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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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THE METHOD'S OF COMPLEX ANALYSIS FOR THE SOLUTION OF PLANE PROBLEMS OF THE THEORY OF THERMOCONDUCTIVITY AND THERMOELASTISITY FOR MULTIPLY CONNECTED BODIES.

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Ключевые слова: многосвязанные тела, теория теплопроводности и термоупругости, комплексный анализ, плоские задачи, сингулярные интегральные уравнения, трещины, стрингеры.
Key words: multiply connected bodies, theory of thermoconductivity and thermoelasticcity, complex analysis, singular integral equations, craks, stringers.

Դ.Ի. Բարձոկաս, Պ. Լալոու
Կոմպլեքս անալիզի մեթոդներով ջերմահաղորդականության և ջերմաառաձգականության տեսության հարթ խնդիրների լուծումը բազմակապ մարմինների համար

Կոմպլեքս փոփոխականի ֆունկցիաների տեսության և սինգուլյար ինտեգրալ հավասա-րումների մեթոդների կիրառմամբ զարգացվում է ճաքեր, ինչպես նաև ուղղագիծ և կորագիծ ստրինգերներ պարունակող բազմակապ իզոտրոպ մարմինների համար ջերմահաղորդականության և ջերմաառաձգականության տեսության հարթ խնդիրների լուծման մեթոդ: Դիտարկված են կոնկրետ օրինակներ:

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Решение плоских задач теории теплопроводности и термоупругости для многосвязных тел методами комплексного анализа

Применением методов теории функций комплексного переменного и сингулярных интегральных уравнений развивается метод решения плоских задач теории теплопроводности и термоупргости для многосвязных изотропных тел, содержащих трещины, а также прямолинейные и криволинейные стрингеры. Рассматриваются конкретные примеры.

1. INTRODUCTION

During their service life, engineering structures are subjected not only to static and dynamic loads, but usually they are also affected by the presence of thermal fields. Thermal influences may sometimes alter the physic-mechanical properties of the materials and consequently, they may affect their strength properties and the resistance of the structure to loads. In the general case, the resulting expansions (contractions) are not occurring freely in the continuous medium. Instead, they produce thermal stresses which, in combination with the mechanical ones (due to external loading), can contribute to the initiation and propagation of cracks.

Even if the failure mechanism cannot, in many cases, be completely described only by the propagation of cracks in materials, the investigations of the conditions which trigger the initiation of a crack or a crack system from the pre-existing defects in the material (cracks, inclusions, cavities, welded joints etc.) is of great theoretical and practical importance. For this reason, the present investigation is concerned with the study of the stress-deformation state of a body under the influence of mechanical and thermal fields of forces, in the regions where singularities or stress concentrators exist, by using the complex functions method and the theory of singular integral equations [1, 2, 3]. The methodology applied is based on the theory of linear elasticity and thermoelasticity of the anisotropic medium and is an extension of a previous work [4], since it includes the effects of curvilinear (in the form of circular arcs) thin strip inclusions and holes.

The present development of this general method permits one to use it effectively for studying the interaction between various types of defects in an isotropic or anisotropic material and reinforcements, in the presence of mechanical and thermal fields of forces, and furthermore to extend it for solving many important problems of engineering practice.

2. FUNDAMENTAL EQUATIONS OF PLANE PROBLEMS IN THERMOCONDUCTIVITY AND THERMOELASTICITY

For the formulation of the mathematical theory of the strength of isotropic or anisotropic bodies characterized by defects in the form of cracks, holes, inclusions etc., under the influence of mechanical and thermal fields of forces, the model of the linear thermo elastic body is used [4]. A general theory of this model assumes that:

a) the strain components are infinitesimal;

b) the relationships between various components of stresses and strains are given by the generalized linear Hooke's law, and

c) the elastic and thermal properties of the body are, in general, different in different directions, but they are independent of the temperature and stress.

Furthermore, it is assumed that at any point of an anisotropic body, a plane of elastic and thermal symmetry exists. Also, it is assumed that the temperature $T(x, y, z, t)$ in an anisotropic body is a continuous function of spatial coordinates x, y, z and time t; and that this holds also for the first differential coefficient with respect to t and for the first and second differential coefficients with respect to x, y and z. The body is referred to a Cartesian or curvilinear coordinate system with unit vectors $\vec{i}, \vec{j}$ and $\vec{k}$. Accordingly, an elementary surface characterized by a normal vector $\vec{n}$ which contains a random point of the body is considered. At the point under consideration, the thermo conductivity vector $\overrightarrow{K_n}$ which refers to the elementary surface with normal vector is defined by

$$\overrightarrow{K_n} = a_1 k_{11} \vec{i} + a_2 k_{22} \vec{j} + a_3 k_{33} \vec{k} \tag{1}$$

Where k_{ii} $\left(i = \overline{1,3}\right)$ are the coefficients of thermal conductivity, and a_i are the direction cosines between the vector $\vec{n}$ and the unit vectors $\vec{i}, \vec{j}$ and $\vec{k}$. The surfaces where the thermal conductivity vector $\overrightarrow{K_n}$ coincides with the normal vector $\vec{n}$ are called principal surfaces of thermal conductivity, whereas the directions normal to them are called principal directions of thermal conductivity. Accordingly, the density of the thermal flux q_n across the elementary surface with normal direction is defined as

$$q_n = -(\overrightarrow{K_n} \cdot \overrightarrow{\mathrm{grad} T}) \tag{2}$$

The surface which is crossed by the thermal flux of maximum density is called principal surface of thermal flux, and the direction perpendicular to it is called principal direction of thermal flux at the point under consideration.

In the case where the axes of the reference system coincide with the principal directions of the thermal conductivity, the thermal field is described by the following differential equation

$$k_{11} \frac{\partial^2 T}{\partial x^2} + k_{22} \frac{\partial^2 T}{\partial y^2} + k_{33} \frac{\partial^2 T}{\partial z^2} = c\rho \frac{\partial T}{\partial t} - Q, \tag{3}$$

Where c is the specific heat of the body, ρ expresses its density, and Q is the quantity of heat which is radiated from the unit volume per unit of time.

In order to find the solution of the partial differential equation (3) in space and time domains, the initial and boundary conditions should be known a priori. The boundary or surface conditions of thermal conductivity encountered in practice are [5]:

i. Boundary conditions of the first kind, when the values of temperature are given at all points of the surface of the body,

$$T = f_1(x, y, z, t) \tag{4}$$

ii. Boundary conditions of the second kind, when the values of density of the thermal flux are given at all points of the surface of the body,

$$\overrightarrow{K_n} \cdot \overrightarrow{\mathrm{grad}T} = f_2(x, y, z, t) \tag{5}$$

iii. Boundary conditions of the third kind, called also "radiation boundary conditions", when the conditions of thermal exchange with the surrounding medium (of temperature T_o) are given at all points of the surface of the body,

$$\overrightarrow{K_n} \cdot \overrightarrow{\mathrm{grad}T} = \lambda(T - T_o), \tag{6}$$

where λ is the coefficient of surface heat transfer.

In the sequel, the fundamental equations of thermoelasticity will be given. For this purpose let us consider a cylindrical body with the generatrix of its lateral surface perpendicular to the plane of the plane of the Cartesian coordinate system, and its end faces being thermally insulated. Further it is assumed that the temperature at any point of the body depends on the spatial coordinates x, y and that the body is characterized by linear thermal anisotropy, such that at any point one of the principal directions of thermal conductivity is perpendicular to the plane xOy .If the body is homogeneous and it does not contain any thermal source, then Eq. (3) takes the form

$$\lambda_{11}\frac{\partial^2 T}{\partial x^2} + 2\lambda_{12}\frac{\partial^2 T}{\partial x \partial y} + \lambda_{22}\frac{\partial^2 T}{\partial y^2} = 0, \tag{7}$$

Where $\lambda_{11} = k_{11}\cos^2 a + k_{22}\sin^2 a$

$$\lambda_{22} = k_{11}\sin^2 a + k_{22}\cos^2 a$$
$$\lambda_{12} = (k_{11} - k_{22})\sin a \cos a \tag{8}$$

With a being the angle between the Ox -axis and one of the principal directions of thermal conductivity, and the quantities k_{ij}, λ_{ij} ($i, j = 1,2$) are constant.

The general solution of Eq.(7) is given in the form [6]:

$$T = F(z_3) + \overline{F(z_3)} = 2\,\mathrm{Re}\,F(z_3) \tag{9}$$

Where the overbear denotes complex conjugate, Re denotes the real part of what follows and $F(z_3)$ is an analytic function of the complex variable z_3 .Parameter μ_3 is one of the roots of the characteristic equation

$$\lambda_{22}\mu^2 + 2\lambda_{12}\mu + \lambda_{11} = 0 \tag{10}$$

Where $\mu_3 = -\lambda_{12} + i(k_{11}k_{22})^{1/2} / \lambda_{22}$

With i denoting the usual imaginary unit.

