ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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A DERIVATION OF EVOLUTION EQUATION FOR THE INTERACTION BETWEEN A WEAKLY INTENSITY HIGH FREQUENCY WAVES IN A GAS-FLUIDS MEDIA

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Գ.Ս. Բեզիրգենյան
Էվոլուցիոն հավասարման ստացումը թույլ ինտենսիվության, բարձր հաճախության փոխազդող ալիքների համար գազահեղուկային միջավայրում

Դիտարկվում է հիպերբոլական տիպի առաջին կարգի կվազիգծային դիֆերենցիալ հավասարումների համակարգ: Բարձր հաճախության (կարճ ալիքներ $\lambda<<1$) և թույլ ինտենսիվության $(\varepsilon<<1)$ փոխազդող ալիքների համար (թույլ ոչ գծային պրոցես) օգտագործելով
ա. ոչ գծային վերադրման երևույթը;
բ. դիֆերենցիալ օպերատորների սիմվոլիկ մեթոդը;
գ. համապատասխան որոշիչների ուղղակի հաշվումը հիմնական մոտավորությունում (ε);
նշված մոտավորությունում ստացված է կվազիգծային երկրորդ կարգի մասնական ածանցյալներով էվոլուցիոն կամ "կարճ ալիքների" հավասարումը, որը բնութագրում է փոխազդող ալիքների ճակատների կոնտակտի շրջակայքում ("նեղ" կամ "կարճ" ալիքային տիրույթ) գրգռված գազահեղուկային միջավայրի շարժումը:

Г.С. Безиргенян
Вывод эволюционного уравнения для взаимодействующих волн слабой интенсивности и высокой частоты в газожидкостных средах

Рассматривается гиперболическая система квазилинейных дифференциальных уравнений в частных производных первого порядка. В случае взаимодействующих волн слабой интенсивности $(\varepsilon<<1)$ и высокий частоты $(\lambda<<1)$ (слабый нелинейный процесс) используя:
а. феномен нелинейной суперпозиции;
б. символический метод дифференциальных операторов;
в. прямое вычисление соответствующих операторов в основном порядке (ε);
получено "эволюционное" уравнение или уравнение "коротких" волн в основном порядке ε (приближенное квазилинейное дифференциальное уравнение второго порядка), описывающее нелинейный волновой процесс в окрестности фронтов ("узкая" или "короткая" область) взаимодействующих волн в газожидкостных средах.

Как приложение, указаны задачи, которые решаются на основании полученного уравнения путем конкретизации коэффициентов, входящих в это уравнение.

1. Introduction

On the base of the strictly hyperbolic system of quasilinear first order partial differential equations for the high frequency (waves of short length, $\lambda<<1$) small intensity $(\varepsilon<<1)$ interacting waves (weakly nonlinear process) by using:

a. the phenomena of nonlinear superposition;
b. the symbolic method of differential operators;
c. the direct calculations of corresponding determinants;

in the principal approximation ε the quasilinear second order partial differential approximation equation for every unknown function have obtained.

The method, of an asymptotic expantion of solutions of hyperbolic systems of quasilinear first order partial differential equations, has based on the work Choquet-Bruhat [1], who had considered a single high frequency wave (without shocks) in a slowly varying medium. This method yields the first terms of an asymptotic expantion.

In the general case, for m high-frequency, small-amplitude interacting waves (weakly nonlinear process), the method (which was noted) was generalized by Hunter, Keller [2]. (They called it "weakly nonlinear geometrical optics").

The article [2] has important meaning for mathematical physics. In [2] the solution of the hyperbolic system of quasilinear first order partial differential equations represented as a superposition of small-amplitude, high-frequency waves, each of them undergoes distortion due to nonlinear interaction. This is the phenomena of nonlinear superposition.

It should be noted, that in [2] had explored broad class questions, connected with nonlinear wave processes are going on in a gas-fluids media. In particular, in the main order, authors had derived eiconal and transport equations, found their solutions by means of rays (bicharacteristics) and etc. The work ends in appendix, where results of section 2 are employed to the gas-dynamics equations (for the insentropic flow of a ideal gas; acoustic problem).

The mentioned method was used by Carbonaro [3] for investigation of the one-dimensional (relatively of space coordinate) non-stationery problem about non-linear propagation of an initial disturbance produced in a compressible inviscid fluid.

It should be noted that; the equation (2.7) in [2] and equation (2.3) in [3] is same.

