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EXCITATION OF SHEAR WAVES IN A PIEZOCERAMIC LAYER WITH A TUNNEL CAVITY BY A SYSTEM OF SURFACE ELECTRODES

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Մակերևույթային էլեկտրոդների համակարգի միջոցով թունելային խոռոչով պյեզոկերամիկ շերտում սահքի ալիքների գրգռումը

Հոդվածում կառուցվում է համալուծ էլեկտրոառաձգական դաշտի էլեկտրական տատանումների ժամանակ մասնակիորեն էլեկտրոդավորված թունելային խոռոչով շերտում ներդաշնակ տատանումների հետազոտության ալգորիթմ: Բերվում է թվային օրինակ:

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ВОЗБУЖДЕНИЕ ВОЛН СДВИГА В ПЬЕЗОКЕРАМИЧЕСКОМ СЛОЕ С ТУННЕЛЬНОЙ ПОЛОСТЬЮ ПОСРЕДСТВОМ СИСТЕМЫ ПОВЕРХНОСТНЫХ ЭЛЕКТРОДОВ

В данной статье строится алгоритм исследования гармонических колебаний слоя с частично электродированной туннельной полостью во время электрических колебаний сопряженного электроупругого поля. Приводится численный пример.

Abstract

An antiplane stationary dynamic problem of electroelasticity for a piezoceramic layer weakened by a tunnel cavity with a system of active surface electrodes is studied. Using Green's function for a homogeneous piezoceramic layer integral representations of the solutions automatically satisfying the boundary conditions on its bases and also the conditions of radiation at infinity are constructed. Allowing for these representations the boundary problem of electroelasticity is reduced to a system of singular integrodifferential equations of the second kind with resolvent kernels. Results of parametric investigations characterizing the behaviour of the components of the electroelastic field on the cavity surface in the area of a piecewise-homogeneous layer are given.

An antiplane stationary dynamic problem of electroelasticity for a piezoceramic layer weakened by a tunnel cavity with a system of active surface electrodes is studied. Using Green's function for a homogeneous piezoceramic layer integral representations of the solutions automatically satisfying the boundary conditions on its bases and also the conditions of radiation at infinity are constructed. Allowing for these representations the boundary problem of electroelasticity is reduced to a system of singular integrodifferential equations of the second kind with resolvent kernels. Results of parametric investigations characterizing the behaviour of the components of the electroelastic field on the cavity surface in the area of a piecewise-homogeneous layer are given.

1. Introduction

Many actual scientific and technological problems of modern engineering are connected with the investigations of the process of propagation of waves in piezoelectrics and with the definition of dynamic strength in the vicinity of inhomogeneities of various types. To solve these problems it is necessary to use modern mathematical means and, in particular, methods and approaches of the dynamic theory of elasticity. Development of these methods is reflected in monographs [1-5] which appeared during the last decades.

In piezoelectric media with inhomogeneities the interchange of electric and mechanical fields may bring to electric, mechanical or mixed electromechanical fracture. The edges of the electrodes are the sources of concentration of the components of the electroelastic field and, hence in these areas there may emerge microcracks or a break-down (Bardzokas et al. [6]). Some aspects of the mechanics of fracture of piezoceramic bodies are considered in [7,8].

In the given article an algorithm for investigation of harmonic oscillations of a layer with a partially electrodized tunnel cavity during the electric oscillation of a conjugated electroelastic field is constructed. Numerical examples are given.

2. Statement of the problem

In Cartesian coordinates $Ox_1x_2x_3$ consider a piezoceramic layer $\left(0 \le x_1 \le a, -\infty < x_2 < \infty, -\infty < x_3 < \infty\right)$ weakened by a tunnel along axis x_3 opening, the cross-section of which is limited by smooth contour C (Fig. 1a). Assume that the bases of the layer are free of forces and bounded with vacuum (the direction of polarization of the ceramics is parallel to axis x_3). On the surface of the opening free from mechanical stresses $2n$ infinite in the direction of axis x_3 thin electrodes with prescribed differences of the electric potential are located, and the non-electroded areas of the opening are bounded with vacuum (air). The boundaries of k-th electrode are determined by quantities β_{2k-1} and β_{2k} $\left(k = \overline{1, 2n}\right)$, and the electric potential on it is given by quantity $\phi_k^* = \operatorname{Re}\left(\Phi_k^* e^{-i\omega t}\right)$ (t is the time, ω is the circular frequency). Location of the electrodes and configuration of the cavity cannot be fully arbitrary; the demanded requirements will be given below.

