ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
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RECTANGULAR PLATE, UNDER TANGENTIAL LOADS WHEN TWO OPPOSITE EDGES ARE HINGED

K.L. Martirosyan

Ключевые слова: изгиб, шарнирное закрепление, упругость, пластина
Key words: bending, hinge joined, elasticity, plate

Ք.Լ. Մարտիրոսյան
Ուղղանկյուն սալը շոշափող բեռի առկայության դեպքում, երբ հանդիպակաց կողմերը հոդակապորեն ամրակցված են

Աշխատանքում դիտարկված է ուղղանկյուն սալը շոշափող բեռի առկայության դեպքում, երբ հանդիպակաց կողմերը հոդակապորեն ամրակցված են: Սալի լարվածադեֆորմացված վիճակի խնդիրը լուծված է դասական տեսության, առաջին կարգի ճշգրտված տեսության և բարձր կարգի ճշգրտված տեսության հիման վրա: Կատարված է համեմատություն տեղափոխությունների միջև:

К.Л. Мартиросян
Прямоугольная пластинка под действием касательных нагрузок, при шарнирном закреплении двух противоположных сторон

В работе рассматривается прямоугольная пластинка под действием касательных нагрузок при шарнирном закреплении двух противоположных сторон. Задача напряженного дефформированного состояния пластинки при наличии касательных нагрузок рассматривается на основе классической теории – Кирхгофа, на основе уточненной теории первого порядка – теории Рейснера-Генки-Миндлина по варианту Васильева, на основе уточненной теории высокого порядка – теории Амбарцумяна.

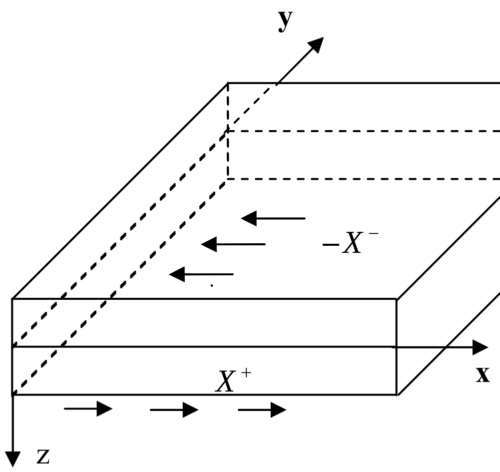
In this work a plate under the action of tangential loads, when two opposite edges are hinged is considered. The problem of stressed deformed state of plate under the action of the tangential loads is considered on the base of classical theory of Kirchhoff , on the base of refined theory of first order Reissner-Genki-Mindlin by Vasilyev variant, on the base of refined theory of high order of A. Ambartsumyan.

In 1957 Vlasov established the exact solution to a three -dimension problem of the bending of rectangular plate under the action of the transversal loads [1]. It was considered that on all edges of plate Navie conditions are given.
The solution is presented in the form of double series of the relative thickness.
It has been presented the comparison of this solution and solution by Kirchhoff theory. For thin plates when the square of the relative thickness is neglected by comparing to one, is coincides with the Kirchhoff solution.
In 1985 Levinson received the solution of the same problem on the base of semi inverse method [2].
In 1999 on the base of Levinson's solution V. Nicotra, P. Podio-Guidugli and A. Tiero considered transversal isotropic cylinder on which for all edges Navier conditions are given [3]. They obtained the exact solution to a three dimensional problem under the action as of the transversal loads and as tangential loads on the face surfaces.
After that in 2003 on the base of [2] the exact solution to a three -dimensional problem for transversely isotropic, linearly elastic body in the form of right cylinder under the action of the transversal load, on edges of which are sliding contact are received by P.Nardinocchi and P. Podio-Guidugli [4].
In 2007 on the base of [1], the problem of plate bending in the form of cylindrical surface under the action of tangential loads on the face planes is considered [5].
It has been considered that sliding contact conditions on the edges of the plate are given and on the face planes only tangential loads act. The problem is solved by using exact equations of theory of elasticity. For comparison with results of approximate theories, the Kirchhoff's

theory for plates and the theory, where the transversal shears are taken into account, it is necessary to produce the solution in the form of decomposition by the parameter of thickness ratio. The problem is also solved by using the refined theory of A. Ambartsumyan. In both cases the deflection decreases when the transversal shears are taken into account. This differs from normal loads action, which increases the deflection.
In this work a plate under the action of tangential loads, when two opposite edges are hinged is considered.

