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ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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HARMONIC OSCILLATIONS OF A PIEZOCERAMIC HALF-SPACE WITH A TUNNEL OPENING EXCITED BY A SYSTEM OF SURFACE ELECTRODES

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Ключевые слова: гармонические колебания, пьезокерамическое полупространство, электроупругость, антиплоская деформация, туннельная полость.

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Մակերևույթային էլեկտրոդների համակարգի միջոցով թունելային անցքով պյեզոկերամիկ կիսատարածության մեջ ներդաշնակ տատանումների գրգռումը

Հոդվածում ընդհանրացվում է նախկին [9] աշխատանքում հետազոտված թունելային անցքով էլեկտրոդավորված անսահմանափակ միջավայրի համար էլեկտրոառաձգականության խառը խնդիրը կիսատարածության դեպքում՝ եզրային մակերևույթի վրա տարբեր եզրային պայմանների համար: Բերվում է թվային օրինակ:

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Возбужденные посредством системы поверхностных электродов гармонические колебания пьезокерамического полупространства с туннельным отверстием

В данной статье обобщается ранее исследованная в [9] смешанная задача электроупругости для неограниченной среды с электродированной туннельной полостью на случай полупространства с отличными на его границе краевыми условиями. Приводится численный пример.

An antiplane stationary dynamic problem of electroelasticity in a piezoceramic half-space weakened by a tunnel cavity with a system of active surface electrodes is studied. Two types of boundary conditions on the boundary of a half-space are considered: (1) a half-space boundary free of forces and bounded with vacuum; (2) a half-space boundary connected and covered by grounded electrodes. Applying the ideas of the method of images integral representations of the solutions which automatically satisfy the edge conditions on the boundary of a half-space and also the conditions of radiation at infinity are constructed. Allowing for these representations the boundary problem of electroelasticity is reduced to a system of singular integrodifferential equations of the second kind with explosive kernels. Results of parametric investigations characterizing the behaviour of the components of an electroelastic field on the boundary and in the area of a piecewise-homogeneous halfspace are given.

1. Introduction

In piezoelectric media with failures the interaction of electric and mechanical fields may be brought to electric, mechanical and mixed electromechanical breakages. The edges of the electrodes are the sources of concentration of the components of an electroelastic field and consequently, in these areas microcracks or break-downs may appear [1]. Recently, numerous attempts have been made to analyse a crack in piezoelectric materials. The first attempt to analyze the piezoelectric crack problems was made by [2], who analysed a slit crack in a piezoelectric solid. He assumed that the crack face was traction free, but that the cracks were permeable, i.e. that the electric potential and normal components of the electric displacement are continuous across the crack surface. As discussed by Suo [3] this assumption is not physically realistic as there will clearly be a potential drop across the lower capacitance crack. Pak [4] has also studied extensively the mode III crack problem in a piezoelectric solid and the electroelastic fields in and around a circular piezoelectric inhomogenity subjected to antiplane loading. In the mode III problem, Pak [5] has employed a complex variable approach to solve the stress and electric field intensity factor for various electroelastic loading configurations.

Assuming that the electrodes are weightless and have negligibly small rigidity many static and dynamic boundary problems of electroelasticity of piezoelectrics with surface electrodes were considered Kudryavtsev [6], Parton and Kudryavtsev [7], Bardzokas and Senik [8], Bardzokas [1].

In the given article investigated by Bardzokas and Filshtinsky [9], a mixed antiplane problem of electroelasticity for an unbounded medium with an electroded tunnel cavity is generalized for a case of a halfspace at different edge conditions on its boundary. Numerical examples are given.