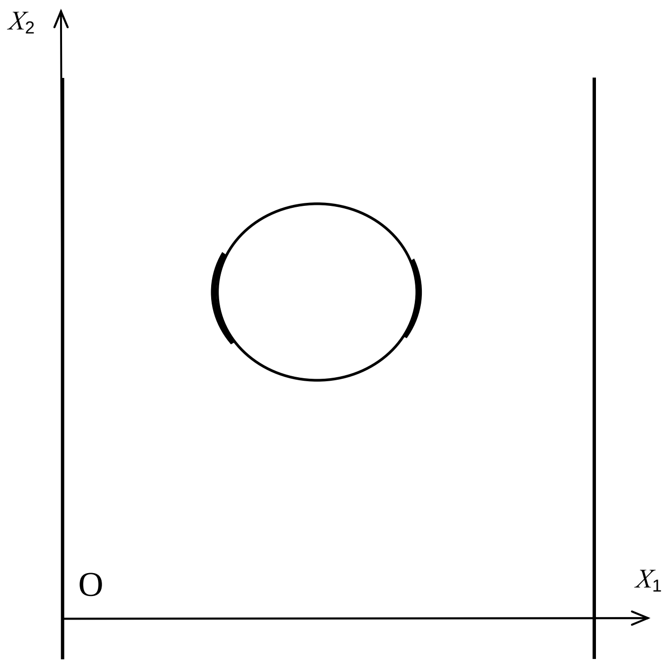


Fig. 1a

In the given conditions in a piecewise-homogeneous layer an electroelastic field corresponding to the state of antiplane deformation occurs. The full system of differential equations in a quasistatic approximation includes the following relations [5] equations of movement

$$\partial_1 \sigma_{13} + \partial_2 \sigma_{23} = \rho \frac{\partial^2 u_3}{\partial t^2}, \qquad \partial_i = \frac{\partial}{\partial x_i} \tag{2.1}$$

constitutive equations of the medium

$$\sigma_{m3} = c_{44}^{E}\partial_m u_3 - e_{15}E_m, \qquad D_m = e_{15}\partial_m u_3 + э_{11}^{\varepsilon}E_m \quad (m = 1, 2) \tag{2.2}$$

and equations of electrostatics

$$\mathrm{div}\mathbf{D} = 0, \qquad \mathbf{E} = -\operatorname{grad}\phi \tag{2.3}$$

In (2.1)-(2.3) σ_{m3} are the components of the stress tensor, u_3 is the component of the displacement vector in the direction of axis x_3; $\boldsymbol{E}$ and $\boldsymbol{D}$ are the vectors of strength and induction of the electric field; ϕ is the electrical potential; c_{44}^{E}, e_{15} and $э_{11}^{\varepsilon}$ is the shear modulus measured at the constant value of the electric field; piezoelectric constant and dielectric permittivity measured at fixed deformations, respectively; ρ is the mass density of the material.

The system of equations (2.1)-(2.3) must be reduced to differential equations referring to displacements u_3 and electric potential ϕ:

$$e_{15}\nabla^2 u_3 - э_{11}^{\varepsilon}\nabla^2\phi = 0 \tag{2.4}$$

From (2.4) we have the equivalent set of equations

$$\nabla^2 u_3 - c^{-2}\frac{\partial^2 u_3}{\partial t^2} = 0, \nabla^2 F = 0$$

$$\phi = \frac{e_{15}}{э_{11}^{\varepsilon}}u_3 + F, \quad c = \sqrt{\frac{c_{44}^{E}\left(1 + k_{15}^2\right)}{\rho}}, \quad k_{15} = \frac{e_{15}}{\sqrt{c_{44}^{E}\,э_{11}^{\varepsilon}}} \tag{2.5}$$

where c is the velocity of a shear wave in a piezoceramic medium, k_{15} is the factor of an electro-mechanical coupling [4].

Mechanical and electric quantities allowing for (2.2), (2.3) and (2.5) may be expressed through functions u_3 and F by formulas

$$\sigma_{13} - i\sigma_{23} = 2\frac{\partial}{\partial z}\left[c_{44}^{E}\left(1 + k_{15}^2\right)u_3 + e_{15}F\right], \tag{2.6}$$

$$D_1 - iD_2 = -2\,э_{11}^{\varepsilon}\frac{\partial F}{\partial z}, \quad E_1 - iE_2 = -2\frac{\partial}{\partial z}\left(F + \frac{e_{15}}{\varepsilon_{11}^{S}}u_3\right), \quad z = x_1 + ix_2$$

Assuming $u_3 = \mathrm{Re}\left(u_3 e^{-i\omega t}\right)$, $\phi = \mathrm{Re}\left(\Phi e^{-i\omega t}\right)$ and $F = \mathrm{Re}\left(e^{-i\omega t}F^*\right)$ we will write down equations (2.5) referring to the amplitude quantities as follows

$$\nabla^2 U_3 + \gamma^2 U_3 = 0, \quad \nabla^2 F^* = 0, \quad \Phi = \frac{e_{15}}{э_{11}^{\varepsilon}}U_3 + F^*, \quad \gamma = \frac{\omega}{c} \tag{2.7}$$

where γ is the wave number.

Mechanical and electric boundary conditions on the surface of the cavity allowing for (2.5), (2.6) we represent in the following form

$$\frac{\partial}{\partial n}\left\{c_{44}^{E}\left(1 + k_{15}^2\right)u_3 + e_{15}F\right\} = 0 \quad \text{on} \quad C$$

$$\phi = F + \frac{e_{15}}{э_{11}^{\varepsilon}}u_3 = \phi^*(\zeta, t), \quad \zeta \in C_\phi \tag{2.8}$$

$$D_n = -\,э_{11}^{\varepsilon}\,\frac{\partial F}{\partial n} = 0 \quad \text{on} \quad C \setminus C_\phi$$

Here C_ϕ is the part of contour C corresponding to the electrodized surface of the cavity; operator $\partial/\partial n$ designates the derivative over normal to contour C.