The problem of stressed deformed state of plate under the action of the tangential loads is considered on the base of classical theory of Kirchhoff (K), on the base of refined theory of first order Reissner-Genki-Mindlin by Vasilyev (V) variant [7], on the base of refined theory of high order of A. Ambartsumyan (A) [6].

1. Let the plate-band occupies area $0 \le x \le a$, $0 \le y \le b, -h \le z \le h$



On the face surfaces of plate are given tangential loads:

$$z = h: \quad \sigma_{33} = 0,\ \sigma_{31} = X^+(x,y),\ \sigma_{32} = 0$$
$$z = -h: \quad \sigma_{33} = 0,\ \sigma_{31} = -X^-(x,y),\ \sigma_{32} = 0 \tag{1.1}$$

Admissions for displacements [8] by the theories (K), (V) and (A) correspondingly:

$$U_1 = U - z\frac{\partial W}{\partial x}, \quad U_2 = V - z\frac{\partial W}{\partial y}, \quad U_3 = W \tag{1.2}$$

$$U_1 = U - z\theta_1, \quad U_2 = V - z\theta_2, \quad U_3 = W \tag{1.3}$$

$$U_1 = U - z\frac{\partial W}{\partial x} + \frac{z}{2G}\left(X_1 + \frac{z}{2h}X_2\right) + \frac{1}{G}g(z)\varphi_1,$$
$$U_2 = V - z\frac{\partial W}{\partial y} + \frac{1}{G}g(z)\varphi_2, \quad U_3 = W \tag{1.4}$$

Here U, V are displacements of median surface, W is deflection of plate and functions $\varphi_1, \varphi_2, \theta_1, \theta_2$, are independent of coordinate z, G is the shear modulus,

$$X_1 = X^+ - X^-, \quad X_2 = X^+ + X^-, \quad g(z) = z\left(1 - \frac{z^2}{3h^2}\right) \tag{1.5}$$

In general the problems of generalized plane deformation and of bending are not separated. They are separated only in special cases when $X_1 = 0$ or $X_2 = 0$.

If $X_2 = 0$ ($X^+ = -X^-$), then the tangential loads don't yield the bending and in this case only the problem of generalized state is supposed to be observe.

If $X_1 = 0$ ($X^+ = X^-$), then only the problem of bending is supposed to be solved.

It should be noted, that tangential loads are given only on one face surface, then the loads are require the solution of both problems.

We are received the following equations for planar displacement of middle surface of the plate by the theories (K) and (V):

$$\Delta U + \theta \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = -\frac{2X_2}{C(1-\nu)}$$
$$\Delta V + \theta \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y} + \frac{\partial U}{\partial x}\right) = 0, \quad \theta = \frac{1+\nu}{1-\nu} \tag{1.6}$$

by the theory (A):

$$\Delta U + \theta \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = -\frac{2X_2}{C(1-\nu)} - \frac{\theta h}{3E}\left(\frac{\partial^2 X_2}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 X_2}{\partial y^2}\right)$$
$$\Delta V + \theta \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y} + \frac{\partial U}{\partial x}\right) = -\frac{\theta h}{12G}\frac{\partial^2 X_2}{\partial x \partial y} \tag{1.7}$$

The equations for bending of plate are received by the theory (K):

$$\Delta^2 W = \frac{h}{D}\frac{\partial X_1}{\partial x} \tag{1.8}$$

by the theory (V):

$$\Delta W - \frac{\partial \theta_1}{\partial x} - \frac{\partial \theta_2}{\partial y} = 0$$
$$D\left[\Delta\theta_1 + \theta\frac{\partial}{\partial x}\left(\frac{\partial\theta_1}{\partial x} + \frac{\partial\theta_2}{\partial y}\right)\right] + \frac{4Gh}{1-\nu}\left(\frac{\partial W}{\partial x} - \theta_1\right) = \frac{2h}{1-\nu}X_1 \tag{1.9}$$
$$D\left[\Delta\theta_2 + \theta\frac{\partial}{\partial y}\left(\frac{\partial\theta_2}{\partial y} + \frac{\partial\theta_1}{\partial x}\right)\right] + \frac{4Gh}{1-\nu}\left(\frac{\partial W}{\partial y} - \theta_2\right) = 0$$

by the theory (A):