The thermal flux as a function of $F(z_3)$ is expressed by a function of β_1, β_2

Which are the direction cosines between the normal vector n and the element ds

$$\overrightarrow{K_n} \cdot \overrightarrow{\mathrm{grad}T} = (\lambda_{11}\frac{\partial T}{\partial x} + \lambda_{12}\frac{\partial T}{\partial y})\beta_1 + (\lambda_{12}\frac{\partial T}{\partial x} + \lambda_{22}\frac{\partial T}{\partial y})\beta_2 \tag{11}$$

By virtue of Eq. (9), relationship (11) assumes the following form

$$\overrightarrow{K_n} \cdot \overrightarrow{\text{grad}T} = A_1^* F'(z_3) + \overline{A_1^* F'(z_3)} \tag{12}$$

where $\left(' \equiv \frac{d}{dz_3} \right)$, and $A_1^* = (\lambda_{12} + \mu_3 \lambda_{22})(-\beta_2 + \mu_3 \beta_1)$

Basing on relationships (9) and (12), we can find the temperature and the thermal flux at any point of the body, if the mathematical form of the thermal potential $F(z_3)$ is a priori known.

At any point of homogeneous and anisotropic body under a plane strain state, where exists a plane of elastic symmetry which is perpendicular to Oz -axis and coincides with one of the principal directions of thermal conductivity. The generalized Hooke's law in this case takes the form

$$\begin{aligned}
\varepsilon_{xx} &= \alpha_{11}\sigma_{xx} + \alpha_{12}\sigma_{yy} + \alpha_{13}\sigma_{zz} + \alpha_{16}\sigma_{xy} + \beta_{11}T, \\
\varepsilon_{yy} &= \alpha_{12}\sigma_{xx} + \alpha_{22}\sigma_{yy} + \alpha_{23}\sigma_{zz} + \alpha_{26}\sigma_{xy} + \beta_{22}T, \\
\gamma_{xy} &= \alpha_{16}\sigma_{xx} + \alpha_{26}\sigma_{yy} + \alpha_{36}\sigma_{zz} + \alpha_{66}\sigma_{xy} - 2\beta_{66}T, \\
\varepsilon_{zz} &= \alpha_{13}\sigma_{xx} + \alpha_{23}\sigma_{yy} + \alpha_{33}\sigma_{zz} + \alpha_{36}\sigma_{xy} + \beta_{33}T = 0, \\
\gamma_{yz} &= \alpha_{44}\sigma_{yz} + \alpha_{45}\sigma_{xz} = 0, \; \gamma_{xz} = \alpha_{45}\sigma_{yz} + \alpha_{55}\sigma_{xz} = 0,
\end{aligned} \tag{13}$$

or alternatively

$$\begin{aligned}
\varepsilon_{xx} &= c_{11}\sigma_{xx} + c_{12}\sigma_{yy} + c_{16}\sigma_{xy} + \alpha_1 T \\
\varepsilon_{yy} &= c_{12}\sigma_{xx} + c_{22}\sigma_{yy} + c_{26}\sigma_{xy} + \alpha_2 T \\
\gamma_{xy} &= c_{16}\sigma_{xx} + c_{26}\sigma_{yy} + c_{66}\sigma_{xy} - 2\alpha_6 T
\end{aligned} \tag{14}$$

where a_{ij}, c_{ij} express the elasticity coefficients and are related by

$$c_{ij} = a_{ij} - \frac{a_{i3}a_{j3}}{a_{33}} \qquad i, j = 1,2,6 \tag{15}$$

In the sequel β_{ij} are the coefficients which given the strain tensor components of a body element free form external tractions, due to a temperature change of one degree. For these coefficients then following relationships hold true:

$$a_i = \beta_{ii} - \frac{\beta_{33}a_{i3}}{a_{33}} \quad (i = 1,2) \qquad a_6 = \beta_{66} + \frac{\beta_{33}a_{36}}{2a_{33}} \tag{15΄}$$

Under the condition that coefficients $c_{ij}, a_i, \lambda_{ij}$ remain constant and independent of the variations of the stress components and the temperature of the body, the relationships giving the stresses and displacements as a function of the complex potentials $\Phi(z_1), \Psi(z_2)$ and $F(z_3)$ are the following:

$$\begin{aligned}
\sigma_{xx} &= 2\,\mathrm{Re}[\mu_1^2\Phi(z_1) + \mu_2^2\Psi(z_2) + \eta_o\mu_3 F(z_3)] \\
\sigma_{yy} &= 2\,\mathrm{Re}[\Phi(z_1) + \Psi(z_2) + \eta_o F(z_3)] \\
\sigma_{xy} &= -2\,\mathrm{Re}[\mu_1\Phi(z_1) + \mu_2\Psi(z_2) + \eta_o\mu_3 F(z_3)] \\
u &= 2\,\mathrm{Re}[p_1\phi(z_1) + p_2\psi(z_2) + p_*\psi(z_3)] \\
v &= 2\,\mathrm{Re}[q_1\phi(z_1) + q_2\psi(z_2) + q_*\psi(z_3)] \\
p_j &= c_{11}\mu_j^2 + c_{12} - c_{16}\mu_j \qquad \mu_j q_j = c_{12}\mu_j^2 + c_{22} - c_{26}\mu_j \qquad j = 1,2
\end{aligned} \tag{16}$$

And $p_* = a_1 + \eta_o(c_{11}\mu_3^2 - c_{16}\mu_3 + c_{12})$ $\quad \mu_3 p_* = a_2 + \eta_o(c_{11}\mu_3^2 - c_{26}\mu_3 + c_{22})$

$$\eta_o = -(a_1\mu_3^2 + 2a_6\mu_3 + a_2)/\Delta(\mu_3)$$
$$\Delta(\mu_3) = c_{11}(\mu_3 - \mu_1)(\mu_3 - \mu_2)(\mu_3 - \overline{\mu_1})(\mu_3 - \overline{\mu_2}) \tag{17}$$
$$\Phi(z_1) = \phi'(z_1) \qquad \Psi(z_2) = \psi'(z_2) \qquad F(z_3) = \psi'(z_3)$$

For the transversely isotropic body, Hooke's law in plane strain conditions takes the following simplified form

$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu_z}{E_z}\sigma_z + \beta_{11}T, \quad \varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y - \frac{\nu_z}{E_z}\sigma_z + \beta_{11}T,$$
$$\varepsilon_z = -\frac{\nu_z}{E_z}\left(\sigma_x + \sigma_y\right) + \frac{1}{E}\sigma_z + \beta_{33}T = 0, \quad \gamma_{xy} = \frac{1}{G_{xy}}\tau_{xy}, \tag{18}$$

and for the generally isotropic body,

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right) + aT, \ \varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) + aT, \ \gamma_{xy} = \frac{1}{G}\tau_{xy}. \tag{19}$$

In the above relations the coefficients β_{11} and β_{33} are, respectively, the thermal coefficients of linear expansion on the plane of isotropy (parallel to the plane xOy), and along the direction perpendicular to the plane of isotropy.

The relationships for the derivation of the stress tensor and displacement vector components are simplified as follows:

$$\sigma_{xx} + \sigma_{yy} = 2[\Phi(z) + \overline{\Phi(z)}]$$
$$(\sigma_{yy} - \sigma_{xx}) + 2i\sigma_{xy} = 2[\bar{z}\Phi'(z) + \Psi(z)] \tag{20}$$
$$2\mu(u + iv) = \kappa\phi(z) - z\overline{\Phi(z)} - \overline{\psi(z)} + \beta\int F(z)dz \quad T(x,y) = \operatorname{Re} F(z)$$

Where for the case of transversely isotropic body

(a) $\kappa = 1 + \dfrac{2E}{1+\nu}\left(\dfrac{1-\nu}{E} - \dfrac{2\nu_z}{E_z}\right)$, $\beta = \dfrac{2E}{1+\nu}\left(\beta_{11} + \nu_z\beta_{33}\right)$, (plane strain case),

(b) $\kappa = \dfrac{3-\nu}{1+\nu}$, $\beta = \dfrac{2E\beta_{11}}{1+\nu}$, (generalized plane stress case)

And for the isotropic body

(c) $\kappa = 3 - 4\nu$, $\beta = aE$, (plane strain case),

(d) $\kappa = \dfrac{3-\nu}{1+\nu}$, $\beta = \dfrac{aE}{1+\nu}$, (generalized plane stress case)

In the case where at the point $\left(x_0, y_0\right)$ inside the orthotropic medium a thermal source of power q_0 exists, the complex potentials in the region enclosing this point take the form $\left(j = \overline{1,3}\right)$

$$\phi\left(z_1\right) = a_0'\left(z_1 - t_1\right)\ln\left(z_1 - t_1\right), \quad \psi\left(z_2\right) = \beta_0'\left(z_2 - t_2\right)\ln\left(z_2 - t_2\right),$$
$$\psi\left(z_3\right) = m_0\left(z_3 - t_3\right)\ln\left(z_3 - t_3\right), \ m_0 = -\frac{q_0}{4\pi\sqrt{k_{11}k_{22}}}, t_j = x_0 + \mu_j y_0, \tag{21}$$

