ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
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MATHEMATICAL SIMULATION OF COLLISION OF ARABIAN AND EUROASIAN PLATES ON THE BASE OF GPS DATA

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Արաբական և Եվրասիական սալերի բախման գոտու մաթեմատիկական մոդելավորումը GPS տվյալների հիման վրա

Օգտագործելով առաձգականության տեսության եզրային խնդրի մաթեմատիկական ճշգրիտ լուծումը շերտի համար, արտածված են բանաձևեր GPS չափումների տվյալների հիման վրա Արաբական և Եվրասիական սալերի բախման գոտում լարումները և տեղափոխությունները որոշելու համար, ընդունելով այդ տարածաշրջանի համար առաձգականության տեսության շրջանակներում միջինացված մոդելի կիրառելիություն: Կառուցված են լարումների թենզորի և տեղափոխման վեկտորի բաղադրիչների համար համապատասխան գրաֆիկները: Կատարված է լարվածա-դեֆորմացիոն վիճակի վերլուծությունը:

Агаловян Л.А., Геворкян Р.С., Саакян А.В.
Математическое моделирование зоны коллизии Аравийской и Евразиатской плит на основе данных GPS

Применяя математически точное решение краевой задачи теории упругости для слоя, выведены формулы для определения напряжений и перемещений зоны коллизии Аравийской и Евразиатской плит, используя данные GPS и считая возможным применение осредненной модели региона по теории упругости. Построены графики для компонентов тензора напряжений и вектора перемещения. Проведен анализ напряженно-деформированного состояния.

Abstract

Applying the mathematically precise solution of the boundary value problem of elasticity theory for a layer [1,2], formulae for determining stresses and displacements of collision zone of Arabian and Euroasian plates applying GPS data and considering possible the application of the average model of region in the frame of elasticity theory are derived. Graphics for displacement vector are built. The analysis of stress-strain state is developed.

Key words: Arabian plate, Seismotectonics, GPS Observation

Introduction

It is known that the armenian mountainous area and the Caucasian region are situated between considered immovable Euroasian and slowly moving to the north (north-west) Arabian plates. In the collision zone during the time big stresses which bring to earthquakes, are accumulated. These phenomena attract the attention of the scientists-seismologists, geophysicists and mechanics. In paper [3] review of modern approaches, qualitative and quantitative analyses of collision zone state is brought. At the same time, using GPS (the Global Positioning System) data by the method of finite element a plane problem of elasticity theory for mathematical simulation of stress-strain state of collision zone taking it for an isotropic plate, is numerically solved.

In the present paper, applying mathematically precise boundary value problems of elasticity theory for a layer [1,2], stress-strain state of the collision zone is determined, taking into account that its facial surface is free from loads and in GPS bench-mark points displacements vector components are given.

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The statement of the problem of elasticity theory and its precise mathematical solution.

Let us have a homogeneous isotropic plate with the width h occupying an area $\Omega = \{x, y, z : 0 \le x, y \le \ell,\ 0 \le z \le h;\ h << \ell\}$ (fig. 1).

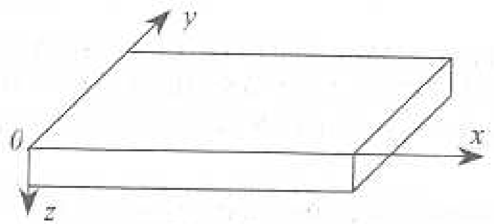

Fig. 1

(axis x is directed along the earth parallel, axis y is in the direction of north and axis z is directed along the radius into the depth of the earth).

On the earth surface $z = 0$ stresses are absent

$$\sigma_{jz}(x, y, 0) = 0 \qquad j = x, y, z \tag{1}$$

and in the depth h (the opposite surface of the plate-layer) the displacements which are represented by the polynomials with yet unknown coefficients are given

$$\begin{aligned} U = U_x(x, y, h) &= \sum_{k=0}^{n}\sum_{i=0}^{k} a_{ik} x^i y^{k-i} \\ V(x, y) = U_y(x, y, h) &= \sum_{k=0}^{n}\sum_{i=0}^{k} b_{ik} x^i y^{k-i} \\ W(x, y) = U_z(x, y, h) &= 0 \end{aligned} \tag{2}$$

It is required to determine the stresses tensor and displacements vector components inside the plate-layer, which would satisfy the equations of the three-dimensional problem of elasticity theory [4,5] (these equations are not shown) and on the surfaces $z = 0$ and $z = h$ would satisfy the boundary conditions (1) and (2).