$$\frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{\partial y} = -\frac{3}{4}\frac{\partial X_1}{\partial x}$$
$$D\frac{\partial}{\partial x}\Delta W - \frac{8h^3}{15}\left[\Delta\varphi_1 + \theta\frac{\partial}{\partial x}\left(\frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{\partial y}\right)\right] + \frac{4h}{3}\varphi_1 =$$
$$= \frac{2h^3}{3(1-\nu)}\left(\frac{\partial^2 X_1}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 X_1}{\partial y^2}\right) \tag{1.10}$$
$$D\frac{\partial}{\partial y}\Delta W - \frac{8h^3}{15}\left[\Delta\varphi_2 + \theta\frac{\partial}{\partial y}\left(\frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{\partial y}\right)\right] + \frac{4h}{3}\varphi_2 = \frac{h^3\theta}{3}\frac{\partial^2 X_1}{\partial x \partial y}$$
$$D\frac{\partial}{\partial y}\Delta W - \frac{8h^3}{15}\left[\Delta\varphi_2 + \theta\frac{\partial}{\partial y}\left(\frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{\partial y}\right)\right] + \frac{4h}{3}\varphi_2 = \frac{h^3\theta}{3}\frac{\partial^2 X_1}{\partial x \partial y}$$

2. Let us consider the rectangular plate which occupies area:

$$0 \le x \le a,\ 0 \le y \le b,\ -h \le z \le h.$$

The tangential loads are given in the following form:

$$X_2 = \tau_0 \sin\lambda_1 y \text{ where } \lambda_1 = \frac{\pi}{b} \tag{2.1}$$

It has been assumed, that on the edges $y = 0,\ b$ of plate are given the hinge joined conditions by the theories (K) и (V):

$$\frac{\partial V}{\partial y} = 0, \quad U = 0 \text{ on } y = 0, b \tag{2.2}$$

by the theory (A):

$$U = -\frac{h}{12G} X_2, \quad \frac{\partial V}{\partial y} = 0 \text{ on } y = 0, b \tag{2.3}$$

and on the face surfaces of plate are given only tangential loads (1.1).

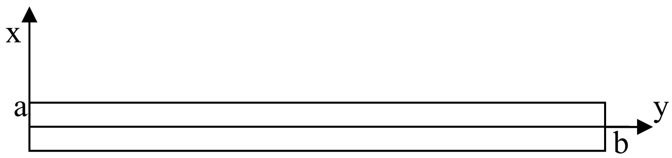
Let on the edges $x = 0, a$ are given the sliding contact by the theories (K) and (V):

$$U = 0, \quad \frac{\partial V}{\partial x} = 0 \text{ on } x = 0, a \tag{2.4}$$

by the theory (A):

$$U = -\frac{h}{12G} X_2, \quad \frac{\partial V}{\partial x} = 0 \text{ on } x = 0, a \tag{2.5}$$

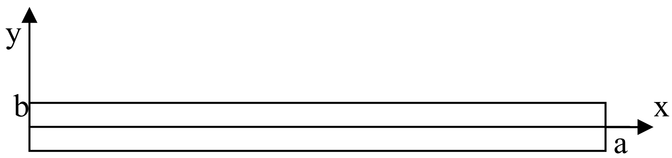
For the thin plate $\beta^2 << 1 \Leftrightarrow a << b$



the statements for efforts by theories (K), (V) and (A), when x=0, are:

$$T_1 = \frac{(1-\nu)}{\lambda_1} \beta\theta\tau_0 \sin\lambda_1 y, \quad T_2 = \frac{\beta}{2\lambda_1}\tau_0\left(\nu - \frac{2\beta^2}{3}\right)\sin\lambda_1 y \tag{2.6}$$

For long plate $\beta^2 >> 1 \Leftrightarrow a >> b$



the statements for efforts by theories (K), (V) and (A), when x=0, are:

$$T_1 = \frac{(3+\nu) b\tau_0}{2\pi} \sin\lambda_1 y, \quad T_2 = \frac{b\tau_0}{2\pi} \sin\lambda_1 y \tag{2.7}$$

We should note that problems, where tangential loads are acting were considered in the references [9], [10], [11].