And the coefficients a_0', β_0' are given from the following relations:

$$\alpha_0' = \frac{m - n\mu_2}{\mu_1 - \mu_2},\ \beta_0' = -\frac{m - n\mu_1}{\mu_1 - \mu_2},\ \mathrm{Im}\left[m(\mu_1 + \mu_2) - n\mu_1\mu_2 - m\lambda_0 / c_{11}\right] = 0,$$

$$\mathrm{Im}\left[m\mu_1\mu_2 + n\mu_1\mu_2(\mu_1 + \mu_2) - m_0\left(a_1\mu_3 - \lambda_0(\mu_3 - \mu_1 - \mu_2)\right)/c_{11}\right] = 0,$$

with $\lambda_0 = \dfrac{\left(a_1\mu_3^2 + 2a_6\mu_3 + a_2\right)}{\left(\mu_3 - \overline{\mu_1}\right)\left(\mu_3 - \overline{\mu_2}\right)}$

For the case of the transversely isotropic or generally isotropic medium, the corresponding complex potentials take the following form:

$$\Phi(z) = A_0 \ln(z - z_0),\quad \psi(z) = -\frac{A_0 \overline{z_0}}{z - z_0},\ F(z) = m_0 \ln(z - z_0), \tag{22}$$

where $A_0 = -\dfrac{\beta m_0}{1 + \kappa}, \qquad m_0 = -\dfrac{q_0}{4\pi\lambda}.$

Recapitulating, the solution of the plane problem of steady state thermoelasticity is derived in two consecutive stages. In the first stage the steady thermal field $T(x, y)$ is derived satisfying one of the boundary conditions (4)-(6) and the differential thermo elasticity equation (7) for the anisotropic medium, or the Laplace equation

$$\frac{\partial^2 T(x, y)}{\partial x^2} + \frac{\partial^2 T(x, y)}{\partial y^2} = 0 \tag{23}$$

for the isotropic medium. The second stage refers to the derivation of the stress tensor and displacement vector components by using relations (16) or (20).

3. THERMAL CONTACT CONDITIONS BETWEEN TWO BODIES.

At the first stage of the solution of the thermo elastic problem for bodies with thin inclusions and cracks it is of great importance to describe correctly the phenomenon of thermal conductivity along the lips of the crack and the contact interfaces of the thin inclusion with the body. This is achieved by properly choosing the representative computational model of thermal contact between the bodies characterized by different elastic and thermal constants. Following [7] approach to the formulation of the model which describes the condition of thermal contact, it is assumed that the contact surfaces are separated by a thin interlayer (inclusion) with the same thermo-physical parameters (Fig.1). If these parameters are assumed to be constant and the thickness of the interlayer tends to zero, it takes the form of a physical separating surface of the two bodies, and the corresponding boundary conditions on this surface correspond to the real contact condition of the two bodies.

The thermo conductivity equation of the embedded layer (isotropic inclusion) referred to the coordinate system (n, s) is the following:

$$\frac{\partial^2 T_c}{\partial n^2} + \frac{\partial^2 T_c}{\partial s^2} = 0 \tag{24}$$

On the separating surfaces $n = \pm h$ of the isotropic inclusion and the anisotropic medium, the following conditions of thermal contact are satisfied

$$T_c(s, \pm h) = T^{\pm}, \qquad -\lambda \left.\frac{\partial T_c}{\partial n}\right|_{n=\pm h} = T_n^{\pm}, \tag{25),(26}$$

where λ is the coefficient of thermal conductivity of the inclusion ,

$$T_n^{\pm} = -\left(\overrightarrow{K_n} \cdot \overrightarrow{\mathrm{grad} T}\right)^{\pm} \tag{27}$$

and $T^{\pm}, T_n^{\pm}$ express the limiting values of the temperature and the thermal flux along the boundary $n = \pm h$ of the anisotropic medium.

Then the following integral representations are introduced:

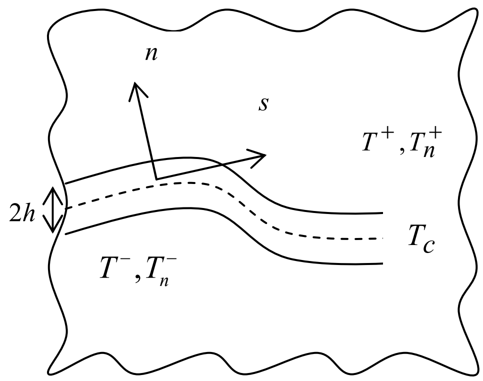


Fig.1. Contact of two infinite elastic bodies, with the contact region to be represented by a thin inclusion characterized by the same thermo physical properties as those of the two bodies.

$$T_c^* = \frac{1}{2h}\int_{-h}^{h} T_c dn, \tag{28}$$

$$T_c^{**} = \frac{3}{2h^2}\int_{-h}^{h} T_c n dn. \tag{29}$$

Multiplying relationship (24) by $1/2h$ and integrating with respect to n in the range $(-h, h)$ and by virtue of the relations (25) and (26), the following equation is derived:

$$\lambda_s \frac{\partial^2 T_c^*}{\partial s^2} + \left(T_n^+ - T_n^-\right) = 0, \qquad \lambda_s = 2\lambda h. \tag{30}$$

Furthermore, multiplying (24) by $3n/2h^2$ and integrating with respect to n in the range $(-h, h)$ we get

$$\lambda_s \frac{\partial^2 T_c^{**}}{\partial s^2} + 3\lambda\left(T_n^+ + T_n^-\right) - 6\lambda_n\left(T^+ - T^-\right) = 0, \quad \lambda_n = \lambda/2h \tag{31}$$

In order to derive the expressions for the quantities T_c^*, T_c^{**} with respect to the limiting values of the temperature $T^{\pm}$ of the anisotropic body, we use the operational expression of the solution of (24) which can be written as follows:

$$\frac{\partial^2 T_c}{\partial n^2} + p^2 T_c = 0 \qquad \left(\mathrm{p}^2 = \frac{\partial^2}{\partial s^2}\right) \tag{32}$$

By considering condition the expression gives the following solution;

$$T_c = \frac{\left(T^+ + T^-\right)}{2\cos ph}\cos pn + \frac{\left(T^+ + T^-\right)}{2\sin ph}\sin pn. \tag{33}$$

By virtue of (33), relationships (28) and (29) become

$$T_c^* = \frac{\left(T^+ + T^-\right)}{2ph}\text{tg}ph, \quad T_c^{**} = \frac{3}{2p^2h^2}\left(T^+ + T^-\right)\left(1 - ph\text{ctg}ph\right). \tag{34}$$

Substituting the values of T_c^* and T_c^{**} found from (34) into relations (30) and (31), respectively, and for $h \to 0$, we get the following relations:

$$\lambda_s \frac{\partial^2}{\partial s^2}\left(T^+ + T^-\right) + 2\left[\left(\overrightarrow{K_n} \cdot \overrightarrow{\text{grad}T}\right)^+ - \left(\overrightarrow{K_n} \cdot \overrightarrow{\text{grad}T}\right)^-\right] = 0, \tag{35}$$

$$\lambda_s \frac{\partial^2}{\partial s^2}\left(T^+ - T^-\right) + 6\left[\left(\overrightarrow{K_n} \cdot \overrightarrow{\text{grad}T}\right)^+ + \left(\overrightarrow{K_n} \cdot \overrightarrow{\text{grad}T}\right)^-\right] - 12\lambda_n\left(T^+ - T^-\right) = 0.$$

The above relations represent the conditions of "non-ideal thermal contact'' at the surface of the anisotropic medium. In the case of the isotropic medium, relations (35) are simplified as follows,

$$\begin{aligned} &\lambda_s \frac{\partial^2}{\partial s^2}(T^+ + T^-) + 2\lambda^*\left[\left(\frac{\partial T}{\partial n}\right)^+ - \left(\frac{\partial T}{\partial n}\right)^-\right] = 0 \\ &\lambda_s \frac{\partial^2}{\partial s^2}(T^+ - T^-) + 6\lambda^*\left[\left(\frac{\partial T}{\partial n}\right)^+ + \left(\frac{\partial T}{\partial n}\right)^-\right] - 12\lambda_n(T^+ - T^-) = 0 \end{aligned} \tag{36}$$

where λ^* is the coefficient of thermal conductivity of the isotropic medium.

In the case when instead of a thin inclusion we have a crack, the values of λ_s and λ_n characterize its thermal conductivity in the longitudinal and transverse directions, respectively. Depending on thermal conductivity we distinguish three categories of cracks:

a) a thermally conducting crack for $\lambda_s \neq 0, \lambda_n \neq 0$;

b) a longitudinally thermally insulating crack for $\lambda_s = 0, \lambda_n \neq 0$;

c) a thermally insulating crack for $\lambda_s = \lambda_n = 0$.

4. STATEMENT OF THE PROBLEM OF A MULTI-CONNECTED BODY.

Let us consider an infinite isotropic plate S containing M internal curvilinear cracks $l_j \ \left(j = \overline{1, M}\right)$, N thin strip inclusions (stringers) $L_j \ \left(j = \overline{1, N} \ \ or \ \ j = \overline{1, n_1^{'} + n_2^{'}}\right)$; $n_1^{'}$ denotes the number of straight stringers, whereas $n_2^{'}$ the number of curvilinear (circular arcs) stringers and L the number of holes indexed as $\gamma_j \ \left(j = \overline{1, L}\right)$. The plate is subjected to biaxial state of stresses $\left(N_1, N_2\right)$ at infinity and is under the influence of a homogeneous thermal flow q_∞. Besides these loading conditions, concentrated forces $P_j + iQ_j$ are acting at the points $z_j^*\left(j = \overline{1, k_1}\right)$, moments M_j at the points $z_j^{**}\left(j = \overline{1, k_2}\right)$, and k_3 thermal sources of powers q_j at the points $a_j\left(j = \overline{1, k_3}\right)$ on the plane of the plate.

