ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 62, №4, 2009 Механика

УДК 539.3

ON BOUNDARY LAYER THE 3D PROBLEM ABOUT FORCED VIBRATIONS OF ORTHOTROPIC PLATE, FREELY-LYING ON THE RIGID FOUNDATION

Sargsyan M. Z.[*)]

Keywords: boundary layer, plate, damping, frequency, Coulomb friction, singularly perturbed system, 3D problem, forced vibration, conjugation of solutions.
Ключевые слова: пограничный слой, пластина, затухание, частота, кулоново трение, сингулярно-возмущенная система, трехмерная задача, вынужденные колебания, сопряжение решений.

Մ. Զ. Սարգսյան
Կոշտ հենարանի վրա ազատ հենված օրթոտրոպ սալի սահմանային շերտի համար ստիպողական տատանման եռաչափ խնդիրը

Դիտարկված է կոշտ հենարանի վրա ազատ հենված օրթոտրոպ սալի սահմանային շերտի խնդիրը, երբ սալի վերին նիստի վրա ազդում է ըստ ժամանակի հարմոնիկ փոփոխվող նորմալ բեռ: Ասիմպտոտիկ մեթոդի կիրառմամբ որոշված է սահմանային շերտի լարվածա-դեֆորմացիոն վիճակը: Ուսումնասիրված է սահմանային շերտի մեծությունների մարումը: Տրված է սահմանային շերտի և ներքին խնդրի լուծումների կարման եղանակ:

М. З. Саргсян
О пограничном слое в трехмерной задаче о вынужденных колебаниях ортотропной пластины, свободно лежащей на жестком основании

Рассмотрена задача пограничного слоя ортотропной пластинки, лежащей на жестком основании, на верхнюю лицевую плоскость которой действует гармонически изменяющаяся во времени нормальная нагрузка. С применением асимптотического метода определено напряженно-деформированное состояние в пограничном слое. Исследовано затухание величин в пограничном слое. Показан способ сопряжения решения внутренней задачи и пограничного слоя.

The problem of boundary layer of the orthotropic plate simply supported on the rigid foundation is considered, when on the upper plane of plate the normal load affects. The stressedly-deformed state boundary layer of plate is determined by using the asymptotic method. The damping of vlues for boundary layer are researched. The method of conjugation of the solutions of inner problem and of boundary layer is showed.

Introduction

As we know, the equations of elasticity theory for thin bodies written in dimensionless coordinates are singularly perturbed by small parameter differential equation. For this kind of equation and systems the asymptotic method is usually applied [1]. Theory of isotropic plates and shells [2], theory of anisotropic plates, shells and beams [3] are built by this method. The asymptotic method effective for solution of dynamic problems of elasticity, particularly, the problems on free and forced vibrations of plate[4].The asymptotic method is also used to solving non-classical boundary layer's problems , i.e. when the boundary conditions are specified on the lateral surfaces of thin bodies. This kind of problems on natural and forced vibrations was solved in References [5, 6]. The review of the papers on the application of the asymptotic method for the solution of static and dynamic problems of beams, plates and shells is included in [7]. Below the boundary layer's problem of orthotropic plate simply-supported on the rigid foundation is researched, when on the upper plane of the plate the normal load acts.

*) It has been reported in FINAL MEETING of INTAS Project "Some nonclassical problems for thin Structures", Rome, Italy, 22-23 Jan 2009.

1. Consider the orthotropic plate $D=\{(x,y,z),\ 0\le x\le a,\ 0\le y\le b,\ |z|\le h,\ h<<l,\ l=\min(a,b)\}$, on which upper plane affects the normal load:

$$F(x,y,t)=P(x,y)\exp(i\Omega t)$$

which changes by time harmonically. Here Ω is the frequency of influencing force. On the lower face of the plate are given the simply supported conditions subject to friction. The conditions on the lateral surface may be arbitrary, whose corresponds the solution of boundary layer. It's supposed, that the friction is Coulomb friction, which means the shear stresses are proportional to the normal stress. So we will have the following boundary conditions:

$$\begin{aligned} &z=h, \quad \sigma_{zz}=-P(x,y)\exp(i\Omega t), \ \ \sigma_{xz}=\sigma_{yz}=0 \\ &z=-h, \quad w=0, \ \ \sigma_{xz}=f_1\sigma_{zz}, \ \ \sigma_{yz}=f_2\sigma_{zz} \end{aligned} \tag{1.1}$$

where f_1 and f_2 are coefficients of friction along coordinate directions x and y.

