ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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On Oscillations of Anisotropic Electroconductive Shallow Shells in Normal Magnetic Fields

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Անիզոտրոպ էլեկտրահաղորդիչ փոքր կորություն ունեցող թաղանթի տատանումները լայնական մագնիսական դաշտում

Անիզոտրոպ թաղանթների տեսության ընդհանուր սկզբունքներին համապատասխան, լայնական սահքերի և բարակ մարմինների մագնիսաառաձգականության հիմնական հիպոթեզի հաշվառմամբ ստացված են ալիքների տարածման ընդհանուր հավասարումները էլեկտրահաղորդ օրթոտրոպ փոքր կորություն ունեցող թաղանթների համար.

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Колебания анизотропной электропроводящей пологой оболочки в поперечном магнитном поле

В соответствии с общими принципами теории анизотропных оболочек с учетом деформаций поперечных сдвигов и с основной гипотезой магнитоупругости тонких тел получены общие уравнения распространения волн для электропроводящих ортотропных пологих оболочек.

В частном случае изотропной весьма пологой прямоугольной сферической панели, без учета деформаций поперечных сдвигов и при наличии начального поперечного магнитного поля, задача преимущественно поперечных колебаний приводится к одному уравнению относительно прогиба оболочки. При граничных условиях шарнирного опирания торцов панели определены частоты колебаний и коэффициент затухания в зависимости от напряженности начального магнитного поля.

1.Introduction

According to the general principles of the theory of anisotropic shells, which considers transverse shear deformations [1-3] and fundamental hypotheses of magnetoelasticity of thin bodies[2,4], the general equations of wave propagation in the electroconductive orthotropic shallow shells are obtained, and the problem of oscillations of extremely shallow shells is investigated.

A certain problem of isotropic and anisotropic electroconductive plates and shells in magnetic fields in [3-9] is considered.

2.Initial Relationships and hypotheses

Let us consider a thin elastic orthotropic shell of uniform thickness h and finite electroconductivity $[\sigma](\sigma_1,\sigma_2,\sigma_3)$, oscillating in the external uniform magnetic field with vector $H_0(0,0,H_{03})$, normal to the middle surface $\alpha 0 \beta$.

Let the body of the shell be expressed by a triorthogonal system of curvilinear coordinates α,β,γ, where α and β coincide with the lines of principal curvature of the middle surface, γ being normal to the coordinate surface $\alpha 0 \beta$ is rectilinear.

The orthotropic material of the shell has three mutually orthogonal planes of elastic symmetry, which at each point are parallel to the coordinate surface $\alpha = \text{const}$, $\beta = \text{const}$, $\gamma = \text{const}$.

The dielectric constants outside and inside the shell are denoted by $\varepsilon^{(e)}$ and ε respectively, and the magnetic permeability coefficients $\mu^{(e)}$ and μ are taken equal to one.

The problem of magnetostatics in an unperturbed case is assumed to be solved [4]. We shall use linearized electromagnetoelastic equations. Then we neglect the shift current.

The present problem of vibrations of a shell is investigated on the base of the following hypotheses:

–The hypotheses of the magnetoelasticity of thin bodies [4] according to which

$$e_\alpha = e_1 = \varphi(\alpha,\beta,t),\ e_\beta = e_2 = \psi(\alpha,\beta,t),\ h_\gamma = h_3 = f(\alpha,\beta,t) \qquad (2.1)$$

where $\mathbf{h}\,(h_1,h_2,h_3)$, $\mathbf{e}\,(e_1,e_2,e_3)$ are the inducted electromagnetic fields components, φ,ψ,f are desired arbitrary functions, which must satisfy the electrodynamic equations, and the conditions on the surfaces of the shell $\left(\gamma = \pm\frac{h}{2}\right)$ [4];