Here we assume that the only deformation, which can be sustained by the straight stringers, is the one directed along their longitudinal axis. Furthermore, both the straight and the curvilinear stringers are taken to be of zero bending stiffness.

In plane thermo conductivity problems of cracked bodies with inclusions, the temperature field $T(x, y)$ is expressed as follows:

$$T(x,y) = T_o(x,y) + T_*(x,y) \tag{37}$$

where $T_0(x,y)$ is the known thermal field induced to the continuous medium, and $T_*(x,y)$ the perturbed thermal field due to the presence of defects in the body.

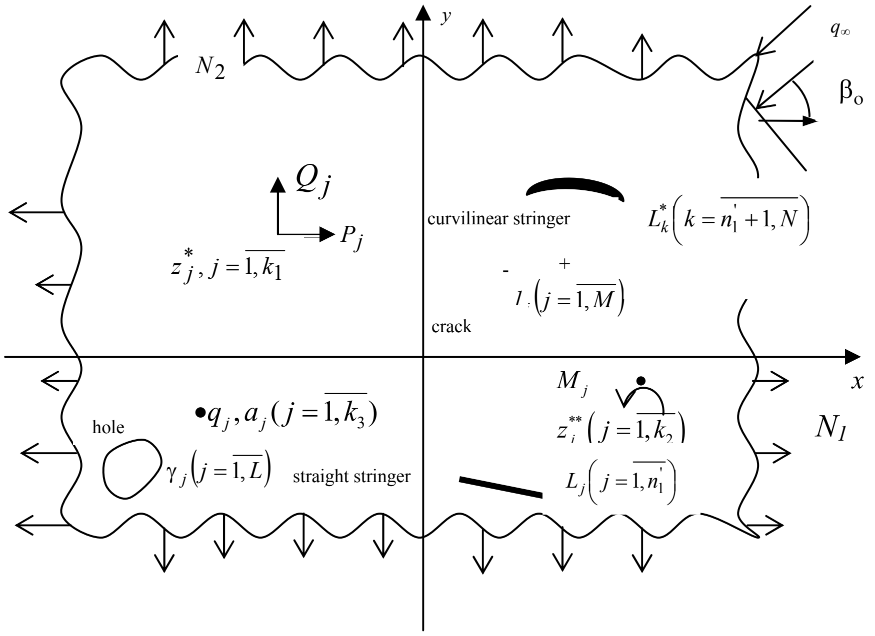


Fig. 2. Infinite isotropic thin plate containing M curvilinear cracks, $N\left(L_j \cup L^*_k\right)$ thin strip inclusions and L holes, which is subjected at infinity to a biaxial state of stress $\left(N_1, N_2\right)$ and to a homogeneous thermal flow q_∞

Depending on the thermal contact conditions at the boundaries of the crack and the thin inclusion, we have the following three relations:

$$T_*^{\pm} = f^{\pm}(t) - T_o(t)\,,\ \left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T_*}\right)^{\pm} = Q^{\pm}(t) - \left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T_0}\right) \tag{38),(39}$$

$$\lambda_s\frac{\partial^2}{\partial s^2}\left(T_*^+ + T_*^-\right) + 2\left[\left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T}\right)^+ - \left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T}\right)^-\right] = -2\lambda_s\frac{\partial^2 T_0}{\partial s^2},$$

$$\lambda_s\frac{\partial^2}{\partial s^2}\left(T_*^+ - T_*^-\right) + 6\left[\left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T}\right)^+ + \left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T}\right)^-\right] - 12\lambda_n\left(T_*^+ - T_*^-\right) =$$

$$= -12\left(\overrightarrow{K_n}\cdot\overrightarrow{\mathrm{grad}T_0}\right). \tag{40}$$

Where $f^{\pm}(t)$ and $Q^{\pm}(t)$ express the know temperatures and thermal fluxes along the boundaries of the crack or the thin inclusion, (n,s) refer to the curvilinear coordinate system; λ is the thermal conductivity. In the case of a crack, the parameters λ_s and λ_n correspond to the thermal conductivity in the longitudinal and transverse directions, respectively, and eq.(38)-(40) read:

$$T_*^{\pm} = f_j^{\pm} - T_0, t\in L^*_{1j},\quad j=\overline{1,n_1}\qquad t\in L^*_{3j}, j=\overline{\left(n_1+n_2+1, N+M+L\right)} \tag{41}$$

$$\lambda\left(\frac{\partial T_*}{\partial n}\right)^{\pm} = \pm Q_j^{\pm} - \lambda\frac{\partial T_0}{\partial n}, \qquad t \in L_{2j}^*, j = \overline{(n_1+1, n_1+n_2)} \tag{42}$$

$$\lambda_s \frac{\partial^2}{\partial s^2}(T_*^+ - T_*^-) + 6\lambda\left[\left(\frac{\partial T_*}{\partial n}\right)^+ + \left(\frac{\partial T_*}{\partial n}\right)^-\right] - 12\lambda_n(T_*^+ - T_*^-) = -12\lambda\frac{\partial To}{\partial n}$$

$$\lambda_s \frac{\partial^2}{\partial s^2}(T_*^+ + T_*^-) + 2\lambda\left[\left(\frac{\partial T_*}{\partial n}\right)^+ - \left(\frac{\partial T_*}{\partial n}\right)^-\right] = -2\lambda_s\frac{\partial^2 T_o}{\partial s^2} \tag{43}$$

where t is the complex coordinate of a point on the contour. When considering a hole, only one of the (41)-(42) thermal conditions is applied.

In addition to the thermal boundary conditions (41)-(43) we also introduce the following mechanical boundary conditions as in Refs. [8,9] on the crack, stringer and hole boundaries:

1. The normal and shear stresses that act along boundaries l_k of the crack and the boundaries γ_j of the holes are considered to be known:

$$\left(\sigma_n^{\pm} - i\sigma_t^{\pm}\right)\Big|_{l_k}, \quad k = \overline{1, M}\ ; \quad \left(\sigma_n - i\sigma_t\right)\Big|_{\gamma_j}, \quad j = \overline{1, L} \tag{44), (45}$$

2. Along the boundary of the straight stringer L_k the stresses are given by:

$$\sigma_n^+ = \sigma_n^-, \varepsilon_0 = \frac{du_t^+}{dx_k} = \frac{du_t^-}{dx_k}, u_n^+ + iu_t^+ = u_n^- + u_t^- on L_k, k = \overline{1, n_1'} \tag{46}$$

Where x_k denotes the abscissa referred to the local coordinate system $x_k O_k y_k$ positioned at the mid-point of the stringer L_k. By virtue of the condition of equilibrium and Hooke's law for the case of generalized plane stress, relations (46) take the form:

$$ih\left[\left(\sigma_n^+ - i\sigma_t^+\right) - \left(\sigma_n^- - i\sigma_t^-\right)\right] + \frac{E^{(k)}S^{(k)}}{E} ie^{i\theta_k}\frac{d}{dt}\left[\left(\sigma_n^+ + \sigma_s^+\right) - (1+\nu)\sigma_n^+\right] = 0$$

$$t \in L_k, \ k = \overline{1, n_1'} \tag{47}$$

where t is the complex coordinate of a point on L_k, h is the thickness of the plate, $E^{(k)}, S^{(k)}$, express the modulus of elasticity and cross-sectional area, respectively, of the stringers L_k and θ_k denotes the angle formed by positive directions of stringer axis; Ox_k and Ox axis; E, ν express the plate's modulus of elasticity and Poisson's ratio, respectively.

3. Finally, along the curvilinear (circular arc) boundary of the stringer $L_k^*\left(k = \overline{n_1' + 1, N}\right)$, the following relationships hold [6,8,9,10]:

$$-\frac{T(\theta)}{R_k} + h\left(\sigma_n^- - i\sigma_n^-\right) = 0, \qquad \frac{1}{R_k}\frac{dT(\theta)}{d\theta} + h\left(\sigma_t^+ - \sigma_t^-\right) = 0 \tag{48}$$

$$u_n^+ + iu_t^+ = u_n^- + iu_t^-, \qquad \varepsilon_0 = \frac{du_t^+}{dt} = \frac{du_t^-}{dt}$$

If we consider Hooke's law in generalized plane stress, the above set of equations read:

$$ER_k h\left[\left(\sigma_n^+ - \sigma_n^-\right) - i\left(\sigma_t^+ - \sigma_t^-\right)\right] - E^{(k)}S^{(k)}\left[1 - \left(t - m_k e^{ib_k}\right)\frac{d}{dt_k}\right] \times$$

$$\times\left[\left(\sigma_n^+ + \sigma_s^+\right) - (1+\nu)\sigma_n^+\right] = 0 \tag{49}$$

where $T(\theta)$ is the circumferential component of the force which acts along the line in the middle plane of the stringer. In generalized plane stress conditions, $T(\theta)$is given by:

$$T(\theta) = E^{(k)} S^{(k)} \varepsilon_0^{str} \tag{50}$$

with ε_θ^{str} being the circumferential component of strain along the line in the middle plane of the stringer:

$$\varepsilon_0^{str} = \frac{1}{E}\left(\sigma_s - \nu\sigma_n\right) \tag{51}$$

5. FORMATION OF THE STATE EQUATIONS.

The derivation of the singular integral equations is based on the method of complex potentials.