The precise mathematical solution of the stated problem under any numerical values a_{ik}, b_{ik} are expressed by recurrent formulae [1,2], which have the following form for an isotropic plate-layer

$$\begin{aligned} &\xi = x/\ell, \quad \eta = y/\ell, \quad \zeta = z/h = \varepsilon^{-1} z/\ell \\ &U = U_x/\ell, \quad V = U_y/\ell, \quad W = U_z/\ell \end{aligned} \tag{3}$$

$$Q(x, y, z) = \varepsilon^{\chi} \sum_{s=0}^{N} \varepsilon^{s} Q^{(s)}(\xi, \eta, \zeta) \tag{4}$$

$$\chi_\sigma = -1, \qquad \chi_u = 0$$

$$\sigma_{zz}^{(s)}(\xi,\eta,\zeta)=\sigma_{zz0}^{(s)}(\xi,\eta)+\sigma_{zz^*}^{(s)}(\xi,\eta,\zeta)$$

$$\sigma_{zz^*}^{(s)}=-\int_0^{\zeta}\left(\frac{\partial\sigma_{xz}^{(s-1)}}{\partial\xi}+\frac{\partial\sigma_{yz}^{(s-1)}}{\partial\eta}\right)d\zeta \tag{5}$$

$$\sigma_{xz}^{(s)}(\xi,\eta,\zeta)=\sigma_{xz0}^{(s)}(\xi,\eta)+\sigma_{xz^*}^{(s)}(\xi,\eta,\zeta)$$

$$\sigma_{xz^*}^{(s)}=-\int_0^{\zeta}\left(\frac{\partial\sigma_{xx}^{(s-1)}}{\partial\xi}+\frac{\partial\sigma_{xy}^{(s-1)}}{\partial\eta}\right)d\zeta \tag{6}$$

$$\sigma_{yz}^{(s)}(\xi,\eta,\zeta)=\sigma_{yz0}^{(s)}(\xi,\eta)+\sigma_{yz^*}^{(s)}(\xi,\eta,\zeta)$$

$$\sigma_{yz^*}^{(s)}=-\int_0^{\zeta}\left(\frac{\partial\sigma_{yy}^{(s-1)}}{\partial\eta}+\frac{\partial\sigma_{xy}^{(s-1)}}{\partial\xi}\right)d\zeta \tag{7}$$

$$\sigma_{xx}^{(s)}=\frac{\nu}{1-\nu}\sigma_{zz}^{(s)}+\frac{2G}{1-\nu}\left(\frac{\partial U^{(s-1)}}{\partial\xi}+\nu\frac{\partial V^{(s-1)}}{\partial\eta}\right) \tag{8}$$

$$\sigma_{yy}^{(s)}=\frac{\nu}{1-\nu}\sigma_{zz}^{(s)}+\frac{2G}{1-\nu}\left(\frac{\partial V^{(s-1)}}{\partial\eta}+\nu\frac{\partial U^{(s-1)}}{\partial\xi}\right) \tag{9}$$

$$\sigma_{xy}^{(s)}=G\left(\frac{\partial U^{(s-1)}}{\partial\eta}+\frac{\partial V^{(s-1)}}{\partial\xi}\right) \tag{10}$$

$$U^{(s)}(\xi,\eta,\zeta)=U_0^{(s)}(\xi,\eta)+\frac{\zeta}{G}\sigma_{xz0}^{(s)}+U_*^{(s)}(\xi,\eta,\zeta)$$

$$U_*^{(s)}=\int_0^{\zeta}\left(\frac{1}{G}\sigma_{xz^*}^{(s)}-\frac{\partial W^{(s-1)}}{\partial\xi}\right)d\zeta \tag{11}$$