Carbonaro's work has some shortcomings, main of which are:

first; the author's requirement, "... Γ^k depends only a single phase φ_k ..." isn't correct;

second; $h^i = \left(h_1^i, h_2^i\right)$ isn't parallel vector for $u_1^i = \left(u_1^1, u_1^2\right)$;

third; the requirement $E\left(\Gamma^m\right) = 0$ not sufficiently grounded. (See p. 45.)

The first time, the conical flow of a inviscid gas, at near triple point (local zone at near fronts of interacting waves) was explored by Kuo [4] and had obtained quasilinear second order partial differential equation for function $F(\xi,\eta)(f = (1/\beta)\alpha' x^m F(\xi,\eta)$, $\varphi = u_\infty z[1 + f(r,\theta)]$, $\beta = \sqrt{M_\infty^2 - 1}$, φ is potential of velocity, ξ, η are expressed by conical coordinates: r, θ. (See details in the article). This equation author called as similarity rule, which is good in agreement with similarity law in the gasofluiddynamics.

In former Soviet Union, the narrow wave region for problem about regular reflection of the weakly plane shock wave from angle above $\pi/2$, first was explored by Ryzhov and Kristianovich [5]. They had introduced special moving coordinate system and slow-time ($\tau = \ln t$?, non stationer problem) and concerning of these coordinates and time had obtained the system of , "short waves equations" for isentropic axisymmetric motion of an inviscid gas.

It should be noted, that the system of equation (1.12) and (1.14) (see p. 588 [5]) isn't connected; this system is equivalent to one quasilinear second order partial differential equation. It isn't clear; as a moving coordinate system with waves fronts is connected.

A derivation of the system of approximation quasilinear first order partial differential equations, describing non-stationar motion of a viscosity thermal-conducting inhomogeneous liquid at near of the front of weakly shock waves is main result of the work [6]. The system, which was obtained concerning of the cartesian coordinates (fastening with fronts of waves) and dimensionless (independent and unknown) variables, is equivalent to one of the quasilinear third order partial differential equation.

It isn't clear; the passage to the dimensionless coordinates, in particular,

$x_1 = L\Lambda x_1^*$, L, Λ have dimension of long.

In the work [7], which had been written in the style; "much in little" (conglomeration), had given the general description theory of "short waves". On the base of a general form hyperbolic system of quasilinear first order partial differential equations in the work had described evolution of weakly non-linear process of interacting waves in a small vicinity of the contact point (for plane problems) or the contact line (for space problems) of wave fronts in continuous media (solid deformation bodies or gas-fluids media), where there are solid boundaries or solid bodies. Authors, by using symbolic operators method [8], and some heuristic calculations in principal approximation $(O(\gamma) = O(\varepsilon))$ the quasilinear second order partial differential equation ("short waves equation") concerning every unknown function in the system of moving beams coordinates have been obtained. Of course, these beams aren't bicharacteristics of initial hyperbolic system.

Author of the article [9] had described in more detail, strictly and well-grounded some results about building solutions of the system of linear and quasilinear first order partial differential equations in vicinity of contact (in particular, tangency) point of waves fronts, which was published in his early works (in particular, in the book "Propagation of waves in continuous media". Yerevan AS of Armenian SSR, 1981, 303p. See chapter I, §2, [7], §3 and etc.)

It's necessary to indicate the article [10] also, where a propagation of weakly magnetogasodynamics perturbations in a inhomogeneous conducting viscous medium is considered, in the presence of initial movement. Author had introduced same moving coordinate system that in [6] and the orders of perturbations had estimated as there, in spite of, in this article there is a applied magnetic field.

It should be noted, that the main drawback of works [5-7,9,10] is separation of interacting weakly waves from each other, without of additional condition.

In an end of this introduction, it's necessary to describe the essence of nonlinear superposition.

It's well known that, the nonlinear (in particularly quasilinear) hyperbolic system doesn't permit the superposition of single natural modes, each of them propagats independently. Due to the interaction between different modes (nonlinear wave process), takes place cumulative distortion of a wave profile. Despite on this fact, for a weakly high frequency waves, as showed in the work [2], the cumulative solution one can represented as superposition of single modes in the presence of additional condition (3.2) (see p.7)and obtaining separate equation for every modes in the main order ε.

