ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 60, №1, 2007 Механика

УДК 539.43

THE FREQUANCY DEPENDANCE OF THE FATIGUE FRACTURE CRITERION

Alexander R. Arutyunyan, Robert A. Arutyunyan

Ключевые слова: усталость, разрушение, частота,
Key words: fatigue, fracture, frequency.

Ա.Ռ.Հարությունյան, Ռ.Ա Հարությունյան

Հոգնածության ամրության սկզբունքի կախվածությունը հաճախությունից

Որոշ Նորագույն (վերջին) հոգնածության փորձարկումներ ցույց են տալիս, որ հոգնածության կորերը ունեն` երկու հոգնածության սահման: Փորձարկումները ցույց են տալիս նաև հոգնածության քայքայման պրոցեցի ուժեղ կախումը հաճախությունից: Այս երկու արդյունքները աշխատանքի հիմնական նյութերն են:

А.Р.Арутюнян, Р.А.Арутюнян

Зависимость критерия усталостной прочности от частоты

Некоторые новейшие испытания на усталость показывают, что кривые усталости имеют два предела усталости. Эксперименты также показывают сильную зависимость процесса устлостного разрушения от частоты. Эти два эффекта являются основными темами работы.

Some recent fatigue tests [1, 2] indicate that the fatigue curves have two fatigue limits. Experiments [3] also indicate the strong frequency dependence of the fatigue fracture process. These two effects are the main topics of the paper.

During cycling loading, for example, under the pulsating stress the plastic deformation is accumulated [4]. The accumulation curves are similar with those received in creep experiments. So they can be described by the following (Norton) power equation $\dot{\varepsilon} = B\sigma^m$ expressed through the effective time $z = t\, f^{-\alpha}$ (f is the frequency of loading),

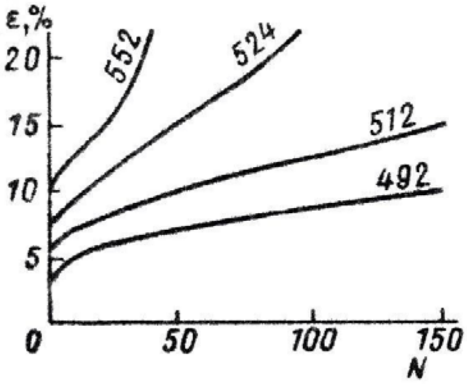

Fig. 1.

$$\frac{1}{l}\frac{dl}{dz} = B\,\sigma_0^m\,(F/F_0)^m \tag{1}$$

where $t = N/f$ (N is the number of loading cycles, t is the real time), B, m, α are constants, $\dot{\varepsilon} = \frac{1}{l}\frac{dl}{dz}$, $\varepsilon = \ln\left(l/l_0\right)$, F_0, l_0 are initial, F, l – are current values of cross section area and length of the specimen.

As it is shown in fig. 1, the plastic strain accumulation depends on the stress level [4]. The two limiting cases can be considered.

In the low cycle regime the value of strain is high and material may be assumed as incompressible. In the high cycle regime due to metal embrittlement the deformation is small and equal to the value of damage parameter, which is defined as the relative change of material density.

To solve the equation (1) we will use the mass conservation law: $\rho_0 F_0 l_0 = \rho F l$ (ρ_0, ρ are the initial and current values of specimen density).

For the incompressible material $\rho = \rho_0$ and from the mass conservation law follows $F_0 l_0 = F l$. Introducing this relation into equation (1) and solving it for the initial conditions $z=0$, ($t = 0$), $l = l_0$ we will have

$$\frac{1}{m}\left[1-(l/l_0)^{-m}\right] = B\,\sigma_0^m\,z \tag{2}$$

According to Hoff [5] the ductile fracture condition follows from (2) when $l \to \infty$

$$\sigma_0^m\,t_P\,f^{-\alpha} = \frac{1}{mB} \tag{3}$$

where t_P is time to fracture.

Introducing relation $t = N / f$ into equation (3) we will receive Coffin's empirical frequency fatigue criterion for low cycle fatigue [6]

$$\sigma_0^m \, N_P \, f^{-(1+\alpha)} = \frac{1}{m\,B} \tag{4}$$

where N_P is the number of cycles to fracture.

Let's consider the case of brittle fracture, when $F \cong F_0$, so the mass conservation law is expressed in the form $l / l_0 = \rho / \rho_0$.

In this case the solution of equation (1) is

$$\ln\,(l / l_0) = B\,\sigma_0^m\, t\, f^{-\alpha} \tag{5}$$

The brittle fracture criterion follows from (5) when $t = t_P$, $l = l_*$, $\rho = \rho_*$, $\varepsilon = \varepsilon_*$

$$\sigma_0^m\, t_P\, f^{-\alpha} = \frac{\varepsilon_*}{\mathrm{B}} \tag{6}$$

Criterion (6) can be expressed through the number of cycles

$$\sigma_0^m\, N_P\, f^{-(1+\alpha)} = \frac{\varepsilon_*}{\mathrm{B}} \tag{7}$$

For the common case of ductile-brittle fracture we will use the current value of the coefficient of lateral deformation $\upsilon = -\varepsilon_y / \varepsilon_x = -\varepsilon_z / \varepsilon_x$.