–The hypothesis of improved theory of anisotropic shells [1,3,10] according to which

$$u_1 = u_\alpha = (1+k_1\gamma)u - \gamma\frac{\partial w}{\partial\alpha} + \gamma\frac{h^2}{8}\left(1+\frac{k_1}{2}\gamma - \frac{4}{3}\frac{\gamma^2}{h^2}\right)a_{55}\Phi$$

$$u_2 = u_\beta = (1+k_2\gamma)v - \gamma\frac{\partial w}{\partial\beta} + \gamma\frac{h^2}{8}\left(1+\frac{k_2}{2}\gamma - \frac{4}{3}\frac{\gamma^2}{h^2}\right)a_{44}\Psi$$

$$u_3 = u_\gamma = w \qquad (2.2)$$

where $u(\alpha,\beta,t), v(\alpha,\beta,t), w(\alpha,\beta,t)$ are the desired displacements of the shells middle surface, $\Phi(\alpha,\beta,t), \Psi(\alpha,\beta,t)$ are desired functions which characterize shear deformations of the shell, $k_1 = k_1(\alpha,\beta), k_2 = k_2(\alpha,\beta)$ are principal curvatures of the coordinate surface $\alpha 0 \beta$, (For shallow shells it is assumed that the k_i upon differentiation behave as constants[1]), $a_{55} = G_{31}^{-1}, a_{44} = G_{23}^{-1}$ are the elasticity coefficients, G_{31}, G_{32}, are shear moduli.

The equations of motion of the shell are [1,2,4]

$$\frac{\partial T_1}{\partial\alpha} + \frac{\partial S_{12}}{\partial\beta} = -\int_{-h/2}^{h/2} k\rho K_1 d\gamma + \int_{-h/2}^{h/2} k\rho\frac{\partial^2 u_\alpha}{\partial t^2}d\gamma$$

$$\frac{\partial T_2}{\partial\beta} + \frac{\partial S_{12}}{\partial\alpha} = -\int_{-h/2}^{h/2} k\rho K_2 d\gamma + \int_{-h/2}^{h/2} k\rho\frac{\partial^2 u_\beta}{\partial t^2}d\gamma$$

$$-(k_1T_1 + k_2T_2) + \frac{\partial N_1}{\partial\alpha} + \frac{\partial N_2}{\partial\beta} = \rho h\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_1}{\partial\alpha} + \frac{\partial H}{\partial\beta} - N_1 = -\int_{-h/2}^{h/2} k\gamma\rho K_1 d\gamma + \int_{-h/2}^{h/2} k\gamma\rho\frac{\partial^2 u_\alpha}{\partial t^2}d\gamma$$

$$\frac{\partial M_2}{\partial\beta} + \frac{\partial H}{\partial\alpha} - N_2 = -\int_{-h/2}^{h/2} k\gamma\rho K_2 d\gamma + \int_{-h/2}^{h/2} k\gamma\rho\frac{\partial^2 u_\beta}{\partial t^2}d\gamma \qquad (2.3)$$

where $k = (1+k_1\gamma)(1+k_2\gamma)$, T_i, S_{ik}, N_i, M_i are the internal forces and moments, ρ is the shell material density, t is the time, ρK_i are the components of the "cargo" term for which we have generally [2,4]

$$\rho K(K_1,K_2,K_3) = [\sigma_i]\frac{1}{c}\left(e + \frac{1}{c}\frac{\partial u}{\partial t}\times B_0\right)\times B_0 \qquad (2.4)$$

B is the magnetic induction vector in shell, $u(u_1,u_2,u_3) = u(u_\alpha,u_\beta,u_\gamma)$ is the displacement vector, c is the electrodynamic constant.