$$V^{(s)}(\xi,\eta,\zeta)=V_0^{(s)}(\xi,\eta)+\frac{\zeta}{G}\sigma_{yz0}^{(s)}+V_*^{(s)}(\xi,\eta,\zeta)$$

$$V_*^{(s)}=\int_0^{\zeta}\left(\frac{1}{G}\sigma_{yz^*}^{(s)}-\frac{\partial W^{(s-1)}}{\partial\eta}\right)d\zeta \tag{12}$$

$$W^{(s)}(\xi,\eta,\zeta)=W_0^{(s)}(\xi,\eta)+\frac{1-2\nu}{2(1-\nu)G}\sigma_{zz0}^{(s)}+W_*^{(s)}(\xi,\eta,\zeta)$$

$$W_*^{(s)}=\int_0^{\zeta}\left(\frac{1-2\nu}{2(1-\nu)G}\sigma_{zz^*}^{(s)}-\frac{\nu}{1-\nu}\left(\frac{\partial U^{(s-1)}}{\partial\xi}+\frac{\partial V^{(s-1)}}{\partial\eta}\right)\right)d\zeta \tag{13}$$

$$\sigma_{jz}^{(0)}=0,\quad j=x,y,z;\quad U_0^{(0)}=U(\xi,\eta),\quad V^{(0)}=V(\xi,\eta),\quad W^{(0)}=0 \tag{14}$$

Iteration process (4)-(14) in case of polynomial loading (2) after $n+1$ step terminates and reduces to a closed (precise) mathematical solution of the stated boundary value problem for a layer under any numerical values a_{ik}, b_{ik}.

Determination of stress-strain state of collision zone by GPS data

Taking into account that we have GPS data in 37 bench-mark points in geographical coordinates [3], we pass to Cartesian coordinates presuming point #10 as the origin of the system (Table 1).

Table 1

N	GPS bench-mark point	longitude	latitude	GPS Data		Cartesian coordinates	
				U_x mm/year	U_y mm/year	x km	y km
1	KAL2	43.34	38.55	-5.3	12	535.4	183.3
2	GORI	46.37	39.51	3.3	9.6	808.9	290.0
3	IJEV	45.14	40.91	4	7.4	693.7	445.6
4	KRES	44.49	42.45	1.3	2.8	632.9	616.7
5	MATS	43.75	42.98	1.5	0.3	567.4	675.6
6	BEUG	42.79	44.01	-0.6	1	483.9	790.1
7	ZELB	41.56	43.79	0.5	0.8	378.3	765.6
8	AKTO	39.7	40.97	0.5	1.7	209.8	452.3
9	SINC	37.96	39.45	-18.3	9.9	47.9	283.4
10	GAZI	37.57	36.9	-8.5	12.3	0	0
11	KIZI	40.65	37.25	-6.9	16.1	287.2	38.9
12	NICH	44.53	41.83	1.1	5.8	637.7	547.8
13	ADYI	38.23	37.75	-7.6	13.4	64.9	94.4
14	KRCD	39.81	37.85	-7.6	13.9	210.9	105.6
15	MLTY	38.22	38.46	-12	10.9	67.1	173.3
16	KMAN	39.16	39.61	-19.6	9.1	157.0	301.1
17	MERC	40.25	39.73	-2.7	4.9	255.8	314.5
18	KRKT	41.79	38.75	-5.1	14.4	393.9	205.6
19	PATN	42.91	39.24	-2.5	8.7	495.9	260.0
20	ARGI	43.03	39.72	1.2	6.7	506.7	313.4
21	ERZU	41.3	39.97	-0.9	5	350.8	341.1
22	ISPI	40.81	40.44	0.2	2.5	307.5	393.4
23	OLTU	41.99	40.55	2.3	4.3	413.2	405.6
24	HOPA	41.34	41.37	-0.1	2.6	356.2	496.7
25	KARS	43.17	40.69	0.7	5.2	518.6	421.1
26	ARTI	43.95	40.61	2.2	7.2	588.3	412.2
27	MMOR	44.11	40.18	2.5	7	603.3	364.5
28	NSSP	44.5	40.23	3.6	8.1	638.2	370.0
29	GARN	44.74	40.15	2.5	9.1	659.9	361.1
30	JERM	45.66	39.84	4.6	10.1	743.6	326.7
31	NINO	43.89	41.54	0.9	4.2	581.7	515.6
32	VANI	42.47	42.02	1.6	4.1	456.3	568.9
33	SACH	43.4	42.35	2.6	4.8	537.7	605.6
34	INGU	42.06	42.72	0.9	2.7	420.6	646.7
35	BALK	43.35	43.06	-2	-0.4	532.7	684.5
36	ULKA	42.19	43.35	-0.9	-1.2	432.3	716.7
37	SHAT	42.67	43.74	0.3	1.7	473.7	760.1

