}\lambda_3\cos 2\lambda_3}\sin\omega t,\quad \bar{u} = \bar{v} = 0$$

The approximation $s = 1$ corresponds to

$$\tau_{\theta\theta}^{(1)} = \left(b_{11} + \frac{r\sin\xi}{B}b_{12}\right)w + \zeta\left(1+\frac{r\sin\xi}{B}\right)b_{13}\frac{\partial w}{\partial\zeta} + b_{13}\frac{\partial w^{(1)}}{\partial\zeta} - \zeta\tau_{\theta\theta}$$

$$\tau_{\varphi\varphi}^{(1)} = \left(b_{12} + \frac{r\sin\xi}{B}b_{22}\right)w + \zeta\left(1+\frac{r\sin\xi}{B}\right)b_{23}\frac{\partial w}{\partial\zeta} + b_{23}\frac{\partial w^{(1)}}{\partial\zeta} - \zeta\tau_{\varphi\varphi} \tag{7.3}$$

$$\tau_{\gamma\gamma}^{(1)} = \left(b_{13} + \frac{r\sin\xi}{B}b_{23}\right)w + \zeta\left(1+\frac{r\sin\xi}{B}\right)b_{33}\frac{\partial w}{\partial\zeta} + b_{33}\frac{\partial w^{(1)}}{\partial\zeta}$$

$$\tau_{\varphi\gamma}^{(1)} = \tau_{\theta\varphi}^{(1)} = 0,\quad \tau_{\theta\gamma}^{(1)} = b_{55}\frac{\partial u^{(1)}}{\partial\zeta}$$

$$\bar{\tau}_{ij} = \tau_{ij}\sin\omega t,\quad \bar{w} = w\sin\omega t$$

$$w^{(1)} = M_w^{(1)}\sin\lambda_3(1-\zeta) + N_w^{(1)}\cos\lambda_3(1-\zeta) + \frac{W}{2b_{33}\lambda_3}\zeta\cos\lambda_3(1-\zeta)$$

$$u^{(1)} = M_u^{(1)}\sin\lambda_1(1-\zeta) + N_u^{(1)}\cos\lambda_1(1-\zeta) + \frac{U}{\lambda_3^2\left(b_{33}-b_{55}\right)}\sin\lambda_3(1-\zeta)$$

$$W = -\frac{\sigma}{\sin 2\lambda_3}\left(1+\frac{r\sin\xi}{B}\right),\quad U = \frac{\sigma r\sin\xi}{Bb_{33}\sin 2\lambda_3}\left(b_{23}-b_{13}\right)$$

For boundary conditions (6.1), (6.2) we have

$$M_w^{(1)} = W^*(\zeta=1)\big/\left(b_{33}\lambda_3\right)$$

$$N_w^{(1)} = \left[W^*(\zeta=-1) - W^*(\zeta=1)\cos 2\lambda_3 - \left(1+\frac{r\sin\xi}{B}\right)\varepsilon\sigma\right]\Big/\left(b_{33}\lambda_3\sin 2\lambda_3\right)$$

$$M_u^{(1)} = \frac{U}{\lambda_3\left(b_{55}-b_{33}\right)},\quad N_u^{(1)} = \frac{U}{\lambda_3\left(b_{55}-b_{33}\right)}\frac{\cos 2\lambda_3 - \cos 2\lambda_1}{\sin 2\lambda_3},\quad \sin 2\lambda_1 \neq 0 \tag{7.4}$$

$$W^*(\zeta) = \frac{W}{2\lambda_3}\left(\cos\lambda_3(1-\zeta) + \zeta\lambda_3\sin\lambda_3(1-\zeta)\right) + \zeta b_{33}\left(1+\frac{r\sin\xi}{B}\right)\frac{\partial w}{\partial\zeta} +$$
$$+\left(b_{13} + b_{23}\frac{r\sin\xi}{B}\right)w$$

and for conditions (6.1), (6.3)

$$M_w^{(1)} = \left[W \sin 2\lambda_3 + 2\left(W^*(\zeta=-1) - \left(1+\frac{r\sin\xi}{B}\right)\varepsilon\sigma \right) \right] \Big/ \left(b_{33}\lambda_3 \cos 2\lambda_3 \right)$$

$$N_w^{(1)} = -W \big/ \left(2b_{33}\lambda_3 \right), \quad M_u^{(1)} = \frac{U\cos 2\lambda_3}{\lambda_3 \left(b_{55} - b_{33} \right) \cos 2\lambda_1}, \quad N_u^{(1)} = 0, \; \cos 2\lambda_1 \neq 0 \qquad (7.5)$$

Hence, after the two step of iteration the components of nonsymmetrical tensor of stresses and vector of displacement are

$$\tau_{\theta\theta} = \left(\overline{\tau}_{\theta\theta} + \frac{h}{R}\tau_{\theta\theta}^{(1)} \right) \sin\omega t, \quad \tau_{\varphi\varphi} = \left(\overline{\tau}_{\varphi\varphi} + \frac{h}{R}\tau_{\varphi\varphi}^{(1)} \right) \sin\omega t, \quad \tau_{\theta\varphi} = \frac{h}{R}\tau_{\theta\varphi}^{(1)} \sin\omega t$$

$$\tau_{\gamma\gamma} = \left(\overline{\tau}_{\gamma\gamma} + \frac{h}{R}\tau_{\gamma\gamma}^{(1)} \right) \sin\omega t, \quad \tau_{\varphi\gamma} = \frac{h}{R}\tau_{\varphi\gamma}^{(1)} \sin\omega t, \quad \tau_{\theta\gamma} = \frac{h}{R}\tau_{\theta\gamma}^{(1)} \sin\omega t$$

$$u_\theta = \frac{h^2}{R} u^{(1)} \sin\omega t, \quad u_\varphi = \frac{h^2}{R} \mathrm{v}^{(1)} \sin\omega t, \quad u_\gamma = h\overline{w} + \frac{h^2}{R} w^{(1)} \sin\omega t$$

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