From these boundary conditions the complete solution of the inner problem is obtained [8], which will not satisfy to conditions on lateral surface. For satisfying them it is necessary to construct the solution of the boundary layer.

Now consider the boundary layer of plate near the lateral face $x=0$. The boundary conditions for boundary layer have the following forms:

$$\begin{aligned} &z=h \qquad \sigma_{zz}=\sigma_{xz}=\sigma_{yz}=0, \\ &z=-h \qquad \sigma_{xz}=f_1\sigma_{zz}, \ \ \sigma_{yz}=f_2\sigma_{zz}, \ \ W=0 \end{aligned} \tag{1.2}$$

For constructing the solution, which corresponds to the boundary layer, we have to pass to dimensionless coordinates and components of displacement vector:

$$\gamma=\frac{x}{h},\ \eta=\frac{y}{l},\ \zeta=\frac{z}{h},\ U=\frac{u_x}{l},\ V=\frac{u_y}{l},\ W=\frac{u_z}{l} \tag{1.3}$$

Where h is the thickness of the plate, $l=\min(a,b)$ and $h<<l$. Consequently, we will get singularly perturbed with small parameter ε system:

$$\begin{aligned} &\varepsilon^{-1}\frac{\partial\sigma_{11}}{\partial\gamma}+\frac{\partial\sigma_{12}}{\partial\eta}+\varepsilon^{-1}\frac{\partial\sigma_{13}}{\partial\zeta}+\varepsilon^{-2}\Omega_*^2U=0 \qquad (1,2,3;\ U,V,W) \\ &\varepsilon^{-1}\frac{\partial U}{\partial\gamma}=a_{11}\sigma_{11}+a_{12}\sigma_{22}+a_{13}\sigma_{33} \qquad (\gamma,\zeta;\ U,W;\ 1,2) \\ &\frac{\partial V}{\partial\eta}=a_{12}\sigma_{11}+a_{22}\sigma_{22}+a_{23}\sigma_{33} \\ &\frac{\partial U}{\partial\eta}+\varepsilon^{-1}\frac{\partial V}{\partial\gamma}=a_{66}\sigma_{12} \\ &\varepsilon^{-1}\frac{\partial W}{\partial\gamma}+\varepsilon^{-1}\frac{\partial U}{\partial\zeta}=a_{55}\sigma_{13} \\ &\frac{\partial W}{\partial\eta}+\varepsilon^{-1}\frac{\partial V}{\partial\zeta}=a_{44}\sigma_{23}, \qquad \Omega_*^2=\rho h^2\Omega^2,\ \ \varepsilon=h/l \end{aligned} \tag{1.4}$$

The solution of this system will be sought in form of the following asymptotic expansion:

$$\begin{aligned} &\sigma_{ijb}=\varepsilon^{-1+s}\sigma_{ijb}^{(s)}(\eta,\zeta)\exp(-\lambda\gamma), \\ &U_b=\varepsilon^{s}U_b^{(s)}(\eta,\zeta)\exp(-\lambda\gamma), \quad (U,V,W) \ \ i,j=1,2,3, \quad s=\overline{0,S} \end{aligned} \tag{1.5}$$

Notation $s=\overline{0,S}$ means that by dummy index *s* summation from 0 up to the number of approximation S takes place. Substituting the forms (1.5) into the system (1.4) we get the system for determining components of the stress tensor and the displacement vector:

$$-\lambda\sigma_{11b}^{(s)}+\frac{\partial\sigma_{13b}^{(s)}}{\partial\zeta}+\Omega_*^2U_b^{(s)}=R_{1\sigma}^{(s)},\quad -\lambda\sigma_{12b}^{(s)}+\frac{\partial\sigma_{23b}^{(s)}}{\partial\zeta}+\Omega_*^2V_b^{(s)}=R_{2\sigma}^{(s)}$$