The bench-mark points and their displacements (multiplied by 10^7) in dimensionless coordinate system are shown (Fig. 2).

Calculating the stresses tensor components and displacements vector components by formulae (4)-(14) under $n = 7$, as the number of the unknown coefficients a_{ik}, b_{ik} coincide with the number of GPS data, we require that on daily surface of the collision zone (i.e. under $z = 0$) the displacements vector components U_x and U_y in the bench-mark points were equal to the corresponding values of GPS data.

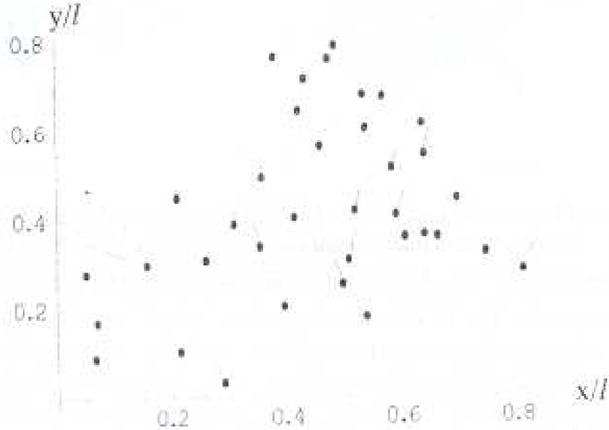

Fig. 2

We obtain a system from 36 linear algebraic equations relative to unknown coefficients a_{ik}, b_{ik} by the solution of which, functions (2) are uniquely determined.

$$U(\xi,\eta) = 3.51876\times10^{-4} - 0.0000313862\,\eta + 0.000194433\,\eta^2 - 0.000568706\,\eta^3 - 0.000172992\,\eta^4 + 0.00114821\,\eta^5 - 0.00129734\,\eta^6 + 0.000494792\,\eta^7 - 0.0000719652\,\xi + 0.000254709\,\eta\xi - 0.000787758\,\eta^2\xi + 0.00480306\,\eta^3\xi - 0.0074754\,\eta^4\xi + 0.00306291\,\eta^5\xi - 0.000732831\,\eta^6\xi - 0.000620365\,\xi^2 - 0.0017512\,\eta\xi^2 - 0.00241075\,\eta^2\xi^2 - 0.00202097\,\eta^3\xi^2 + 0.00970518\,\eta^4\xi^2 - 0.00148646\,\eta^5\xi^2 - 0.00243986\,\xi^3 + 0.00941016\,\eta\xi^3 + 0.00524678\,\eta^2\xi^3 - 0.00942483\,\eta^3\xi^3 - 0.0047618\,\eta^4\xi^3 + 0.00410064\,\xi^4 - 0.0214747\,\eta\xi^4 + 0.00313998\,\eta^2\xi^4 + 0.00961436\,\eta^3\xi^4 - 0.00220122\,\xi^5 + 0.019655\,\eta\xi^5 + 0.00769818\,\eta^2\xi^5 - 0.00115491\,\xi^6 - 0.00516484\,\eta\xi^6 + 0.000982346\,\xi^7$$