2. Statement of problem.

Let waves of high frequency ($\lambda << 1$) and small intensity ($\varepsilon << 1$) (ultrasonic or electromagnetic) are propagating in a gas-fluids media, where can be solid bodies or boundaries, which as usuall excitent the interaction between different modes. In general case, the movement (supersonic or supercritical) of inviscid inhomogeneous gas-fluids media one can describe by hyperbolic system of quasilinear first order partial differential equations:

$$A_{ij}^{(k)}(U)\frac{\partial U_j}{\partial x_k} + B_i(U) = 0 \tag{2.1}$$

where $i, j = 1,...,m$; x_k $(k = 0,1,...,n)$ are independent variables, U_j are components of vector U and characterize the properties and movement of media, B_j are components of vector B and characterize as usual forces excitent a movement; $A_{ij}^{(k)}$ are scalars. As usually $x_0 = t$ is a time and x_N $(N = 1,...,n)$ denote the hyperspace cartesian (Euler's) coordinates.

It isn't necessary to adduce some special conceptions: the characteristic determinant and the condition of hyperbolisity of system (2.1), bicharacteristics or rays and etc. These conceptions have been described very good in the famous book-monograph [11], which is very difficult for reading.

A solution of the system (2.1) one can be represented in the form

$$U = U^{(0)}(x) + V(x)$$

where $x = (x_0, x_1, \ldots, x_n)$ is vector, $U^{(0)}$ is exact solution of (2.1), which isn't singular and describes an initial (basic) flow of the media, $V(x)$ is perturbation.

Since the intensity of propagating waves is small, consequently

$$\overline{U} = \left|U - U^{(0)}\right| / \left|U^{(0)}\right| << 1$$

Where $\overline{U}$ is surplus volume of U.

After substituting this solution in the system (2.1) and expanding $A_{ij}^{(k)}\left(U^{(0)}+V\right)$, $B_j\left(U^{(0)}+V\right)$ the power series and taking into account "0" approximation, in first approximation we obtain

$$a_{ij}^{(k)}(U^{(0)})\frac{\partial V_j}{\partial x_k} + b_{ij}(U^{(0)})V_j = C_{ijl}^{(k)}V_l\frac{\partial V_j}{\partial x_k} \tag{2.2}$$

where $a_{ij}^{(k)} = A_{ij}^{(k)}\left(U^{(0)}\right)$, $b_{jl} = \left.\frac{\partial A_{ij}^{(k)}}{\partial U_l}\right|_{U_l^{(0)}}\frac{\partial U_i^{(0)}}{\partial x_k} + \left.\frac{\partial B_i}{\partial U_l}\right|_{U_l^{(0)}}$, $C_{ijl}^{(k)} = -\left.\frac{\partial A_{ij}^{(k)}}{\partial U_l}\right|_{U_l^{(0)}}$

The orders of the terms included in the system (2.2) in cartesian coordinates it is impossible to find, since they are fixed coordinates and describing the global disturbance of medium. Therefore it is necessary to introduce moving coordinate system is fastened with waves fronts.

For magnetohydrodynamic problems the number of independent variables (coordinates) (nonstationar, nonlinear space problem) is four. The formulas of transformation to moving coordinates (beams coordinates) are:

$$\tau = \tau(t, x_1, x_2, x_3),\ \zeta = \zeta(x_1, x_2, x_3),\ \vartheta = \vartheta(x_1, x_2, x_3) \tag{2.3}$$

where τ is directed along normal to front of the wave $\tau = 0$, ϑ, ζ are directed along tangents to surface $\tau = 0$.

The origin of moving curvilinear coordinate system is located in the contact line S(t) or contact point $P(t)$ of front waves. (see Fig.)

It should be noted that, in general case ϑ, ζ depends on time also, but for many magnetohydrodynamics practical problems (propagation contrary ultrasonic or electromagnetic waves in continuous media, diffraction of gravitation waves on rigid obstacles and etc.) ϑ, ζ are stationer beams.

From physical aspect of the under consideration problem are following:

first, the intensive changing of volues of disturbing gasohydrothermodynamic parameters of a gas-fluid flow are going on in the vicinity of tangential point $P(t)$ or tangential line $S(t)$ (see Fig.);

second, in the noted region, more fast changing of these parameters take place in direction τ for scalars and for normal components of vectors, i.e.

$$V_j = V_j' + V_j'',\ V_j' = u_j,\ u_j \sim \varepsilon,\ \left|V_j''\right| << \left|u_j\right| \tag{2.4}$$

(V_j' are scalars or normal components of vectors, V_j'' are their tangential components)

third, the character size Λ of narrow short waves region to normal τ=0 is strictly small, than the character size L along to the tangential direction to the front of wave, i.e.