3. The Equations of Magnetoelasticity for a Thin Orthotropic Shell

Integrating electrodynamic equations with the account of the surface conditions $h_i = h_i^+$, when $\gamma = \frac{h}{2}$ and $h_i = h_i^-$, when $\gamma = -\frac{h}{2}$, for h_i we obtain [2,4]

$$h_1 = \frac{h_1^+ + h_1^-}{2} + \gamma\left(\frac{\partial f}{\partial \alpha} + \frac{4\pi\sigma_2}{c}\psi\right) - \frac{4\pi\sigma_2 B_0}{c^2}\left(a_1\frac{\partial u}{\partial t} - b\frac{\partial^2 w}{\partial\alpha\partial t} + c_1 a_{55}\frac{\partial \Phi}{\partial t}\right)$$

$$h_2 = \frac{h_2^+ + h_2^-}{2} + \gamma\left(\frac{\partial f}{\partial \beta} - \frac{4\pi\sigma_1}{c}\varphi\right) - \frac{4\pi\sigma_1 B_0}{c^2}\left(a_2\frac{\partial v}{\partial t} - b\frac{\partial^2 w}{\partial\beta\partial t} + c_2 a_{44}\frac{\partial \Psi}{\partial t}\right) \quad (3.1)$$

where

$$a_i = \gamma + \frac{\gamma^2}{2}k_i - \frac{h^2}{8}k_i,\ b = \frac{\gamma^2}{2} - \frac{h^2}{8}$$

$$c_i = \frac{\gamma^2 h^2}{16} - \frac{\gamma^4}{24} - \frac{5h^4}{384} + \frac{\gamma^3 h^2}{48}k_i$$

Then we have

$$\sigma_3 e_3 = -\gamma\left(\sigma_1\frac{\partial\varphi}{\partial\alpha} + \sigma_2\frac{\partial\psi}{\partial\beta}\right) - \frac{B_0}{c}\left[\sigma_1\left(a_2\frac{\partial^2 v}{\partial\alpha\partial t} - b\frac{\partial^3 w}{\partial\alpha\partial\beta\partial t} + c_2 a_{44}\frac{\partial^2\Psi}{\partial\alpha\partial t}\right) - \right.$$

$$\left. - \sigma_2\left(a_1\frac{\partial^2 u}{\partial\beta\partial t} - b\frac{\partial^3 w}{\partial\alpha\partial\beta\partial t} + c_1 a_{55}\frac{\partial^2\Phi}{\partial\beta\partial t}\right)\right] \quad (3.2)$$

Thus, we have all the components of excited electromagnetic field in the shell, given by eight functions $u, v, w, \Phi, \Psi, \varphi, \psi, f$ and by induced magnetic field's values h_1 and h_2 on the shell's surfaces $\left(\gamma = \pm\frac{h}{2}\right)$.

Then from (2.4) for the components of the "cargo" term we obtain

$$\rho K_1 = \frac{\sigma_2}{c}\left\{B_0\psi - \frac{B_0^2}{c}\left[\left(1 + k_1\gamma\right)\frac{\partial u}{\partial t} - \gamma\frac{\partial^2 w}{\partial\alpha\partial t} + \frac{\gamma}{2}\left(\frac{h^2}{4} - \frac{\gamma^2}{3} + \frac{\gamma h^2}{8}k_1\right)a_{55}\frac{\partial\Phi}{\partial t}\right]\right\}$$

$$\rho K_2 = -\frac{\sigma_1}{c}\left\{B_0\varphi + \frac{B_0^2}{c}\left[\left(1 + k_2\gamma\right)\frac{\partial v}{\partial t} - \gamma\frac{\partial^2 w}{\partial\beta\partial t} + \frac{\gamma}{2}\left(\frac{h^2}{4} - \frac{\gamma^2}{3} + \frac{\gamma h^2}{8}k_2\right)a_{44}\frac{\partial\Psi}{\partial t}\right]\right\} \quad (3.3)$$

