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THERMOMAGNETOELASTIC MODULATION WAVES IN A NON-LINEAR PLATE

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Модуляция термомагнитоупругих волн в нелинейной пластине

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Ջերմամագնիսաառաձգական ալիքների մոդուլյացիան ոչ գծային սալում

Դասական դրվածքով դիտարկվում է ֆիզիկորեն ոչ գծային սալում ալիքների տարածումը և մոդուլյացիան: Սալը վերջավոր հաղորդիչ է, գտնվում է երկայնական մագնիսական և կապակցված ջերմային դաշտերում: Ստացված է մոդուլյացիայի կայունության պայմանը: Ցույց է տրվում, որ դիսիպացիան, որը պայմանավորված է վերջավոր հաղորդականությամբ և ջերմային կապակցվածությամբ, կարող է ոչ դիսիպացիոն անկայուն համակարգը դարձնել կայուն:

In the present paper the modulation equation for a non-linear magnetoelastic plate with finite conductivity in longitudinal magnetic field accounting thermal effects is derived. The obtained equation is investigated on stability.

1.Introduction.

The modulation waves in physics and hydrodynamics are investigated rather well [1,2]. During the last period the non-linear modulation waves for plates and shells are studied [3,4].

These waves are proved to be not stable in materials of metal type (soft non-linearity). In ideal conducting medium [4] for rather great magnetic field instability is shown to yield to stability and vice versa. In the case of finite conductivity the unstable ideal problem in the presence of dissipation term can be reduced to stable one. In this paper dissipation effects connected with finite electrothermoconductivity are studied.

The problems of non-linear vibrations of magnetoelastic plate in longitudinal and transverse fields are considered [5].

2.Dispersion relation.

Let us consider a non-linear elastic [6] plate in the longitudinal magnetic field $\overline{H}_0(H_0,0,0)$. It is assumed that the vibration of the plate gives rise to thermal effects. The equations of classical theory of plates motion and thermoconductivity can be written in the form [7-9].

$$D\left[\frac{\partial^4 w}{\partial x^4}+D_1\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}\right)^3+\alpha(1+\nu)\frac{\partial^2 T}{\partial x^2}\right]+\rho h\frac{\partial^2 w}{\partial t^2}-$$

$$-\frac{h}{2}\frac{\partial}{\partial x}\left(\sigma_{xz}^{+}+\sigma_{xz}^{-}\right)-\left(\sigma_{zz}^{+}-\sigma_{zz}^{-}\right)=\rho\int_{-h/2}^{h/2}\left(K_z+\frac{\partial K_x}{\partial x}z\right)dz \qquad (2.1)$$

$$\frac{\partial T}{\partial t}-\chi\frac{\partial^2 T}{\partial x^2}+\left(\frac{12\chi}{h^2}+\frac{6k_1^2}{\rho h c_p}\right)T-\chi\eta\frac{\partial^3 w}{\partial x^2\partial t}=0 \qquad (2.2)$$

It is assumed that the wave propagates along x axis.

$D=\dfrac{Eh^3}{12(1-\nu^2)}$, $D_1=\dfrac{4h^2}{45}\nu_1\gamma_2$, $\nu_1=\dfrac{(1-\nu+\nu^2)^2}{(1-\nu)^3}$ and other notations are taken from [6-9].

The volume Lorents force is determined by formula $\overline{K} = \mu_0\left(\text{rot}\overline{h} \times \overline{H}_0\right)$ and magnetic field is $\overline{H} = \overline{H}_0 + \overline{h}$ and takes place

$$K_x = 0,\ K_z = \frac{H_0\mu_0}{\rho}\left(\frac{\partial h_z}{\partial x} - \frac{\partial h_x}{\partial z}\right) \tag{2.3}$$

The terms corresponding to magnetic field are taken in linear approximation:

The induction equation in plate has the form

$$\frac{\partial \overline{h}}{\partial t} = \left(\overline{H}_0\nabla\right)\overline{\text{v}} - \overline{H}_0 \text{div}\overline{\text{v}} + \frac{1}{\sigma\mu_0}\Delta\overline{h} \tag{2.4}$$

where μ_0 is the magnetic constant, σ is the electroconductivity and $\overline{\text{v}}\left(-z\frac{\partial^2 w}{\partial x\partial t}, \frac{\partial w}{\partial t}\right)$ is the particles velocity.