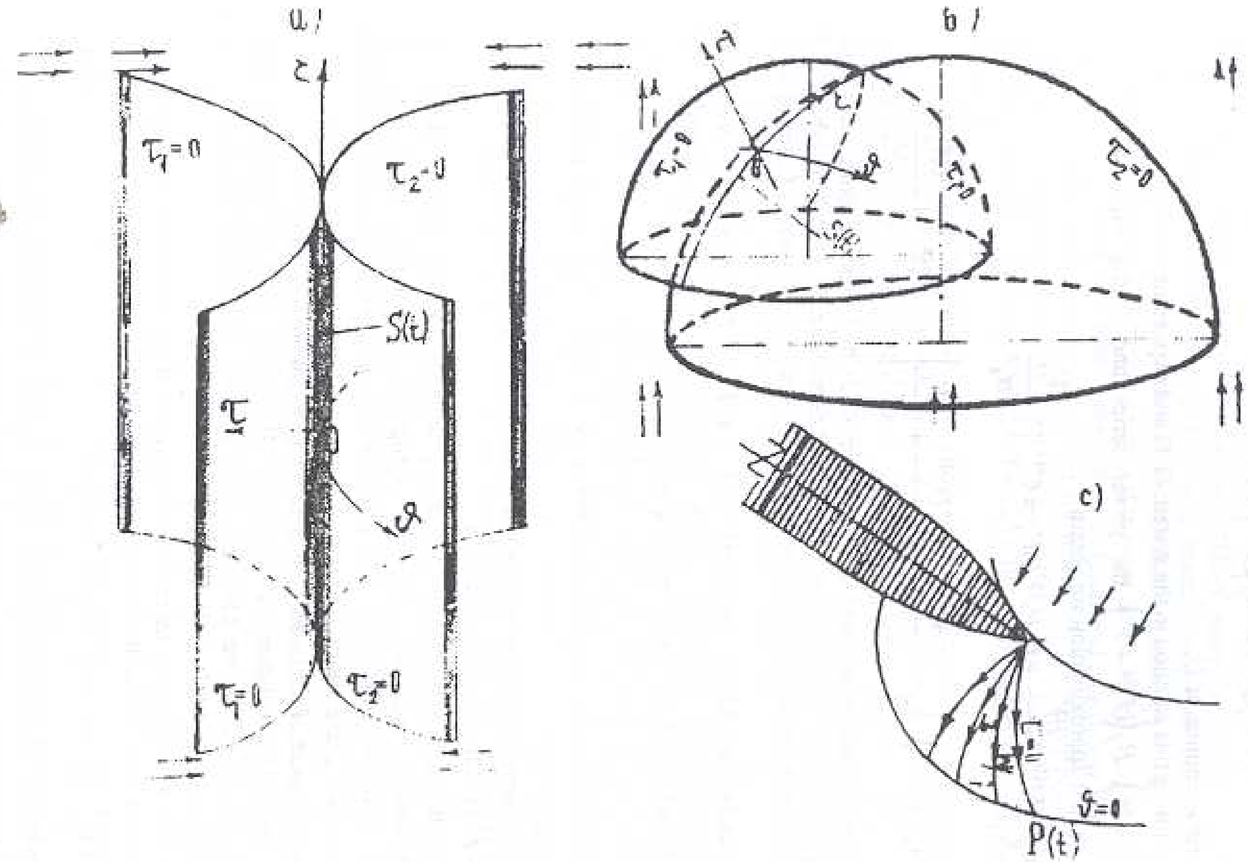

Fig 1 Some possible schemas for interaction between waves P(t) in fixed moment a) cylindrical waves; b) spherical waves; c) diffraction plane problem

$\Lambda << L$.

As it was show in the work [12] (see p.46, formula (1.1)) $\Lambda \sim L^2$.

In accordance with [5-7,9,10], Λ has order ε, consequently

$$\Lambda \sim \varepsilon,\ L \sim \varepsilon^{1/2},\ \frac{\partial u_j}{\partial \vartheta} \sim \frac{\partial u_j}{\partial \zeta} \sim \varepsilon^{1/2},\ \frac{\partial u_j}{\partial \tau} \sim 1 \tag{2.5}$$

3. The system (2.2) in the main order and its some solution.

The transformation beams coordinates are described by formulas

$$\frac{\partial u_j}{\partial x_N} = \alpha_N \frac{\partial u_j}{\partial \tau} + \beta_N \frac{\partial u_j}{\partial \vartheta} + \gamma_N \frac{\partial u_j}{\partial \zeta},\quad N = 1, 2, 3 \tag{3.1a}$$

where $\alpha_N = \dfrac{\partial \tau}{\partial x_N}$, $\beta_N = \dfrac{\partial \vartheta}{\partial x_N}$, $\gamma_N = \dfrac{\partial \zeta}{\partial x_N}$ and α_N, β_N, γ_N are components of normals respectively to the surfaces: $\tau, \vartheta, \zeta = \text{const}$.