Substituting the values of internal forces and moments, components of displacements and components of the "cargo" term in (2.3) we get the following equations of motion [1,2,4]

$$C_{11}\frac{\partial^2 u}{\partial\alpha^2} + C_{66}\frac{\partial^2 u}{\partial\beta^2} + \left(C_{12} + C_{66}\right)\frac{\partial^2 v}{\partial\alpha\partial\beta} + \left(k_1 c_{11} + k_2 c_{12}\right)\frac{\partial w}{\partial\alpha} = \rho h\frac{\partial^2 u}{\partial t^2} -$$

$$-\frac{\sigma_2}{c}\left\{B_0 h\psi - \frac{B_0^2}{c}\left[h\frac{\partial u}{\partial t} - \frac{h^3}{12}\left(k_1 + k_2\right)\frac{\partial^2 w}{\partial\alpha\partial t} + \frac{h^5}{120}\left(1.625k_1 + k_2\right)a_{55}\frac{\partial\Phi}{\partial t}\right]\right\}$$

$$C_{22}\frac{\partial^2 v}{\partial\beta^2} + C_{66}\frac{\partial^2 v}{\partial\alpha^2} + \left(C_{12} + C_{66}\right)\frac{\partial^2 u}{\partial\alpha\partial\beta} + \left(k_2 C_{22} + k_1 C_{12}\right)\frac{\partial w}{\partial\beta} = \rho h\frac{\partial^2 v}{\partial t^2} +$$

$$+\frac{\sigma_1}{c}\left\{B_0 h\varphi + \frac{B_0^2}{c}\left[h\frac{\partial v}{\partial t} - \frac{h^3}{12}\left(k_1 + k_2\right)\frac{\partial^2 w}{\partial\beta\partial t} + \frac{h^5}{120}\left(1.625k_2 + k_1\right)a_{44}\frac{\partial\Psi}{\partial t}\right]\right\}$$

$$-\left(k_1 C_{11} + k_2 C_{12}\right)\frac{\partial u}{\partial\alpha} - \left(k_2 C_{22} + k_1 C_{12}\right)\frac{\partial v}{\partial\beta} - \left(k_1^2 C_{11} + 2k_1 k_2 C_{12} + k_2^2 C_{22}\right)w -$$

$$-k_1\left(k_1 D_{11}+k_2 D_{12}\right)\frac{\partial^2 w}{\partial\alpha^2}-k_2\left(k_2 D_{22}+k_1 D_{12}\right)\frac{\partial^2 w}{\partial\beta^2}+\frac{h^3}{12}\left(\frac{\partial\Phi}{\partial\alpha}+\frac{\partial\Psi}{\partial\beta}\right)=\rho h\frac{\partial^2 w}{\partial t^2}$$

$$D_{11}\frac{\partial^3 w}{\partial\alpha^3}+(D_{12}+2D_{66})\frac{\partial^3 w}{\partial\alpha\partial\beta^2}-\frac{h^2}{10}\left[a_{55}\left(D_{11}\frac{\partial^2}{\partial\alpha^2}+D_{66}\frac{\partial^2}{\partial\beta^2}\right)\Phi+\right.$$

$$\left.+a_{44}(D_{12}+D_{66})\frac{\partial^2\Psi}{\partial\alpha\partial\beta}\right]+\frac{h^3}{12}\Phi=\rho\frac{h^3}{12}\frac{\partial^3 w}{\partial\alpha\partial t^2}-\rho\frac{h^5}{120}a_{55}\frac{\partial^2\Phi}{\partial t^2}-\rho\frac{h^3}{12}k_1\frac{\partial^2 u}{\partial t^2}+$$

$$+\frac{\sigma_2}{c}\left\{\frac{h^3}{12}B_0(k_1+k_2)\psi-\frac{B_0^2}{c}\left[\frac{h^3}{12}(2k_1+k_2)\frac{\partial u}{\partial t}-\frac{h^3}{12}\frac{\partial^2 w}{\partial\alpha\partial t}+\frac{h^5}{120}a_{55}\frac{\partial\Phi}{\partial t}\right]\right\}$$