The equation (2.4) with respect to components is as follows

$$\text{div}\overline{h} = 0$$

$$\frac{\partial h_z}{\partial t} = H_0\frac{\partial^2 w}{\partial x\partial t} + \frac{1}{\sigma\mu_0}\Delta h_z \tag{2.5}$$

In dielectric medium (out of the plate) the equation of electrodynamics has the form

$$\text{div}\overline{h} = 0,\ \text{rot}\overline{h} = 0 \tag{2.6}$$

The solution of (2.1), (2.2) and (2.5) is sought as

$$w = \frac{1}{2}\left(Ae^{i\tau} + c.c.\right),\ T = \frac{1}{2}\left(Be^{i\tau} + c.c.\right)$$

$$h_x = \frac{1}{2}\left(Ce^{i\tau} + c.c.\right)\sin\lambda z,\ h_z = \frac{1}{2}\left(Ge^{i\tau} + c.c.\right)\cos\lambda z \tag{2.7}$$

$$\tau = kx - \omega t,\ \omega = \omega_1 + i\omega_2$$

and of (2.6) in the form

$$h_x = \frac{1}{2}\left(Me^{i\tau\mp kz} + c.c.\right),\ h_z = \frac{1}{2}\left(Ne^{i\tau\mp kz} + c.c.\right) \tag{2.8}$$

Satisfying the boundary conditions $h_j = h_j$ on $z = \pm\frac{h}{2}$ and (2.5) we obtain

$$C = -\frac{2i}{\lambda h}G,\ \lambda = \sqrt{\frac{2k}{h}} \tag{2.9}$$

Keeping the terms of main order kh and taking into account (2.3), (2.5), one can obtain

$$G\left[\frac{1}{\sigma\mu_0}\left(\lambda^2 + k^2\right) - i\omega\right] = k\omega H_0 A$$

$$K_z = \frac{2iH_0\mu_0}{\rho h}Ge^{i\tau} \tag{2.10}$$

and the following dispersion equation

$$\left[D\left(k^4 + D_2 k^8 A\overline{A}\right) - \rho h\omega^2 + \frac{2\mu_0 H_0^2 k\omega}{\omega + \frac{2ik}{h\sigma\mu_0}}\right] \times$$

$$\times\left(\chi_1 - i\omega\right) - iD\alpha(1+\nu)\chi\eta\omega k^4 = 0 \tag{2.11}$$

where

$$D_2 = 3D_1 e^{2\omega_2 t},\ \chi_1 = \frac{12\chi}{h^2} + \frac{6k_1^2}{\rho h c_p}$$

The equation (2.11) with the account of smallness of amplitudes and dissipation connected with thermo and electroconductivity can be written in the form

$$\omega = \omega^0 + \frac{\partial \omega}{\partial a^2} a^2,\ a = |A| \tag{2.12}$$

where the linear part $\omega^0 = \omega_1^0 + i\omega_2^0$, ω_1^0 is linear frequency taking into account the fact that dissipation is small

$$\omega_1^0 = \frac{1}{\rho h}\left[Dk^4 + 2H_0^2\mu_0 k + D\alpha(1+\nu)\chi\eta k^4\right]^{1/2} \tag{2.13}$$

ω_2^0 is the damping coefficient

$$\omega_2^0 = -\frac{k^2}{\rho h(\omega_1^0)^2}\left[\frac{2H_0^2}{\sigma h} + \frac{1}{2}D(1+\nu)\chi\eta\chi_1\alpha k^2\right] \tag{2.14}$$

and non-linear coefficient

$$\frac{\partial \omega}{\partial a^2} = D_3 + iD_4 \tag{2.15}$$

$$D_3 = \frac{DD_2k^8}{8\rho h\omega_1^0},\ D_4 = -D_3\frac{\omega_2^0}{\omega_1^0}$$

To clarify the contribution of magnetic field and thermoconductivity in ω_1^0 and ω_2^0, the example of cuprum is considered [8] in the case of absence of thermal currents with external medium $(k_1 = 0)$.