Mechanical and electric boundary conditions on the bases of the layer may formally be represented as

$$\sigma_{13} = 0\,, \quad D_1 = 0 \quad \left(x_1 = 0, a\right) \tag{2.9}$$

Thus, the boundary problem of electroelasticity is reduced to the determinations of functions U_3 and F^* from differential equations of Helmholtz and Laplace (2.7), boundary conditions (2.8), (2.9) and conditions at infinity.

3. Green’s function for a piezoceramic layer

To solve the stated problem it is expedient to have integral representations of the solution automatically satisfying the boundary conditions (2.9), and the conditions of radiation at infinity. To this purpose let us construct a Green’s function for a homogeneous piezoceramic layer.

Boundary problems (2.7), (2.9) allowing for relations (2.6) may be written down as follows

$$\nabla^2 U_3 + \gamma^2 U_3 = 0\,; \quad \partial_1 U_3 = 0 \quad \left(x_1 = 0, a\right) \tag{3.1}$$

$$\nabla^2 F = 0; \quad \partial_1 F^* = 0 \quad \left(x_1 = 0, a\right) \tag{3.2}$$

We find Green’s function corresponding to problems (3.1), (3.2) in the form of [16]:

$$G\left(\zeta, z\right) = \sum_{\nu=0}^{\infty} b_\nu \left(x_2 - \xi_2\right) \cos\alpha_\nu \xi_1 \cos\alpha_\nu x_1$$

$$E\left(\zeta, z\right) = \sum_{\nu=1}^{\infty} d_\nu \left(x_2 - \xi_2\right) \cos\alpha_\nu \xi_1 \cos\alpha_\nu x_1$$

$$\nabla^2 G + \gamma^2 G = \delta\left(x_1 - \xi_1, x_2 - \xi_2\right)\,,\ \alpha_\nu = \frac{\pi\nu}{a}$$

$$\nabla^2 E = \delta\left(x_1 - \xi_1, x_2 - \xi_2\right) = \delta\left(x_1 - \xi_1\right)\delta\left(x_2 - \xi_2\right)$$

$$z = x_1 + i x_2\,,\ \zeta = \xi_1 + i\xi_2 \tag{3.3}$$

where $\delta\left(x\right)$ is $2a$ periodical Dirac δ-function.

Applying the representation

$$\delta\left(x_1 - \xi_1\right) = \frac{1}{a} + \frac{2}{a}\sum_{\nu=1}^{\infty} \cos\alpha_\nu \xi_1 \cos\alpha_\nu x_1 \tag{3.4}$$

dividing the variables in equations (3.1), (3.2) and then using the procedure of determination of the fundamental solution of a common differential equation we find that

$$b_\nu = -\frac{1}{a\lambda_\nu} e^{-\lambda_\nu \left(x_2 - \xi_2\right(}\,, \quad b_0 = \frac{1}{2ia\gamma} e^{i\gamma\left|x_2 - \xi_2\right|}$$

$$d_\nu = -\frac{1}{a\alpha_\nu} e^{-\alpha_\nu (x_2 - \xi_2(}, \quad d_0 = 0, \quad \lambda_\nu = \begin{cases} \sqrt{\alpha_\nu^2 - \gamma^2}\,, \gamma < \alpha_\nu \\ -i\sqrt{\gamma^2 - \alpha_\nu^2}\,, \gamma > \alpha_\nu \end{cases} \quad (\nu = 1, 2, ...) \quad (3.5)$$

The series for functions $E(\zeta, z)$ in (3.3) allowing for (3.5) may be easily summed up by using equation [10]

$$\sum_{m=1}^{\infty} \frac{e^{-m|x|}}{m} \cos my = \frac{|x|}{2} - \frac{1}{2} \ln\left[2\left(\operatorname{ch} x - \cos y\right)\right] \quad (3.6)$$

and has the following form

$$E(\zeta, z) = -\frac{|x_2 - \xi_2|}{2a} + \frac{1}{2\pi} \ln\left|4 \sin\frac{\pi(\zeta - z)}{2a} \sin\frac{\pi(\zeta + \bar{z})}{2a}\right|$$

$$\bar{z} = x_1 - ix_2 \quad (3.7)$$

To separate the main part of function $G(\zeta,z)$ we will write down Green's function G_0 of the prime operator in Helmholtz equation (3.1). Summing up the corresponding series and using (3.6) we obtain

$$G_0 = -\frac{1}{a} \sum_{m=1}^{\infty} a_m (x_1, \xi_1) e^{-\alpha_m |x_2 - \xi_2|} = -\frac{|x_2 - \xi_2|}{2a} + \frac{1}{2\pi} \ln\left|4 \sin\frac{\pi(\zeta - z)}{2a} \sin\frac{\pi(\zeta + \bar{z})}{2a}\right|$$

$$a_m (x_1, \xi_1) = \frac{\cos \alpha_m \xi_1 \cos \alpha_m x_1}{\alpha_m}. \quad (3.8)$$

Due to (3.3), (3.5), (3.8) we represent function $G(\zeta,z)$ in its final form

$$G(\zeta, z) = G_0 + G_1, \quad G_1 = \frac{1}{2ia\gamma} e^{i\gamma|x_2 - \xi_2|} - \frac{1}{a} \sum_{m=1}^{\infty} c_m (x_2 - \xi_2) \cos \alpha_m \xi_1 \cos \alpha_m x_1 \quad (3.9)$$

$$c_m (x_2 - \xi_2) = \frac{1}{\lambda_m} e^{-\lambda_m |x_2 - \xi_2|} - \frac{1}{\alpha_m} e^{-\alpha_m |x_2 - \xi_2|} \quad (m = 1, 2, ...).$$

Thus, function $E(\zeta, z)$ and $G(\zeta, z)$ determined by formulas (3.7)-(3.9) are Green's functions of boundary problem (3.1), (3.2) for a piezoceramic layer. The conditions of radiation in problem (3.1) and damping in problem (3.2) are satisfied. After separation of the main singularity in (3.3) the common term of series in (3.9) decays at point $z = \zeta$ as m^{-3}.