$$-\lambda\sigma_{13b}^{(s)}+\frac{\partial\sigma_{33b}^{(s)}}{\partial\zeta}+\Omega_*^2W_b^{(s)}=R_{3\sigma}^{(s)}$$

$$\sum_{j=1}^{3}a_{1j}\sigma_{jjb}^{(s)}+\lambda U_b^{(s)}=0,\quad \sum_{j=1}^{3}a_{j2}\sigma_{jjb}^{(s)}=R_V^{(s)},\quad \sum_{j=1}^{3}a_{j3}\sigma_{jjb}^{(s)}-\frac{\partial W_b^{(s)}}{\partial\zeta}=0 \tag{1.6}$$

$$\frac{\partial U_b^{(s)}}{\partial\zeta}-\lambda W_b^{(s)}=a_{55}\sigma_{13b}^{(s)},\ -\lambda V_b^{(s)}=a_{66}\sigma_{12b}^{(s)}+R_U^{(s)},\ \frac{\partial V_b^{(s)}}{\partial\zeta}=a_{44}\sigma_{23b}^{(s)}+R_W^{(s)}$$

where

$$R_{i\sigma}^{(s)}=-\frac{\partial\sigma_{i2b}^{(s-1)}}{\partial\eta},\quad i=1,2,3$$

$$R_U^{(s)}=-\frac{\partial U_b^{(s-1)}}{\partial\eta},\quad (U,V,W)$$

Using (1.6) it is possible to express all the required values in terms of components of displacement vector by formulae:

$$\sigma_{11b}^{(s)}=-A_{23}\frac{\partial W_b^{(s)}}{\partial\zeta}-A_{22}\lambda U_b^{(s)}+A_{12}R_V^{(s)}\qquad \sigma_{12b}^{(s)}=-\frac{1}{a_{66}}(\lambda V_b^{(s)}-R_U^{(s)})$$

$$\sigma_{22b}^{(s)}=-A_{13}\frac{\partial W_b^{(s)}}{\partial\zeta}+A_{12}\lambda U_b^{(s)}-A_{33}R_V^{(s)}\qquad \sigma_{13b}^{(s)}=\frac{1}{a_{55}}(\frac{\partial U_b^{(s)}}{\partial\zeta}-\lambda W_b^{(s)}) \tag{1.7}$$

$$\sigma_{33b}^{(s)}=A_{11}\frac{\partial W_b^{(s)}}{\partial\zeta}+A_{23}\lambda U_b^{(s)}+A_{13}R_V^{(s)}\qquad \sigma_{23b}^{(s)}=\frac{1}{a_{44}}(\frac{\partial V_b^{(s)}}{\partial\zeta}-R_W^{(s)})$$

and from the system (1.6) for components of displacements vector the following equations are obtained:

$$\frac{\partial^2U_b^{(s)}}{\partial\zeta^2}-\lambda(1-a_{55}A_{23})\frac{\partial W_b^{(s)}}{\partial\zeta}+a_{55}(A_{22}\lambda^2+\Omega_*^2)U_b^{(s)}=T_U^{(s)}$$

$$\frac{\partial^2W_b^{(s)}}{\partial\zeta^2}-\lambda\frac{1-a_{55}A_{23}}{A_{11}}\frac{\partial U_b^{(s)}}{\partial\zeta}+\frac{1}{A_{11}}(\frac{\lambda^2}{a_{55}}+\Omega_*^2)W_b^{(s)}=T_W^{(s)} \tag{1.8}$$

$$\frac{\partial^2V_b^{(s)}}{\partial\zeta^2}+a_{44}(\frac{\lambda^2}{a_{66}}+\Omega_*^2)V_b^{(s)}=T_V^{(s)} \tag{1.9}$$

$$T_U^{(s)}=A_{12}a_{55}\lambda R_V^{(s)}+a_{55}R_{1\sigma}^{(s)},\ T_V^{(s)}=-\frac{a_{44}}{a_{66}}\lambda R_U^{(s)}+a_{44}R_{2\sigma}^{(s)}+\frac{\partial R_W^{(s)}}{\partial\zeta},$$

$$T_W^{(s)}=-\frac{A_{13}}{A_{11}}\lambda R_V^{(s)}+\frac{1}{A_{11}}R_{3\sigma}^{(s)} \tag{1.10}$$