For thin $\beta^2 << 1$ and long plates $\beta^2 >> 1$ the statement for efforts by theories (K), (V) when $x = 0$ is:

$$S = -\frac{b\tau_0}{\pi}\cos\lambda_1 y \tag{2.8}$$

For thin $\beta^2 << 1$ and long plates $\beta^2 >> 1$ the statement for efforts by theory (A) when $x = 0$ is:

$$S = -\frac{b\tau_0}{\pi}\left(1 - \frac{h^2\lambda_1^2}{6}\right)\cos\lambda_1 y \tag{2.9}$$

3. Here the problem of bending of rectangular plate is considered.
It is supposed that the tangential loads are given in the following form:

$$X_1 = \tau_0 \sin\lambda_1 y \text{ where } \lambda_1 = \frac{\pi}{b} \tag{3.1}$$

We assume, that on the edges $y = 0, b$ of plate are given the hinge joined conditions by the theories (K):

$$W = 0, \quad \frac{\partial^2 W}{\partial y^2} = 0 \text{ on } y = 0, b \tag{3.2}$$

by the theory (V):

$$W = 0, \quad \theta_1 = 0, \quad \frac{\partial\theta_2}{\partial y} = 0 \text{ on } y = 0, b \tag{3.3}$$

by the theory (A):

$$\mathrm{W} = 0, \quad \varphi_1 = -\frac{5}{8}X_1, \quad \frac{\partial^2 W}{\partial y^2} - \frac{4}{5G}\frac{\partial\varphi_2}{\partial y} = 0 \text{ on } y = 0, b \tag{3.4}$$

and on the face surfaces of plate only tangential loads are given (1.1).
Let us consider, that on the edges $x = 0, a$ are given the conditions of the sliding contact by the theories (K):

$$\frac{\partial W}{\partial x} = 0, \quad \frac{\partial^3 W}{\partial x^3} = \frac{h}{D}X_1 \text{ on } x = 0, a \tag{3.5}$$

by the theory (V):

$$\frac{\partial W}{\partial x} = 0, \quad \theta_1 = 0, \quad \frac{\partial\theta_2}{\partial x} = 0 \text{ on } x = 0, a \tag{3.6}$$

by the theory (A):

$$\varphi_1 = -\frac{3}{4}X_1 \quad \frac{\partial W}{\partial x} + \frac{1}{10G}X_1 = 0, \quad \frac{\partial\varphi_2}{\partial x} - \frac{1}{8}\frac{\partial X_1}{\partial y} = 0 \text{ on } x = 0, a \tag{3.7}$$

For thin plate $\frac{a}{b} << 1$ the statements for bending by theories (K) and (A) when x=0 are:

When $(a\lambda)^2 << 1$

$$\text{(K)}\, W = \frac{ha^3(1 - a\lambda_1)\tau_0}{6D\,(2 + a\lambda_1)^2}\sin\lambda_1 y \tag{3.8}$$

$$\text{(A)}\, W = \frac{ha^3(1 - a\lambda_1)\tau_0}{6D\,(2 + a\lambda_1)^2}\left(1 + \frac{2h^2\lambda_1^2}{15} + \frac{8h^2}{5a^2(1-\nu)}\,\frac{1 + a\lambda_1}{1 - a\lambda_1}\right)\sin\lambda_1 y \tag{3.9}$$

We should note that when $\frac{2h^2\lambda_1^2}{15}+\frac{8h^2}{5a^2(1-\nu)}\frac{1+a\lambda_1}{1-a\lambda_1}<<1$ the deflections by theories (K) and (A) are coincide. $a\lambda<<1$

$$(\text{K})W=\frac{h\,a^3\tau_0}{24D}\sin\lambda_1 y\,,\ (\text{A})W=\frac{ha^3\tau_0}{24D}\left(1+\frac{2h^2\lambda_1^2}{15}+\frac{8h^2}{5a^2(1-\nu)}\right)\sin\lambda_1 y \quad (3.10)$$

and when $\frac{2h^2\lambda_1^2}{15}+\frac{8h^2}{5a^2(1-\nu)}<<1$ the deflections by theories (K) and (A) are coincide.

For long plates $a/b>>1$ the deflections by theories (K) and (A) when $x=0$ are:

$$(\text{K})W=\frac{h\,\tau_0}{2D\lambda_1^3}\sin\lambda_1\,,\ (\text{A})W=\frac{h\,\tau_0}{2D\lambda_1^3}\left(1-\frac{2h^2\lambda_1^2\,\theta}{15}\right)\sin\lambda_1 y \quad (3.11)$$

We should note that when $2h^2\lambda_1^2\,\theta/15<<1$ the deflections by theories (K) and (A) are coincide.

We received also the deflection by the theory (V), which has a significant difference from the deflection by the theory (A) since by the theory (A) the tangential stresses satisfy the conditions on the face planes, in contrast to theory (V) as the theory (A) has higher order, then theory (V).

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Институт механики
НАН Армении

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