The thermal potential $F(z)$ $\left[T(x,y) = 2\operatorname{Re}F(z)\right]$ of the thermal field $T(x,y)$ is expressed as follows:

$$F(z) = \frac{q_\infty}{2} z e^{-i\beta_o} - \sum_{j=1}^{k_3} \frac{q_j}{2\pi\lambda^*} \ln(z - a_{j)}) + F_*(z) \tag{52}$$

$F_*(z)$ represents the thermal potential that refers to the perturbed thermal field and is given by:

$$F_*(z) = \sum_{j=1}^{n_1} \frac{1}{2\pi i} \int_{L_{1J}^*} \frac{f_j^*(\tau) + i\phi_j^{(2)}(\tau)}{\tau - z} d\tau + \sum_{j=n_1+1}^{n_1+n_2} \left[\frac{1}{2\pi i} \int_{L_{2j}^*} f_j^{**}(\tau) e^{-ia_j} \ln(\tau - z) d\tau + \right.$$

$$\left. + \frac{1}{2\pi i} \int_{L_{2j}^*} \frac{\phi_j^{(1)}(\tau)}{\tau - z} d\tau \right] + \sum_{j=n_1+n_2+1}^{N+M+L} \frac{1}{2\pi i} \int_{L_{3J}^*} \frac{\phi_j^{(1)} + i\phi_j^{(2)}}{\tau - z} d\tau \tag{53}$$

$\left(\phi_j^{(1)} + i\phi_j^{(2)}\right)$ denote the densities along the crack, stringer and hole boundaries.

The quantities $f_j^*(t)$ and $f_j^{**}(t)$ are expressed as:

$$f_j^*(t) = \frac{1}{2}\left[f_j^+(t) - f_j^-(t)\right], \qquad f_j^{**}(t) = -\frac{1}{2\lambda^*}\left[Q^+(t) + Q^-(t)\right] \tag{54}$$

By substitution of the limiting values of Eq. (53), to the boundary conditions Eqs (41)-(43) we get the following system of integro-differential equations. $t \in L_{1k}^*$, $k = \overline{1, n_1}$

$$2\operatorname{Re}\left[\left[\frac{1}{\pi} \int_{L_{1K}} \frac{\phi_k^{(2)}(\tau)}{\tau - t} dt\right] + \sum_{\substack{j=1 \\ j\neq k}}^{n_1} \frac{1}{\pi} \int_{L_{1j}^*} \frac{\phi_j^{(2)}(\tau)}{\tau - t} dt + \sum_{j=n_1+1}^{n_1+n_2} \frac{1}{\pi i} \int_{L_{2j}^*} \frac{\phi_j^{(1)}(\tau)}{\tau - t} d\tau + \right.$$

$$\left. + \sum_{j=n_1+n_2+1}^{N+M+L} \frac{1}{\pi i} \int_{L_j} \frac{\phi_j^{(1)}(\tau) + i\phi_j^{(2)}(\tau)}{\tau - t} d\tau \right] = f_k^+(t) + f_k^-(t) - 2\operatorname{Re}\left[\sum_{j=1}^{n_1} \frac{1}{\pi i} \int_{L_{2J}^*} \frac{f_j^*(\tau)}{\tau - z} d\tau + \right.$$

$$\left. + \sum_{j=n_1+1}^{n_1+n_2} \frac{1}{\pi i} \int_{L_{2j}^*} f_j^{**} e^{-ia_j} \ln(\tau - t) d\tau + \frac{q_\infty}{2} t e^{-i\beta_0} + \sum_{j=1}^{k_3} \frac{q_j}{2\pi\lambda} \ln(t - a_j) \right] \tag{55}$$

$$2\operatorname{Re}\left\{e^{ia_k(t)}\left[\frac{1}{\pi i}\int_{L_{2k}^*}\frac{\phi^{*(1)}(\tau)}{\tau-t}d\tau+\sum_{\substack{j=n_1+1\\j\neq k}}^{n_1+n_2}\frac{1}{\pi i}\int_{L_{2j}^*}\frac{\phi^{*(2)}(\tau)}{\tau-t}d\tau+\sum_{j=1}^{n_1}\frac{1}{\pi i}\int_{L_{1j}^*}\frac{\phi^{*(2)}(\tau)}{\tau-t}d\tau+\right.\right.$$

$$\left.\left.+\sum_{j=n_1+n_2+1}^{N+M+L}\frac{1}{\pi i}\int_{L_{3j}^*}\frac{\phi_j^{(1)}(\tau)+i\phi_j^{(2)}(\tau)}{\tau-t}d\tau\right]\right\}=\frac{Q_k^+(t)-Q_k^-(t)}{\lambda^*}-2\operatorname{Re}\left[e^{ia_k(t)}ia_k(t)\times\right.$$

$$\left.\times\left[\left(\frac{q_\infty}{2}e^{-i\beta_0}-\sum_{j=1}^{k_3}\frac{q}{2\pi\lambda}\frac{1}{t-a_j}\right)+\sum_{j=1}^{n_1}\frac{1}{\pi i}\int_{L_{1j}^*}\frac{f_j^*(\tau)}{(\tau-t)^2}d\tau+\sum_{j=n_1+1}^{n_1+n_2}\frac{1}{\pi i}\int_{L_{2l}^*}\frac{f_j^{**}(\tau)e^{-ia_j(\tau)}}{\tau-t}d\tau\right]\right],$$

$$t\in L_{2k}^*,\qquad k=\overline{n_1+1,n_1+n_2}\tag{56}$$

$$\operatorname{Re}\left[\omega_k(s)e^{ia_k(t)}\left[\frac{1}{\pi i}\int_{L_{3k}^*}\frac{\phi_k^{'(1)}(\tau)+i\phi_k^{'(2)}(\tau)}{\tau-t}d\tau-\frac{e^{2ia_k(t)}}{\pi i}\int_{L_{3K}^*}\frac{\phi_k^{'(1)}(\tau)+i\phi_k^{'(2)}}{\tau-t}d\tau\right]+\right.$$

$$+\sum_{\substack{j=n_1+n_2+1\\j\neq k}}^{N+M+L}\left[\frac{1}{\pi i}\int_{L_{3j}^*}\frac{\phi_j^{'(1)}(\tau)+i\phi_j^{'(2)}(\tau)}{\tau-t}d\tau-\frac{e^{2ia_k(t)}}{\pi i}\int_{L_{3j}^*}\frac{\phi_j^{'(1)}(\tau)+i\phi_j^{'(2)}}{\tau-t}d\tau\right]+$$

$$+\sum_{j=1}^{n_1}\left[\frac{1}{\pi i}\int_{L_{1j}^*}\frac{i\phi_j^{'(2)}(\tau)}{\tau-t}d\tau-\frac{e^{2ia_k(t)}}{\pi i}\int_{L_{1j}^*}\frac{i\phi_j^{'(2)}(\tau)}{\tau-t}d\tau\right]+\sum_{j=n\ +1}^{n_1+n_2}\left[\frac{1}{\pi i}\int_{L_{2j}^*}\frac{\phi_j^{'(1)}(\tau)}{\tau-t}d\tau-\right.$$

$$\left.\left.-\frac{e^{2ia_k(t)}}{\pi i}\int_{L_{2j}^*}\frac{\phi_j^{'(1)}(\tau)}{\tau-t}d\tau\right]+\frac{\lambda}{\lambda_s}\left[e^{ia_k(t)}\phi_\kappa^{'(1)}(t)+i\phi_k^{'(2)}(t)\right]\right]=-2\operatorname{Re}\left[\omega_k(s)e^{ia_k(t)}\times\right.$$

$$\times\left[\frac{q_\infty}{2}e^{-i\beta_0}-\sum_{j=1}^{k_3}\frac{q}{2\pi\lambda}\left[\frac{1}{t-a_j}-\frac{e^{2ia\ (t)}}{(t-a_j)^2}\right]\right]-\frac{1}{2}\left[\sum_{j=1}^{k_1}\frac{1}{\pi i}\int_{L_{1j}^*}\frac{f_j^*(\tau)}{\tau-t}d\tau-\frac{e^{2ia_k(t)}}{\pi i}\times\right.$$

$$\times\int_{L_{1j}^*}\frac{f_j^*(\tau)}{\tau-t}d\tau+\sum_{j=n_1+1}^{n_1+n_2}\frac{1}{\pi i}\int_{L_{1J}^*}\frac{f_j^*(\tau)e^{-ia_j(\tau)}}{\tau-t}d\tau-\frac{e^{2ia_k(t)}}{\pi i}\times$$