$$D_{22}\frac{\partial^3 w}{\partial\beta^3}+(D_{12}+2D_{66})\frac{\partial^3 w}{\partial\beta\partial\alpha^2}-\frac{h^2}{10}\left[a_{44}\left(D_{22}\frac{\partial^2}{\partial\beta^2}+D_{66}\frac{\partial^2}{\partial\alpha^2}\right)\Psi+\right.$$

$$\left.+a_{55}(D_{12}+D_{66})\frac{\partial^2\Phi}{\partial\alpha\partial\beta}\right]+\frac{h^3}{12}\Psi=\rho\frac{h^3}{12}\frac{\partial^3 w}{\partial\beta\partial t^2}-\rho\frac{h^5}{120}a_{44}\frac{\partial^3\Psi}{\partial t^2}-\rho\frac{h^3}{12}k_2\frac{\partial^3 v}{\partial t^2}+$$

$$+\frac{\sigma_1}{c}\left\{\frac{h^3}{12}B_0(k_1+k_2)\varphi-\frac{B_0^2}{c}\left[\frac{h^3}{12}(2k_2+k_1)\frac{\partial v}{\partial t}-\frac{h^3}{12}\frac{\partial^2 w}{\partial\beta\partial t}+\frac{h^5}{120}a_{44}\frac{\partial\Psi}{\partial t^2}\right]\right\} \quad (3.4)$$

where

$$C_{ik}=hB_{ik}\,,D_{ik}=\frac{h^3}{12}B_{ik}$$

$$B_{11}=\frac{E_1}{1-\nu_1\nu_2},\ B_{22}=\frac{E_2}{1-\nu_1\nu_2},\ B_{66}=G_{12}$$

$$B_{12}=\frac{\nu_2 E_1}{1-\nu_1\nu_2}=\frac{\nu_1 E_2}{1-\nu_1\nu_2},\ a_{55}=G_{13}^{-1},\ a_{44}=G_{23}^{-1}$$

E_i are Young's moduli in the directions $\alpha,\beta,\gamma(1,2,3);G_{ik}$ are shear moduli for the planes parallel at each point to the coordinate surfaces $\alpha=\text{const},\ \beta=\text{const},\ \gamma=\text{const};\ \nu_1,\ \nu_2$ are Poisson's ratios.

The equations of electrodynamics averaged over the shell's thickness we obtain [1,2,4,5]

$$\frac{\partial f}{\partial\alpha}=\frac{h_1^+-h_1^-}{h}-\frac{4\pi\sigma_2}{c}\left[\psi-\frac{B_0}{c}\left(\frac{\partial u}{\partial t}+\frac{h^4}{192}k_1 a_{55}\frac{\partial\Phi}{\partial t}\right)\right]$$

$$\frac{\partial f}{\partial\beta}=\frac{h_2^+-h_2^-}{h}-\frac{4\pi\sigma_1}{c}\left[\varphi+\frac{B_0}{c}\left(\frac{\partial v}{\partial t}+\frac{h^4}{192}k_2 a_{44}\frac{\partial\Psi}{\partial t}\right)\right]$$

$$\frac{\partial\psi}{\partial\alpha}-\frac{\partial\varphi}{\partial\beta}+\frac{1}{c}\frac{\partial f}{\partial t}=0 \quad (3.5)$$

The equations of electrodynamics for outside regions (vacuum) [4] are

$$\text{rot}h^{(e)}=\frac{1}{c}\frac{\partial e^{(e)}}{\partial t},\ \text{div}h^{(e)}=0$$

$$\text{rot}e^{(e)}=-\frac{1}{c}\frac{\partial h}{\partial t},\ \text{div}he^{(e)}=0 \quad (3.6)$$

The systems of equations (3.5), (3.6) are to be added to the system (3.4)

The hypothesis of magnetoelasticity of thin bodies reduces the problem of magnetoelasticity for the region occupied by the thin body itself to a two-dimensional problem of shell. However, the obtained equations of perturbed motion contain unknown boundary (surface) values of the inducted electromagnetic field components. Consequently, the problem of magnetoelasticity in most of the cases remains a spatial one. Therefore, the obtained two- dimensional equations of motion have to be solved with the equations of electrodynamics for the medium surrounding the thin body [4,11].