It should be noted, that

$$H_\tau = \sqrt{\sum_{N=1}^{3}[(\alpha_N)^{-1}]^2},\ H_\vartheta = \sqrt{\sum_{N=1}^{3}[(\beta_N)^{-1}]^2},\ H_\zeta = \sqrt{\sum_{N=1}^{3}[(\gamma_N)^{-1}]^2}$$

are Lame's coefficients.

One can put $\tau = T(x_1, x_2, x_3) - t$. Consequently

$$\frac{\partial u_j}{\partial t} = \frac{\partial u_j}{\partial t}\Big|_{x_N} + \frac{\partial u_j}{\partial \tau}\frac{\partial \tau}{\partial t} = \frac{\partial u_j}{\partial t}\Big|_{x_N} - \frac{\partial u_j}{\partial \tau} \tag{3.1b}$$

Comment; some interpretations by diffraction problem:

τ is time of runing from the initial gravitation wave till to the current point M along a diffraction rays, $\tau = t_\phi$ = const is time of waves propagation, t is time of runing from point M till current front of waves (fig.).

After of the passage in the system (2.2) from independent variables: t, x_1, x_2, x_3 to news variables: t, τ, ϑ, ζ by formulas (3.1a), (3.1b) and retaining only the terms which have orders no more than ε in obtained system and using estimations (2.4), (2.5), one can obtain

$$a_{ij}^{(0)} \frac{\partial u_j}{\partial t} + (\alpha_N\, a_{ij}^{(N)} - a_{ij}^{(0)}) \frac{\partial u_j}{\partial \tau} + a_{ij}^{(N)} (\beta_N \frac{\partial u_j}{\partial \vartheta} + \gamma_N \frac{\partial u_j}{\partial \zeta}) =$$
$$= (\alpha_N\, C_{ijl}^{(N)} - C_{ijl}^{(0)}) u_l \frac{\partial u_j}{\partial \tau} - b_{ij} u_j$$

Using the phenomena of nonlinear superposition for high frequency weakly waves, in case of two interacting waves the solution u can be represented as

$$u = u^{(1)}(t, \tau_1, \vartheta, \zeta) + u^{(2)}(t, \tau_2, \vartheta, \zeta) \text{ or } u_j = u_j^{(1)}(t, \tau_1, \vartheta, \zeta) + u_j^{(2)}(t, \tau_2, \vartheta, \zeta)$$

each of them satisfies the condition

$$\frac{1}{T_t}\int_0^{T_t} u_j^{(1,2)} d\tau_{1,2} = 0 \tag{3.2}$$

Where $T_t = 2\pi/\omega_t$, $\omega_t >> 1$ is typical frequency of waves.

It's very important to note, that the condition (3.2) is right condition for high-frequency, weakly intensity interacting waves, which permits to separate different waves from each other in main order (ε).

Comment; without the condition (3.2) it's inpossible to separate high-frequency, weakly intensity interacting waves from each other and to obtain quasilinear first order partial differential equation for each wave. The essence of the non-linear superposition consists just in this. (In the general case, the interaction process is strictly non-linear.)

By substituting the last solution in the last system and producing average to τ_2 with respect to the condition (3.2) for wave with eiconal τ_1 one can obtain

$$a_{ij}^{(0)}\frac{\partial u_j^{(1)}}{\partial t}+(\alpha_N\, a_{ij}^{(N)}-a_{ij}^{(0)})\frac{\partial u_j^{(1)}}{\partial \tau_1}+a_{ij}^{(N)}(\beta_N^{(1)}\frac{\partial u_j^{(1)}}{\partial \vartheta}+\gamma_N^{(1)}\frac{\partial u_j^{(1)}}{\partial \zeta})=$$

$$=(\alpha_N\, C_{ijl}^{(N)}-C_{ijl}^{(N)})u_l^{(1)}\frac{\partial u_j^{(1)}}{\partial \tau_1}-b_{ij}u_j^{(1)} \qquad (3.3)$$

It's easy to show that, the last term in the right side of this system which was neglected, has order ε^2. It's well known that, each wave one can be presented as a superposition of harmonic waves, i.e.

$$u_{i,l}^{(2)}=\operatorname{Re}[\sum_{k=1}^{\infty}\varphi_{lk,ik}^{(2)}e^{ik\tau_2}];$$

Of course, in general case it should be better to represent the function as Fourier integral, but this will not influence on result.

$$\frac{1}{T_t}\int_0^{T_t}u_l^{(2)}\frac{\partial u_j^{(2)}}{\partial \tau_2}\partial\tau_2\approx\sum_{m,n=1}^{M_0}\varphi_{lm}^{(2)}\varphi_{jn}^{(2)}\frac{n}{(m+n)T_t}\sin(m+n)\tau_2\Big|_0^{T_t}\approx\varepsilon^2$$

since $T_t<<1$ and $\varphi_{lm}^{(2)}$, $\varphi_{jn}^{(2)}\sim\varepsilon$.