$$\left.\left.\times\int_{L_{2J}^*}\left(\omega_j(s)e^{-ia_j(s)}f_j^{**}(\tau)+f_j^{**}(\tau)\right)e^{-ia_j(\tau)}\frac{d\tau}{\tau-t}\right]\right]\tag{57}$$

$$\operatorname{Re}\left\{\omega_k(s)e^{ia_k(t)}\left[\phi_k^{'(1)}(t)+i\phi_k^{'(2)}(t)-e^{2ia_k(t)}\left(\phi_k^{'(1)}(t)+i\phi_k^{'(2)}(t)\right)\right]+\right.$$

$$+6\frac{\lambda}{\lambda_s}e^{ia_k(t)}\left[\frac{1}{\pi i}\int_{L_{3k}^*}\frac{\phi_k^{'(1)}(\tau)+i\phi_k^{'(2)}(\tau)}{\tau-t}d\tau\right]+\sum_{\substack{j=n_1+n_2+1\\j\neq k}}^{N+M+L}\left[\frac{1}{\pi i}\int_{L_{3j}^*}\frac{\phi_j^{'(1)}(\tau)+i\phi_j^{'(2)}(\tau)}{\tau-t}d\tau\right]+$$

$$\left.+\sum_{j=1}^{n_1}\frac{1}{\pi i}\int_{L_{1j}^*}\frac{i\phi_j^{'(2)}(\tau)}{\tau-t}d\tau+\sum_{j=n_1+1}^{n_1+n_2}\frac{1}{\pi i}\int_{L_{2j}^*}\frac{\phi_j^{'(1)}(\tau)}{\tau-t}d\tau-12\frac{\lambda_n}{\lambda_s}\left(\phi_k^{(1)}(t)+i\phi_k^{(2)}(t)\right)\right\}=$$

$$-12\frac{\lambda}{\lambda_s}\mathrm{Re}\left[e^{ia_k(t)}\left(\frac{q_\infty}{2}e^{-i\beta_0}-\sum_{j=1}^{k_3}\frac{q_j}{2\pi\lambda}\frac{1}{\tau-a_j}\right)-\frac{1}{12}\left(\sum_{j=1}^{n_1}\frac{1}{\pi i}\int_{L_{1J}^*}\frac{f_j^{'*}(\tau)}{\tau-t}d\tau+\sum_{j=n_1+1}^{n_1+n_2}\frac{1}{\pi i}\times\right.\right.$$

$$\left.\left.\times\int_{L_{2j}^*}f_j^{**}\frac{e^{-ia_j(t)}}{\tau-t}d\tau\right)\right]\quad t\in L_{3k}^*,\ k=\overline{n_1+n_2+1,N+M+L},\ \omega_k(s)=a_k{}'(s)\qquad(58)$$

In the description of the boundary conditions (44)-(48) the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ are defined as follows:

$$\Phi_o(z)=\Gamma-\sum_{j=1}^{k_1}\frac{P_j+iQ_j}{2\pi(1+\kappa)}\frac{1}{z-z_j^*}+\frac{\beta}{1+\kappa}\sum_{j=1}^{k_3}\frac{q_j}{2\pi\lambda}\ln(z-a_{j)})+\Phi(z)\qquad(59)$$

$$\Psi_o(z)=\Gamma'+\sum_{j=1}^{k_1^*}\left[\frac{\kappa(P_j-iQ_{j)})}{2\pi(1+\kappa)}\frac{1}{z-z_j{}^*}-\frac{\overline{z_j^*}(P_j+iQ_{j)})}{2\pi(1+\kappa)}\frac{1}{(z-z_j^*)^2}\right]-$$
$$\sum_{j=1}^{k_2}\frac{M_j}{2\pi}\frac{1}{(z-z_j^{**})^2}-\frac{\beta}{1+\kappa}\sum_{j=1}^{N}\frac{q_j}{2\pi\lambda^*}\frac{\overline{a_j}}{z-a_j}+\Psi(z)\qquad(60)$$

where $\Gamma=\frac{1}{4}(N_1+N_2)\quad\Gamma'=-\frac{1}{2}(N_1-N_2)$ (61)

$$\Phi(z)=\sum_{j=1}^{M}\frac{1}{2\pi i}\int_{l_j}\frac{G_{1j}(\tau)}{\tau-z}d\tau+\sum_{j=1}^{N}\frac{1}{2\pi i}\int_{Lj}\frac{G_{2j}(\tau)}{\tau-z}d\tau+\frac{1}{2\pi i}\oint_{\gamma_j}\frac{G_{3j}(\tau)}{\tau-z}d\tau$$

$G_{1j}(t)$, $G_{2j}(t)$, $G_{3j}(t)$ denote the densities on l_j, L_j and γ_j, respectively.

By virtue of the boundary conditions Eqs. (44)-(46) and Eq. (48), as well as Muskhelishvilli formulae, we get the following integral representation for $\Psi(z)$:

$$\Psi(z)=\sum_{j=1}^{M}\left[\frac{1}{2\pi i}\int_{l_l}\frac{q_{1j}(\tau)}{\tau-z}\overline{d\tau}-\frac{1}{2\pi i}\int_{l_j}\frac{\overline{G_{1j}(\tau)}}{\tau-z}\overline{d\tau}-\frac{1}{2\pi i}\int_{l_j}\frac{\bar{\tau}G_{1j}(\tau)}{(\tau-z)^2}d\tau\right]+$$
$$\sum_{j=1}^{N}\left[\frac{\kappa}{2\pi i}\int_{L_J}\frac{\overline{G_{2j}(\tau)}}{\tau-z}\overline{d\tau}-\frac{1}{2\pi i}\int_{L_j}\frac{\bar{\tau}G_{2j}}{(\tau-z)^2}d\tau\right]+\qquad(62)$$
$$\sum_{j=1}^{L}\left[\frac{1}{2\pi i}\oint_{\gamma_j}\frac{q_{1j}^*(\tau)}{\tau-z}\overline{d\tau}-\frac{1}{2\pi i}\oint_{\gamma_j}\frac{\overline{G_{3j}(\tau)}}{\tau-z}\overline{d\tau}-\frac{1}{2\pi i}\oint_{\gamma_j}\frac{\bar{\tau}G_{3j}}{(\tau-z)^2}d\tau\right]$$

where: $q_{1j}(t)=(\sigma_n^+-\sigma_n^-)-i(\sigma_t^+-\sigma_t^-),\quad t\in l_j,\quad j=\overline{1,M}$

$q_{1j}^*(t)=(\sigma_n-i\sigma_t),\quad t\in\gamma_j,\quad j=\overline{1,L}$

The combination of boundary conditions (44) and (46), with the Muskhelishvilli formulae and Sohotsky-Plemelj formulae [1] for the integral representations of $\Phi(z)$, $\Psi(z)$ and $F_*(z)$, gives the following system of singular equations: $t\in l_k$, $k=\overline{1,M}$

$$\frac{1}{\pi i}\int_{l_k}\frac{G_{1k}(\tau)}{\tau-t}d\tau-\frac{1}{\pi i}\int_{l_k}\frac{\overline{G_{1k}(\tau)}}{\bar{\tau}-\bar{t}}\overline{d\tau}-\frac{\overline{dt}}{dt}\left[\frac{1}{\pi i}\int_{l_k}\frac{\overline{G_{1k}(\tau)}}{\tau-t}\overline{d\tau}+\frac{1}{2\pi i}\int_{l_k}\frac{\bar{\tau}-\bar{t}}{(\tau-t)^2}G_{1k}(\tau)d\tau\right]+$$

$$+\sum_{\substack{j=1\\j\neq k}}^{M}\left[\frac{1}{\pi i}\int_{l_{jk}}\frac{G_{1j}(\tau)}{\tau-t}d\tau-\frac{1}{\pi i}\int_{l_j}\frac{\overline{G_{1j}(\tau)}}{\bar{\tau}-\bar{t}}\overline{d\tau}-\frac{\overline{dt}}{dt}\left[\frac{1}{\pi i}\int_{l_j}\frac{\overline{G_{1j}(\tau)}}{\tau-t}\overline{d\tau}+\right.\right.$$

$$\left.\left.+\frac{1}{\pi i}\int_{l_j}\frac{\bar{\tau}-\bar{t}}{(\tau-t)^2}G_{1j}(\tau)d\tau\right]+\sum_{j=1}^{N}\left[\frac{1}{\pi i}\int_{L_{jk}}\frac{G_{2j}(\tau)}{\tau-t}d\tau-\frac{1}{\pi i}\int_{L_j}\frac{\overline{G_{2j}(\tau)}}{\bar{\tau}-\bar{t}}\overline{d\tau}-\right.$$

$$\left.-\frac{\overline{dt}}{dt}\left[-\frac{\kappa}{\pi i}\int_{L_j}\frac{\overline{G_{2j}(\tau)}}{\tau-t}\overline{d\tau}+\frac{1}{\pi i}\int_{L_j}\frac{\bar{\tau}-\bar{t}}{(\tau-t)^2}G_{2j}(\tau)d\tau\right]\right]+$$

$$+\sum_{j=1}^{L}\left[\frac{1}{\pi i}\oint_{\gamma_j}\frac{G_{3j}(\tau)}{\tau-t}d\tau-\frac{1}{\pi i}\oint_{\gamma_j}\frac{\overline{G_{3j}(\tau)}}{\bar{\tau}-\bar{t}}\overline{d\tau}-\right.$$

$$\left.-\frac{\overline{dt}}{dt}\left[\frac{1}{\pi i}\oint_{\gamma_j}\frac{\overline{G_{3j}(\tau)}}{\tau-t}\overline{d\tau}-\frac{1}{\pi i}\int_{\gamma_j}\frac{\bar{\tau}-\bar{t}}{(\tau-t)^2}G_{3j}(\tau)d\tau\right]\right]=$$

$$=A_{1k}(t,\bar{t})-\frac{\overline{dt}}{dt}\left[\frac{1}{\pi i}\int_{l_k}\frac{q_{1k}(\tau)}{\tau-t}\overline{d\tau}+\sum_{\substack{j=1\\j\neq k}}^{M}\frac{1}{\pi i}\int_{l_j}\frac{q_{1j}(\tau)}{\tau-t}\overline{d\tau}+\sum_{j=1}^{L}\frac{1}{\pi i}\oint_{\gamma_j}\frac{q_{1j}^{*}(\tau)}{\tau-t}\overline{d\tau}\right], \tag{63}$$