For $kh = 10^{-1}$ the effects of magnetic field and thermoconductivity in ω_1^0 are of the same order when $H_0 \sim 3.5\cdot 10^3$ Gauss. . On the other hand in the expression of ω_2^0 the mentioned effects are of the same order $H_0 \sim 25$ Gauss, i.e. magnetic field plays essential role in damping.

3.The stability of modulation waves.

Using equation (2.12) one can write down the modulation equation [1]. For this one has to put

$\omega \to i\frac{\partial}{\partial t}$, $k \to i\frac{\partial}{\partial x}$ and seek w in the form

$$w = \frac{1}{2}\left(\psi e^{i\tau_0} + \overline{\psi} e^{-i\tau_0}\right),\ \tau_0 = kx - \omega_0 t,\ |\psi| = |A| \tag{3.1}$$

Substituting $\omega^0\left(-i\frac{\partial}{\partial x}\right)$ instead of $\omega^0(k)$ one can write [1]

$$\omega^0\left(-i\frac{\partial}{\partial x}\right)\psi e^{i\tau_0} = \omega^0(k)\psi - (\omega^0(k))\ i\frac{\partial \psi}{\partial x} - \frac{1}{2}(\omega^0(k))\ \frac{\partial^2 \psi}{\partial x^2}$$

and obtain the following modulation equation

$$\frac{\partial \psi}{\partial t} + \frac{d\omega_0}{dk}\frac{\partial \psi}{\partial x} - \frac{1}{2}i\frac{d^2\omega_0}{dk^2}\frac{\partial^2 \psi}{\partial x^2} - (D_4 - iD_3)\psi|\psi|^2 = 0 \tag{3.2}$$

For investigation on stability of modulation equation one can write $\psi = \Psi\exp(i\varphi)$.

Then, with respect to Ψ and φ one can obtain

$$\frac{\partial\Psi}{\partial t}+\frac{d\omega_1^0}{dk}\frac{\partial\varphi}{\partial x}-\frac{d\omega_2^0}{dk}\Psi\frac{\partial\varphi}{\partial x}+\frac{1}{2}\frac{d^2\omega_2^0}{dk^2}\left[\frac{\partial^2\Psi}{\partial x^2}-\Psi\left(\frac{\partial\varphi}{\partial x}\right)^2\right]+$$

$$+\frac{1}{2}\frac{d^2\omega_1^0}{dk^2}\left(2\frac{\partial\Psi}{\partial x}\frac{\partial\varphi}{\partial x}+\Psi\frac{\partial^2\varphi}{\partial x^2}\right)-D_4\Psi^3=0 \qquad (3.3)$$

$$\Psi\frac{\partial\varphi}{\partial t}+\frac{d\omega_1^0}{dk}\Psi\frac{\partial\varphi}{\partial x}+\frac{d\omega_2^0}{dk}\frac{\partial\Psi}{\partial x}-\frac{1}{2}\frac{d^2\omega_1^0}{dk^2}\left[\frac{\partial^2\Psi}{\partial x^2}-\Psi\left(\frac{\partial\varphi}{\partial x}\right)^2\right]+$$

$$+\frac{1}{2}\frac{d^2\omega_2^0}{dk^2}\left(\Psi\frac{\partial^2\varphi}{\partial x^2}+2\frac{\partial\Psi}{\partial x}\frac{\partial\varphi}{\partial x}\right)+D_3\Psi^3=0$$

For investigation on stability one must write

$$\Psi=\Psi_0(t)+\delta\Psi(x,t),\ \varphi=\varphi_0(t)+\delta\varphi(x,t) \qquad (3.4)$$

Then in initial approximation it is obtained

$$\frac{d\Psi_0}{dt}=D_4\Psi_0^3,\ \frac{d\varphi_0}{dt}+D_3\Psi_0^2=0 \qquad (3.5)$$