The corresponding system of differential equations for wave $\tau_2=0$ is written similarly.

In null approximation from system (3.3) we obtain

$$[\alpha_N\, a_{ij}^{(N)}-a_{ij}^{(0)}]\frac{\partial u_j^{(1)}}{\partial \tau_1}=0$$

and the general solution of last system one can represent as

$$u_1^{(1)}=\sum_{s=0}^{\infty}u_j^{(1,s)}(\zeta,\vartheta)f_s(\tau_1)$$

where $f_s(\tau_1)$ are a powering functions; $f_s\sim\tau_1^{s+\alpha}$ behind of the wave front, and are 0 is before them, $\alpha>0$ is a parameter characterizing singularity of functions f_s (f_s are "fast" changing functions nearby of the wave front: $\tau_1(t,x_1,x_2,x_3)=0$.

After the substituting this solution in the linear system, in null approximation we obtain

$$(\alpha_N a_{ij}^{(N)}-a_{ij}^{(0)})u_j^{(1,0)}=a_{ij}^{(K)}\frac{\partial\tau_1}{\partial x_k}u_j^{(1,0)}=0$$

Since we are seeking non trivial solution of this system, consequently its the determinant must be 0, i.e.

$$D_0=\det\|a_{ij}^{(k)}\,\partial\tau_1/\partial x_k\|=\det\|\alpha_N a_{ij}^{(N)}-a_{ij}^{(0)}\|=\sum_{\nu_1,\dots,\nu_m}\pm\prod_1^m(a_{i\nu_j}^{(N)}\alpha_N-a_{i\nu_j}^{(0)})=0 \qquad (3.4)$$

where each $a_{i\nu_j}^{(N)}\alpha_N - a_{i\nu_j}^{(0)}$ is element from of ith line and of ν_jth column of D_0. $\nu_1,\ldots,\nu_m$ are different possible permutations of natural numbers: $1,\ldots,m$. We should take the sign + before terms, if substitution

$$\begin{pmatrix} 1,2,\ldots,m \\ \nu_1,\nu_2,\ldots,\nu_m \end{pmatrix}$$

is even, and sign – otherwise.

It should be noted, that D_0 and $\tau_1=0$ are characteristic determinant and characteristic surface for system (3.3) in the null approximation.

Thus! In the null approximation the solution of the system (3.3) is written in the form: $u_j^{(1)} = u_j^{(1,0)}R_j^{(1)}(x)$, where $R_j^{(1)}(x)$ is right null vector of matrix $\| a_{ij}^{(k)}\ \partial\tau_1/\partial x_k\|$. Since, the convection terms in the right side (3.3) have higher small orders, than other terms (except of the local term), therefore this solution must be substituted in right side of (3.3). Consequently, the system (3.3) can be rewritten for eiconal τ_1 in the next form.

$$a_{ij}^{(0)}\frac{\partial u_j}{\partial t} + (\alpha_N\, a_{ij}^{(N)} - a_{ij}^{(0)})\frac{\partial u_j}{\partial \tau} + \beta_N\, a_{ij}^{(N)}\frac{\partial u_j}{\partial \vartheta} + \gamma_N\, a_{ij}^{(N)}\frac{\partial u_j}{\partial \zeta}) =$$

$$= K_i u^{(1,0)}\frac{\partial u^{(1,0)}}{\partial \tau} - b_{ij}R_j^{(1)}u^{(1,0)} \qquad (3.5)$$

where $K_i = (C_{ijl}^{(N)}\alpha_N - C_{ijl}^{(0)})R_l R_j$.

(Here and future the index (1) omits at u, τ, K_i and R_i or R_j).

4. Evolution or short waves equation

Subsequent development of material of this item will consist in obtaining the quasilinear second order partial differential equation from (3.5) by using symbolic method of differential operators [8].

In the book [8] (chapter 7, item 7.2, p.372) have been confirmed

"... In the some sense, operations of differentiation and integration satisfies the arithmetical rules...". On the page 383 have been written "... The process of obtaining the operator solution of the system: $a_{rs}dy_s/dt + b_{rs}y_s = S_r(x)$ coincides with process of solving of the system of algebraic equation relatively to y_s ($l_{rs}y_s = S_r$; $l_{rs} = a_{rs}d/dt + b_{rs}$)...".