4. Singular Integrodifferential Equations of a Boundary Value Problem

Applying the above constructed Green's functions we will write down the integral representations of the solutions in the following form

$$U_3 (x_1, x_2) = \int_C p(\zeta) G(\zeta, z) ds, \quad F^* (x_1, x_2) = \int_C f(\zeta) \frac{\partial E(\zeta, z)}{dn_\zeta} ds, \quad \zeta \in C \quad (4.1)$$

Here ds is the element of the arc length of contour C. Representations (4.1) satisfy differential equations (2.7), boundary conditions (2.9) on the bases of the layer and the conditions of radiation at infinity.

Allowing for expressions (3.7) the representation for function $F^*\left(x_1,x_2\right)$ is transformed into the form

$$F^*\left(x_1,x_2\right)=\int_C f\left(\zeta\right)K\left(\zeta,z\right)ds$$

$$K\left(\zeta,z\right)=\frac{\sin\psi}{2a}\operatorname{sign}\left(x_2-\xi_2\right)+\frac{1}{4a}\operatorname{Re}\left\{e^{i\psi}\left[\operatorname{ctg}\frac{\pi\left(\zeta-z\right)}{2a}+\operatorname{ctg}\frac{\pi\left(\zeta+\bar{z}\right)}{2a}\right]\right\} \quad (4.2)$$

Here $\psi=\psi(\zeta)$ is the angle between the normal to contour C and axis Ox_1, at point $\zeta\in C$.

Expanding into simple fractions [10]

$$\operatorname{ctg}\pi x=\frac{1}{\pi x}+\frac{2x}{\pi}\sum_{m=1}^{\infty}\frac{1}{x^2-m^2}$$

and using Sohotsky-Plemmelj formulas [11] we find the expressions for the limiting values of the integrals with a kernel of Hilbert type at $z\to\zeta_0\in C$ appearing in (4.2).

$$\begin{aligned}\left\{\int_C f\left(\zeta\right)\operatorname{ctg}\frac{\pi\left(z-\zeta\right)}{2a}d\zeta\right\}^{\pm}&=\mp 2iaf\left(\zeta_0\right)+\int_C f\left(\zeta\right)\operatorname{ctg}\frac{\pi\left(\zeta_0-\zeta\right)}{2a}d\zeta,\\ \left\{\int_C f\left(\zeta\right)\operatorname{ctg}\frac{\pi\left(\bar{z}-\bar{\zeta}\right)}{2a}d\bar{\zeta}\right\}^{\pm}&=\pm 2iaf\left(\zeta_0\right)+\int_C f\left(\zeta\right)\operatorname{ctg}\frac{\pi\left(\bar{\zeta}_0-\bar{\zeta}\right)}{2a}d\bar{\zeta}.\end{aligned} \quad (4.3)$$

Differentiating function $F^*\left(x_1,x_2\right)$ in (4.1) we find

$$\begin{aligned}\frac{\partial F^*}{\partial z}&=\frac{\pi}{16a^2}\int_C f\left(\zeta\right)\left\{e^{i\psi}\sec^2\frac{\pi\left(\zeta-z\right)}{2a}-e^{-i\psi}\sec^2\frac{\pi\left(\bar{\zeta}+z\right)}{2a}\right\}ds,\\ \frac{\partial F^*}{\partial \bar{z}}&=\frac{\pi}{16a^2}\int_C f\left(\zeta\right)\left\{e^{-i\psi}\sec^2\frac{\pi\left(\bar{\zeta}-\bar{z}\right)}{2a}-e^{i\psi}\sec^2\frac{\pi\left(\zeta+\bar{z}\right)}{2a}\right\}ds.\end{aligned} \quad (4.4)$$

At $z\to\zeta_0\in C$ the integrals in (4.4) become divergent. In order to regulate them it is necessary to carry out the integration by parts allowing for the conditions of periodicity of function $f\left(\zeta\right)$.

Substituting the limiting values of functions (4.1) and their derivatives at $z\to\zeta_0\in C$ into boundary conditions (2.8) allowing for (4.3) we come to the system of singular integrodifferential equations of the second kind

$$p\left(\zeta_0\right)+\int_C p\left(\zeta\right)g_1\left(\zeta,\zeta_0\right)ds+\int_C f'\left(\zeta\right)g_2\left(\zeta,\zeta_0\right)ds=N_1\left(\zeta_0\right)$$

$$-\frac{1}{2}f\left(\zeta_0\right)+\int_C\left\{p\left(\zeta\right)g_3\left(\zeta,\zeta_0\right)+f\left(\zeta\right)g_4\left(\zeta,\zeta_0\right)\right\}ds=N_2\left(\zeta_0\right),\ \zeta\in C_\phi \quad (4.5)$$

$$\int_C f'\left(\zeta\right)g_5\left(\zeta,\zeta_0\right)ds=0,\ \zeta_0\in C\setminus C_\phi$$