2. Statement of a problem

Referring to Cartesian coordinates $Ox_1x_2x_3$, let the piezoceramic half-space be weakened by a tunnel along the symmetry axis of material x_3 opening, the cross-section of which is limited by an arbitrary (in some way) smooth contour C (Fig.1a). On a surface which is free from mechanical stress, there are positioned $2n$ infinite (in the direction of the axis x_3) thin electrodes with given differences of electric potentials and the unelectroded areas of the opening are conjugated with vacuum (air). The boundaries of the k-th electrode are determined by quantities α_{2k-1} and α_{2k} $\left(k=\overline{1,2n}\right)$ and the electric potential on it is prescribed by quantity $\phi_k^* = \mathrm{Re}\left(\Phi_k^* e^{-i\omega t}\right)$. It is assumed that the cross-section of the cavity is symmetrical to the axis x_2 and the electrodes are weightless and have negligibly small rigidity. The disposition of the electrodes cannot be quite arbitrary; the conditions of matching will be given below.

In quasistatic approximation the system of equations of an antiplane boundary problem of electroelasticity is reduced to differential equations with respect to displacement $u_3 = \mathrm{Re}\left(U_3 e^{-i\omega t}\right)$ and electric potential $\phi = \mathrm{Re}\left(\Phi e^{-i\omega t}\right)$ Parton and Kufdryavtsev [7].

$$c_{44}^E \nabla^2 u_3 + e_{15}\nabla^2 \phi = \rho \frac{\partial^2 u_3}{\partial t^2}, \quad e_{15}\nabla^2 u_3 - \varepsilon_{11}^S \nabla^2 \phi = 0 \tag{2.1}$$

Here $c_{44}^E, \varepsilon_{11}^S, e_{15}$ and ρ are the shear modulus measured at constant electric field, the dielectric permeability measured at constant deformation, the piezoelectric constant and the mass density of the material, respectively, t is time.

From system (2.1) the following relations follow

$$\nabla^2 u_3 - c^{-2}\frac{\partial^2 u_3}{\partial t^2} = 0, \nabla^2 F = 0$$

$$\phi = \frac{e_{15}}{\varepsilon_{11}^S} u_3 + F, c = \sqrt{\frac{c_{44}^E\left(1+k_{15}^2\right)}{\rho}}, k_{15} = \frac{e_{15}}{\sqrt{c_{44}^E \varepsilon_{11}^S}} \tag{2.2}$$

where c is the shear wave velocity in a piezoelectric medium, k_{15} is the factor of an electromechanical connection.

The components of the electroelastic field are expressed by functions u_3 and F according to the formulas

$$\sigma_{13} - i\sigma_{23} = 2\frac{\partial}{\partial z}\left[c_{44}^{E}\left(1+k_{15}^{2}\right)u_3 + e_{15}F\right] \quad (2.3)$$

$$D_1 - iD_2 = -2\varepsilon_{11}^{S}\frac{\partial F}{\partial z}, \quad E_1 - iE_2 = -2\frac{\partial}{\partial z}\left(F + \frac{e_{15}}{\varepsilon_{11}^{S}}u_3\right), z = x_1 + ix_2$$

Here σ_{ij} are stresses of longitudinal shear, D_j and E_j are the components of the vector induction and strengths of the electric field, respectively.

Mechanical and electric boundary conditions on the surface of the cavity allowing for (2.2), (2.3) may be represented in the form

$$\frac{\partial}{\partial n}\left\{c_{44}^{E}\left(1+k_{15}^{2}\right)u_3 + e_{15}F\right\} = 0 \text{ on } C$$

$$\phi = F + \frac{e_{15}}{\varepsilon_{11}^{S}}u_3 = \phi^{*}\left(\zeta, t\right), \zeta \in C_{\phi}$$

$$D_n = -\varepsilon_{11}^{S}\frac{\partial F}{\partial n} = 0 \text{ on } C \setminus C_{\phi} \quad (2.4)$$

Here C_{ϕ} is a part of the contour C corresponding to the electrodized surface of the cavity; differential operator $\partial/\partial n$ designates a derivative along the normal to the contour C.

Equations (2.2) recorded for the peak values of functions u_3 and F obtain the form

$$\nabla^2 U_3 + \gamma^2 U_3 = 0, \quad \nabla^2 F^{*} = 0, \Phi = \frac{e_{15}}{\varepsilon_{11}^{S}}U_3 + F^{*}, \gamma = \frac{\omega}{c} \quad (2.5)$$

where γ is the wave number.