The essence of this method consists in following: the linear differential operators are represented as polynomials and the operations of differentiation can be produced as arithmetical multiplication with them. Such formalities provides exactitude only eldest derivations.

On the base of this method, the operatoring solutions of system (3.5) one can be written in the form

$$u_j = D^{-1}D_j \text{ or } Du_j = A_{ij}d_i^{(1,0)} \qquad (4.1a,b)$$

where $D(D_{ij}) = \det\|D_{ij}\|$ is operatoring determinant of the system (3.5),

$$D_{ij} = a_{ij}^{(N)}(\alpha_N\ \partial/\partial\tau + \beta_N\partial/\partial\vartheta + \gamma_N\partial/\partial\zeta) + a_{ij}^{(0)}(\partial/\partial t - \partial/\partial\tau),$$

D_j is also operatoring determinant which can be obtained from D by changing jth column in it to $d_i^{(1,0)} = K_i u^{(1,0)}\ \partial u^{(1,0)}/\partial\tau - b_{ij}R_j u^{(1,0)}$.

A_{ij} are algebraical supplements of jth column of D.

It should be noted that, the equations (4.1 a,b) are exactly relatively eldest derivation

u_j. But these equations aren't evolution equations. Therefore, in their it is necessary to retain only the terms which have orders not more than ε [5-7,10].

The operatoring determinant D can be presented as sum

$$D=\sum_{\nu_1,\ldots,\nu_m}\pm\prod_{1}^{m}D_{i\nu_j}$$

where each $D_{i\nu_j}$ is $a_{i\nu_j}^{(N)}(\alpha_N\,\partial/\partial\tau+\beta_N\partial/\partial\vartheta+\gamma_N\partial/\partial\zeta)+a_{i\nu_j}^{(0)}(\partial/\partial t-\partial/\partial\tau)$.

Here we are introducing the notations:

$$D_{i\nu_j}^{(0)}=(\alpha_N a_{i\nu_j}^{(N)}-a_{i\nu_j}^{(0)})\partial/\partial\tau\,,D_{i\nu_j}=D_{i\nu_j}^{(0)}+\overline{D_{i\nu_j}}\,,\ \overline{D_{i\nu_j}}=\overline{D'_{i\nu_j}}+\overline{D''_{i\nu_j}}\,,$$

$$\overline{D'_{i\nu_j}}=a_{i\nu_j}^{(N)}(\beta_N\partial/\partial\vartheta+\gamma_N\partial/\partial\zeta),\ \overline{D''_{i\nu_j}}=a_{i\nu}^{(0)}\partial/\partial t \tag{4.2}$$

On the base the estimations (2.4), (2.5):

$D_{i\nu_j}^{(0)}u_j\sim O(1)$, $\overline{D'_{i\nu_j}}\,u_j\sim O(\varepsilon^{1/2})$, $\overline{D''_{i\nu_j}}\,u_j\sim O(\varepsilon)$

By realizing corresponding calculations, taking into account these orders by using condition of compatibility (3.4), one can show that

$$Du_j=\{\sum_{\nu_1,\ldots,\nu_m}\pm[\sum_{i=1}^{m}\prod_{q=1,q\neq i}^{m}D_{q\nu_q}^{(0)}(\overline{D'_{i\nu_i}}+\overline{D''_{i\nu_i}})+$$

$$+\sum_{k=1}^{m-1}\sum_{i=k-1}^{m}\prod_{s=0}^{k-1}D_{s\nu_s}^{(0)}\prod_{q=k+1,q\neq i}^{m}D_{q\nu_q}^{(0)}\overline{D'_{k\nu_q}D''_{i\nu_i}}\,]\}\,u_j+O(\varepsilon^{3/2});\ (D_{0\nu_0}\equiv 1) \tag{4.3}$$

The common view of elements A_{ij} is $D_{1\nu_1}\ldots D_{i-1\nu_{i-1}}D_{i-1\nu_{i+1}}\ldots D_{j\nu_j}\ldots D_m$

and $$A_{ij}d_i^{(1,0)}=\sum_{i=1}^{m}\Big(\sum_{\nu_1,\ldots,\nu_{i-1},\nu_{i+1},\ldots,\nu_m}\prod_{\mu=1,\mu\neq i}^{m}D_{\mu\nu_\mu}^{(0)}\Big)d_i^{(1,0)}+0(\varepsilon^{3/2}) \tag{4.4}$$

where $\nu_j\neq j,\ j\neq i\text{-}1,\ i,\ i+1$. (It's clear that when $j=i\text{-}1,\ i,\ i+1$ multiplier in the right side of last sum is absend). The indexes: $\nu_1,\ \nu_2,\ldots,\ \nu_{i-1},\ \nu_{i+1},\ \ldots,\ \nu_j,\ \ldots,\ \nu_m$ are possibily difference permutations natural numbers: $1,2,\ldots,\ \nu\text{-}1,\ \nu+1,\ldots,j,\ldots,m$.