Now we will consider only approximation $s = 0$, because the next approximations don't represent any practical interest.
So in approximation $s = 0$ the right-hand members (1.10) of equations (1.8) (1.9) is equal to zero:

$$T_U^{(0)} = T_V^{(0)} = T_W^{(0)} = 0$$

and boundary conditions will get the following forms:

$$\begin{aligned} &\zeta = -1 \quad \sigma_{13b}^{(0)} = f_1 \sigma_{33b}^{(0)}, \quad W_b^{(0)} = 0 \\ &\zeta = 1 \quad \sigma_{13b}^{(0)} = \sigma_{33b}^{(0)} = 0 \end{aligned} \tag{1.11}$$

$$\begin{aligned} &\zeta = -1 \quad \sigma_{23b}^{(0)} = f_2 \sigma_{33b}^{(0)} \\ &\zeta = 1 \quad \sigma_{23b}^{(0)} = 0 \end{aligned} \tag{1.12}$$

The solution of the system (1.8) will be sought in the following form:

$$U_b^{(0)} = G_b^{(0)}(\eta) \exp k\zeta, \quad W_b^{(0)} = L G_b^{(0)}(\eta) \exp k\zeta, \tag{1.13}$$

where L is an indefinite multiplier for a present. Substituting the forms of components of displacement vector (1.13) into the system of equations (1.8) we will get the following system:

$$\begin{cases} k^2 - \lambda(1 - a_{55}A_{23})Lk + A_{22}a_{55}\lambda^2 + a_{55}\Omega_*^2 = 0 \\ k^2 - \lambda \dfrac{1 - a_{55}A_{23}}{A_{11}} k + \dfrac{1}{A_{11}a_{55}} L\lambda^2 + L\dfrac{\Omega_*^2}{A_{11}} = 0 \end{cases} \tag{1.14}$$

from which multiplier L will have the following form:

$$L = \frac{k^2 + A_{22}a_{55}\lambda^2 + a_{55}\Omega_*^2}{(1 - a_{55}A_{23})\lambda k} \tag{1.15}$$

and for k the following characteristic equation is obtained.

$$k^4 + (B_1\lambda^2 + B_2\Omega_*^2)k^2 + B_3\lambda^4 + B_4\lambda^2\Omega_*^2 + B_5\Omega_*^2 = 0 \tag{1.16}$$

where

$$B_1 = A_{22}a_{55} - \frac{1 - a_{55}A_{23}}{A_{11}} + \frac{1}{a_{55}A_{11}}, \; B_2 = a_{55} + \frac{1}{A_{11}},$$

$$B_3 = \frac{A_{22}}{A_{11}}, \; B_4 = \frac{1 + a_{55}A_{22}}{A_{11}}, \; B_5 = \frac{a_{55}}{A_{11}}$$

The equation (1.16) has four roots:

$$\begin{aligned} &k_{1,2}^2 = \frac{-B_1\lambda^2 - \Omega_*^2 B_2 \pm \sqrt{D}}{2} \\ &D = (B_1^2 - 4B_3)\lambda^4 + \Omega_*^2(2B_1B_2 - 4B_4)\lambda^2 + \Omega_*^2 B_2^2 - 4B_5\Omega_*^2 \end{aligned} \tag{1.17}$$

Therefore, for any k_i there will be a one multiplier L_i:

$$L = L(k_i) \;\Rightarrow\; L_i = \frac{k_i^2 + A_{22}a_{55}\lambda^2 + a_{55}\Omega_*^2}{(1 - a_{55}A_{23})\lambda k_i} \tag{1.18}$$

So the solution of system (1.8) will have the following form:

$$U_b^{(0)} = \sum_{i=1}^{4} G_{ib}^{(0)}(\eta) \exp k_i\zeta, \quad W_b^{(0)} = \sum_{i=1}^{4} L_i G_{ib}^{(0)}(\eta) \exp k_i\zeta, \tag{1.19}$$

in $s=0$ approximation. Satisfying the boundary conditions (1.11) and in consideration of (1.7) we will get a system of homogeneous equations, for existence of which nonzero solution it is necessary that the determinant of matrix of coefficients of the system to be equal to zero.