Consider two types of boundary conditions on the boundary of a half-space $\left(x_2 = 0\right)$

a) a half-space is fixed rigidly and covered by grounded electrodes along the boundary

$$u_3 = 0, \phi = 0 \quad (2.6)$$

b) a halfspace is free from forces and is bounded with vacuum

$$\sigma_{23} = 0, D_2 = 0 \quad (2.7)$$

Hence, the edge problem of electroelasticity is reduced to the definition of functions U_3 and F^{*} from differential equations of Helmholtz and Laplace (2.5) and boundary conditions (2.4), (2.6) or (2.7).

3. Singular Integrodifferential Equations of a Boundary Value Problem

To solve this problem it is necessary to have integral representations of the solutions which satisfy conditions (2.6) or (2.7) and also conditions of radiation at infinity automatically. Using the conception of the method of images More and Feshbah [9] we represent the sought-for functions in the form

$$U_3\left(x_1, x_2\right) = \frac{1}{c_{44}^{E}}\int_{C} p\left(\zeta\right)\left[H_0^{(1)}\left(\gamma r\right) - AH_0^{(1)}\left(\gamma r_1\right)\right]ds \quad (3.1)$$

$$F^{*}\left(x_1, x_2\right) = \int_{C} f\left(\zeta\right)\frac{\partial}{\partial n_{\zeta}}\left(\ln r - A\ln r_1\right)ds, \qquad r = \left|\zeta - z\right|, r_1 = \left|\overline{\zeta} - z\right|, \zeta \in C$$

Here $H_\nu^{(1)}(x)$ is the Hankel-function of the first kind of order ν, ds is an element of the arc length of contour C; value $A=-1$ corresponds to a halfspace free from force and bounded with vacuum; $A=1$ corresponds to a connected halfspace which is covered by grounded electrodes. At $A=0$ we have an unlimited space with a tunnel cavity.
Substituting the limiting values of functions (3.1) at $z \to \zeta_0 \in C$ in boundary conditions (2.4) and using the procedure of integrating by parts of divergency integrals we come to the system of singular integrodifferential equations of the second kind

$$2ip(\zeta_0)+\int_C \{p(\zeta)g_1(\zeta,\zeta_0)+f'(\zeta)g_2(\zeta,\zeta_0)\}ds = N_1(\zeta_0) \tag{3.2}$$

$$-\pi f(\zeta_0)+\int_C \{p(\zeta)g_3(\zeta,\zeta_0)+f(\zeta)g_4(\zeta,\zeta_0)\}ds = N_2(\zeta_0),\ \zeta_0 \in C_\phi$$

$$\int_C f'(\zeta)g_5(\zeta,\zeta_0)\,ds = 0,\ \zeta_0 \in C \setminus C_\phi$$

where kernels g_m $(m=1,2,...,5)$ and the right parts are determined by expressions

$$g_1(\zeta,\zeta_0)=\frac{2}{\pi i}\mathrm{Re}\frac{e^{i\psi_0}}{\zeta-\zeta_0}+\gamma\left[H_1(\gamma r_0)\cos(\psi_0-\alpha_0)-AH_1^{(1)}(\gamma r_{10})\cos(\psi_0-\alpha_{10})\right]$$

$$g_2(\zeta,\zeta_0)=\frac{e_{15}}{1+k_{15}^2}g_5(\zeta,\zeta_0),\ g_3(\zeta,\zeta_0)=\frac{k_{15}^2}{e_{15}}\left[H_0^{(1)}(\gamma r_0)-AH_0^{(1)}(\gamma r_{10})\right]$$

$$g_4(\zeta,\zeta_0)=\mathrm{Re}\left\{\frac{e^{i\psi}}{\zeta-\zeta_0}-\frac{Ae^{i\psi}}{\zeta-\overline{\zeta}_0}\right\},\ f'(\zeta)=\frac{df}{ds}$$