It should be noted that, in the operatoring determinant A_{ij} it is retained only the terms which have orders 1, since $d_i^{(1,0)}$ have orders ε.

After of the substituting the expressions $D_{i\nu_i}^{(0)}$, $\overline{D'_{i\nu_i}}$, $\overline{D''_{i\nu_i}}$ from (4.2) in the (4.3), (4.4) we obtain corresponding expressions for Du_j and $A_{ij}d_i^{(1,0)}$. By substituting have obtained expressions for Du_j and $A_{ij}d_i^{(1,0)}$ in the equations (4.1b), taking into account that $u_j=\varepsilon u_{(0)}^{(1,0)}+\varepsilon v^{(1)}+\ldots$ one can receive the final form of evolution equation or short waves equation:

$$\sum_{\nu_1,\ldots,\nu_m}\pm\left\{\left[\sum_{i=1}^{m}(\varphi_{q\nu_q}|_1^m\,q\neq i)a_{i\nu_i}^{(0)}\right]\frac{\partial^2 u_j}{\partial t\partial\tau}+\sum_{k=1}^{m-1}\sum_{i=k+1}^{m}\left[\left(\varphi_{s\nu_s}|_0^{k-1}\varphi_{q\nu_q}|_{k+1}^{m};\,q\neq i\right)a_{i\nu_i}^{(N)}a_{k\nu_k}^{(N)}\right]\times\right.$$

$$\times(\beta_N^2 \frac{\partial^2 u_j}{\partial \vartheta^2} + 2\beta_N \gamma_N \frac{\partial^2 u_j}{\partial \vartheta \partial \varsigma} + \gamma_N^2 \frac{\partial^2 u_j}{\partial \varsigma^2}\Bigg\} = A'_j \frac{\partial}{\partial \tau}\left(u_j \frac{\partial u_j}{\partial \tau}\right) + A''_j \frac{\partial u_j}{\partial \tau} \quad (4.5)$$

where $\varphi_{\mu\nu_\mu}\Big|_p^m = \prod_p^m (a_{\mu\nu_\mu}^{(N)} - a_{\mu\nu_\mu}^{(0)})$, A'_j, A''_j are $\det\left\|a_{ij}^{(N)}\alpha_N - a_{ij}^{(0)}\right\|$ in which the elements of jth column changed through K_j and $b_{ij}R_i$ respectively.

It's very important to note, that the difference between in "short waves equations" (2.9) have been obtained in work [7] (in which the differential operator $\Delta(p,q,s) = \Delta(p_0 + \overline{p},\ q_0 + \overline{q},\ s_0 + \overline{s})$ is substituted by expression (2.11); see pp.1519, 1520) and this article consists in difference of their coefficients. This is the consequence of applying methods. In [7] $\Delta(p_0 + \overline{p},\ q_0 + \overline{q},\ s_0 + \overline{s})$ is expended Taylor's symbolic series and are calculated by heuristic way its coefficients in the main order $(O(\gamma)) = O(\varepsilon)$. In this article by using symbolic method of differential operators was produced direct calculations of corresponding determinants in the principal approximation $(O(\varepsilon))$.

Conclusions

First, the quasilinear second order partial differential equations (4.5) which are called evolution or short waves equation, for each u_j are exact in the principal approximation (ε);

Second, for each magnetogasfluiddynamic problems: propagation contrary axisymmetric ultrasonic beams in acoustic resonator filled by magnetic fluids with gas bubbles with presence of magnetic field or diffraction of gravition waves on the rigid obstacle and etc., the coefficients including in this equation one can be define concretely. This will be application of this theory and are themes for next works;

Third, in the right side of equation (4.5) one can be add the terms: $E\partial^2 u_j / \partial\tau^2, F d^3 u_j / d\tau^3$ which are describing the small dissipative and dispersion effects and have smaller high orders than principal approximation;

Fourth, equation (4.5) can be simplificated by using the condition: of compatibility (3.4), equations of bicharacteristics and etc. on the base approximative and heuristic calculations. But I am not supporter such methods in present modern powerful computers.

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