According to the basic statements of the hypothesis of magnetoelasticity of thin bodies, effective methods for reducing the general problem of magnetoelasticity to a two-dimensional one are proposed [4,11].

Introducing the concept of a boundary layer around the shell (with thickness λ_0, where λ_0 is the length of the halfwave of the shell elastic oscillations), we obtain the following differential relations between the components of the interface of the media (shell and boundary layer) [3,11]

$$\square\left(h_1^+ - h_1^-\right) = \frac{2}{\lambda_0}\left(\frac{\partial f}{\partial\alpha} + \frac{1}{c}\frac{\partial\psi}{\partial t}\right),\ \square\left(h_1^+ + h_1^-\right) = 0$$

$$\square\left(h_2^+ - h_2^-\right) = \frac{2}{\lambda_0}\left(\frac{\partial f}{\partial\beta} - \frac{1}{c}\frac{\partial\varphi}{\partial t}\right),\ \square\left(h_2^+ + h_2^-\right) = 0 \qquad (3.7)$$

where

$$\square = \frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\beta^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

When these equations are added to the equations (3.4), (3.5), a closed system of two-dimensional equations of magnetoelasticity of thin shell for a general case is obtained.

Adding the corresponding boundary conditions, initial conditions at infinity [1,4,11], we must solve the problems of oscillations of the orthotropic shallow shells and investigate the wave propagation in shells [1-11].

4. On the Transverse Oscillations of a Rectangular Spherical Panel

As an example let us consider a problem of basically transverse magnetoelastic oscillations of an isotropic $\left(E_1 = E_2 = E,\ \nu_1 - \nu_2 = \nu,\ G_{ik} = E/2(1+\nu),\ \sigma_1 = \sigma_2 = \sigma\right)$ rectangular spherical panel $\left(k_1 = k_2 = k = R^{-1}\right)$ without taking into account the rotatory inertia and transverse shears. According to (3.7) we assume that $h_1^- = -h_1^+$, $h_2^- = -h_2^+$. Here we neglect the inertia terms $\partial^2 u/\partial t^2$, $\partial^2 v/\partial t^2$, $\partial u/\partial t$, $\partial v/\partial t$ and consequently the energy dissipation caused by longitudinal oscillation.

In this case the following solution

$$f \equiv 0,\ \varphi \equiv 0,\ \psi \equiv 0,\ h_1^+ \equiv 0,\ h_2^+ \equiv 0 \qquad (4.1)$$

satisfies the equations (3.5), (3.7).

The equations of mainly transverse oscillations can be rewritten in the form

$$\frac{1-\nu}{2}\Delta u + \frac{1+\nu}{2}\frac{\partial}{\partial\alpha}\left(\frac{\partial u}{\partial\alpha} + \frac{\partial v}{\partial\beta}\right) + \frac{1+\nu}{2}\frac{1}{R}\frac{\partial w}{\partial\alpha} + \frac{\left(1-\nu^2\right)h^2\sigma B_0^2}{6ERc^2}\frac{\partial^2 w}{\partial\alpha\partial t} = 0$$

$$\frac{1-\nu}{2}\Delta v + \frac{1+\nu}{2}\frac{\partial}{\partial\beta}\left(\frac{\partial u}{\partial\alpha} + \frac{\partial v}{\partial\beta}\right) + \frac{1+\nu}{2}\frac{1}{R}\frac{\partial w}{\partial\beta} + \frac{\left(1-\nu^2\right)h^2\sigma B_0^2}{6ERc^2}\frac{\partial^2 w}{\partial\beta\partial t} = 0 \qquad (4.2)$$