$$g_5(\zeta,\zeta_0)=\mathrm{Im}\left\{\frac{e^{i\psi_0}}{\zeta-\zeta_0}+\frac{Ae^{i\psi_0}}{\overline{\zeta}-\zeta_0}\right\}$$

$$N_1(\zeta_0)=0,\quad N_2(\zeta_0)=\Phi^*(\zeta_0),\quad \psi=\psi(\zeta),\ \psi_0=\psi(\zeta_0),\ \zeta,\zeta_0 \in C$$

$$r_0=(\zeta-\zeta_0(,\ \alpha_0=\arg(\zeta-\zeta_0),\ r_{10}=(\overline{\zeta}-\zeta_0(,\ \alpha_{10}=\arg(\overline{\zeta}-\zeta_0)$$

Here ψ is the angle between the normal to contour C and axis x_1, $\Phi^*(\zeta_0)$ is the piecewise constant function determining the values of electric potentials on the system of electrodes. Kernels $g_2(\zeta,\zeta_0)$, $g_5(\zeta,\zeta_0)$ are singular, the other kernels due to the assumption of smoothness of contour C may possess not more than slight singularities.
Calculating functions $p(\zeta)$ and $f(\zeta)$ from system (3.2) by formulas (2.3) and introducing integral representations (3.1) it is possible to define all the components of the electroelastic field in the area of the halfspace.

4. Definition of components of an electroelastic field in a halfspace

Let us find an expression for the amplitude of density distribution of electric charges $q_k(\beta)$ on k-th electrode. Introducing the parametrization of contour C with the help of equality $\zeta=\zeta(\beta)$ $(0\le\beta\le 2\pi)$ and allowing for the fact that the opening surface is bounded with vacuum we write down

$$q_k(\beta) = D_n^{(k)}(\beta), \alpha_{2k-1} < \beta < \alpha_{2k} \tag{4.1}$$

Here $D_n^{(k)}(\beta)$ represents the amplitude of the normal component of the electric induction vector on the corresponding electroded area of contour C.

Due to (2.3), (3.1), (4.1) we find

$$q_k(\beta_0) = -\varepsilon_{11}^S \int_C f'(\zeta)\operatorname{Im}\left\{\frac{e^{i\psi_0}}{\zeta-\zeta_0} + \frac{Ae^{i\psi_0}}{\bar{\zeta}-\zeta_0}\right\} ds, \zeta_0 \in C_{\phi_k} \tag{4.2}$$

where C_{ϕ_k} is a part of contour C which k-th electrode is located on.

Integrating expression (4.2) by variable β_0 in the limits from α_{2k-1} to α_{2k} we obtain the peak value of total charge Q_k of k-th electrode referring to the unit of its length. The current flowing through the given electrode and equal to the conduction current in the generator circuit may be defined by formula

$$I_k(t) = \operatorname{Re}\left\{i\omega e^{-i\omega t} \int_{\alpha_{2k-1}}^{\alpha_{2k}} q_k(\beta_0) s'(\beta_0) d\beta_0\right\}, s'(\beta_0) = \frac{ds}{d\beta_0} \tag{4.3}$$

By analogy we find the expressions for the peak values of the other mechanical and electric quantities in the area of a piecewise-homogeneous halfspace. We have