where kernels g_m $(m=1,2,...,5)$ and the right parts are determined by expressions

$$g_1(\zeta,\zeta_0)=\frac{1}{2a}\operatorname{Re}\left\{e^{i\psi_0}\left[\operatorname{ctg}\frac{\pi(\zeta_0-\zeta)}{2a}+\operatorname{ctg}\frac{\pi(\zeta_0+\bar{\zeta})}{2a}\right]\right\}+P_1e^{i\psi_0}+P_2e^{-i\psi_0}$$

$$g_2(\zeta,\zeta_0)=\frac{2e_{15}}{c_{44}^E\left(1+k_{15}^2\right)}g_5(\zeta,\zeta_0)$$

$$g_3(\zeta,\zeta_0)=\frac{e_{15}}{э_{11}^{\varepsilon}}\left\{-\frac{|\xi_{20}-\xi_2|}{2a}+\frac{1}{2\pi}\ln\left|4\sin\frac{\pi(\zeta-\zeta_0)}{2a}\sin\frac{\pi(\zeta+\bar{\zeta}_0)}{2a}\right|+\right.$$

$$\left.+\frac{1}{2ia\gamma}e^{i\gamma|\xi_{20}-\xi_2|}-\frac{1}{a}\sum_{m=1}^{\infty}c_m(\xi_{20}-\xi_2)\cos\alpha_m\xi_1\cos\alpha_m\xi_{10}\right\}$$

$$g_4(\zeta,\zeta_0)=\frac{1}{4a}\operatorname{Re}\left\{e^{i\psi}\left[\operatorname{ctg}\frac{\pi(\zeta-\zeta_0)}{2a}+\operatorname{ctg}\frac{\pi(\zeta+\bar{\zeta}_0)}{2a}\right]\right\}+\frac{\sin\psi}{2a}\operatorname{sign}(\xi_{20}-\xi_2)$$

$$g_5(\zeta,\zeta_0)=\frac{1}{4a}\operatorname{Im}\left\{e^{i\psi_0}\left[\operatorname{ctg}\frac{\pi(\zeta-\zeta_0)}{2a}+\operatorname{ctg}\frac{\pi(\bar{\zeta}+\zeta_0)}{2a}\right]\right\}$$

$$P_1=S-\frac{1}{a}(A_0-iB_0),\quad P_2=-S-\frac{1}{a}(A_0+iB_0)$$

$$S=\frac{1}{2ia}\operatorname{sign}(\xi_2-\xi_{20})\left(1-e^{i\gamma|\xi_2-\xi_{20}|}\right)$$

$$A_0=\sum_{k=1}^{\infty}\beta_{1k}\alpha_k\cos\alpha_k\xi_1\sin\alpha_k\xi_{10},\ B_0=\sum_{k=1}^{\infty}\beta_{0k}\operatorname{sign}(\xi_{20}-\xi_2)\cos\alpha_k\xi_1\cos\alpha_k\xi_{10}$$

$$\beta_{mk}=\frac{1}{\alpha_k^m}e^{-\alpha_k|\xi_2-\xi_{20}|}-\frac{1}{\lambda_k^m}e^{-\lambda_k|\xi_2-\xi_{20}|}$$

$$c_m(\xi_{20}-\xi_2)=\frac{1}{\lambda_m}e^{-\lambda_m|\xi_{20}-\xi_2|}-\frac{1}{\alpha_m}e^{-\alpha_m|\xi_{20}-\xi_2|}\quad N_1(\zeta_0)=0,\quad N_2(\zeta_0)=\Phi^*(\zeta_0)$$

$$\psi=\psi(\zeta),\quad \psi_0=\psi(\zeta_0),\quad \zeta,\zeta_0\in C$$

Here $\Phi^*(\zeta_0)$ is the piecewise-constant function determining the value of the electric potential on the system of electrodes. Kernels $g_2(\zeta,\zeta_0)$, $g_5(\zeta,\zeta_0)$ are singular (Hilbert type), the remaining kernels may have not more than light singularities on the assumption that contour C is smooth.

It should be noted here that originating in the process of oscillation reflected from boundary $x_1=0$ and $x_1=a$ shear waves cause the appearance of additional charges on active electrodes. Therefore the configuration of the cavity cross-section, its location and also position of pair electrodes (supplied from a separate generator) should have a certain symmetry in relation to the bases of the layer, i.e. the brought on the given electrodes charges by an absolute quantity were similar. If this requirement is not satisfied system (4.5) becomes unsolvable.

Calculating functions $p(\zeta)$ and $f(\zeta)$ out of system (4.5) by formulas (2.6) using integral representations (4.1) it is possible to determine all the components of the electroelastic field in the layer.

Let us find the expression for the amplitude of density distribution of electric charges $q_k(\beta)$ on k -th electrode. Introducing the parameterization of contour C with the help of equation $\zeta = \zeta(\beta)$ $(0 \le \beta \le 2\pi)$ and taking into account the fact that the surface of the opening is in contact with vacuum we write

$$q_k(\beta) = D_n^{(k)}(\beta), \; \beta_{2k-1} < \beta < \beta_{2k} \; \left(k = \overline{1, 2n}\right) \tag{4.6}$$

Here $D_n^{(k)}(\beta)$ is the amplitude of the normal component of the vector of electric induction on k -th electrode.