Introducing the following geometrical relation $F_0 / F = (l / l_0)^{2\upsilon m}$ into equation (1) we will have

$$\frac{1}{l}\frac{dl}{dz} = B\,\sigma_0^m\,(l / l_0)^{2\upsilon m} \tag{8}$$

Assuming $\upsilon \cong \upsilon_*\left(\sigma_0\right)$ the solution of the equation (8) can be received in the form

$$\frac{1}{2\,m\upsilon_*}\left[1-(l / l_0)^{-2\,\upsilon_*\,m}\right] = B\sigma_0^m\, t\, f^{-\alpha} \tag{9}$$

From (9) follows the ductile-brittle fracture criterion

$$\sigma_0^m\, t_P\, f^{-\alpha} = \frac{1-e^{-2m\upsilon_*\,\varepsilon_*}}{2mB\upsilon_*} \tag{10}$$

Criterion (10) can be expressed through the number of cycles

$$\sigma_0^m \, N_P \, f^{-(1+\alpha)} = \frac{1-e^{-2m\upsilon_* \varepsilon_*}}{2mB\upsilon_*} \tag{11}$$

Relations for ductile (3)-(4) and brittle (6)-(7) fracture follow from (10) and (11) as a limiting cases, when $\upsilon_* = 1/2$, $\varepsilon_* \to \infty$, and $\upsilon_* \to 0$.

The theoretical fatigue curves according to the criterion (11) for the frequencies $f_1 = 35 \ Hz$ and $f_2 = 500 \ Hz$ are shown in fig. 2. As follow from fig. 2, the fatigue curves shift into the domain of great longevities with the increase of loading frequency. This effect is in agreement with the experimental results [3].

Conclusions

A mechanical model describing the frequency dependent damage evolution in metallic materials is developed and the fatigue fracture criterion is formulated.
Two limiting processes are considered.
In the ductile region it coincides with the Coffin empirical frequency criterion
For compressible materials it corresponds to the brittle failure criterion.
Theoretical fatigue curves are plotted. According to the experiments, the fatigue curves shift into the domain of great longevities with the increase of loading frequency.

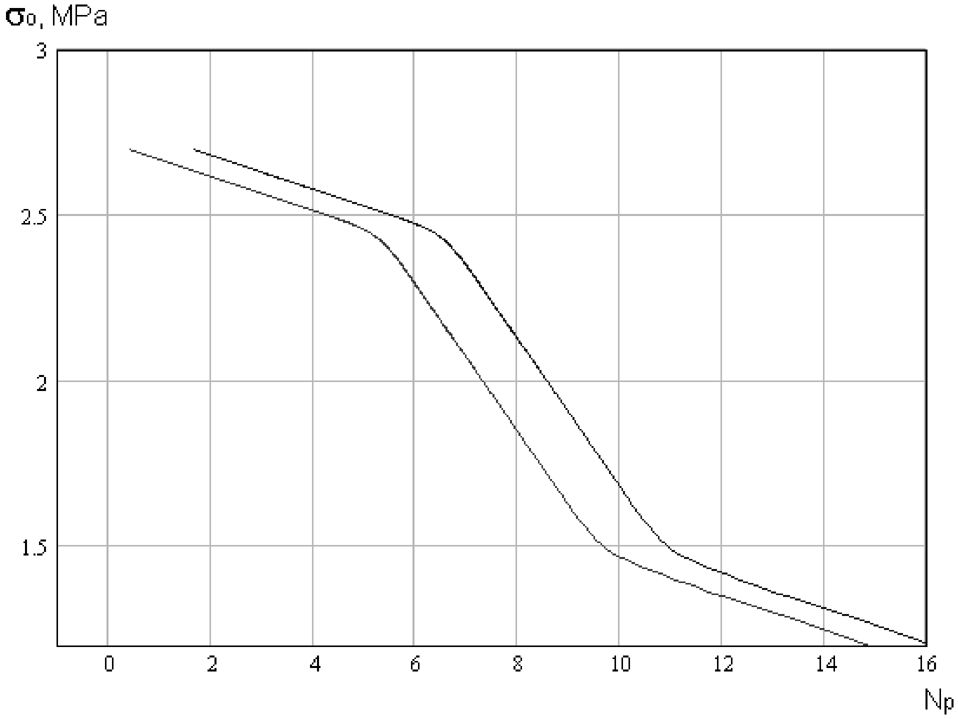

Fig. 2.

Financial support of the Russian Foundation for Basic Research (Grant № 06-08-01003) is gratefully acknowledged.

References

1. Sakai T., Takeda M., Shiozawa K., Ohi Y., Nakajiama M., Nakamura T., Oguma N. Experimental evidence of duplex $S-N$ characteristics in wide life region for high strength steels // Proc. 7th Intern. Fatigue Congress “Fatigue 99”, 8-12 June 1999, Beijing, P.R. China. Beijing: Higher Education Press. 1999. V.1. P. 573-578.
2. Bathias C., Baudry G. Application de la rusistance ≥ la fatigue gigacyclique au calcul de structure // Bulletin S.F.M. Revue Frannaise de Mucanique. 2000. N 1. P. 13-18.
3. The fatigue tests in high frequency loading / V.A. Kuzmenko, L.E. Machnotyuk, G.S. Pisarenko et al. Kiev: Naukova dumka. 1981. 241p. (in Russian).
4. Arutyunyan R.A. The problem of deformation aging and prolonged fracture in material sciences. Sankt-Petersburg: S.-Petersburg University Press. 2004. 252 p.
5. Hoff N.J. The necking and rupture of rods subjected to constante tensile loads // Jour. appl. mech. 1953. Vol.20. №1. P. 105-108.
6. Coffin L.F. The effect of frequency on the cyclic strain and fatigue behavior of Cast Rene at $1600^0\ F$ // Met. Trans. 1974. Vol. 5. № 5. P. 1053-1060.

S.-Petersburg University Received 14.11.2006