$$\begin{aligned}
\sigma_{13}^* &= \left(1+k_{15}^2\right)\gamma \int_C p(\zeta)\left\{H_1^{(1)}(\gamma r)\cos\alpha - AH_1^{(1)}(\gamma r_1)\cos\alpha_1\right\} ds + \\
&+ e_{15}\int_C f'(\zeta)\operatorname{Im}\left[\frac{1}{\zeta - z} + \frac{A}{\bar{\zeta} - z}\right] ds \\
\sigma_{23}^* &= \left(1+k_{15}^2\right)\gamma \int_C p(\zeta)\left\{H_1^{(1)}(\gamma r)\sin\alpha - AH_1^{(1)}(\gamma r_1)\sin\alpha_1\right\} ds + \\
&+ e_{15}\int_C f'(\zeta)\operatorname{Re}\left[\frac{1}{\zeta - z} + \frac{A}{\bar{\zeta} - z}\right] ds \\
E_1^* &= -\frac{k_{15}^2\gamma}{e_{15}} \int_C p(\zeta)\left\{H_1^{(1)}(\gamma r)\cos\alpha - AH_1^{(1)}(\gamma r_1)\cos\alpha_1\right\} ds - \\
&- \int_C f'(\zeta)\operatorname{Im}\left[\frac{1}{\zeta - z} + \frac{A}{\bar{\zeta} - z}\right] ds \\
E_2^* &= -\frac{k_{15}^2\gamma}{e_{15}} \int_C p(\zeta)\left\{H_1^{(1)}(\gamma r)\sin\alpha - AH_1^{(1)}(\gamma r_1)\sin\alpha_1\right\} ds - \\
&- \int_C f'(\zeta)\operatorname{Re}\left[\frac{1}{\zeta - z} + \frac{A}{\bar{\zeta} - z}\right] ds \\
D_1^* &= -\varepsilon_{11}^S \int_C f'(\zeta)\operatorname{Im}\left[\frac{1}{\zeta - z} + \frac{A}{\bar{\zeta} - z}\right] ds \\
D_2^* &= -\varepsilon_{11}^S \int_C f'(\zeta)\operatorname{Re}\left[\frac{1}{\zeta - z} + \frac{A}{\bar{\zeta} - z}\right] ds
\end{aligned} \tag{4.4}$$

$$\alpha = \arg(\zeta - z), \alpha_1 = \arg(\overline{\zeta} - z), \zeta \in C$$

5. A direct piezoelectric effect in a halfspace (space) with an electroded tunnel cavity

Let us apply the above described approach to a situation where a fixed and grounded along boundary $x_2 = 0$ piezoceramic halfspace with a tunnel opening is used as a generator of electric energy. In this case as mechanical exciters are considered two flat monochromatic shear waves which propagate in positive and negative directions of axis x_1 and have the following values of displacement amplitude u_3 and electric potential ϕ, respectively

$$U_3^{(1)} = \tau_1\left(e^{-i\gamma x_1} - Ae^{i\gamma x_1}\right), U_3^{(2)} = \tau_2\left(e^{i\gamma x_1} - Ae^{-i\gamma x_1}\right)$$
$$\Phi^{(j)} = \frac{e_{15}}{\varepsilon_{11}^S}U_3^{(j)} \; (j = 1,2) \tag{5.1}$$

Here value $A = 1$ corresponds to the fixed halfspace with zero potential on the boundary, value $A = 0$ corresponds to space.

For definiteness, assume that the cross-section of the cavity has vertical and horizontal axes of symmetry and on its surface there are two symmetrically located infinitely long electrodes (Fig. 1b). To obtain potential differences $2V(t)$ in the process of the medium deformation there should appear electric charges of different signs on the electroded platings which require matching of displacement amplitude in monochromatic waves. Therefore in (5.1) it is necessary $\tau_1 = -\tau_2 = \tau$.

The generating energy is used in the external electric circuit closing the electrodes and in the form of a model it may be represented by losses on an element with conductivity Y (Fig. 1b). In this case the value of the potential difference on electrodes $2V(t)$ and the current in circuit $I(t)$ are unknowns. To obtain the electric boundary condition of the considered problem it is necessary to involve Ohm's law for external circuit [10].

$$I(t) = 2YV(t) \tag{5.2}$$

Construction of the solution of the boundary problem consists of assignment of unknown electric potential differences $2V(t)$ on the electrodes, i.e. in application of boundary conditions (2.4) under the action of harmonic waves. Thus from equalities (4.2), (4.3) and (5.2) we can define unknown potential amplitude $V(t)$ on the electrode

$$V^*(\omega) = \frac{i\tau\omega\varepsilon_{11}^S B_1}{2Y - i\omega\varepsilon_{11}^S B_2} \tag{5.3}$$

$$B_m = \int_{\alpha_1}^{\alpha_2} A_m(\beta_0)s'(\beta_0)d\beta_0 \; (m = 1,2)$$

$$A_m(\beta_0) = -\int_C f_m'(\zeta)\,\mathrm{Im}\left\{\frac{e^{i\psi_0}}{\zeta - \zeta_0} + \frac{Ae^{i\psi_0}}{\overline{\zeta} - \zeta_0}\right\}ds$$