$$\frac{\partial N_1}{\partial\alpha} + \frac{\partial N_2}{\partial\beta} - \frac{Eh}{(1-\nu)R}\left(\frac{\partial u}{\partial\alpha} - \frac{\partial v}{\partial\beta}\right) - \frac{2Eh}{1-\nu}\frac{w}{R^2} = \rho h\frac{\partial^2 w}{\partial t^2}$$

$$D\frac{\partial}{\partial\alpha}\Delta w + N_1 = \frac{h^3\sigma B_0^2}{12c^2}\frac{\partial^2 w}{\partial\alpha\partial t}$$

$$D\frac{\partial}{\partial\beta}\Delta w+N_2=\frac{h^3\sigma B_0^2}{12c^2}\frac{\partial^2 w}{\partial\beta\partial t}$$

$$\Delta\equiv\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\beta^2},D=\frac{Eh^3}{12(1-\nu^2)} \quad (4.3)$$

Introducing the notations:

$$\theta=\frac{1+\nu}{1-\nu},\ \ \gamma_1^2=\frac{h^3\sigma B_0^2}{6c^2},\ \ \gamma_2^2=\frac{(1+\nu)h^3\sigma B_0^2}{3ERc^2}$$

we rewrite the system (4.2), (4.3) in a more convenient form

$$\Delta u+\theta\frac{\partial}{\partial\alpha}\left(\frac{\partial u}{\partial\alpha}+\frac{\partial v}{\partial\beta}\right)+\frac{\theta}{R}\frac{\partial w}{\partial\alpha}+\gamma_2^2\frac{\partial^2 w}{\partial\alpha\partial t}=0$$

$$\Delta v+\theta\frac{\partial}{\partial\beta}\left(\frac{\partial u}{\partial\alpha}+\frac{\partial v}{\partial\beta}\right)+\frac{\theta}{R}\frac{\partial w}{\partial\beta}+\gamma_2^2\frac{\partial^2 w}{\partial\beta\partial t}=0 \quad (4.4)$$

$$D\Delta^2 w-\gamma_1^2\frac{\partial}{\partial t}\Delta w+\rho h\frac{\partial^2 w}{\partial t^2}+\frac{2Eh}{1-\nu}\frac{w}{R^2}+\frac{Eh}{(1-\nu)R}\left(\frac{\partial u}{\partial\alpha}+\frac{\partial v}{\partial\beta}\right)=0 \quad (4.5)$$

By analogy with the wave propagation theory we introduce φ_1,ψ_1 functions

$$u=\frac{\partial\varphi_1}{\partial\alpha}+\frac{\partial\psi_1}{\partial\beta},\ v=\frac{\partial\varphi_1}{\partial\alpha}-\frac{\partial\psi_1}{\partial\beta} \quad (4.6)$$

which allow to separate the equation for ψ_1, which determines the shears of middle surface. Using the transform (4.6) we bring the system (4.4)-(4.5) into the form

$$\Delta\psi_1=0\,,\ \ (1+\theta)\Delta\varphi_1+\frac{\theta}{R}w+\gamma_2^2\frac{\partial w}{\partial t}=0$$

$$D\Delta^2 w-\gamma_1^2\frac{\partial}{\partial t}\Delta w+\rho h\frac{\partial^2 w}{\partial t^2}+\frac{2Eh}{1-\nu}\frac{w}{R^2}+\frac{Eh}{(1-\nu)R}\Delta\varphi_1=0 \quad (4.7)$$

The equation for ψ_1 is independent from the system of two other equations for φ_1 and w_0. However, in general case w,φ_1 and ψ_1 are connected by means of the boundary conditions.