$$\begin{vmatrix} \alpha_1 e^{k_1} & \alpha_2 e^{k_2} & \alpha_3 e^{k_3} & \alpha_4 e^{k_4} \\ \beta_1 e^{k_1} & \beta_2 e^{k_2} & \beta_3 e^{k_3} & \beta_4 e^{k_4} \\ L_1 e^{-k_1} & L_2 e^{-k_2} & L_3 e^{-k_3} & L_4 e^{-k_4} \\ \mu_1 e^{-k_1} & \mu_2 e^{-k_2} & \mu_3 e^{-k_3} & \mu_4 e^{-k_4} \end{vmatrix} = 0 \tag{1.20}$$

$\alpha_i = k_i - L_i\lambda,\ \beta_i = A_{11}L_i k_i + A_{23}\lambda,\ \mu_i = k_i(1/a_{55} - f_1 A_{11} L_i) - \lambda(L_i / a_{55} + f_1 A_{23}),\ i = \overline{1,4}$

The equation (1.20) is the characteristic equation for λ. The roots of this equation we will denote by λ_{pn}. Coefficient of system of homogeneous equations will be expressed in term of one of them, for example by $G_{1b}^{(1)}(\eta)$. Taking into account, that to each λ_p, with $\operatorname{Re}\lambda_p > 0$, corresponds its conjugate $\overline{\lambda_p}$, we will represent the $G_{1bn}^{(s)}(\eta)$ in the following form:

$$G_{1bn}^{(s)}(\eta) = \left(A_{1n}^{(s)} - iA_{2n}^{(s)}\right)/2 \tag{1.21}$$

Consequently, in the final forms of required quantities, we will have the real expression in the following general form:

$$Q_b^{(s)}\left(\gamma,\eta,\zeta\right) = \operatorname{Re}\tilde{Q}_{bn}^{(s)} A_{1n}^{(s)} + \operatorname{Im}\tilde{Q}_{bn}^{(s)} A_{2n}^{(s)} \tag{1.22}$$

where we are used the following notations:

$$\tilde{Q}_{bn}^{(s)} = Q_{bn}\exp\left(-\lambda_{pn}\gamma\right),\ Q_b^{(s)} = Q_{bn} G_{1bn}^{(s)} \tag{1.23}$$

Now return to equation (1.9). In the approximation $s=0$ the solution of this equation will be sought in form of

$$V_b^{(0)} = C_b^{(0)}(\eta)\exp\theta\zeta \tag{1.24}$$

After substituting this form into equation (1.9), we will obtain the following form of $V_b^{(0)}$:

$$V_b^{(0)} = C_{1b}^{(0)}\sin\theta\zeta + C_{2b}^{(0)}\cos\theta\zeta \tag{1.25}$$

where $\theta = \sqrt{(\lambda^2/a_{66} + \Omega_*^2)a_{44}}$.

In the plane problem when $\lambda = \lambda_p$, satisfying the following boundary conditions:

$$\zeta = -1 \quad \sigma_{23b}^{(0)} = f_2\sigma_{33b}^{(0)},\ \ \zeta = 1 \quad \sigma_{23b}^{(0)} = 0 \tag{1.26}$$

it is possible to express the coefficients $C_{ib}^{(0)}$ by $G_{ib}^{(0)}$:

$$\begin{aligned} C_{1b}^{(0)} &= \frac{a_{44} f_2}{2 f_1 \cos\theta_p}\sum_{i=1}^{4} G_{ib}^{(0)}(\eta)(k_i - L_i\lambda_p)\exp\left(-k_i\right) \\ C_{2b}^{(0)} &= \frac{a_{44} f_2}{2 f_1 \sin\theta_p}\sum_{i=1}^{4} G_{ib}^{(0)}(\eta)(k_i - L_i\lambda_p)\exp\left(-k_i\right) \end{aligned} \tag{1.27}$$

So, the components of displacement vector $U_b^{(0)}, W_b^{(0)}$ corresponding to the plane problem, give rise to component of displacement vector $V_b^{(0)}$, which corresponds to the out-of-plane problem, because of Coulomb friction.