Here functions $f_m(\zeta)\,(m=1,2)$ represent "standard" solutions of system (3.2) according to the right parts

$$N_1^{(1)}(\zeta_0)=2i\gamma c_{44}^E(1+A)\cos\psi_0\cos\gamma\xi_{10}\ ,\ \ N_2^{(1)}(\zeta_0)=\frac{2e_{15}}{\varepsilon_{11}^S}i(1+A)\sin\gamma\xi_{10} \quad (5.4)$$

$$N_1^{(2)}(\zeta_0)=0\ ,\quad N_2^{(2)}(\zeta_0)=\begin{cases}1, \alpha_1<\beta_0<\alpha_2\,, \\ -1, \alpha_3<\beta_0<\alpha_4\,,\end{cases}\ \zeta_0=\xi_{10}+i\,\xi_{20}\in C$$

where quantities $\alpha_k\,\left(k=\overline{1,4}\right)$ assign the location of the electrodes.

From formula (5.3) we obtain two cases for interrupted circuit $(Y=0)$ and short circuit $(Y\to\infty)$. In the first case the total charge on the electrodes do not change in the process of the medium deformation, and in the second it is obvious that $V(t)=0$.

6. Results of a numerical investigation

As an example of the first case consider a halfspace from ceramic $PZT-4$ Berlincourt [11] with circular opening $\zeta=\mathrm{Re}^{i\beta}+ih(\beta\in[0,2\pi])$ excited by two electrodes with the amplitude difference of electric potentials $2\Phi^*$ located symmetrically to axis x_2 $(\alpha_1=-\pi/7, \alpha_2=\pi/7,\ \alpha_3=6\pi/7, \alpha_4=8\pi/7)$. Solution of the system of integrodifferential equations (3.2) was carried out by the scheme of the quadrature method (see Appendix A).

For the considered case in Fig. 2 the changes of quantity $Q^*=\left|Q_1/\left(\varepsilon_{11}^S\Phi^*\right)\right|$ are shown, characterizing the amplitude of total electric charge Q_1 on the electrode as a function of the normalized wave number γR for different variants of boundary conditions on the boundary halfspace $(h/R=2.5)$. It is seen that in case of restrained halfspace $(A=1)$ quantity Q^* may exceed its static analogue by 26%. Influence of the inertial effect in the space is hardly seen.

The behaviour of quantity $\mu=\left|q_2(\beta)/\left(\varepsilon_{11}^S\Phi^*\right)\right|$ on the electrodes at $h/R=1.5, \gamma R=1$ for various values of the boundary condition identificator A is represented in Fig. 3. As it follows from the last singular equation in (3.2) and expression (4.2), the intensity of the charge distribution (a normal component of the electric induction vector) has singularities of root type on the edge electrodes which are confirmed by curves in Fig. 3.

Fig. 4 illustrates the level line of the module of displacement amplitude $|U_3|$ in the area covering the opening for different conditions on boundary $x_2=0$ at $\gamma R=1,\ h/R=7.5$. The lighter zones conform to the maximum values of quantity $|U_3|$. Fig. a,b and c are given for values of parameter $A=0,1$ and -1, respectively.

Distribution of the moduli of stress amplitude $\left|\sigma_{13}^{*}\right|$ and $\left|\sigma_{23}^{*}\right|$ in the nearest and furthest zones at $\gamma R = 1$, $h/R = 7.5$ for values $A = 0,1$ and -1 is represented in Fig. 5 and 6, respectively. It should be noted here that in statics $(\omega = 0)$ the electric loading of the medium in the condition of antiplane deformation does not cause any mechanical stress in it.

Now consider a case of excitation of conjugated fields by four electrodes, the disposition of which is fixed by the values $\alpha_k = (2k-1)\pi/8 \quad \left(k = \overline{1,8}\right)$.