Due to (2.6), (4.1), (4.6) we find

$$q_k(\beta_0) = -\frac{\text{э}_{11}^{\varepsilon}}{4a} \int_C f'(\zeta) \operatorname{Im}\left\{ e^{i\psi_0} \left[\operatorname{ctg}\frac{\pi(\zeta - \zeta_0)}{2a} + \operatorname{ctg}\frac{\pi(\bar{\zeta} + \zeta_0)}{2a} \right] \right\} ds, \; \zeta_0 \in C_{\phi_k} \tag{4.7}$$

where C_{ϕ_k} is a part of contour C on which k -th electrode is located.

Integrating expression (4.7) on the variable β_0 in the limits from β_{2k-1} to β_{2k}, we obtain the amplitude value of summed charge Q_k of k -th electrode referring to the unit of its length. The current flowing through the given electrode and equal to the conduction current in the generator circuit may be determined by the formula

$$I_k(t) = \operatorname{Re}\left\{ i\omega e^{-i\omega t} \int_{\beta_{2k-1}}^{\beta_{2k}} q_k(\beta_0) s'(\beta_0) \, d\beta_0 \right\}, \quad s'(\beta_0) = \frac{ds}{d\beta_0}. \tag{4.8}$$

5. A direct piezoelectric effect in a layer with a partially electrodized tunnel cavity

Let us use the above described approach to the situation when a piezoceramic layer with a tunnel opening is used as a generator of electric energy. In this case consider as mechanical excitation two plane monochromic shear waves propagating in positive and negative directions of axis x_2 and accordingly having the following values of displacement amplitude u_3 and electric potential ϕ

$$U_3^{(1)} = \tau_1 e^{-i\gamma x_2}, \; U_3^{(2)} = \tau_2 e^{i\gamma x_2}, \quad \Phi^{(j)} = \frac{e_{15}}{\text{э}_{11}^{\varepsilon}} U_3^{(j)} \; (j = 1, 2) \tag{5.1}$$

For definiteness assume that the cross-section of the cavity has a vertical axis of symmetry and on its surface two symmetrically located continuous electrodes (Fig. 1b) are placed. To obtain the difference of electrical potentials $2V(t)$ in the process of the medium deformation it is necessary to have electric charges of different signs on the electrode platings which require the matching of displacement amplitudes in monochromatic waves. Therefore in (5.1) it is necessary to assume $\tau_1 = -\tau_2 = \tau$.

The generating energy is used in the outer electric circuit closing the electrodes and as a model may be represented by losses on an element with conductivity Y (Fig. 1b). In this case the unknowns are the values of the potential differences on electrodes $2V(t)$ and, the current in circuit $I(t)$ as well. To obtain the electric boundary condition of the considered problem it is necessary to apply Ohm's law to the outer circuit

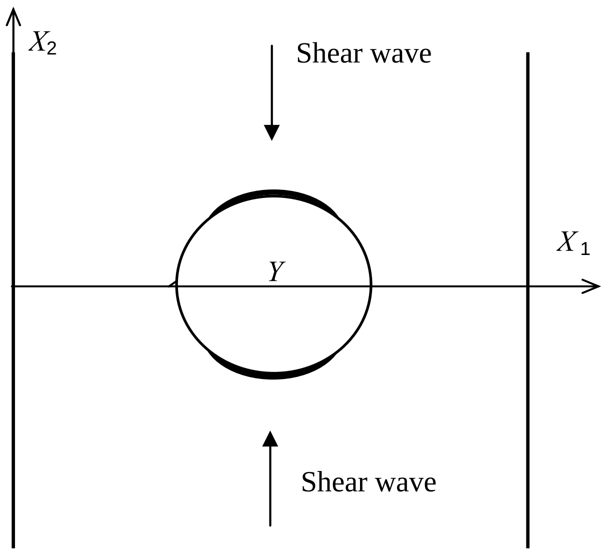


Fig. 1b

$$I(t) = 2YV(t)\,. \tag{5.2}$$

Here solution of the boundary problem consists of prescribing on the electrodes the difference of electric potentials $2V(t)$, i.e. invoking of boundary conditions (2.8) under the action of harmonic waves. Thus, from equations (4.7), (4.8) and (5.2)we may determine the unknown amplitude of potential $V(t)$ on the electrode

$$V^*(\omega) = \frac{i\tau\omega\,э_{11}^{\varepsilon}\,B_1}{2Y - i\omega\,э_{11}^{\varepsilon}\,B_2}, \quad B_m = \int_{\beta_1}^{\beta_2} A_m(\beta_0)\,s'(\beta_0)\,d\beta_0 \;(m = 1,2) \tag{5.3}$$

$$A_m(\beta_0) = -\frac{1}{4a}\int_C f_m'(\zeta)\,\mathrm{Im}\left\{e^{i\psi_0}\left[\mathrm{ctg}\frac{\pi(\zeta-\zeta_0)}{2a} + \mathrm{ctg}\frac{\pi(\bar{\zeta}+\zeta_0)}{2a}\right]\right\}ds$$

Here function $f_m(\zeta)\;(m = 1,2)$ is the "standard" solution of system (4.5) according to the right parts

$$N_1^{(1)}(\zeta_0) = 4i\gamma\cos\gamma\xi_{20}\sin\psi_0,\;\; N_2^{(2)}(\zeta_0) = \frac{2ie_{15}}{э_{11}^{\varepsilon}}\sin\gamma\xi_{20} \tag{5.4}$$