Because Ψ_0 and D_4 are small one can assume $\Psi_0=\text{const}$ and seek the solution of the equation for disturbances in the form

$$\delta\Psi=F\exp(i\theta),\ \delta\varphi=\Phi\exp(i\theta),\ \theta=Kx-\Omega t \qquad (3.6)$$

Then under the assumption that $\exp(2\omega_2^0 t)\approx 1$ which holds for $|\omega_2^0|<<K\frac{d\omega_1^0}{dk}$ we can obtain the following characteristic equation

$$p^2-3D_4\Psi_0^2p+p_1(p_1+2D_3\Psi_0^2)=0 \qquad (3.7)$$

where

$$p=-i\Omega+iK\frac{d\omega_1^0}{dk}-\frac{1}{2}\frac{d^2\omega_2^0}{dk^2}K^2 \qquad (3.8)$$

$$p_1=iK\frac{d\omega_2^0}{dk}+\frac{1}{2}\frac{d^2\omega_1^0}{dk^2}K^2$$

Solution of (3.7) is as follows

$$-i\Omega+iK\frac{d\omega_1^0}{dk}=\frac{3}{2}D_4\psi_0^2\pm\sqrt{\left(\frac{3}{2}D_4\psi_0^2\right)^2-\Delta_0} \qquad (3.9)$$

where

$$\Delta_0=\frac{1}{2}\frac{d^2\omega_1^0}{dk^2}K^2\left(\frac{1}{2}\frac{d^2\omega_1^0}{dk^2}K^2+2D_3\psi_0^2\right) \qquad (3.10)$$

Here is taken into account that $\left|K^2\frac{d^2\omega_2^0}{dk^2}\right|<<|\omega_2^0|$.

The amplitude of w contains multiplier $e^{2\omega_2^0 t}\approx 1$ and if the dissipation essentially affects the stability condition, one must put $|\omega_2^0|<<\frac{3}{4}D_4\Psi_0^2$

The last condition holds for rather short waves

$$\frac{Dhk^8\nu_1}{40\rho(\omega_1^0)^2}\psi_0^2|\gamma_2|>>1 \qquad (3.11)$$

The condition of modulation stability will be

$$\Omega=\Omega\ +i\Omega\ ,\ \Omega\ <0 \qquad (3.12)$$

In the absence of thermoconductivity for ideal conducting medium $\omega_2^0 = D_4 = 0$ the stability condition will be in adiabatic approximation

$$\gamma_2 \frac{d^2\omega_1^0}{dk^2} > 0 \tag{3.13}$$

From (2.13) one can obtain [4]

$$\frac{d^2\omega_1^0}{dk^2} \lessgtr 0, \quad \frac{H_0^2}{4\pi} \gtrless Dk^4\left(4+3\sqrt{2}\right) \tag{3.14}$$

For bounded magnetic fields $\frac{d^2\omega_1^0}{dk^2} > 0$ and stability condition of (3.13) gives $\gamma_2 > 0$.

As it is seen from (3.9), the sign of D_4 is essential for the dissipative problem and one must consider two cases.

a) For small H_0 as shows (3.14) $\frac{d^2\omega_1^0}{dk^2} > 0$ and in adiabatic approximation if $\gamma_2 < 0$ the condition (3.13) is not fulfilled and $\Delta_0 < 0$. From (3.9) one can obtain that the dissipative wave is unstable.

If $\gamma_2 > 0$, $\Delta_0 > 0$ the nondissipative wave is stable, the dissipative wave is again unstable.

b) For strong H_0 if $\gamma_2 < 0$, $\Delta_0 > 0$ (3.13) is fulfilled and $\Omega < 0$. The dissipative wave is stable. If $\gamma_2 > 0$, $\Delta_0 < 0$, $\Omega > 0$ and in both cases there is instability.

It is surprising that for more general case of diffraction approximation where Δ_0 is given by (3.10), consideration on stability is very simple.

If $\gamma_2 < 0$ $(D_4 < 0)$ from (3.9) it is seen that for $\Delta_0 > 0$ one can obtain $\Omega < 0$, nondissipative and dissipative solutions are stable; for $\Delta_0 < 0$ there is instability of both solutions.

If $\gamma_2 > 0$ there is instability of dissipative solution.

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