In Figs. 7a and 7b the behaviour of quantities $Q_1^* = \left|Q_1/\left(\varepsilon_{11}^S V\right)\right|$, $Q_3^* = \left|Q_3/\left(\varepsilon_{11}^S V\right)\right|$ at the most remote and nearest electrodes on the boundary of a halfspace is given, respectively, as a function of γR for various variants of edge conditions for boundary halfspace $\left(h/R = 2.5\right)$. On the electrodes the potentials were assigned as follows $\Phi_1^* = V$, $\Phi_2^* = -V$, $\Phi_3^* = V$, $\Phi_4^* = -V$.

Results of the investigation of the distribution of the level lines of quantities $\left|U_3\right|$, $\left|\sigma_{13}^*\right|$ and $\left|\sigma_{23}^*\right|$ in the vicinity of the circular opening are given in Fig. 8,9 and 10, respectively. In calculations we supposed $\gamma R = 1$, $h/R = 7.5$, $\Phi_1^* = V$, $\Phi_2^* = -V$, $\Phi_3^* = V$, $\Phi_4^* = -V$. Fig. 11 illustrates the level lines of quantities $\left|\sigma_{13}^*\right|$ and $\left|\sigma_{63}^*\right|$ for a free halfspace in case of $\Phi_1^* = V$, $\Phi_2^* = -V$, $\Phi_3^* = -V$, $\Phi_4^* = V$ at $\gamma R = 1$.

The graphs of the amplitude module changing, relating to electrical potential $\left\langle V^* \right\rangle = \left|\varepsilon_{11}^S V^* / \tau e_{15}\right|$ on the electrode, as a function of γR, under the action of harmonic waves type (5.1) are given for the values of parameter $A = 0$ and 1, respectively, in Figs. 12a and 12b $\left(h/R = 2.5\right)$. Calculations were fulfilled by formula (5.3) for the mode of "idle running" (disconnected electrodes). Curves 1-3 conform to the following variants of disposition of the electrodes $\alpha_1 = -\pi/7$, $\alpha_2 = \pi/7$, $\alpha_3 = 6\pi/7$, $\alpha_4 = 8\pi/7$; $\alpha_1 = -\pi/4$, $\alpha_2 = \pi/4$, $\alpha_3 = 3\pi/4$, $\alpha_4 = 5\pi/4$ and $\alpha_1 = -\pi/3$, $\alpha_2 = \pi/3$, $\alpha_3 = 2\pi/3$, $\alpha_4 = 5\pi/3$.

Analysis of the results show that more efficient electroacustic transformation of energy is observed at the smallest area of electroded plating and it must be mentioned here that in a halfspace it is much more higher than in a space.

7. Concluding Remarks

From the given results it follows that in the conditions of reverse piezoelectric effect the pictures of distribution of mechanical quantities in a halfspace substantially change according to the type of edge conditions on the boundary of a halfspace and the assigned electrical potentials on the system of electrodes. In case of antiplane deformation the stresses of longitudinal shear on a free from mechanical loading surface do not have singularities on the edges of electrodes Bardzokas [1]. The numerical investigation based on the constructed here algorithm confirms it.

It is necessary to note that as the reflected from the boundary of a halfspace conjugated wave field introduces appearance of additional charges on pair (connected to a separate generator) electrodes, the latter should be located symmetrically to the axis x_2 (a case when the centres of the electrodes lie on this axis is obviously excluded). Otherwise the system of integral equations (3.2) becomes unsolvable.

The constructed algorithm may be generalized in case of n tunnel openings C_m $\left(m=\overline{1,n}\right)$ with cross-section of canonical form if their symmetry centers are located on axis x_2. For this in (3.2) it should be assumed $p(\zeta)=\left\{p_m(\zeta),\zeta\in C_m\right\}$, $f(\zeta)=\left\{f_m(\zeta),\zeta\in C_m\right\}$, $C=\bigcup\limits_{m=1}^{n}C_m$.