Determining $\Delta\varphi_1$ from the second equation (4.7) and substituting it into the third equation we obtain the equation for w only

$$D\Delta^2 w-\frac{h^3\sigma B_0^2}{12c^2}\frac{\partial}{\partial t}\Delta w+\rho h\frac{\partial^2 w}{\partial t^2}-\frac{(1+\nu)h^3\sigma B_0^2}{6R^2c^2}\frac{\partial w}{\partial t}+\frac{(3-\nu)Eh}{2(1-\nu)}\frac{w}{R^2}=0 \quad (4.8)$$

Although the equation for w is independent, the problem for transverse oscillations does not separate from the problem for φ_1,ψ_1 in the case of general boundary conditions. In the case of rectangular spherical panel when the edges $\alpha=0,a$ and $\beta=0,b$ are free-supported, the separation mentioned above is valid. So the solution for w be expressed as

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{nm}e^{\omega_{mn}t}\sin\lambda_m\alpha\sin\mu_n\beta\,,\ \lambda_m=\frac{m\pi}{a}\,,\ \mu_n=\frac{n\pi}{b} \quad (4.9)$$

where ω_{mn} is determined by the characteristic equation

$$\omega_{mn}^2+\frac{h^2\sigma B_0^2}{12\rho c^2}\left[\lambda_m^2+\mu_n^2-\frac{2(1+\nu)}{R^2}\right]\omega_{mn}+\frac{D}{\rho h}(\lambda_m^2+\mu_n^2)^2+\frac{(3-\nu)E}{2(1-\nu)R^2\rho}=0 \quad (4.10)$$

The imaginary part of the root ω_{mn} determines the oscillation frequency. Similar to the plate oscillations, the influence of the magnetic field on the frequency is weak in case of real limitations for the magnetic field intensity.

From the expression for the coefficient $\mathrm{Re}(\omega_{mn})$

$$\mathrm{Re}(\omega_{mn}) = -\frac{h^2\sigma B_0^2}{24\rho c^2}\left[\lambda_m^2 + \mu_n^2 - \frac{2(1+\nu)}{R^2}\right] \tag{4.11}$$

it follows that the minimum damping occurs for the oscillation form $m=1, n=1$. In this case for shallow spherical panel the following condition is necessary:

$$\frac{1}{a^2} + \frac{1}{b^2} > \frac{2(1+\nu)}{(\pi R)^2} \tag{4.12}$$

From (4.11) it follows, that the curvature of the panel causes reduction of the dissipation. In the case of anisotropic electroconductive shell, i.e. when $\sigma_1 \neq \sigma_2$, the characteristic equation is:

$$\omega_{mn}^2 + \frac{B_0^2 h^2}{12\rho c^2}\left(\sigma_2\lambda_m^2 + \sigma_1\mu_m^2\right)\left[1 - \frac{2(1+\nu)}{R^2\left(\lambda_m^2 + \mu_n^2\right)}\right]\omega_{mn} +$$

$$+\frac{D}{\rho h}\left(\lambda_m^2 + \mu_n^2\right)^2 + \frac{E(3-\nu)}{2(1-\nu)\rho R^3} = 0 \tag{4.13}$$

In genеral case the roots of equation (4.13) are complex ones. Imaginary and real parts of the root represent frequency and damping respectively. Both of these characteristics depend not only on absolute values of electroconductivity coefficients, but also on relations between these values and dimensions of the shell, i.e. on the expression:

$$\frac{\sigma_2 m^2}{a^2} + \frac{\sigma_1 n^2}{b^2} > 0 \tag{4.14}$$

In particular cases it is possible, that

$$\left\{\frac{B_0 h}{24\rho c}\left(\sigma_2\lambda_m^2 + \sigma_1\mu_n^2\right)\left[1 - \frac{2(1+\nu)}{R^2\left(\lambda^2 + \mu^2\right)}\right]\right\}^2 - \frac{D}{\rho h}\left(\lambda_m^2 + \mu_n^2\right)^2 - \frac{E(3-\nu)}{2(1-\nu)\rho R^2} > 0$$

So that $\mathrm{Im}(\omega_{mn}) = 0$ and any disturbances will attenuate without oscillation.

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