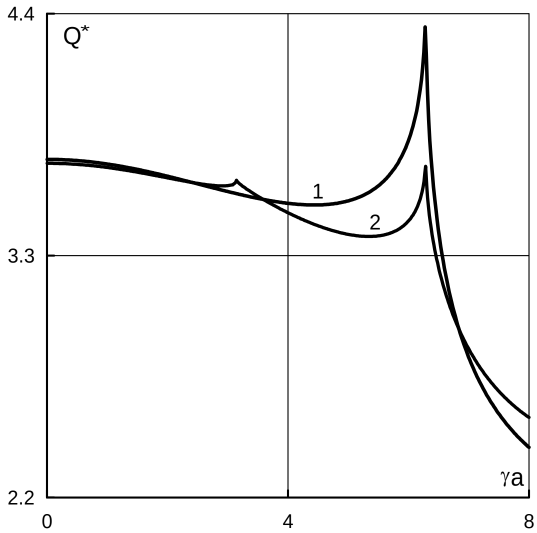
$$N_1^{(2)}(\zeta_0) = 0, \quad N_2^{(2)}(\zeta_0) = \begin{cases} 1, \beta_1 < \beta_0 < \beta_2 \\ -1, \beta_3 < \beta_0 < \beta_4 \end{cases} \quad \zeta_0 = \xi_{10} + i\,\xi_{20} \in C$$

where quantities $\beta_k\;\left(k = \overline{1,4}\right)$ prescribe the location of the electrodes.

From formulas (5.3) we have two limiting cases for interrupted $(Y=0)$ and short $(Y \to \infty)$ circuit, respectively. In the first case the summed charge on the electrodes is not changed in the process of the medium deformation and in the second case it is obvious that $V(t)=0$.

6. Examples of calculations

As a first example we consider a layer from ceramics $PZT-4$ [12] with circular radius R excited by two electrodes, the centers of which are located on its vertical diameter $(\beta_1 = 5\pi/14, \beta_2 = 9\pi/14, \beta_3 = 19\pi/14, \beta_4 = 23\pi/14)$. The system of integrodifferential equations (4.5) was solved numerically by the scheme of the method of quadratures (see Appendix A). The number of interpolation nodes on the cross-section contour of the opening was assumed to be $N = 151, 201$ and 251; the further increase of



parameter N practically did not influence the accuracy of the obtained results.

Fig. 2.

For the considered case in Fig. 2 the change of quantity $Q^* = \left| Q / \left(э_{11}^{\varepsilon} \Phi^* \right) \right|$, which characterizes the amplitude of summed electric charge Q on the electrode with respect to normalized wave number γa ($2\Phi^*$ is the difference of the amplitude of electric potential on the electrodes) is shown. Curve 1 corresponds to the opening displaced from the symmetry axis of the layer at a distance of $0.1a$; curve 2 is to be constructed symmetrical located opening $(R/a = 0.1)$. It is seen that in the first case due to the inertial effect the quantity Q^* may exceed its static analogue by 16%. It should be noted now that by continuing the wave number γ across values $\alpha_m = m\pi/a$ $(m = 1, 2, ...)$, an instability of the solution due to the emergency of a new running wave moving the energy along the waveguide from inhomogeneity to infinity is observed. This circumstance ensures a characteristic "beakwise" form of the curves in the vicinity of points $\gamma = \pi$ and $\gamma = 2\pi(a = 1)$.

If the centers of two active electrodes lie on the lateral diameter of symmetrically located openings $(\beta_1 = -\pi/7, \beta_2 = \pi/7, \beta_3 = 6\pi/7, \beta_4 = 8\pi/7)$ we have a quite different picture. Here (Fig. 3) the phenomenon of resonance is observed. The values of normalized wave numbers corresponding to the first and second natural frequencies of oscillations are equal: $\gamma_{(1)}a \approx 2.95$ and $\gamma_{(2)}a \approx 8.69$. The antiresonance frequency, when the current in the generator circuit is equal to zero, is $\gamma_{(1)}^{*}a \approx 3.1$. In the process of calculations we assumed $R/a = 0.1$.

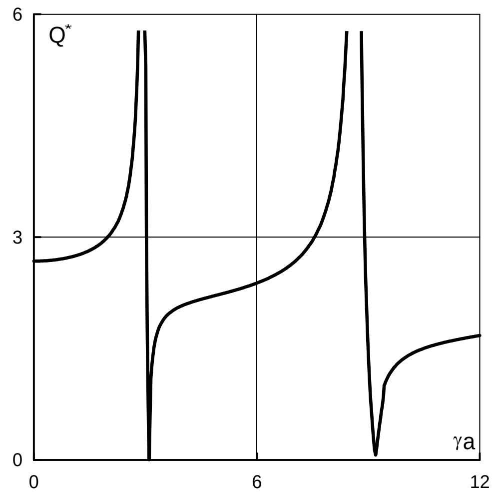


Fig. 3.

The analysis of the results shows that more effective electroacoustic transformation of energy in the considered frequency interval is observed when the area of the electroded plating is smaller.