Appendix A

Let us consider one of the numerical realization of the system (3.2). Let us build the interpolating Lagrange polynomial for the sought-for functions $p(\zeta)$ and $f'(\zeta)$ in the nodes $\beta_j=2\pi(j-1)/N$ $\left(j=\overline{1,N}\right)$. Such polynomial has the form (Ivanov, 1968)

$$L_N\left[\{p_*,f'_*\};\beta\right]=\frac{1}{N}\sum_{j=1}^{N}\left\{p_j^0,f_j^0\right\}\sin\frac{N\left(\beta_j-\beta\right)}{2}\operatorname{cosec}\frac{\beta_j-\beta}{2} \qquad \text{(A1)}$$

$$p(\zeta)=p_*(\beta),\ p_j^0=p_*\left(\beta_j\right),\ f(\zeta)=f_*(\beta),\ f_j^0=f'_*\left(\beta_j\right)$$

It must be mentioned here that the formulas (A1) are valid for odd numbers of the node division of the contour C.

Integration of the formula (A1) for function $f'_*(\beta)$ using the equation Prudnikov [13]

$$\int\frac{\sin(2m+1)x}{\sin x}dx=2\sum_{k=1}^{m}\frac{\sin 2kx}{2k}+x$$

brings to the following expression for the function $f_*(\beta)$

$$M_N\left[f_*(\beta);\beta\right]=\frac{1}{N}\sum_{j=1}^{N}f_j^0\Omega_j(\beta)+A$$

$$\Omega_j(\beta)=-2\sum_{k=1}^{\frac{N-1}{2}}\frac{\sin k\left(\beta_j-\beta\right)-\sin k\beta_j}{k}+\beta \qquad \text{(A2)}$$

Constant A appearing here must be determined from the conditions of the periodicity of the function $f_*(\beta)$ which due to (A2) has the following form

$$\sum_{j=1}^{N}f_j^0=0 \qquad \text{(A3)}$$

Applying (A2) we also find the quadrature formula

$$\int_0^{2\pi}f_*(\beta)G\left(\beta,\beta^*\right)d\beta=\frac{2\pi}{N^2}\sum_{j=1}^{N}f_j^0\sum_{m=1}^{N}\Omega_{jm}G\left(\beta_m,\beta^*\right)+A\frac{2\pi}{N}\sum_{m=1}^{N}G\left(\beta_m,\beta^*\right) \qquad \text{(A4)}$$

where $\Omega_{jm}=\Omega_j\left(\beta_m\right)$. In the node collocations $\beta_\ell^*=\pi(2\ell-1)/N$ $\left(\ell=\overline{1,N}\right)$ the polynomial (A1) has the following value at odd value of N

$$L_N\left[p_*(\beta);\beta_\ell^*\right]=\frac{1}{N}\sum_{j=1}^{N}p_j^0(-1)^{\ell+j}\operatorname{cosec}\frac{\beta_\ell^*-\beta_j}{2}\ \left(\ell=\overline{1,N}\right) \qquad \text{(A5)}$$

For the singular integral in (3.2) the formula analogous to the formula of calculating regular integrals Panasyuk [14] appears

$$\int_0^{2\pi} f_*'(\beta_j)\,\mathrm{Im}\frac{e^{i\psi_0}}{\zeta(\beta)-\zeta_0(\beta_\ell^*)}s'(\beta)\,d\beta=\frac{2\pi}{N}\sum_{j=1}^{N} f_j^0\,\mathrm{Im}\frac{e^{i\psi_0(\beta_\ell^*)}}{\zeta(\beta_j)-\zeta_0(\beta_\ell^*)}s'(\beta_j) \quad \text{(A6)}$$

Now, substituting the integrals in (3.2) by finite sums of the formulas (A.4), (A.6) and using the equalities (A.2), (A.3) and (A.5) we come to the system $2N+1$ of algebraic equations related to the values of functions $p(\zeta)$ and $f'(\zeta)$ in the nodes of interpolation $\beta_j\left(j=\overline{1,N}\right)$ and constant A.

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