Using the boundary conditions (46) we have relative integral equations along the boundaries of the hole γ_j.

and $t\in L_k$, $k=\overline{1,n_1'}$

$$ih[(\kappa+1)G_{2k}(t)+\beta\phi_{2k}(t)]+\frac{E^{(k)}S^{(k)}}{E}ie^{i\theta_k}\times$$

$$\times\frac{d}{dt}\left(\mathrm{Re}\left[\frac{3-\nu-\kappa(1+\nu)}{2}G_{2k}(t)-\frac{\beta(1+\nu)}{2}\phi_{2k}(t)+\right.\right.$$

$$+\frac{1-\nu}{2}\int_{L_k}\frac{G_{2k}(\tau)}{\tau-t}d\tau-(1+\nu)\frac{\overline{dt}}{dt}\left[\frac{\kappa}{2\pi i}\int_{L_k}\frac{\overline{G_{2k}(\tau)}}{\tau-t}\overline{d\tau}-\frac{1}{2\pi i}\int_{L_k}\frac{(\bar{\tau}-\bar{t})G_{2k}(\tau)}{(\tau-t)^2}d\tau\right.$$

$$\left.+\frac{\beta}{2\pi i}\int_{L_k}\frac{\overline{\phi_{2k}(\tau)}}{\tau-t}\overline{d\tau}\right]+\sum_{\substack{j=1\\j\neq k}}^{N}\left[\frac{1-\nu}{\pi i}\int_{L_j}\frac{G_{2j}(\tau)}{\tau-t}d\tau-(1+\nu)\frac{\overline{dt}}{dt}+\right.$$

$$+\left[\frac{\kappa}{2\pi i}\int_{L_j}\frac{\overline{G_{2j}(\tau)}}{\tau-t}\overline{d\tau}-\frac{1}{2\pi i}\int_{L_k}\frac{(\bar{\tau}-\bar{t})G_{2k}(\tau)}{(\tau-t)^2}d\tau+\frac{\beta}{2\pi i}\int_{L_k}\frac{\overline{\phi_{2k}(\tau)}}{\tau-t}\overline{d\tau}\right]\Bigg]+$$

$$+\sum_{\substack{j=1\\ j\neq k}}^{N}\left[\frac{1-\nu}{\pi i}\int_{L_j}\frac{G_{2j}(\tau)}{\tau-t}d\tau-(1+\nu)\frac{dt}{\overline{dt}}\left[\frac{\kappa}{2\pi i}\int_{L_j}\frac{\overline{G_{2j}(\tau)}}{\tau-t}\overline{d\tau}-\frac{1}{2\pi i}\int_{L_j}\frac{(\bar\tau-\bar t)G_{2j}(\tau)}{(\tau-t)^2}d\tau+\right.\right.$$

$$\left.\left.+\frac{\beta}{2\pi i}\int_{L_j}\frac{\overline{\phi_{2j}(\tau)}}{\tau-t}\overline{d\tau}\right]\right]+\sum_{\substack{j=1\\ j\neq k}}^{M}\left[\frac{1-\nu}{\pi i}\int_{L_j}\frac{G_{1j}(\tau)}{\tau-t}d\tau+(1+\nu)\frac{dt}{\overline{dt}}\times\right.$$

$$\left.\times\left[\frac{1}{2\pi i}\int_{L_j}\frac{\overline{G_{1j}(\tau)}}{\tau-t}\overline{d\tau}+\frac{1}{2\pi i}\int_{l_j}\frac{(\bar\tau-\bar t)G_{1j}(\tau)}{(\tau-t)^2}d\tau\right]\right]+\sum_{j=1}^{L}\left[\frac{1-\nu}{\pi i}\oint_{\gamma_j}\frac{G_{3j}(\tau)}{\tau-t}d\tau+(1+\nu)\frac{dt}{\overline{dt}}\times\right.$$

$$\left.\left.\left.\left[\frac{1}{2\pi i}\oint_{\gamma_j}\frac{\overline{G_{3j}(\tau)}}{\tau-t}\overline{d\tau}+\frac{1}{2\pi i}\oint_{\gamma_j}\frac{\bar\tau-\bar t}{(\tau-t)^2}G_{3j}(\tau)d\tau\right]\right]\right]\right)=A_{2k}(t,\bar t) \tag{64}$$

$$A_{1k}(t,\bar t)=(\sigma_n^+ +\sigma_n^-)-i(\sigma_t^+ +\sigma_t^-)-2\left(\Gamma+\bar\Gamma+\frac{dt}{\overline{dt}}\Gamma'\right)+$$

$$+\sum_{j=1}^{k_1}\left[2\,\mathrm{Re}\frac{P_j+iQ_j}{\pi(1+\kappa)}\frac{1}{t-z_j^*}-\frac{dt}{\overline{dt}}\left[\frac{\kappa(P_j+iQ_j)}{\pi(1+\kappa)}\frac{1}{t-z_j^*}+\frac{P_j+iQ_j}{\pi(1+\kappa)}\frac{\bar t-\overline{z_j^*}}{t-z_j^*}\right]\right]-$$

$$-i\sum_{j=1}^{k_2}\frac{dt}{\overline{dt}}\frac{M_j}{\pi}\frac{1}{(t-z_j^{**})^2}-\frac{\beta}{1+\kappa}\sum_{j=1}^{k_3}\frac{q_j}{2\pi\lambda}\left[\mathrm{Re}\ln(t-a_j)+\frac{dt}{\overline{dt}}\frac{\bar t-\overline{a_j}}{t-a_j}\right]$$

$$A_{2k}(t,\bar t)=\frac{E^{(k)}S^{(k)}}{E}ie^{i\theta_k}\frac{d}{\overline{dt}}\mathrm{Re}\left[\frac{dt}{\overline{dt}}\left(\sum_{j=1}^{M}\frac{1}{\pi i}\int_{l_j}\frac{q_{1j}(\tau)}{\tau-t}\overline{d\tau}+\sum_{j=1}^{L}\frac{1}{\pi i}\oint_{\gamma_j}\frac{q_{1j}^*(\tau)}{\tau-t}\overline{d\tau}\right)\right]+$$

$$+\sum_{j=1}^{k_1}(1-\nu)\frac{P_j+iQ_j}{\pi(1+\kappa)}\frac{1}{t-z_j^*}-\frac{dt}{\overline{dt}}\left(\frac{\kappa(P-iQ_j)}{2\pi(1+\kappa)}\frac{1}{t-z_j^*}+\frac{P_j+iQ_j}{2\pi(1+\kappa)}\frac{\bar t-\overline{z_j^*}}{t-z_j^*}\right)-$$

$$-i\sum_{j=1}^{k_2}\frac{dt}{\overline{dt}}\frac{M_j}{2\pi}\frac{1}{(t-z_j^{**})^2}-\frac{\beta}{1+\kappa}\sum_{j=1}^{k_3}\frac{q_j}{2\pi\lambda}\left[2(1-\nu)\ln(t-a_j)+\frac{dt}{\overline{dt}}\frac{\bar t-\overline{a_j}}{t-a_j}\right]$$

If we use the boundary conditions (48)-(49) for the curvilinear stringer, we take another two relative integral equations.