Now consider the out-of-plane problem. In this case we have $\lambda \neq \lambda_p$, from which follows that all coefficients $G_{ib}^{(0)}$ equal to zero. Therefore, the components of displacement $U_b^{(0)}$ and $W_b^{(0)}$ are equal to zero too:

$$U_b^{(0)} = W_b^{(0)} = 0$$

So we will have the following boundary conditions:

$$\zeta = \pm 1 \quad \sigma_{23b}^{(0)} = 0 \tag{1.28}$$

satisfying to which we will get a system of homogeneous equations for existence of which nonzero solution it is necessary the fulfillment of the following equality:

$$\begin{vmatrix} \cos\theta_a & -\sin\theta_a \\ \cos\theta_a & \sin\theta_a \end{vmatrix} = 0 \;\Rightarrow\; \sin 2\theta_a = 0 \tag{1.29}$$

We will denote the roots of this equation by λ_{an}, which will have the following form:

$$\lambda_a = \pm\sqrt{a_{66}\left(\frac{\pi^2 n^2}{4a_{44}} - \Omega_*^2\right)}, \quad n = \overline{0, N} \tag{1.30}$$

In this case $V_b^{(0)}$ will have the following form:

$$V_{b2}^{(0)} = H_{1b}^{(0)} \sin\theta_2\zeta + H_{2b}^{(0)} \cos\theta_2\zeta \tag{1.31}$$

where from coefficients $H_{1b}^{(0)}, H_{2b}^{(0)}$ only a one, for example $H_{1b}^{(0)}$, is independent because of equation (1.29), and other is expressed by it. The roots of equation (1.29) are real. Therefore, for stresses we will get the following real expressions:

$$\sigma_{11b}^{(0)} = \sigma_{22b}^{(0)} = \sigma_{33b}^{(0)} = \sigma_{13b}^{(0)} = 0, \quad \sigma_{12b}^{(0)} = -\frac{\lambda_2}{a_{66}}\left(H_{1b}^{(0)} \sin\theta_2\zeta + H_{2b}^{(0)} \cos\theta_2\zeta\right)$$
$$\sigma_{23b}^{(0)} = -\frac{\theta_2}{a_{44}}\left(H_{1b}^{(0)} \cos\theta_2\zeta - H_{2b}^{(0)} \sin\theta_2\zeta\right) \tag{1.32}$$

So the out-of-plane problem is completely separated from the plane problem. As we know the all required quantities of boundary layer are proportional to $\exp(-\lambda\gamma)$:

$$Q_b \sim \exp(-\lambda\gamma)$$

where the real positive part of λ is attributed the rate of damping of boundary layer.

plastic SVAM 10:1

	λ_p	λ_a
1	$0.17174 - 0.293819\,i$	0.896929
2	$0.17174 + 0.293819\,i$	2.29646
3	$0.2191 - 0.0528\,i$	3.56686
4	$0.729788 + 0.0528\,i$	4.81152
5	0.8789	6.04636
6	0.9984	7.27639
7	1.41	8.50368

glass plastic STET

	λ_p	λ_a
1	$0.016-0.3448\,i$	1.24176
2	$0.016+0.3448\,i$	2.77092
3	$0.1817-0.3328\,i$	4.2314
4	$0.1817+0.3328\,i$	5.67646
5	0.5605	7.1155
6	1.2111	8.55155
7	1.32946	9.98592

In tables the first seven values of λ_p and λ_a are reduced for the plastic SVAM 10:1 ($E_1 = 38.259 10^9 \times \text{Pa}$, $E_2 = 17.658 \times 10^9 \text{Pa}$, $E_3 = 9.6138 \times 10^9 \text{Pa}$, $G_{12} = 5.1993 10^9 \text{Pa}$, $G_{13} = 3.8357\ 10^9 \text{Pa}$, $G_{23} = 3.1392\ 10^9 \text{Pa}$, $\nu_{12} = 0.22$, $\nu_{23} = 0.31$, $\nu_{31} = 0.07$, $h = 0.5\text{m}$, $\rho = 1900 \text{kg/m}^3$, $f_1 = 0.2$, $\Omega = 2\pi / 0.1$) and glass plastic STET