7. Concluding Remarks

The represented approach to the solution of the mixed stationary dynamic problem of electroelasticity permits to investigate the influence of the inertial effect on the behaviour of the components of an electric field in a layer with an opening of rather arbitrary configuration for different number of electrodes and their disposition. For the numerical solution of the system of integrodifferential equations (4.5) by the prescribed scheme of the method of quadratures due to the fact that some of its kernels undergo fractures, and the densities have root singularities on the edges of electrodes, in order to reach the satisfactory accuracy it is necessary to use a considerable number of the nodes subdivision of the cavity contour section which yields an increase of the computation time. Nevertheless, the considered seems to be universal, permitting to investigate various variants of electric excitation of conjugated fields without any basic change of the algorithm.

From the given results it follows that in the condition of the inverse piezoelectric effect the distribution of the displacement in a layer considerably depends on the frequency of harmonic loading, the configuration of the transverse section of the tunnel cavity, and the prescribed in the system of electric potentials electrodes. In case of an antiplane deformation the stresses of a longitudinal shear on a surface free from mechanical loading do not have singularities on the edges of the electrodes [6]. The numerical investigation proceeding from the presently constructed algorithm, confirms it.

System (4.5) may be generalized to the case of several tunnel openings C_m $\left(m=\overline{1,n}\right)$, if we assume $p(\zeta)=\left\{p_m(\zeta),\zeta\in C_m\right\}$, $f(\zeta)=\left\{f_m(\zeta),\zeta\in C_m\right\}$, $C=\bigcup\limits_{m=1}^{n} C_m$. The configuration of the openings, their location and the position of the pair surface electrodes warrant its solvability.

Appendix A

Consider one of the methods of numerical implementation of the system (4.5). Let us build the interpolating Lagrange polynomial for the sought-for functions $p(\zeta)$ and $f'(\zeta)$ in of the nodes $\beta_j=2\pi(j-1)/N$ $\left(j=\overline{1,N}\right)$. Such a polynomial has the form [13]

$$L_N\left[\{p_*,f_*'\};\beta\right]=\frac{1}{N}\sum_{j=1}^{N}\left\{p_j^0,f_j^0\right\}\sin\frac{N\left(\beta_j-\beta\right)}{2}\operatorname{cosec}\frac{\beta_j-\beta}{2} \qquad \text{(A1)}$$

$$p(\zeta)=p_*(\beta),\ p_j^0=p_*\left(\beta_j\right),\ f(\zeta)=f_*(\beta),\ f_j^0=f_*'\left(\beta_j\right)$$

It must be mentioned here that formulas (A1) are valid for odd numbers of the node division of the contour C.

Integration of the formula (A1) for function $f_*'(\beta)$ using the equation [19]

$$\int\frac{\sin(2m+1)x}{\sin x}dx=2\sum_{k=1}^{m}\frac{\sin 2kx}{2k}+x$$

leads to the following expression for the function $f_*(\beta)$

$$M_N\left[f_*(\beta);\beta\right]=\frac{1}{N}\sum_{j=1}^{N}f_j^0\Omega_j(\beta)+A$$

$$\Omega_j(\beta)=-2\sum_{k=1}^{\frac{N-1}{2}}\frac{\sin k\left(\beta_j-\beta\right)-\sin k\beta_j}{k}+\beta \qquad \text{(A2)}$$

The constant A must be determined from the conditions of the periodicity of the function $f_*(\beta)$ which due to (A2) has the following form

$$\sum_{j=1}^{N} f_j^0 = 0 \tag{A3}$$

Applying (A2) we also find the quadrature formula

$$\int_0^{2\pi} f_*(\beta) G(\beta, \beta^*) d\beta = \frac{2\pi}{N^2} \sum_{j=1}^{N} f_j^0 \sum_{m=1}^{N} \Omega_{jm} G(\beta_m, \beta^*) + A \frac{2\pi}{N} \sum_{m=1}^{N} G(\beta_m, \beta^*) \tag{A4}$$

where $\Omega_{jm} = \Omega_j(\beta_m)$. In the node collocations $\beta_\ell^* = \pi(2\ell - 1)/N \quad (\ell = \overline{1, N})$ the polynomial (A1) has the following value at odd values of N

$$L_N\left[p_*(\beta); \beta_\ell^*\right] = \frac{1}{N} \sum_{j=1}^{N} p_j^0 (-1)^{\ell+j} \operatorname{cosec} \frac{\beta_\ell^* - \beta_j}{2} \quad (\ell = \overline{1, N}). \tag{A5}$$

For the singular integral in (4.5) the formula is analogous to the formula for calculating regular integrals [14]

$$\begin{aligned} &\int_0^{2\pi} f_*'(\beta_j) \operatorname{Im}\left\{ e^{i\psi_0} \operatorname{ctg} \frac{\pi(\zeta - \zeta_0)}{2a} \right\} d\beta = \\ &= \frac{2\pi}{N} \sum_{j=1}^{N} f_j^0 \operatorname{Im}\left\{ e^{i\psi_0(\rho_\ell^*)} \operatorname{ctg} \frac{\pi\left[\zeta(\beta_j) - \zeta_0(\beta_\ell^*)\right]}{2a} \right\} \end{aligned} \tag{A6}$$

Now, substituting the integrals in (4.5) by finite sums of the formulas (A.4), (A.6) and using the equalities (A.2), (A.3) and (A.5) we arrive to a system $2N+1$ of the algebraic equations related to the values of the functions $p(\zeta)$ and $f'(\zeta)$ at the nodes of the interpolation $\beta_j(j = \overline{1, N})$ and the constant A.

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