$$V(\xi,\eta) = 3.85638\times10^{-4} - 0.000053583\,\eta + 0.000418385\,\eta^2 - 0.00149779\,\eta^3 + 0.0017753\,\eta^4 + 0.000726678\,\eta^5 - 0.00239369\,\eta^6 - 0.00106913\,\eta^7 - 0.0000522178\,\xi + 0.000190163\,\eta\xi - 0.000966174\,\eta^2\xi + 0.00731688\,\eta^3\xi - 0.0165612\,\eta^4\xi + 0.0122199\,\eta^5\xi - 0.00266616\,\eta^6\xi + 0.000435682\,\xi^2 - 0.00109834\,\eta\xi^2 - 0.00460338\,\eta^2\xi^2 + 0.00688864\,\eta^3\xi^2 + 0.00657357\,\eta^4\xi^2 - 0.00570417\,\eta^5\xi^2 - 0.00164373\,\xi^3 + 0.00756846\,\eta\xi^3 + 0.000986512\,\eta^2\xi^3 - 0.0172764\,\eta^3\xi^3 - 0.0049901\,\eta^4\xi^3 + 0.00226691\,\xi^4 - 0.0149408\,\eta\xi^4 + 0.0130193\,\eta^2\xi^4 + 0.00336324\,\eta^3\xi^4 - 0.0006254\,\xi^5 + 0.00873813\,\eta\xi^5 - 0.0075214\,\eta^2\xi^5 - 0.000624649\,\xi^6 - 0.00103845\,\eta\xi^6 + 0.000263062\,\xi^7 \qquad (15)$$

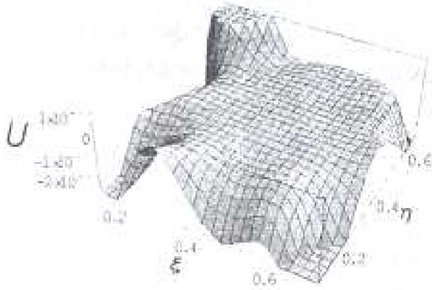

Fig. 3

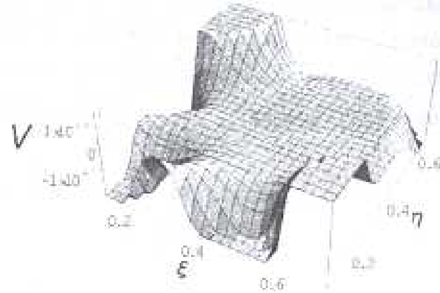

Fig. 4

Thus, the obtained solution (1)-(15) mathematically precisely satisfies the equations of elasticity theory and boundary conditions (1), (2), and in the bench-mark points the displacements values U_x, U_y coincide with the values of GPS data (Table 1.).

The more the density of the bench-mark points will be the more precisely the obtained solution will express the true picture of the stress-strain state of the collision zone.

As we are not aware of the conditions on the border of the considered zone, for great exactness it is necessary to be restricted by the area inside this zone. It is proved by the authors [4,6], that in this case the effect influence of the denoted zone (boundary layer) on the stress-strain state inside plate-layer is negligibly small.

In fig. 3, fig. 4 the 3D plots of the tangential displacements U and V in the depth $z = h = 10$ km are brought.

The full energy of the deformation chosen in the collision zone of the plate with the size of 700x700x10 km determined by the formula

$$E_{def} = \frac{1}{4(1+\nu)G}\iiint_{\Omega}[\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2(1-\nu)(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2) - \\ - 2\nu(\sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz})]dxdydz \quad (16)$$

$0 \le z \le 10^4 m, \quad 0 \le x, y \le 7\cdot10^5 m. \qquad \nu = 0{,}2, \quad G = 2{,}5\cdot10^{10} Pa$

equal to

$$E_{def} = 3{,}7\cdot10^{22} erg = 3{,}7\cdot10^{15} J$$

Analytical solution (1)-(15) of the stated problem permits us to conduct other investigations as well when collaborating with geophysicists and seismologists.

The algorithm of the problem solution developed by the authors permits in the future to take into account in homogeneity and anisotropy of the collision zone, layered, the existence of reonomic layers, relief change of the area (variability h), and also dependence of physical-mechanical constants from the depth of the layer [4,7,8]. We have opportunity to take into account the curvature of the region surface, i.e. to admit the region layer as part of the spherical surface [9]. Other the earthquake-hazard regions may be considered too.

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