($E_1 = 35.2179\times10^9$ Pa , $E_2 = 28.7433\times10^9$ Pa , $E_3 = 17.9523\times10^9$ Pa, $G_{12} = 7.4556\times10^9$ Pa , $G_{13} = 6.4746\times10^9$ Pa , $G_{23} = 6.1803\times10^9$ Pa , $\nu_{12} = 0.177$, $\nu_{31} = 0.157$, $h = 0.5$m , $\rho = 1900\text{kg/m}^3$, $f_1 = 0.2$, $\Omega = 2\pi / 0.1$):

From the table it is obvious that the boundary layer damps in out-of-plane problem faster, then in the plane problem.

2. Now we consider the first boundary condition on the lateral face $\gamma = 0$:

$$\sigma_{xx} = \varphi(\eta,\zeta),\ \sigma_{xy} = \psi(\eta,\zeta),\ \sigma_{xz} = \chi(\eta,\zeta) \tag{2.1}$$

The general solution of the formulated problem is the sum of the solution of the inner problem and of boundary layer. It will be written in the following form:

$$I = I^{\text{int}} + I_b \tag{2.2}$$

For determination of the 3 unknown coefficients of solution corresponding to boundary layer, we will use the quadratic error on lateral face $\gamma = 0$, which will have following form:

$$J = \int_{-1}^{1}\left[\left(\sigma_{xx}^{\text{int}} + \sigma_{xxb} - \varphi(\eta,\zeta)\right)^2 + \left(\sigma_{xy}^{\text{int}} + \sigma_{xyb} - \psi(\eta,\zeta)\right)^2 + \left(\sigma_{xz}^{\text{int}} + \sigma_{xzb} - \chi(\eta,\zeta)\right)^2\right]d\zeta \tag{2.3}$$

It will be minimum on the lateral face $\gamma = 0$ if following derivatives are equal to zero:

$$\frac{\partial J}{\partial A_{1n}^{(s)}} = \frac{\partial J}{\partial A_{2n}^{(s)}} = \frac{\partial J}{\partial H_{1b}^{(s)}} = 0 \tag{2.4}$$

So, from this system of equation it is possible to find required coefficients and, consequently, to find the complete solution corresponding to boundary layer.

REFERENCES

1. Nayfeh A. Perturbation Methods. New York: John Willey and Sons. 1973. 455p.
2. Goldenveizer A. L. The Theory of Thin Elastic Shells. Moscow: Nauka. 1976. 510p.
3. Aghalovyan L. A. The Asymptotic Theory of Anisotropic Plates and Shells. Moscow: Nauka. Fizmatlit. 1997. 414p.
4. Aghalovyan L. A. A class of problems on the forced vibrations of anisotropic plates. //Problems of the Mechanics of Thin Deformable Bodies. Yerevan. Armenia. 2002. pp. 8-19.
5. Aghalovyan L. A. An asymptotic method of solving dynamic mixed problems of anisotropic strips and plates. //Izv Vuzov Severo-Kavkaz Region Yestesv Nauki 2000. Vol. 3. pp.8-11.
6. Aghalovyan L.A. Determination of frequencies of natural vibrations and eigenfunction in a three-dimensional mixed boundary-value problem for plates. //In: Proc. Conf. “Present Problems of the Optimal Control and Stability Systems“. Yerevan: Izd YeGU. 1997. pp. 128-31.
7. Aghalovyan L.A. The asymptotic form of the solutions of classical and non-classical boundary-value problems of the statics and dynamics of thin bodies.// Int. Appl. Mech. 2002. Vol.38. №7. pp. 3-24.
8. Sargsyan M. Z. The 3D problem on forced vibrations of orthotropic plate free situated on the rigid foundation.// In coll.: ”The problems of dynamics of interaction of deformable media”. Inst. of Mech. of NAS of RA. Yerevan. 2008. pp.399-403. 456p.

Институт механики
НАН Армении

Поступила в редакцию
9.03.2009