Finally, the system of integro-differential equations (60), (61) is augmented with the conditions for singlevaluedness of the displacement along l_k $\left(k=\overline{1,M}\right)$:

$$\int_{l_k}G_{1k}(t)\,dt=\frac{1}{1+\kappa}\int_{l_k}\overline{q_{1k}(t)}\,dt-\frac{\beta}{1+\kappa}\int_{l_k}\varphi_{1k}(t)\,dt,\quad t\in l_k,\quad k=\overline{(1,M)} \tag{65}$$

The densities $G_{1k}(t)$, $G_{2k}(t)$, $G_{3k}(t)$ on the cracks, stringers and holes, respectively, are expressed as follows:

$$G_{1k}(t)=\frac{q_{1k}(t)}{1+\kappa}+g_{1k}(t)-\frac{\beta}{1+\kappa}\phi_{1k}(t),\qquad t\in l_k,\quad k=\overline{1,M}, \tag{66}$$

$$G_{2k}(t)=\frac{i(\sigma_t^+-\sigma_t^-)}{1+\kappa}-\frac{\beta}{1+\kappa}\phi_{2k}(t), \qquad t\in L_k, \quad k=\overline{1,N},$$

$$G_{3k}(t)=\frac{q_{1k}^*(t)}{1+\kappa}+g_{3k}(t)-\frac{\beta}{1+\kappa}\phi_{3k}(t), \qquad t\in \gamma_k, \quad k=\overline{1,L},$$

$$g_{1k}(t)=\frac{2\mu}{1+\kappa}\frac{d}{dt}\left[\left(u^+(t)-u^-(t)\right)+i\left(\upsilon^+(t)-\upsilon^-(t)\right)\right],\ g_{3k}(t)=\frac{2\mu}{1+\kappa}\frac{d}{dt}\left(u(t)+i\upsilon(t)\right).$$

6. NUMERICAL APPLICATION

As examples of application of the method developed we present two different problems. We assume an infinite plate which contains one rectilinear crack of a length l .The Ox axis coincides with the axis of the crack and its origin with the mid-point of the crack. The plate is under the influence of homogeneous flux heat q_∞ at infinity. Furthermore, heat sources $+q_1,-q_1$, $+q_2,-q_2$ act at points $(a_{1,0})$, $(-a_{1,}0)$ $(0,a_2)$ and $(0,-a_2)$ respectively. The behavior of the stress intensity factors is given below:

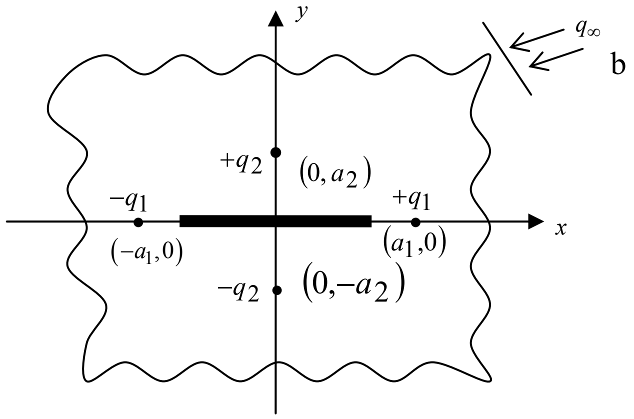
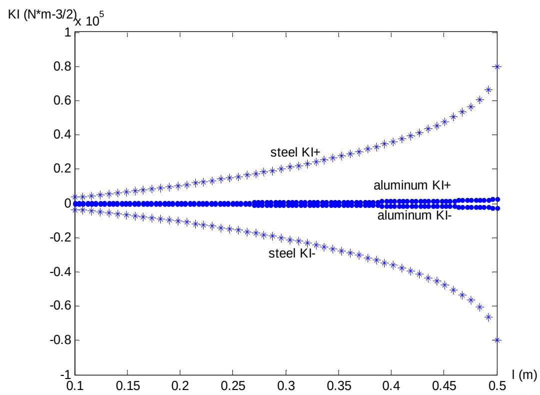
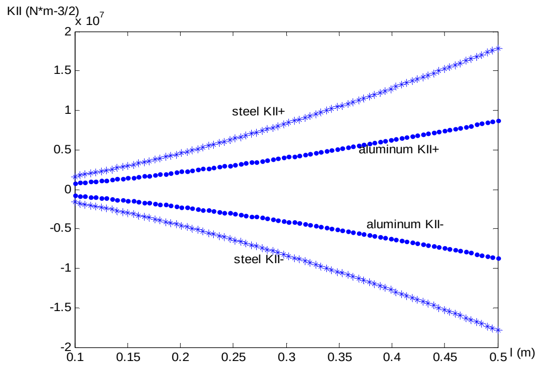


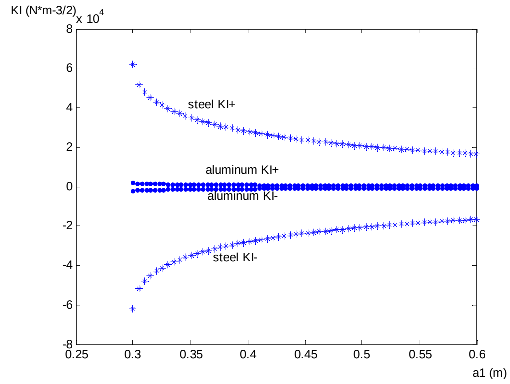
Fig. 3 Infinite plate under the influence of thermal field



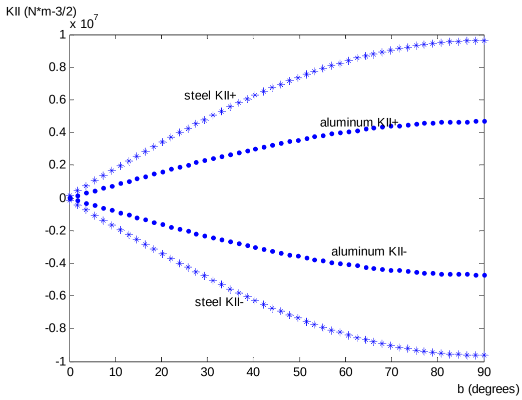
Fig(3a): The variation of K_I stress intensity factor when the length of the crack is increasing



Fig(3b):The variation of K_{II} stress intensity factor when the length of the crack is increasing



Fig(3c): The variation of K_I intensity factor when the distance of the heat source of the point O, is increasing



Fig(3d): The variation of K_{II} intensity factor, when the angle b is increasing

In sequel we assume an infinite plate which contains three rectilinear cracks of lengths $l_1 = 1m, l_2 = 0.2m, l_3 = 0.2m$. The Ox axis coincides with the axis of the crack l_1 and its origin with the mid-point of the crack. The crack l_2 is perpendicular to Ox axis at 0.5. The crack l_3 is parallel to Ox axis, and its mid-point is at point $(0.5, -0.2)$. The plate is submitted to stress $N_2 = 9.81 \cdot 10^4 N/m^2$ at infinity. Heat sources act near the lips of each of the cracks l_2 and l_3.

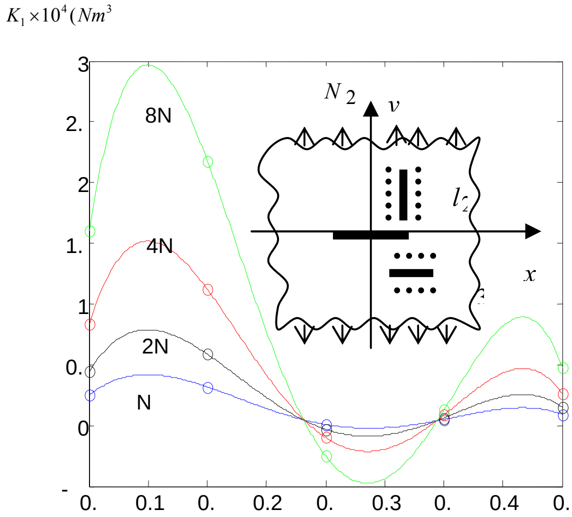
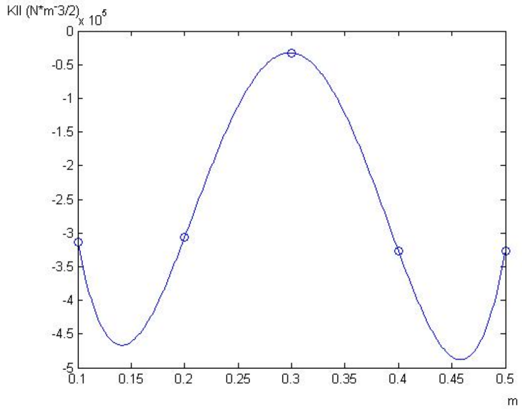


Fig (4a): The variation of K_I stress intensity factor , when the distance of the mid-point of l_2 from Ox axis is increasing.



Fig(4b): The variation of K_{II} stress intensity factor, when the distance of the mid-point of l_2 from Ox axis increasing.

The numerical solution of the singular integral equations, based on the substitution of the integrals by a discrete analogue. The discrete points used (both integration and collocation points) are the notes of an approximate formulae and they are determined on the basis of some functional relation.

7. CONCLUDING REMARKS

Basing on the method of complex functions and the theory of singular integral equations, a general method was proposed for solving plane thermoelasticity and thermo conductivity problems for cracked, isotropic or anisotropic, multiply connected bodies with linear and curvilinear stringers.

Many important engineering problems can be solved by the above general method, such as the body with a partially of fully supported hole and periodic linear and circularly symmetric arrays of cracks, stringers, inclusions etc., as well as other plane elastic problems of a generic geometry which may be encountered in actual engineering applications. It is obvious that the important aspect of prediction of the behavior of a body under the influence of existing singularities inside the mechanical and thermal fields of forces can be considered by using the proposed method.

Furthermore, the principles and procedures of the method can be effectively applied to extend it to a large category of problems, such as bodies with inclusions and bodies in contact containing or not a s system of cracks.

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